Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 178 (2017) 289 – 297

Procedia Engineering

www.elsevier.com/locate/procedia

16th Conference on Reliability and Statistics in Transportation and Communication, RelStat'2016, 19-22 October, 2016, Riga, Latvia

Methodology of OD Matrix Estimation Based on Video Recordings and Traffic Counts

Mihails Savrasovs*, Irina Pticina

Transport and Telecommunication Institute, Riga, LV-1019, Latvia

Abstract

Microscopic traffic modelling as the input data for traffic could use two types of data 1) OD matrices; 2) input intensities for each entering point in the network. The goal of the paper is to describe in detail and demonstrate application of the methodology of OD matrix evaluation based on video recording and traffic counts done in the study area. The approbation of the methodology was done in Riga city in the area which covers part of few city districts. The study area was surrounded by the video recording devices. In the same time manual counting of the traffic was performed. Next the manual decoding of the video recording was performed, recording the type of the vehicle and licence plate number for each recording site with split on incoming and outgoing traffic. This gave the information about origin and destination transport zones of the traffic flow and allows to construct initial OD matrix. The evaluated initial OD matrix was used to estimate the probability matrix, which describes the probability of driving to the different destinations from the origin. The obtained updated OD matrix was calibrated based on TFlowFuzzy approach. The manual counting data were used for the calibration of OD matrix. Finally the OD matrix was manually checked by the group of experts. The proposed methodology allows to estimate the OD matrix, which could be used for microscopic traffic modelling in case of dynamic routing application in the model.

Peer-review under responsibility of the scientific committee of the International Conference on Reliability and Statistics in Transportation and Communication

Keywords: OD matrix evaluation, traffic counts, video recording, TFlowFuzzy

* Corresponding author.
E-mail address: savrasovs.m@tsi.lv

Peer-review under responsibility of the scientific committee of the International Conference on Reliability and Statistics in Transportation and Communication
doi:10.1016/j.proeng.2017.01.116

1. Introduction

Traffic simulation is a powerful tool to forecast the impact of introduction changes in the transport network to the traffic flow circulation in the selected area. A first stage of the traffic flow simulation is related with input data collection and processing. The simulation results are highly dependent from the quality of the collected data. Microscopic traffic modelling as the input data for traffic could use two types of data 1) OD matrices; 2) input intensities for each entering point in the network. The second approach is widely used by practitioners in case if the simulation area is not wide, includes number of crossroads and decisions which should be tested by the simulation are local. In case of the wider network, significant quantity of routes between origin and destination points and testing of the solutions which could lead to the distribution of the traffic flows across the network the OD matrices is a feasible solution. In this case it is necessary to use in the model not the static routing, but the dynamic routing. The input for dynamic routing is OD matrix. Usually the OD matrix could be obtained from macroscopic model of the city or a district, but if the study area is too small and fits in one or few transport zones (defined by existing macroscopic model of the city) or there is no macroscopic model, the practitioners are faced to the problem of OD matrix evaluation for the current study area. The usual case, when OD matrix is estimated based on traffic counts done in the study area. Table 1 represents the summary of research works dedicated to the OD matrix estimation problem based on traffic counts.

Table 1. Review OD matrix estimation approaches based on traffic counts.

Authors	Approach
Robillard (1975), Hogberg (1976)	Gravity (GR) model based approach
Tamin and Willumsen (1989); Tamin*et al.* (2003)	Gravity-Opportunity (GO) based models
Van Zuylen (1978), Willumsen (1978)	Information Minimization (IM) and Entropy Maximization (EM) Approach
Bell (1991), Dey and Fricker (1994), Maher (1983), Bierlaire and Toint (1995), Spiess (1987), Cascetta and Nguyen (1988), Hazelton (2000)	Statistical Approaches (Maximum Likelihood (ML), Generalized Least Squares (GLS) and Bayesian Inference (BI)
Gong (1998)	Neural Networks
Xu and Chan (1993a, b), Reddy and Chakroborty (1998)	Fuzzy Based Approach
Baek *et al.* (2004), Wong *et al.*(2005)	Multi-Vehicle ODM Estimation
Kryger and Ottesen (1956), Brenner *et al.* (1957), Maher (1985), Shewey (1983), Evans *et al.* (1993), Asakura, Y., Hato and Kashiwadani (2000)	OD-estimation from license plate surveys, automatic vehicle identification data

As could be seen from the Table 1 there are a variety of approaches, which could be used to estimate the OD matrix from the counting data, but usually it is a subject of the quality of the final results and existing particularities of the existing transport network (is it congested or not, number of paths etc). As could be seen the idea of using licence plate numbers as a source of information to estimate the OD matrix is not new, but the development of ITS gave a new life to this approach. But still there are a number of issues, which should be taken into account applying licence plate survey. Current amount of vehicles foreseen in urban area is rather high to make a manual recording of the plate numbers (or part of it). But in many cities still ITS technologies are not very well presented to use such data for OD matrix estimation, for any selected study area. That is why use of video recording with manual transcription could be a feasible and very flexible solution.

In frame of this research it is proposed to use video with manual transcription with additional procedures, which should insure the raise of the final OD matrix quality. Under additional procedures it is proposed to use matrix of probabilities to avoid losing of the data during survey and use of TFlowFuzzy approach for the calibration and validation of the OD matrix.

Table 2. Comparison of instrumentation levels for the licence plate matching techniques (Fhwa.dot.gov. 2016).

Instrumentation level	Costs			Skill level		Accuracy	Automation potential
	Capital	Data collection	Data Reduction	Data collection	Data reduction		
Manual	Very Low	Moderate	High	Low	Low	Low	Low
Portable Computer	Moderate	Low to moderate	Low	Low	Moderate	Moderate	High
Video with manual transcription	Low	Moderate	High	Moderate	Low	Moderate to High	Moderate
Video with character recognition	High	Moderate	Low	Moderate to High	Moderate to High	Moderate to High	High

The second section of the paper demonstrates and describes in details the OD matric estimation methodology, while third section provides example of the developed methodology use.

2. OD matrix estimation methodology

This section of the paper is dedicated to the description of the proposed OD matrix estimation methodology which is based on video recording data. Graphically methodology is presented on Fig. 1. Before describing steps of the proposed methodology it is necessary to complete preliminary steps, which are not part of the methodology. The goal of preliminary stage is to define conditions necessary to apply the methodology. In this stage the following tasks should be completed:

A. Transport zones (TZs) should be defined; in this case TZ corresponds to the entering and exit point of the traffic flow. Number of TZ defines sizes of OD matrix.
B. Next analysis of the traffic recording points must be done, taking into account following factors: a) number of lanes on road; b) allowed traffic flow speed; c) safety of the recording device and controller (person, who is responsible for recording); presence of the artificial and/or natural obstacles (like trees, bushes, public transport stops, etc.).
C. Presence of the needed amount of recording devices. Here must be noted, that not only video cameras could be used, but also mobile phones, cameras, tablets etc. Before use of the mentioned above devices following should be checked: capacity of the memory card, capacity of the batteries, video recording quality. It is also important to have special stands to put devices on best traffic observation high (to note: artificial and/or natural objects could be used also). Also in case if traffic flow intensity is weak, it is possible to make photos of the vehicles or do manual recording of the license plate numbers.
D. Finally it is necessary to define traffic counting points in the network. They are important for the calibration of OD matrix. Here different approaches could be used: manual traffic counting, video recording etc.

The methodology consists of the following steps which are described in details below and are represented in Fig. 1.

1. The recorded video is decoded by means of vehicle type and registration number at license plate. The decoding is done in manual way, using additional video filters in case if it is necessary. All data is recorded in Excel table by traffic counting point. In case, if it is not possible to recognize the license plate, the symbol "*" is used to mark such vehicles.
2. Next according to the developed algorithm the data are processed by searching exit point for each entering vehicle (by license plate number). If record is found the initial origin-destination matrix is updated, by adding plus 1 in cell, which corresponds to the origin and destination zones, if not, the record is marked as not located. The data processing should be done individually by vehicles types (if it is necessary).
3. The initial versions of origin-destination matrix is analysed and validated in manual mode. Based on validated origin-destination matrix, the probability matrix is calculated. The probability matrix shows the

distribution of arriving traffic flow among destinations. Next, the rows of matrix are multiplied by total arriving quantity for the zone. This operation is necessary to distribute vehicles not recognized by the matching algorithm. By this, the 2nd version of the matrix is obtained.

4. Next, the calibration of the matrices is performed by application of TFlowFuzzy method, which is implemented in macroscopic traffic flow simulation software PTVVISUM. TFlowFuzzy is a method used to adjust a given origin-destination matrix in such a way that the result of the assignment closely matches observed volumes at points within the network. In other words, TFlowFuzzy results were calibrated to develop origin-destination matrix that matches existing conditions. The general view on TFlowFuzzy approach is presented in Fig. 2. The method uses 3 types of the input data: 1) traffic counts – obtained during traffic survey; 2) allowed error for each traffic count point (the standard value is 5%); 3) initial origin-destination matrix, which should be calibrated. Based on input data the TFlowFuzzy method calibrates the initial origin-destination matrix.
5. After performing calibration of the matrix, the calibration results must be validated. It is proposed to use a NOVEL approach, which is based on regression analysis application. If validation results are acceptable the matrix are treated as valid and final, if not, the calibration procedure is repeated. The quality of the solution produced by TFlowFuzzy is best illustrated through a goodness-of-fit plot. The R-square statistic measures how well TFlowFuzzy was able to match the input volumes. Also the RMSNE are used to estimate the quality of the calibration.

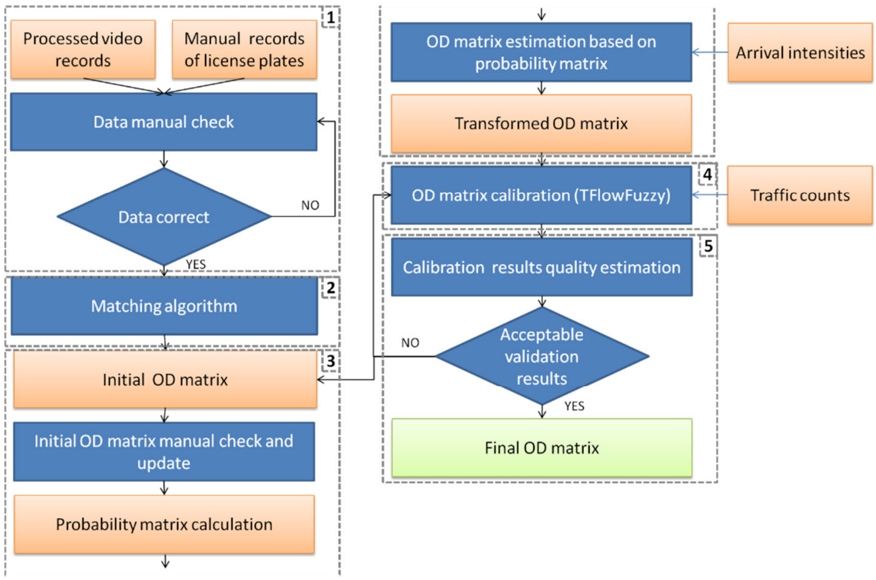


Fig. 1. Methodology of OD matrix estimation.

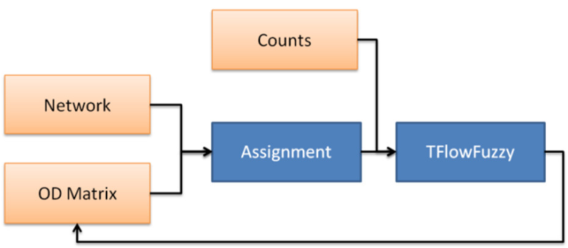


Fig. 2. TFlowFuzzy approach.

3. Approbation of the methodology

3.1. Survey process

The approbation of the methodology was done for the fragment of the transport network of the Riga city. The study object is illustrated in Fig. 5 and includes number of signalised and not signalised crossroads and three, two level flyovers. The territory is a part of Riga transport system, which connects several residential districts of Riga city: Purvciems, Kengarags, Centre and the left riverside. Another important point, which should be taken into account, the territory includes parts of the following main streets: Krasta, Maskavas, Lubanas, Katlakalna, Krustpils, Darzciema, Piedrujas, Ilukstes and A. Saharova street. Maskavas and Krasta streets are highly used by the traffic from city centre to neighboring cities. The study area is much congested (see Fig. 3 and Fig. 4).

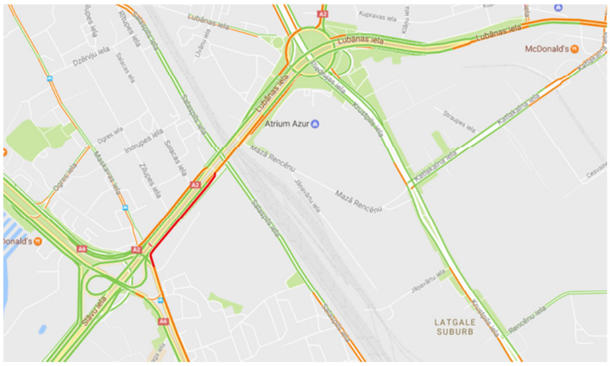


Fig. 3. Congestions during morning peak hour.

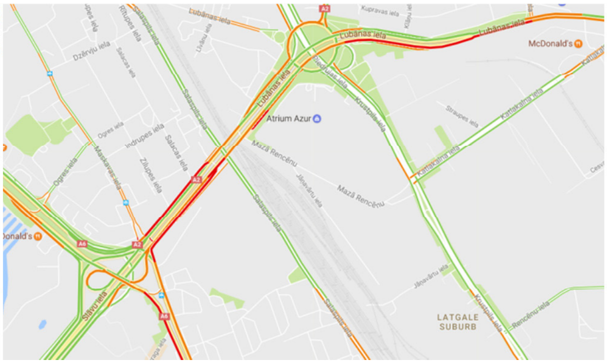


Fig. 4. Congestions during evening peak hour.

As could be seen from Fig. 5 in total 14 TZ were defined for the study area (red circles), 37 locations (black dots with traffic flow fixing direction) for video recording were selected, in 6 of them the manual license plate number registration were used, because of low traffic intensity (corresponds to counting point in TZ Nr12, Nr13 and Nr14). Five counting points of intensities were defined and represented in Fig. 5 as blue bold lines.

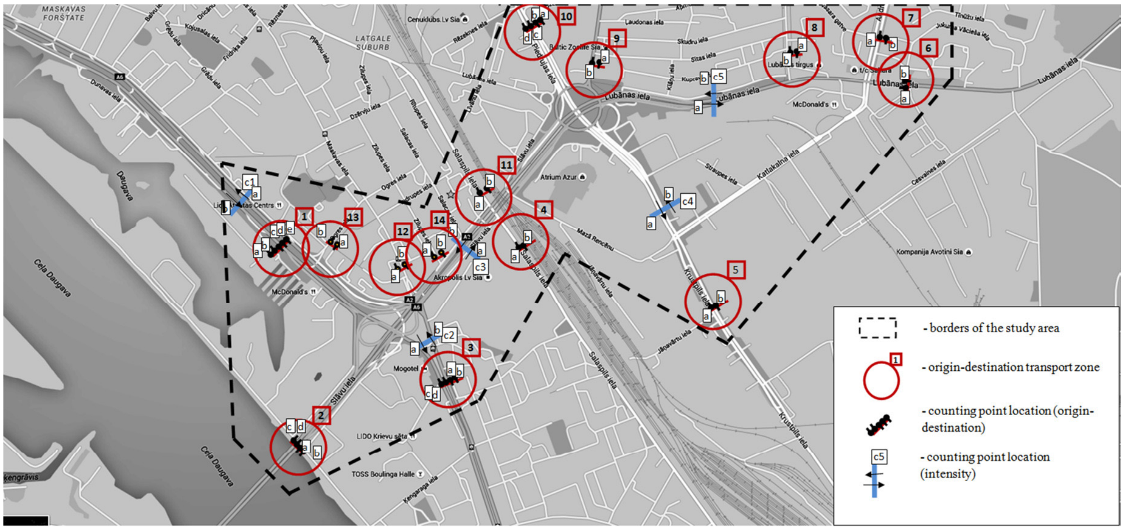
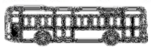


Fig. 5. Study object, transport zones and counting points location.

Taking into account particularities of the study object the following types of vehicles were taking into account during the study process (see Table 3).

Table 3. Types of vehicles.

Graphical representation	Code	Description
	V	Passenger vehicles
	C1	Light cargo vehicles
	C2	Mid cargo vehicles
	C3	Cargo vehicles
	C4	Cargo vehicles with trailer
	S	Buses (without public transport of Riga city)

The survey was conducted during evening peak hour from 17:15 till 18:30. Survey was carried out with the help of 50 observers recruited from Transport and Telecommunication Institute student community. The meeting with observers was done 4 days before traffic survey. Observers were instructed and the information about the process of traffic survey was provided. Each observer got the individual instruction and task in printed form. The observers who do the manual traffic counting also got the printed counting forms. Three inspectors from the TSI have monitored the process of the traffic survey.

3.2. OD matrix evaluation process

In total more than 36 000 records were included into the Excel file for processing and OD matrix evaluation. The Fig. 6 shows a part of the Excel file with the data and demonstrates the structure of the processing file. The license plate numbers are blurred (as this data is private and cannot be distributed).

TZ:7				TZ:8				TZ:9				TZ:10			
type	a	type	b	type	b	type	a	type	b	type	a	type	d,c	type	a,b
V	[illegible]	S	[illegible]	V	[illegible]	V	*	C3	[illegible]	S	[illegible]	C1	[illegible]	V	[illegible]
C1	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	S	*	V	[illegible]	V	[illegible]	v	[illegible]	V	*	C1	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	*	v	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	*	v	[illegible]	V	[illegible]
V	[illegible]	V	*	V	[illegible]	V	*	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	S	[illegible]	V	[illegible]	V	*	v	*	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	C4	[illegible]	V	[illegible]
C1	[illegible]	V	*	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	V	*	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
C1	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	v	*	V	[illegible]	v	[illegible]	C1	[illegible]
V	[illegible]	V	[illegible]	V	[illegible]	V	[illegible]	C1	[illegible]	V	[illegible]	v	[illegible]	C1	[illegible]
C1	[illegible]	V	*	V	[illegible]	V	*	C1	[illegible]	V	*	v	[illegible]	V	[illegible]
C2	*	V	[illegible]	V	[illegible]	V	[illegible]	C1	[illegible]	V	[illegible]	v	[illegible]	V	[illegible]

Fig. 6. The fragment of the Excel file with the data.

Next the run of the Matching algorithm was completed and initial OD matrix was constructed. During the Matching algorithm execution the table with initial data was coloured in order to visualise the results of matching procedure (see Fig. 6). The red coloured cell means that the license plate number was not found in table, the blue means that the license plate number was found at the exit from transport network. The cells are coloured only for the entering directions into the study area. This information is useful for manual controlling of the Matching algorithm and for data validation. The initial matrix was checked manually and confirmed for further use. Based on initial data the probability matrix was calculated. The probabilities shows, how entering volume of traffic is distributed by destination TZs. The Fig. 7 demonstrates the probability matrix, arrival volume per TZ and obtained OD matrix after multiplying arrival amount.

TZ	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.033	0.207	0.350	0.018	0.047	0.128	0.030	0.005	0.076	0.073	0.003	0.002	0.026	0.002
2	0.160	0.073	0.206	0.015	0.017	0.181	0.008	0.004	0.145	0.178	0.003	0.000	0.009	0.001
3	0.511	0.205	0.090	0.039	0.006	0.021	0.009	0.002	0.101	0.011	0.003	0.001	0.001	0.000
4	0.003	0.015	0.116	0.226	0.006	0.006	0.009	0.003	0.000	0.009	0.591	0.009	0.000	0.009
5	0.007	0.217	0.060	0.056	0.247	0.043	0.079	0.022	0.112	0.140	0.000	0.017	0.000	0.000
6	0.327	0.328	0.070	0.042	0.027	0.060	0.032	0.063	0.015	0.011	0.009	0.012	0.002	0.002
7	0.371	0.169	0.061	0.040	0.172	0.028	0.104	0.034	0.006	0.003	0.000	0.009	0.003	0.000
8	0.269	0.374	0.095	0.050	0.041	0.064	0.022	0.045	0.005	0.012	0.002	0.019	0.000	0.002
9	0.356	0.354	0.090	0.058	0.075	0.017	0.003	0.000	0.005	0.015	0.003	0.018	0.002	0.005
10	0.537	0.227	0.057	0.046	0.023	0.035	0.014	0.034	0.002	0.008	0.006	0.009	0.002	0.001
11	0.011	0.021	0.005	0.894	0.007	0.019	0.011	0.000	0.000	0.007	0.011	0.012	0.001	0.003
12	0.036	0.107	0.250	0.071	0.000	0.143	0.000	0.000	0.000	0.036	0.000	0.107	0.214	0.036
13	0.759	0.074	0.028	0.019	0.009	0.000	0.000	0.000	0.028	0.019	0.009	0.028	0.028	0.000
14	0.083	0.042	0.056	0.264	0.028	0.139	0.097	0.000	0.000	0.083	0.000	0.083	0.125	0.000

*

TZ	Arrival volume
1	5315
2	2054
3	1568
4	781
5	1266
6	972
7	504
8	806
9	997
10	2190
11	1147
12	57
13	196
14	163

=

TZ	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	175	1099	1860	98	251	680	159	27	405	387	16	13	136	11
2	329	150	423	31	35	372	17	7	298	366	6	0	19	1
3	802	322	142	61	9	34	14	3	158	17	4	2	1	0
4	2	12	90	176	5	5	7	2	0	7	461	7	0	7
5	9	274	76	71	312	54	99	28	142	177	0	21	0	0
6	318	319	68	41	26	59	31	62	15	10	9	12	1	1
7	187	85	31	20	87	14	53	17	3	2	0	5	2	0
8	217	302	76	40	33	51	18	36	4	10	1	15	0	1
9	355	353	90	58	75	17	3	0	5	15	3	18	2	5
10	1176	497	124	101	50	77	31	74	4	18	13	20	4	2
11	12	24	6	1025	8	21	12	0	0	8	12	14	2	3
12	2	6	14	4	0	8	0	0	0	2	0	6	12	2
13	149	15	5	4	2	0	0	0	5	4	2	5	5	0
14	14	7	9	43	5	23	16	0	0	14	0	14	20	0

Fig. 7. Probability OD matrix, arrival volumes and OD matrix.

After receiving the second version of the OD matrix, the TFlowFuzzy approach was used to calibrate the obtained OD matrix. The simplified model of the transport network was constructed in PTVVISUM simulation software, OD matrix was provided as the demand, and counted volumes of traffic, with possible 5% error were declared.

As could be seen (Fig. 8) the evaluated OD matrix provides the data, which leads to difference in modelled data and observed data. Not looking to the fact, that R-square is 0.87 and Mean Relative Error is 18%, it was decided to complete a TFlowFuzzy calibration of the OD matrix. The TFlowFuzzy procedure was completed and Fig. 9 represents validation results after the calibration. The adjustment of the origin-destination matrix gave a positive effect, the R-Square become close to 1, the calculated Mean Relative Error is decreased to 2%. Also decrease from 25% to 3% was observed for RMSNE. The obtained results were considered as valid.

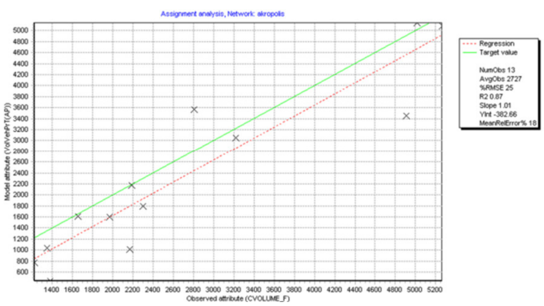


Fig. 8. Validation of OD matrix before calibration.

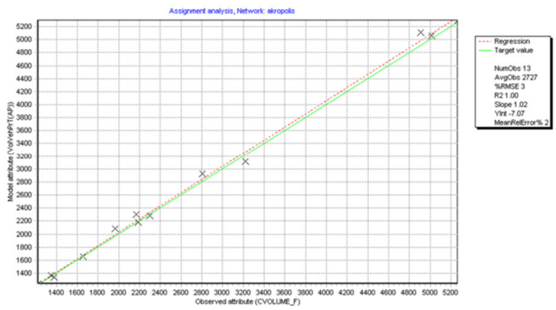


Fig. 9. Validation of OD matrix after calibration.

4. Conclusions and results

4.1. Results and discussion

The goal of the paper was to describe in detail and demonstrate application of the methodology of OD matrix evaluation based on video recording and traffic counts done in the study area. The paper describes in detail the proposed methodology with additional recommendation, which should be taken into account implementing the methodology and demonstrates the case-study of methodology application in Riga city. The advantage of the proposed methodology are following: a) in case of using dynamic routing in traffic flow micro model for wide areas the OD matrix could be estimated in acceptable time (the case-study presented in a paper was completed in two weeks: data collection, data processing and OD matrix evaluation); b) the quality of the obtained OD matrix is high, which is confirmed by high level of R-square (close to 1), low Mean Relative Error (2%) and low RMSNE (3%); c) from practical point of view it is much easier to explain the customers of how OD matrix were obtained, rather than explaining them methods which are based on mathematical models or heuristic procedures.

4.2. Conclusions and recommendations

Of course there are some limitations, which should be taken into account applying methodology: a) the transport network particularities in the study area defines number of entering and exit points, which could be rather high, it means that to carry out such survey, the significant number of resources will be necessary (recording devices, staff etc); b) the high number of the vehicles with not recognized license plate numbers will influence on final OD matrix quality; c) in case of congested area careful planning of video recording sites locations should be done in order, to catch incoming volume of vehicles.

Additionally some practical recommendation could be drawn from practical application of the methodology and its preliminary steps:

- Planning the survey, be sure to have reserve video recording devices and staff, in case of technical problems or problems with staff you will have the opportunity to continue the survey.
- Ask staff to check and test all video recording devices before survey. The quality of recording, battery capacity and capacity of the memory are subjects to be checked and tested. Note, that in order to get video with acceptable quality it is recommended to use HD or Full HD mode of recording. In the survey the HD mode was used and the memory allocation was around 10GB. Please check the technical specification of the video recording devices, as some of the devices has time-limited recording. Put the autofocus mode during recording to get best recording quality of the object.
- Locating video recording devices take care of sun position. If sun is in front of camera it could significantly influence on quality of video recording and transcription quality. If it is a case, change the direction of the video recording device; record the back license plates not the front.
- Give necessary amount of time to do the manual transcription. In this survey, for 1h15 video 4 days were granted to the staff to do the decoding. This one is necessary to get good quality of transcription.
- Use the video playing software, which allows to zoom selected part of the screen, showing the license plate with zoom. In this survey the VLC media player was used, as it allows to configure such option.
- The recommended video playing speed is 0.12X, but it depends from the staff qualification and intensity of the traffic flow.

Acknowledgements

This work was supported by Latvian state research program 2014-2017 project "The next generation of information and communication technologies (NexIT)"

References

Asakura, Y., Hato, E. and Kashiwadani, M. (2000) Origin-destination matrices estimation model using automatic vehicle identification data and its application to the Han-Shin expressway network, Transportation 27: 419. DOI:10.1023/A:1005239823771.

Baek, S., Kim, H. and Lim, Y. (2004) Multiple-vehicle origin-destination matrix estimation from traffic counts using genetic algorithm, Journal of Transportation Engineering, ASCE, May/June: 339-347.

Bell, M. (1991) The estimation of origin-destination matrices by constrained generalized least squares, Transportation Research, Part B: Methodological 25, 13–22. Robillard, P. (1975) “Estimating the O-D matrix from observed link volumes”, Transportation Research, 9, 123–128.

Bierlaire, M. and Toint, Ph. L. (1995) Meuse: an origin-destination matrix estimator that exploits structure, Transportation Research, Part B: Methodological, 29, 47–60.

Brenner, R., Mathewson, J.H. and Gerlough, D.L. (1957) A General Method for Estimating Through Traffic in a Road Network, Highway Research Abstracts, 27 (8).

Cascetta, E. and Nguyen, S. (1988) A unified framework for estimating or updating origin/ destination matrices from traffic counts, Transportation Research, Part B: Methodological, 22, 437–455.

Dey, S.S. and Fricker, J.D. (1994) Bayesian updating of trip generation data: combining national trip generation rates with local data, Transportation, 21(4), 393–403.

Gong, Z. (1998) Estimating the urban o-d matrix: a neural network approach, European Journal of OperationalResearch, 106, 108–115.

Evans, R., Martin, P.T. and Bell, M.C. (1993) A Method for Analysing Partial Registration-plate Data, Traffic Engineering and Control, 34, 76–79.

Fhwa.dot.gov. (2016) [online] Available at: https://www.fhwa.dot.gov/ohim/handbook/chap4.pdf [Accessed 4 Aug. 2016].

Hazelton, M.L. (2000) Estimation of origin–destination matrices from link flows on uncongested networks, Transportation Research, Part B: Methodological, 34, 549–566.

Hogberg, P. (1976) Estimation of parameters in models for traffic prediction: a non-linear regression approach, Transportation Research, 10, 263–265.

Kryger, P. and OttesenK.A. (1956) Can the License Plate Method be Used for Traffic Studies?, Traffic Quarterly, 18 Vol.10, pp.377–386.

Maher, M.J. (1983) Inferences on trip matrices from observations on link volumes: a bayesian statistical approach, Transportation Research, Part B: Methodological 17, 435–447.

Maher, M.J. (1985) The Analysis of Partial Registration-plate Data, Traffic Engineering and Control, 26, 495–497.

Shewey, P.J.H. (1983) An Improved Algorithm for Matching Partial Registration Numbers, Transportation Research B, 17B(5), .391–397.

Spiess, H. (1987) A maximum-likelihood model for estimating origin-destination matrices, Transportation Research, Part B: Methodological, 21, 395–412.

Tamin, O.Z. and Willumsen, L.G. (1989) Transport demand model estimation from traffic counts, Transportation, 16, 3–26.

Tamin, O.Z., Hidayat, H. and Indriastuti, A.K. (2003) The development of maximum-entropy (ME) and bayesian-inference (BI) estimation methods for calibrating transport demand models based on link volume information, Proceedings of the Eastern Asia Society for Transportation Studies, 4, 630–647.

Van Zuylen, J.H. (1978) The information minimizing method: validity and applicability to transport planning, In: New Developments in Modelling Travel Demand and Urban Systems (edited by G. R. M. Jansen et al).

Willumsen, L.G. (1978) Estimation of O-D matrix from traffic counts: a review, Working Paper 99, Institute for Transport Studies, University of Leeds.

Xu, W. and Chan, Y. (1993a) Estimating an origin-destination matrix with fuzzy weights. Part 1: Methodology, Transportation Planning and Technology, 17, 127–144.

Xu, W. and Chan, Y. (1993b) Estimating an origin-destination matrix with fuzzy weights. Part 2: Case Studies, Transportation Planning and Technology, 17, 145–164.