Thesis

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July 2016

Distribution of duration

Let F be the cdf of a gev random variable (write this properly). Let a be the threshold level, then define T_a as the duration between jumps that have been thresholded(?) at the a-level. From (insert reference here) we know that (explain proof here?)

$$T_a \sim (-\log F(a))^{\frac{-1}{\beta}} X^{\frac{1}{\beta}} D(1).$$

Where $D(1)$ is a random variable of the stable distribution with stability parameter β and skewness parameter 1 and where X is a standard exponential random variable. After taking the logarithms of both sides we arrive at

$$\log T_a \sim \frac{1}{\beta} \log X + \log D(1) - \frac{1}{\beta} \log(-\log F(a)).$$

In order to find the distribution for $\log T_a$ we will first need to find the distributions of $\frac{1}{\beta} \log X$ and $\log D$. We have that

$$\begin{aligned} f_{\frac{1}{\beta}\log X}(x) &= \frac{d}{dx}\mathbb{P}\left(\frac{1}{\beta}\log X \leq x\right) \\ &= \frac{d}{dx}\mathbb{P}(X \leq e^{x\beta}) \\ &= f_X(e^{x\beta})\beta e^{x\beta}. \qquad (a) \end{aligned}$$

Similarly we have

$$\begin{aligned} f_{\log D}(x) &= \frac{d}{dx}\mathbb{P}(\log D \leq x) \\ &= \frac{d}{dx}\mathbb{P}(D \leq e^{x}) \\ &= f_D(e^{x})e^{x}. \qquad (b) \end{aligned}$$

Using convolution we know that

$$f_{\frac{1}{\beta}\log X + \log D}(x) = \int_{-\infty}^{\infty} f_{\frac{1}{\beta}\log X}(x-y) f_{\log D}(y) dy$$

and thus

$$f_{\log T_a}(x) = \int_{-\infty}^{\infty} f_{\frac{1}{\beta}\log X}\left(x - y + \frac{1}{\beta}\log(-\log F(a))\right) f_{\log D}(y) dy \qquad (1)$$

as $\frac{1}{\beta}\log(-F(a))$ is a constant with respect to x. Substituting (a) and (b) into (1) we get

$$f_{\log T_a}(x) = \int_{-\infty}^{\infty} f_X(e^{\beta(x-y+\frac{1}{\beta}\log(-\log F(a)))})\beta e^{\beta(x-y+\frac{1}{\beta}\log(-\log F(a)))} f_D(e^y)e^y dy$$

which simplifies to

$$f_{\log T_a}(x) = \int_{-\infty}^{\infty} -\log F(a) f_X(-\log F(a)e^{\beta(x-y)})\beta e^{\beta(x-y)} f_D(e^y)e^y dy.$$