CTRM STATISTICS

Abstract. We fit a max-renewal process, (aka 'CTRM', Continuous Time Random Maxima) to data.

1. Introduction

2. The Model

The stochastic model $X(t)$ below has been studied under the various names "Shock process" [EM73, SS83, SS84, SS85, And87, Gut99], "max-renewal process", [Sil04, ST04, BŠ14] and "CTRM (continuous time random maxima) [BSM07, MS09, Hee14, HS15]: Assume i.i.d. pairs of random variables (W_k, J_k), $k = 1, 2, \dots$ where $W_k > 0$ represents an inter-arrival time of certain events and $J_k \in \mathbb{R}$ the corresponding event magnitude. Write $N(t) = \max\{n \in \mathbb{N} : W_1 + \ldots + W_n \leq t\}$ for the renewal process associated with the W_k (where the maximum of the empty set is set to 0), and define

$$X(t) = \bigvee_{k=1}^{N(t)} J_k = \max\{J_k : k = 1, \dots, N(t)\}.$$

If W_k is interpreted as the time leading up to the event with magnitude J_k, then $X(t)$ is the largest magnitude up to time t. The alternative case where W_k is the inter-arrival time *following* J_k is termed "second type" (in the shock model literature) or OCTRM (overshooting CTRM), and the largest magnitude up to time t is then given by

$$Y(t) = \bigvee_{k=1}^{N(t)+1} J_k.$$

Finally, the model is called coupled when W_k and J_k are not independent; in this article we focus on the uncoupled case. For this case, $X(t)$ and $Y(t)$ have the same limiting behaviour for large times and magnitudes [HS15] and hence we focus on the CTRM $X(t)$.

Any probability distribution with cumulative distribution function (CDF) F on $\mathbb{R}$ lies in the max-domain of attraction of a generalized extreme value (GEV) distribution with CDF Λ. That is, there exist sequences $a_n \in \mathbb{R}^+$ and $b_n \in \mathbb{R}$ such that

$$F(a_n x + b_n) \to \Lambda(x), \quad x \in (x_{min}, x_{max}).$$

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