Anderson approach

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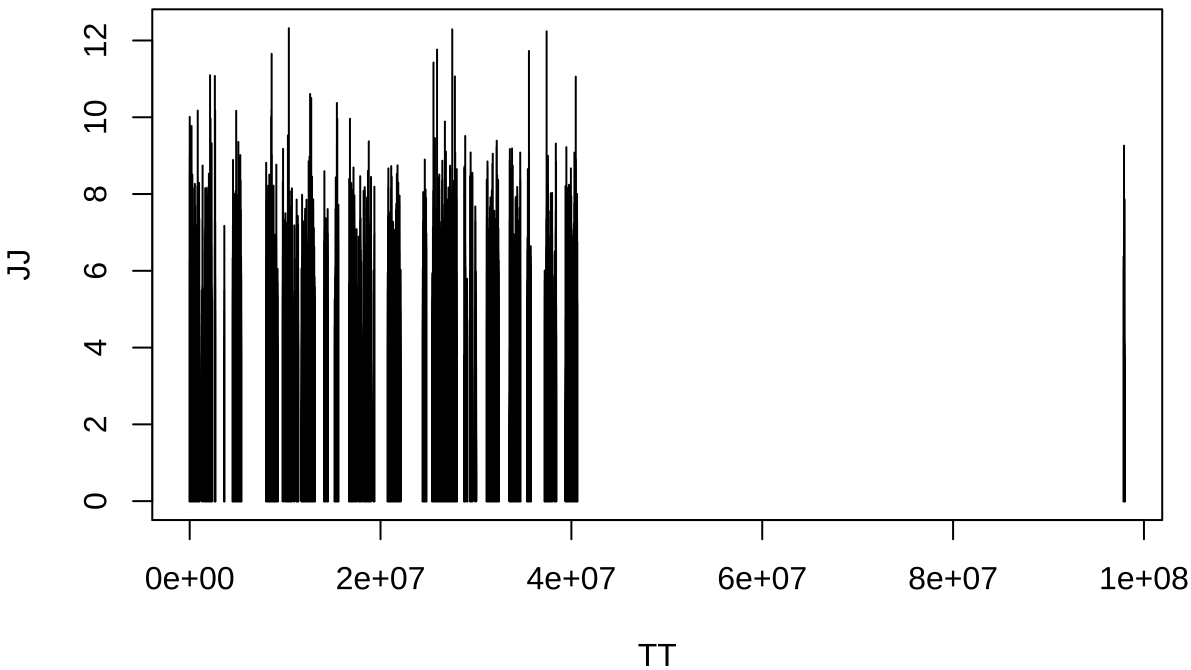
Suppose we are given a time series of observations with i.i.d. inter-event times W_k with Pareto distribution

$$\mathbf{P}[W_k > t] = 1 - F_W(t) = \begin{cases} \left(\frac{t}{\sigma}\right)^{-\beta}, & x \geq \sigma \\ 0, & x < \sigma \end{cases}$$

where $\beta \in (0,1)$ and σ is a scale parameter. With e.g. event magnitudes of unit exponential size, this defines a marked renewal process:



Now we vary the threshold ℓ so that there are between 10 and 10^{4} exceedances. For each choice of $k = 10$, 11, . . . , 10^{4}, use the dataset of k exceedances to estimate the shape parameter β and scale parameter δ of the Mittag-Leffler distribution. First β:

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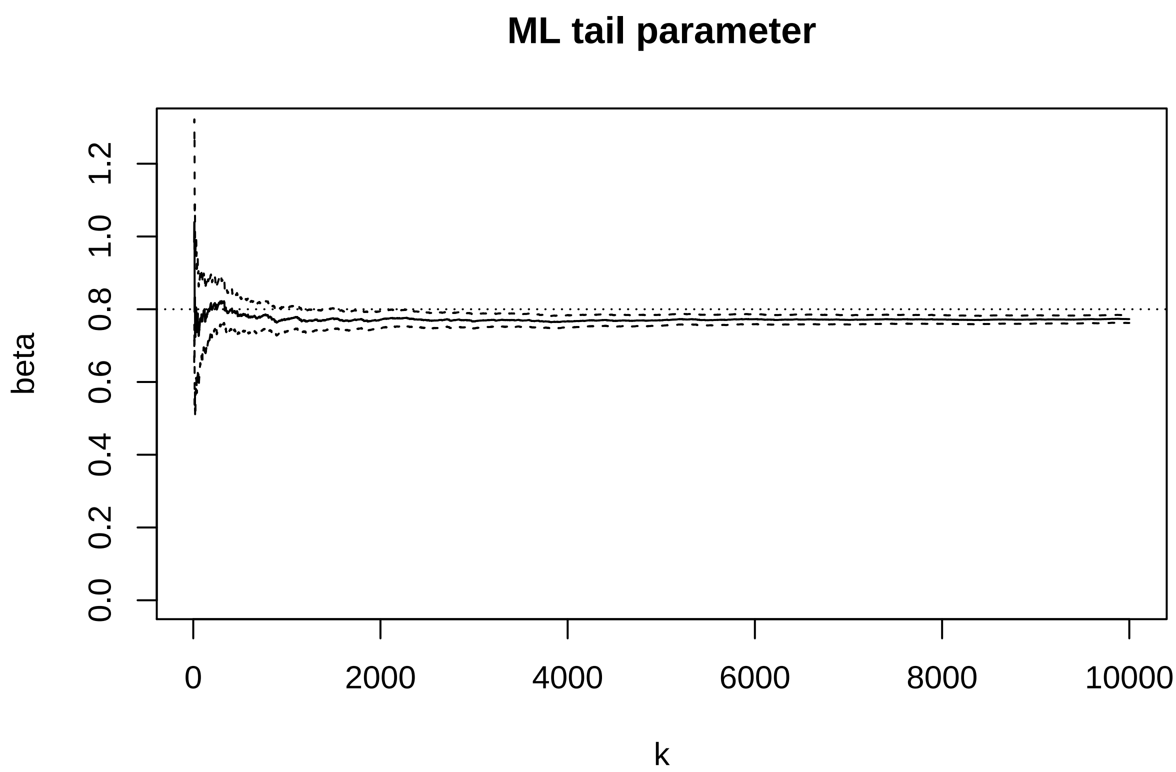


Now δ. From Anderson, we have

$$\frac{T_\ell}{n(1/(1-F_J(\ell)))} \Rightarrow W_\beta, \quad \ell \uparrow x_* \tag{1}$$

where W_β is Mittag-Leffler with scale 1 (and x_* denotes the right end of the support of the distribution of J). Write $v \in [1, \infty)$ for the number of recurrence epochs; i.e. $v = 100$ means that an event occurs once every 100 times. Choose the level $\ell(v)$ in such a way that an exceedance occurs only once every v observations (s. Beirlant, Ch.2, where $\ell = U$). Then

$$1 - F_J(\ell(v)) = 1/v$$

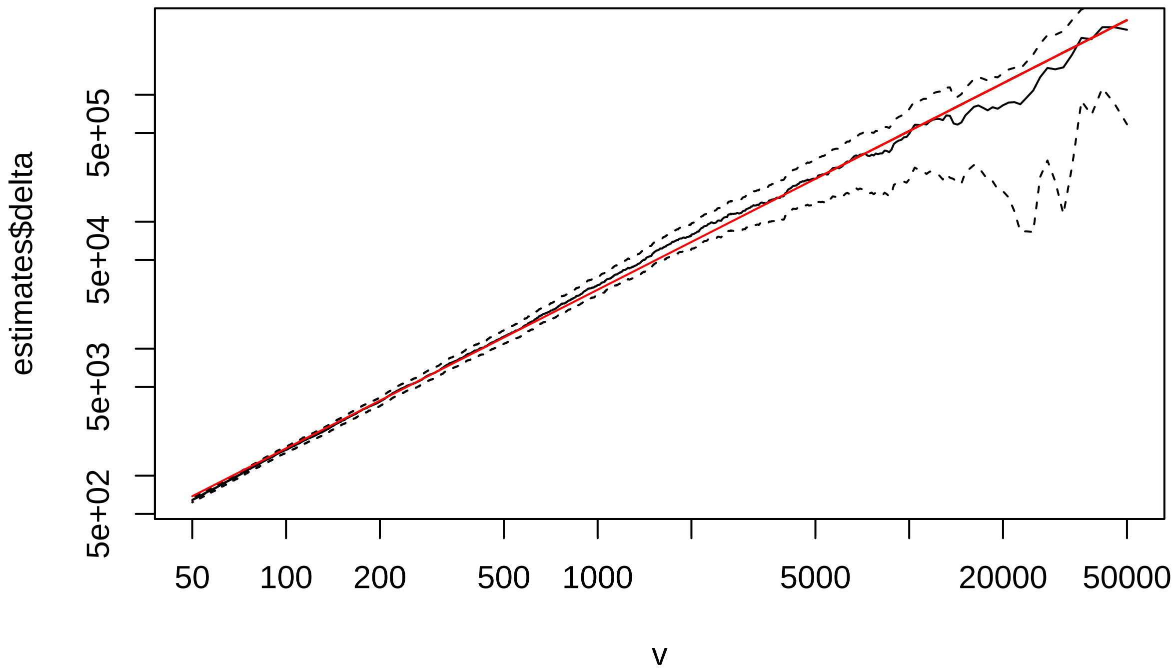
and the above weak convergence result becomes

$$\frac{T_{\ell(v)}}{n(v)} \Rightarrow W_\beta, \quad v \to \infty. \tag{2}$$

Since $T_{\ell(v)} \approx n(v) W_\beta$, we may identify the scale parameter, depending on the threshold via v:

$$\delta = n(v).$$

Recall that the function $n(v)$ is regularly varying at ∞ with parameter $1/\beta$. Assume that $n(v) = C \times v^{1/\beta}$. On a logarithmic scale, the parameter β appears as the slope $1/\beta$:



To estimate C, we calculate

$$\hat{C} = n(v)v^{-1/\beta}$$

for various values of v. The actual value of C can be easily calculated: Recall that $n(t)$ is (asymptotically) inverse to

$$g(t) := \frac{t}{\Gamma(2-\beta)\int_0^t (1 - F_W(u))\, du} = \frac{t^\beta}{\Gamma(2-\beta)\sigma^\beta}, \tag{3}$$

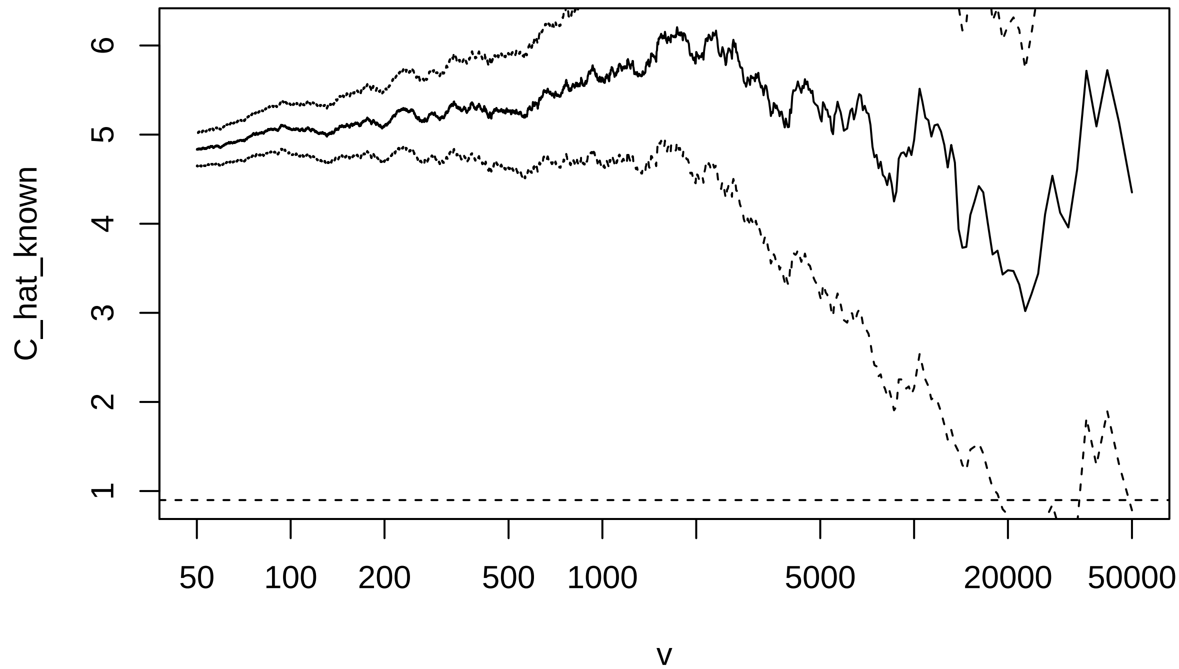
which has the exact inverse

$$n(v) = \sigma \left(\Gamma(2-\beta)v\right)^{1/\beta}. \tag{4}$$

Thus

$$C = \sigma(\Gamma(2-\beta)) = 0.8987794.$$

Below are the estimates $\hat{C}$, where β is assumed known / unknown. The dashed line corresponds to the actual value of C.



and where β is plugged in from the previous estimate: