An Assignment Model on Traffic Matrix Estimation

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Abstract. It is important to acquire accurate knowledge of traffic matrices of networks for many traffic engineering or network management tasks. Direct measurement of the traffic matrices is difficult in large scale operational IP networks. One approach is to estimate the traffic matrices statistically from easily measured data. The performance of the statistical methods is limited due to they rely on the limited information and require large amount of computation, which limits the convergence of such computation. In this paper, we present an alternative approach to traffic matrix estimation. This method uses *assignment model*. The model is based on the link characters and includes a fast algorithm. The algorithm combines statistical and optimized tomography. The algorithm is evaluated by simulation and the simulation results show that our algorithm is robust, fast, flexible, and scalable.

1 Introduction

Many decisions that IP network operators make depend on how the traffic flows in their network. *Traffic Matrix* (TM) reflects the volume of traffic that flows between all possible pairs of origins and destinations in a network. When used together with routing information, the traffic matrix gives the network operator valuable information about the current network state, routing protocol configuration and management of network congestion, etc. It is impossible to set up internal monitors at all routers in the networks due to cost and deployment problems so that directly measuring TMs is very difficult. In another hand such measurement would impose on the regular behavior of the network because of large amounts of data to be collected. The challenge is then to obtain TMs using limited data measured and information obtained from the network.

Previous work on obtaining traffic matrix usually relied on statistical inference methods that use limited measurements to estimate TM. The term *Network Tomography* [3] was coined for this problem when the partial data come from repeated measurements of the traffic flowing along directed links in the network. In order to handle the estimation problem, Vardi [3] assumes a Poisson model for the traffic matrix estimation in which the covariance of the link loads is used as an additional constraint condition. The traffic matrix is estimated through Maximum Likelihood (ML) estimation. Cao et al. [4] proposed a modified EM algorithm to estimate TM in which a more general scaling law between means and variances of traffic is adopted to find maximal likelihood parameters based on Gaussian model and convergence is hastened by using second-order methods. Tebaldi and West [1] presented a solution which is

L. Jiao et al. (Eds.): ICNC 2006, Part II, LNCS 4222, pp. 295 – 304, 2006.

also based on the Poisson model and the Bayesian approach. Since posterior distributions are hard to calculate, they use a Markov Chain Monte Carlo simulation to simulate the posterior distribution. It is also possible to formulate the traffic matrix estimation as a constrained optimization problem that could be solved by using the methods such as Linear Programming [5]. Those statistic techniques produce unacceptable error rates to ISPs. Medina et al. [6] made a comparison between [3] and [4] and proposed a novel choice model to estimates the probability of sending a packet from an origin node to a destination node in the network. Zhang et al. [7] introduced a gravity model and assumed that the amount of traffic from or to a given node was only related to this node. The estimation of TM by this model is consistent with measured link loads at the network edge, but not necessarily so in the interior links. Zhang et al. [8] presented an information-theoretic approach with the Kullback-Leibler distance to minimize the mutual information between source and destination. Medina et al.[10] proposed a two-step approach to infer network traffic demands. First, alternative models, for instance choice model [6] or gravity model [7], are evaluated to generate good starting points for iterative statistical inference techniques. Second, the generated starting point is provided to a statistical inference technique. The division of the TM estimation process into two steps offers great flexibility for combining and evaluation different strategies. In all papers mentioned above, the routing is considered to be constant. Nucci et al [11] presented an approach of changing routing and shifting link loads to infer the traffic matrix.

In this paper, we present an assignment model based on the link characteristics and inspired by Transportation and Traffic Theory [12], propose two algebraic operations to analyze the network and a new algorithm *Expectation Error Rectify Algorithm (*EERA), which combines statistical and optimized tomography.

The remaining parts of this paper are organized as follows. Section 2 introduces the problem and notation. Section 3 introduces the estimation model. Section 4 describes the details of EERA. Section 5 gives the simulation results. Finally, Section 6 makes conclusion.

2 Problem Descriptions

The problem of inferring network traffic matrix can be formulated as follows. Consider a network with N nodes and L directed links. Such a network has $P = N\times(N-1)$ pairs of origin-destination pairs. Although conceptually traffic matrix is represented in a matrix X, with the amount of data transmitted from node n to node m as element x_{nm}, it is more convenient to use a vector representation. Thus, we enumerate all P origin-destination pairs, and let x_p denote the point-to-point demand of node pair p. For simplicity, we will assume that each point-to-point demand is routed on a single path. The paths are represented by a routing matrix R. R is a {0, 1} matrix with rows representing the links of the network and columns representing the OD pairs. Element r_{ij}=1 if link i belongs to the path associated to OD pair j, and r_{ij}=0 otherwise. Let $Y=(y_1,\cdots,y_L)$ be the vector of link counts where y_j gives the link count for link j. Then, X and Y are related via:

$$RX = Y \quad \text{or} \quad \sum_{j=1}^{P} r_{ij} x_j = y_i \ . \tag{1}$$

The matrix R can be obtained by gathering the OSPF or IS-IS links weights and computing the shortest-paths between all OD pairs. The link counts Y are available from the SNMP data. The traffic matrix estimation problem is simply the one of estimating the non-negative vector x based on knowledge of R and y. The challenge in this problem comes from the fact that this system of equations tends to be highly underdetermined: there are typically many more origin-destination pairs than links in a network, and the formula (1) has many more unknowns than equations. So some sort of side information or assumptions must then be added to make the estimation problem well-posed.

3 Assignment Model

The traffic matrix is necessary for many network planning functions. It is clearly necessary to know the volume of expected demand in order to plan the network adequately to handle that traffic with satisfactory quality (low delay and loss of transmitted data). Traffic on any link of the network depends on the way traffic is routed. In current IP networks, the path of a given origin-destination pair is the shortest one, in which the "length" of a link is an administratively assigned weight. Routing for a given origin-destination demand is thus fixed. It is therefore very important to plan route assignments carefully to avoid the demand on any link from overload. Our proposed assignment model is one of such solutions.

The traffic matrix estimation is equivalent to finding a reasonable OD matrix X which reproduces count data of the observed links when X is assigned to the network. In a practical application the reproduction might not be exactly achieved for all traffic counts because of different representation of traffic collection at different times or in the different aggregated network. Therefore, we believe that it is crucial to use the assignment technique in OD matrix estimation based on link counts.

3.1 Link Assignment Coefficient

The link counts are the sum of the OD pairs that are routed across that link. If a link is shared by a large number of OD pairs, then it may be hard to disambiguate how much bandwidth belongs to each OD pair. Each value of OD pair is different proportion out of the link load. We define *assignment coefficient* l_{kz} and let l_{kz} denote the fraction that OD pair x_z is in the link load y_k. Thus, the traffic of OD pair x_z can be described by: $x_z = l_{kz}\, y_k$.

$$y_k = \sum_{z=1}^{p} r_{kz} x_z \quad \text{and} \quad \sum_{z} l_{kz} = 1 \ . \tag{2}$$

Thus, T values can be got from each OD pair x_z ($0 \le T \le L$).

Assignment coefficient usually depends on the policies of traffic exogenous or endogenous determination in the network, for example, congestion in the network and route characteristics. Assuming independence between the traffic volumes and proportion $l_{kz,}$ the link counts are proportional to the OD volume. In order to reduce the computational complexity of the OD matrix estimation model, we define another coefficient to dispose the data.

3.2 Weight Coefficient

It is true that some OD pairs are more difficult to estimate than others. It would be interesting to investigate the properties of the paths associated with such “troublesome” OD pairs. We assign some weights to each OD pair depend on links, so that we can dispose different values of the OD pair. Let x_{zt} denote one of the values of OD pair x_z. Note that $t \in T$. Each of x_{zt} had different weight during estimating x_z. So we introduce a new coefficient — *weight coefficient*, w_{zt}. The ultimate value of OD pair x_z can be described by:

$$x_z = \sum_t w_{zt} x_{zt} \quad \text{and} \quad \sum_t w_{zt} = 1 \quad . \tag{3}$$

That is, estimate volume of OD pair x_z is expectation of all of x_{zt}.

The values of l_{kz} and w_{zt} can be determined before estimation of the OD matrix is done and taken as exogenous given. The “all-or-nothing” assignment method can be used: all-or-nothing assignment of traffic is obtained when all traffic, for all OD pairs, is assigned to the cost minimizing path. Or, the equilibrium assignment is also a more realistic approach. We have made use of Wardrop’s first equilibrium principle in our model. It is that the traffic is in “equilibrium” when no POP can achieve a lower cost by switching to another POP.

Once we have specified the value of assignment coefficient l_{kz} and weight coefficient w_{zt}, the estimation problem is reduced to the combine of constraint program problem and statistical problem. We named it as *assignment model*. The use of assignment model was motivated by wonder which OD pair would influence the load through the link and which OD pair is difficult to estimate. We introduce assignment coefficient and weight coefficient, because we think that the two coefficients are much more stable than the OD matrix itself during the measurement period. What’s more, the additional data or assumption about the travel behavior can be used to find a unique traffic matrix.

3.3 Algebraic Operation

The routing matrix summarizes the network traffic structure in useful way. In order to realize the network framework and its characteristic, we present two types multiplication operation to the routing matrix R.

First: $$S = R \times R^T . \tag{4}$$

R multiplied by R^T (matrix transpose) is an L by L matrix $S_{L\times L}$. Entries s_{ij} of matrix S have different signification. While $i = j$, s_{ij} is the total of OD pair through the link i or j, otherwise, s_{ij} is the total of OD pair through the link both i and j.

Second, $$Q = R^T \times R. \tag{5}$$

R^T multiplied by R is a P by P matrix $Q_{P\times P}$. Element q_{ij} of matrix Q has also different meaning. While $i = j$, q_{ij} is the path length of the OD pair i or j. While $i \neq j$, q_{ij} is the sum of the link counts that is shared by both OD pair i and OD pair j.

Applying the two operations, we can obtain the correlation between the OD pairs and find the vital link that impacts the accuracy of estimated method.

4 Algorithm

Any method must provide a solution to the traffic matrix estimation from limited data. This indicates that a new approach incorporating additional knowledge about the network is needed. Indeed non-statistical knowledge about how networks are designed is available to network operators. It would achieve more accurate traffic matrix estimation if combined with statistical data.

The goal of our algorithm is to find the optimization of the following equation:

$$\text{minimize } \left\|RX - Y\right\|_2^2 . \tag{6}$$

$$\text{sub} \qquad x_{ij} \geq 0 . \tag{7}$$

That is, the distance between the estimated OD matrix and the real OD matrix is minimized subject to the observed link counts.

The algorithm contains three nested iterative processes. The first is to compute the OD pairs based on the link counts. The second is to calculate the expectation of OD pairs. The third is to validate the constrained term and adjust the traffic matrix. When the total measured and total estimated link loads in each direction converge, this reckoned process ends.

The pseudo-code of our algorithm is listed following:

① Network topology, link vector Y and error threshold e are inputs;

② Computer shortest paths for all OD pairs. Using the operation described in the section 3.3, find the “vital” link and the “troublesome” OD pairs;

③ Iterate as follow to determine the OD pairs.

(Ⅰ) compute OD pair by: $x_{zk}= l_{kz}\, y_k$;

(Ⅱ) calculate expectation of OD pairs by: $x_z = \sum_t w_{zt} x_{zt}$;

(Ⅲ) test the acquired OD pair by: $RX = Y'$.

④ Let $\varepsilon=Y- Y'$ denotes the error vector. If $\varepsilon<e$, the algorithm end. Otherwise, make ε an relaxation variable turn to the step ③.

The results estimated must satisfy equation (1),. Therefore it is vital that there is no error in measurement of the link loads. If the link count data are wrong, it is impossible to get accurate traffic matrix by any algorithm based on link count data. Our approach is not to incorporate additional constraint, but rather to use assignment model to obtain initial estimate, which needs to rectify to satisfy the constraint.

5 Performance

It is not possible to obtain an entire "real" traffic matrix via direct measurement. Therefore assessing the quality of TM estimations and validating TM models is difficult because one cannot compare an estimated TM to "the real thing". There would be no inference problem if real TMs could be obtained.

In order to evaluate TM estimation methods, we need the data about routing, traffic matrix elements and link loads to be consistent. We get the data by the simulate tool OPNET. Our algorithm assumes that Dijkstra's Minimum Weight Path Algorithm is used to the traffic routing. We also assume that all nodes can originate and terminate traffic.

5.1 Topology

We consider a small scale network with 4 nodes topology, as depicted in Figure 1.

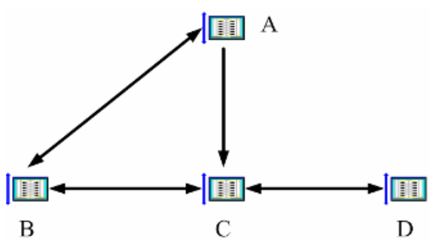

Fig. 1. 4-Node Topology

This network has L = 7 directed links and P = 12 OD pairs; the ordered sequence of nodes comprising these links and OD pairs appears in the flowing. The corresponding 7×12 routing matrix R is:

$$R=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&1&0&0&1&0&0\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&1&1&0\\ 0&0&1&0&0&1&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&1 \end{pmatrix}$$

We use this simple case because it allows us to enumerate all the OD pairs and link counts, which is useful for illustrating how the method behaves. It had also been used in many pioneer papers [3], [6].

observed link: y_1 : A→B, y_2 : B→A, y_3 : A→C, y_4 : B→C, y_5 : C→B, y_6 : C→D, y_7 : D→C;

OD pair: x_1 : A→B, x_2 : A→C , x_3 : A→C→D, x_4 : B→A , x_5 : B→C , x_6 : B→C→D, x_7 : C→B→A, x_8 : C→B, x_9 : C→D, x_{10} : D→C→B→A, x_{11} : D→C→B, x_{12} : D→C .

5.2 Comparison of Algorithms

In this section, we evaluate our method and three methods of others. The optimization approach [2] presents a linear program (LP). VPossion and CGauss separately described in [3] and [4]. The evaluation results of the four methods are presented in Table 1.

Table 1. The compared result among four algorithms

OTM	LP		VPoisson		EERA		CGauss		
	ETM	E%	ETM	E%	ETM	E%	OTM	ETM	E%
AB:318	318	0	318	0	318	0.0	318.65	318.65	0
AC:289	601	107	342	18	297	2.8	329.48	286.98	13
AD:312	0	100	259	17	304	2.6	277.18	318.36	15
BA:294	579	96	334	14	300	2.0	298.14	298.14	0
BC:292	559	91	310	6	271	7.2	354.81	360.97	1.6
BD:267	0	100	249	7	288	7.9	355.39	347.94	2
CA:305	303	0.6	291	5	293	3.9	327.20	317.34	3
CB:289	0	100	361	25	290	0.3	330.04	373.65	13
CD:324	903	178	395	22	311	4.0	253.01	217.32	14
DA:283	0	100	257	9	289	2.1	320.50	329.07	3
DB:277	851	207	245	12	282	1.8	291.52	246.60	15
DC:291	0	100	349	20	280	3.1	310.40	344.82	11

The table 1 shows the original traffic matrix (OTM), the estimated value (ETM) for each OD pair and the relative error (E%). The average error was 98% for the LP method, 13% for the VPoisson method, 3.2% for the EERA method and 7.6% for the CGauss method. The LP method clearly is the worst in the four methods. Although this approach may have worked in the 3-node topology considered in [2], it seems not seem to be effective in the classical 4-node topology. The reason can be that LPs are indeed sensitive to the topology. The data show that EERA is the best method in terms of both the average error and the biggest error. What's more important, our method computation complexity is proportion to n^3, however, other approaches computation complexity is proportion to n^5.

5.3 Sensitivity

We have compared these techniques with respect to the estimation errors yielded. Next we will analyze the sensitivity to prior information and modeling assumptions on OD pairs of our algorithm.

We use synthetic traffic matrices since real TMs are not available. The best TM subjected to certain random model can be obtained by generating synthetic TMs based on what the properties of real TMs are. We need also to generate synthetic TMs that exhibit certain properties that expose the strengths and the weaknesses of the evaluated techniques. With this goal in mind, we generate six types of synthetic TMs that differ in the distribution used to generate their elements. Specifically, we consider *constant*, *Exponential*, *Poisson*, *Gaussian*, *Uniform* and *Mixture* TMs. Up to now, human being have not discovered the random model of OD pair distributions. We suspect that these are no distributions that may properly reflect the distributions of OD traffic demands in the Internet backbone. So, we design the mixture TMs, which each node have different attribute of delivering packets. We consider the Uniform scenario because this is what is often used by researchers who need a traffic matrix in evaluating algorithms. We include the Constant case because this should be the easiest for these methods to estimate. To compare with [3], [4], we also include poisson and gauss case. All of scenario can be obtained by assigning a attribute of node, constant(1.0), uniform(0.5,1.5) , poisson (1.0), exponential (1.0), normal(1.0,0.2).

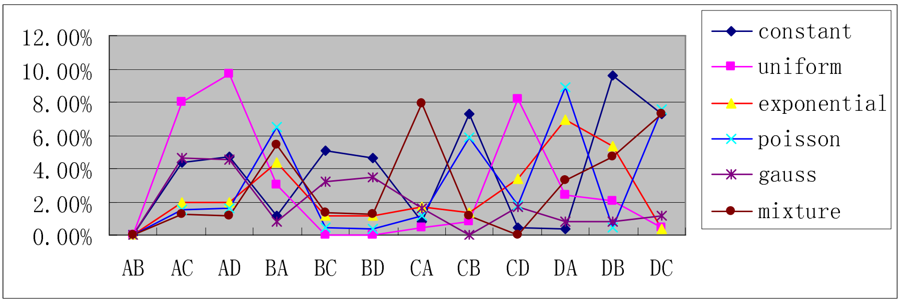

Fig. 2. A comparison of the error to the traffic matrix elements estimated by our algorithm. The curve lines show the error of different scenarios.

Figure 2 shows the error of each OD pairs in different scenario. Obviously, the error is related to the prior information. That is, the maximum error of OD pair is different in the different case. For example, the AD pair is the worst in the Uniform scenario; however, the DB pair is the worst in the Constant scenario. From the Figure 2 we can think that estimation errors may be correlated to heavily shared links, because the assignment model is based on the links. For a given traffic matrix, there are always some OD pairs that can be estimated very closely, while at the same time, other OD pairs are estimated very poorly. What's more, the assumption of independence between the OD pairs can be inaccurate. So, we think that it will be a new approach to find the correlation between the OD pairs. In order to evaluate our method sensitivity to prior information, we show the average error in Figure 3.

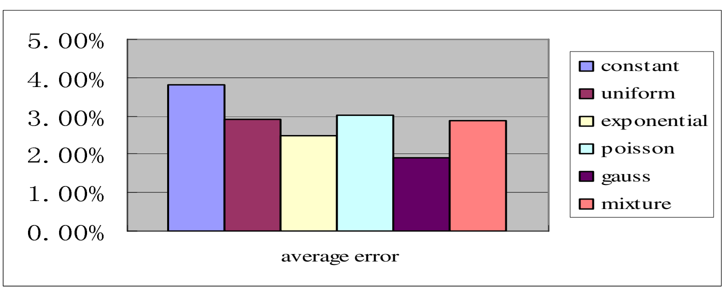

Fig. 3. The average error of different scenarios

Figure 3 shows the results of our method. These results indicate that our method is not sensitive to the distribution, and is more robust to various types of OD pair distributions. From the both figure, we can get a result that the "troublesome" OD pairs are related to not only topology configure but also "real" traffic.

6 Conclusion and Future Work

Availability of an O-D traffic matrix is essential if network "what if" analyses are desired. Estimation techniques based on partial information are used to populate traffic matrix because amassing sufficient data from direct measurements to populate a traffic matrix is typically prohibitively expensive. To handle the problem, we consider the links attribute. For example, how is the link loads aggregated by OD pairs that are routed through the link.

We have presented a new model and introduce a new algorithm. We have validated the method through simulate tool. The results show remarkable accuracy in the small scale network. Our study leaves many important issues unexplored. For example, the data set does not contain measurement errors or component failures and we have not evaluated the effect of such events on the estimation. We have not considered how sensitive traffic engineering tasks are to estimation errors in different demands, and how such information could be incorporated in the estimation procedures. We will expand our method to large network. Another interesting topic for future work would be to understand the nature of the worst-case bounds, and see if they could be exploited in other ways.

Acknowledgement

This paper is supported by *Chunhui* project funded by Ministry of Education, Nature Science Foundation of Chongqing(2005BB2067) and 4G-research special fund of CQUPT.

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