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The Conditional Capital Asset Pricing Model Revisited: Evidence from High-Frequency Betas

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Abstract. When using high-frequency data, the conditional capital asset pricing model (CAPM) can explain asset-pricing anomalies. Using conditional betas based on daily data, the model works reasonably well for a recent sample period. However, it fails to explain the size anomaly as well as three out of six of the anomaly component excess returns. Using high-frequency betas, the conditional CAPM is able to explain the size, value, and momentum anomalies. We further show that high-frequency betas provide more accurate predictions of future betas than those based on daily data. This result holds for both the time-series and the cross-sectional dimensions.

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Keywords: beta estimation • conditional CAPM • high-frequency data

1. Introduction

Shortly after its introduction by Sharpe (1964), Lintner (1965), and Mossin (1966), the unconditional capital asset pricing model (CAPM) was heavily challenged by empirical studies (e.g., Black et al. 1972). However, even though the unconditional CAPM seems to be rejected in the data, the model may well hold conditionally period by period. The market portfolio may be on the conditional mean–variance frontier but not on the unconditional one (Hansen and Richard, 1987).

In this paper, we revisit the conditional CAPM. We use a large sample of more than 2,500 stocks with both daily and high-frequency data for the period from 1996 until 2014 to empirically test whether the conditional CAPM can explain asset pricing anomalies.

We make several contributions to the literature. First, we perform a test of the conditional CAPM following the approach of Lewellen and Nagel (2006) for our recent sample period. We find that the conditional CAPM performs reasonably well when using daily data. Examining the anomaly component portfolios (i.e., the portfolios of small, big, growth, value, loser, and winner stocks), the model can explain the excess returns of three out of these six, namely those of the big, loser, and winner stocks. Overall, the conditional CAPM is able to explain the returns of the value (value minus growth) and momentum (winners minus losers) anomaly portfolios, whereas it fails to explain the size (small minus big) anomaly return.

Second, we go beyond the approach of the previous literature and estimate conditional betas with high-

frequency (30-minute) data. We find that with high-frequency data, the conditional CAPM performs even better than on the basis of daily data. The model can explain the excess returns of five out of the six anomaly component portfolios, namely those of the small, big, growth, value, and winner stocks. More important, the conditional CAPM is able to explain the returns of the size anomaly, the value anomaly, and the momentum anomaly portfolios. Thus, our results suggest that the inaccurate measurement of (conditional) betas weakens the empirical performance of the conditional CAPM. Once we measure betas more accurately with high-frequency data, the model performs even better than with betas based on daily data.

Third, we examine whether market-timing and volatility-timing biases documented in the previous literature can account for the magnitudes of the unconditional alphas we observe. On the basis of the calibrations of Lewellen and Nagel (2006) and Boguth et al. (2011), we conclude that the magnitude of the failure of the unconditional CAPM is potentially consistent with underconditioning biases.

Our main results are robust to several tests. When using different window lengths between 1 and 12 months to estimate the conditional betas, alternative frequencies for the high-frequency beta estimator, and when accounting for a potential bias based on the use of ex post betas, we obtain qualitatively similar results. For the 6- and 12-month windows, the conditional CAPM based on high-frequency data can even explain the returns of

all six component portfolios, along with those of the anomaly return portfolios.

In the final part of this paper, we examine the precision of high-frequency betas. We compare the predictive ability of the high-frequency beta estimator for future betas with that of the estimator based on daily data. We find that high-frequency betas have superior predictive power for future realized betas. Sorting stocks into five portfolios, we find that the root mean squared error (RMSE) is significantly lower for high-frequency betas compared with betas based on daily return data. This pattern holds for each portfolio. We find that high-frequency betas also yield a lower average RMSE than those based on daily data for alternative realized beta estimators based on lag-adjusted high-frequency or daily data. Furthermore, we find that high-frequency betas perform at the very least as well as the option-implied approach of Buss and Vilkov (2012). We find that the predicted and realized betas line up very well when using high-frequency data, whereas the other estimators tend to yield high prediction errors, especially for low-beta and high-beta stocks. Furthermore, we obtain similar results when using coarser sampling frequencies. We also confirm that betas based on high-frequency data outperform those based on daily data in Mincer–Zarnowitz regressions and not only in the time-series but also in the cross-sectional dimension. Finally, we obtain similar results when using the mean absolute error (MAE) criterion as an alternative statistical loss function.

Our work relates to a large literature that tests the CAPM. Black et al. (1972) and Fama and French (1992) find that unconditional betas are unrelated to average stock returns. However, Jagannathan and Wang (1996) argue that the CAPM may hold period by period even if it does not hold unconditionally. That is, the market portfolio could be on the conditional mean–variance frontier each period but not on the unconditional mean–variance frontier. In this vein, Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005), and Santos and Veronesi (2005) find empirical support for the conditional CAPM with different conditioning variables. However, Lewellen and Nagel (2006) use daily data recorded over short windows to directly test the conditional CAPM and reject the model. We complement these studies by using betas estimated with high-frequency data. Our results suggest that the more accurate measurement of betas helps to improve the performance of the conditional CAPM.

Our paper is also related to Gilbert et al. (2014). The authors find that the CAPM pricing errors are lower when estimating betas with quarterly data compared with using daily data. However, by using quarterly data along with a long historical window of five years or more, one essentially gives up on trying to obtain a truly conditional estimate for beta. Our results indicate that

conditional betas estimated with intraday data outperform those based on lower-frequency data both economically and statistically.

We also add to the literature on beta estimation. Buss and Vilkov (2012), Chang et al. (2012), and Baule et al. (2016) propose different estimators that exploit the information content of option prices. Hollstein and Prokopczuk (2016) compare these approaches and find that the hybrid estimator of Buss and Vilkov (2012) outperforms the low-frequency historical beta estimator as well as all other approaches that use options or daily return data. We depart from these studies by exploiting high-frequency return data to more accurately estimate beta. To the best of our knowledge, we are the first to compare these approaches and show that high-frequency beta performs at least as well as the option-implied approach. Furthermore, we add to Hollstein and Prokopczuk (2016) by evaluating different beta estimators primarily from an economic perspective by testing the conditional CAPM.

This study uses the concept of realized beta in the spirit of Bollerslev and Zhang (2003), Barndorff-Nielsen and Shephard (2004), Andersen et al. (2006), Bollerslev and Todorov (2010), and Patton and Verardo (2012). Cenesizoglu et al. (2016) study the stocks that make up the Dow Jones Industrial Average and show that beta measured with high-frequency data yields statistically more accurate predictions than the historical estimator based on daily or monthly return data. Although our results are consistent with theirs, there are important differences with our study. First, we focus on the asset pricing implications of high-frequency beta. Second, we study a much broader cross section of stocks. Third, we analyze the predictability for realized beta not only from a time-series perspective but also in the cross-sectional dimension.

Finally, our paper belongs to a broader research area analyzing the benefits of high-frequency data. Andersen and Bollerslev (1998) highlight the importance of intraday data for accurately measuring realized volatility. Andersen et al. (2005b) demonstrate the impact of measurement error in the variance of an asset on the classic Mincer–Zarnowitz regressions. Patton (2011) studies the impact of measurement error on the ranking of different volatility forecasting models. Amaya et al. (2015) establish the importance of intraday data for the accurate measurement of skewness and kurtosis. We complement these studies by showing that high-frequency data matter for the estimation of betas. Based on this finding, our recommendation to the academic and professional literature is to generally use high-frequency data for beta estimation whenever possible.

The remainder of this paper is organized as follows. Section 2 describes our data set and methodology and presents summary statistics for the different

beta estimators. In Section 3, we test the conditional CAPM. Section 4 examines the precision of high-frequency betas. Finally, Section 5 concludes.

2. Data and Methodology

2.1. Data

Our data set covers U.S. stocks for the sample period from January 1996 to December 2014. We use daily and monthly prices and returns as well as data on dividend payments and shares outstanding from the Center for Research in Security Prices (CRSP). In addition, we collect high-frequency price data from the Thomson Reuters Tick History (TRTH) database.¹ We use all stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotations (NASDAQ) that are classified as ordinary common shares (CRSP share codes 10 or 11) and for which both daily and high-frequency data are available. Following Amihud (2002) and Zhang (2006), we exclude very illiquid stocks. To be more precise, we expunge firm-month observations with prices below 1 U.S. dollar or a market capitalization below 10 million U.S. dollars (Cohen et al. 2002, Hou and Loh 2016). On average, the stocks for which high-frequency data are available represent 87% of the entire market capitalization of eligible U.S. stocks.

To process the final high-frequency data set, we follow the data-cleaning steps outlined in Barndorff-Nielsen et al. (2009). First, we use only data with a time stamp during the exchange trading hours (i.e., between 9:30 a.m. and 4:00 p.m. Eastern Standard Time). Second, we remove recording errors in prices. To be more specific, we filter out prices that differ by more than 10 mean absolute deviations from a rolling centered median of 50 observations. Afterward, we assign prices to every five-minute interval using the most recent entry recorded that occurred at most one day before. Finally, we follow Bollerslev et al. (2016) and supplement the TRTH data with data on stock splits and distributions from CRSP to adjust the TRTH overnight returns. To guard against the potential effects of non-trading in smaller stocks (Gorodnichenko and Weber 2016), for our main tests, we rely on a 30-minute frequency for our main analysis.

The Options data set comes from IvyDB Option-Metrics. We obtain data from the Volatility Surface that directly provides implied volatilities over standardized times to maturity for certain levels of delta.² We select out-of-the-money options, namely puts with deltas larger than -0.5 and calls with deltas smaller than 0.5 . To compute the model-free option-implied moments, we use the formulas provided by Bakshi et al. (2003). A more detailed outline of the procedure is

provided in the appendix. Data on the interest-rate term structure come from the IvyDB zero curve file.

We collect balance sheet data from the Compustat database. The book equity of each stock is computed as stockholders' equity plus balance sheet deferred taxes and investment tax credit plus postretirement benefit liabilities minus the book value of preferred stock. Data on the risk-free (one-month Treasury Bill) rate are from Kenneth French's data library.

2.2. Beta Estimation

2.2.1. Historical Beta. We consider historical estimates (HIST) following, for example, Fama and MacBeth (1973), regressing an asset's daily excess return on a constant and the (contemporaneous) market excess return:

$$\beta_{j,t}^{\text{HIST}} = \frac{\text{cov}(r_j - r_f, r_M - r_f)}{\text{Var}(r_M - r_f)}, \quad (1)$$

where $\beta_{j,t}^{\text{HIST}}$ denotes the estimator for the historical beta of asset j using data from time $t - k$ to t . Here k is the length of the estimation window, r_j is the vector of returns on asset j , r_M denotes the return vector of the market, and r_f is the vector of the risk-free rate. All returns are observed up to time t . Alternatively, $\text{HIST}^{(l)}$ denotes a Dimson (1979)-adjusted historical estimator, which we obtain through the following regression:

$$\begin{aligned} r_{j,\tau} - r_{f,\tau} = & \alpha_{j,t} + \beta_{j,t}^{(0)}(r_{M,\tau} - r_{f,\tau}) + \beta_{j,t}^{(1)}(r_{M,\tau-1} - r_{f,\tau-1}) \\ & + \beta_{j,t}^{(2)} \left(\sum_{n=2}^l r_{M,\tau-n} - r_{f,\tau-n} \right) + \epsilon_{j,\tau}. \end{aligned} \quad (2)$$

We incorporate $l = 1$ up to $l = 4$ lagged returns. In the case of $l = 1$, the term associated with $\beta_{j,t}^{(2)}$ drops. The estimator for beta is then $\beta_{j,t}^{\text{HIST}^{(l)}} = \sum_{i=0}^{\min(2,l)} \beta_{j,t}^{(i)}$, where $\min(\cdot)$ is the minimum operator.

2.2.2. Realized Beta. Following Andersen et al. (2006), we use realized beta (HF_{freq}). We use intraday high-frequency (log) returns, sampled at intervals of freq minutes to estimate³

$$\beta_{j,t}^{\text{HF}} = \frac{\sum_{\tau=1}^N r_{j,\tau} r_{M,\tau}}{\sum_{\tau=1}^N r_{M,\tau}^2}, \quad (3)$$

where $r_{j,\tau}$ and $r_{M,\tau}$ refer to the return of asset j and the market return at time τ , respectively. Also, N is the number of high-frequency return observations during the time period under investigation. Note that we use realized beta both as estimator for high-frequency historical beta (HF_{freq}) and for the ex post evaluation of estimators ($\beta_{j,t,T}^{\text{R}} = \beta_{j,t}^{\text{HF}}$, using data between t and $T = t + m$, with m denoting the length of the evaluation horizon).

An important point relates to the sampling frequency (*freq*). As Patton and Verardo (2012) point out, there is a delicate trade-off. On the one hand, using low-frequency data could result in noisy estimates of beta (Andersen et al. 2005a). On the other, pushing the analysis to a very high frequency introduces a number of microstructure issues (Scholes and Williams 1977, Epps 1979). We focus our main analysis on a sampling frequency of 30 minutes. One might worry that the sampling frequency we use is quite high and that stocks may not have synchronous trading times, introducing a bias in the estimation of beta. To address this concern, we also use an estimator that accounts for lag effects, as in Dimson (1979).⁴ In addition, we use several alternative sampling frequencies, including 75 and 130 minutes and obtain similar results.

2.2.3. Hybrid (Option-Implied) Beta. We also consider the hybrid estimator of Buss and Vilkov (2012) (BV), which has been shown by Hollstein and Prokopczuk (2016) to yield the most accurate predictions among models relying on daily or monthly return data. Essentially, the approach combines model-free option-implied volatilities and historical correlations to estimate beta. Buss and Vilkov (2012) use (1) the identity that the implied variance of the market index has to be the same as the implied variance of the value-weighted portfolio of all index constituents and (2) a technical condition that maps physical correlations ($\rho_{ji,t}^P$) into risk-neutral correlations ($\rho_{ji,t}^Q$), namely $\rho_{ji,t}^Q = \rho_{ji,t}^P - \alpha_t(1 - \rho_{ji,t}^P)$.⁵ Combining these two relations and solving for α_t , the authors recover implied correlations. Thus, a beta estimate under the risk-neutral probability measure is obtained by

$$\rho_{j,t}^{BV} = \frac{\sigma_{j,t,T}^Q \sum_{i=1}^N \omega_{i,t} \sigma_{i,t,T}^Q \rho_{ji,t}^Q}{\left(\sigma_{M,t,T}^Q\right)^2}, \quad (4)$$

where $\sigma_{j,t,T}^Q$ and $\sigma_{M,t,T}^Q$ denote the option-implied volatilities at time t from options of asset j and the market index, respectively. The implied volatilities needed for the approach are extracted from options whose time to expiration $T - t$ matches the evaluation horizon m . Thus, for our main tests, we use options with a time to maturity of 30 days. Then $\omega_{i,t}$ denotes the weight of asset i in the market index at time t . We use all daily returns observed in the measurement period to obtain the correlations needed for the standard BV estimator.

We also use high-frequency correlations, estimated with 30-minute data, combined with option-implied volatility (BV₃₀). By comparing BV and BV₃₀, we can shed light on the value of high-frequency data for the hybrid estimator. Notice that because of condition (1), we need the full correlation matrix of all index

constituents. Therefore, it is not clear whether using high-frequency data is beneficial because the problem of infrequent and nonsynchronous trading is likely much more severe than for the historical realized beta estimator, where for each stock only one correlation (i.e., that of an asset with the market) has to be estimated.

2.3. Summary Statistics and Correlation Analysis

Table 1 reports summary statistics on the different beta estimation approaches.⁶ The number of total observations is substantially higher for all historical and high-frequency estimators compared with BV and BV₃₀ mainly because the hybrid estimator can only be implemented for the constituents of the Standard & Poor's (S&P) 500 Index at each point in time, whereas we can estimate the historical and high-frequency betas independently of whether the stock is part of a market index or not.

We first examine the value-weighted average beta over all stocks (Mean_{vw}). This quantity should be equal to 1 when examining a complete market index. Because we do not examine a full market index but a much broader universe than the S&P 500 stocks, this condition is not binding, but it can serve as a reference value to assess whether the beta estimates are reasonable. We find that the value-weighted averages for all approaches are close to 1. For HIST, for example, the value-weighted average beta is 1.00. Adding lags in HIST⁽¹⁾ and HIST⁽²⁾, the value-weighted average increases slightly to 1.02. However, adding lags increases the standard deviation and decreases the AR(1) coefficient. Both observations indicate that adding lagged betas substantially increases the measurement error in HIST. Noise in the estimator naturally increases the standard deviation and creates an attenuation bias in the AR(1) coefficient. Because the value-weighted average of HIST is 1.00, there is little evidence to suggest that HIST is systematically biased. Thus, we focus our main discussion on the estimator without lag adjustment.

By contrast, the value-weighted average beta from high-frequency estimators is just under 1. For our main estimator HF₃₀, the value-weighted beta reaches 0.95. For the lower 75- and 130-minute frequencies, this quantity amounts to 0.96. These figures deliver some indication that infrequent and nonsynchronous trading may bias beta toward zero for some stocks. However, adding only one lag to HF₃₀ (HF₃₀⁽¹⁾), the value-weighted average rises to 0.99. With five lags (HF₃₀⁽⁵⁾), the value-weighted average is 1.02. Similar to HIST, when adding lags, the standard deviation of HF₃₀ increases, and the AR(1) coefficient decreases, which points toward increased measurement error when adding lags. Nevertheless, because it is possible that there is a bias in HF₃₀, we focus our main

Table 1. Summary Statistics and Sample Correlations

	Summary statistics					Correlations												
	Number of observations	Mean _{n_w}	Standard deviation	AR(1)	q ^{0.05}	q ^{0.25}	q ^{0.75}	q ^{0.95}	HIST ⁽¹⁾	HIST ⁽²⁾	HF ₃₀	HF ₃₀ ⁽¹⁾	HF ₃₀ ⁽⁵⁾	HF ₅	HF ₇₅	HF ₁₃₀	BV	BV ₃₀
HIST	221,842	1.00	0.37	0.97	0.44	0.73	1.27	1.64	0.88	0.80	0.78	0.78	0.73	0.74	0.79	0.80	0.74	0.67
HIST ⁽¹⁾	221,842	1.02	0.42	0.96	0.44	0.77	1.34	1.80	—	0.92	0.70	0.73	0.74	0.65	0.72	0.74	0.69	0.64
HIST ⁽²⁾	221,842	1.02	0.44	0.95	0.41	0.77	1.37	1.87	—	—	0.65	0.68	0.69	0.60	0.67	0.68	0.65	0.61
HF ₃₀	221,842	0.95	0.36	0.97	0.32	0.57	1.11	1.46	—	—	—	0.92	0.79	0.95	0.95	0.92	0.72	0.78
HF ₃₀ ⁽¹⁾	221,842	0.99	0.37	0.95	0.40	0.68	1.20	1.60	—	—	—	—	0.90	0.83	0.92	0.91	0.73	0.73
HF ₃₀ ⁽⁵⁾	221,842	1.02	0.40	0.93	0.43	0.75	1.28	1.72	—	—	—	—	—	0.70	0.82	0.84	0.71	0.66
HF ₅	221,842	0.93	0.35	0.97	0.26	0.47	1.01	1.33	—	—	—	—	—	—	0.89	0.86	0.67	0.74
HF ₇₅	221,842	0.96	0.37	0.96	0.35	0.62	1.15	1.52	—	—	—	—	—	—	—	0.95	0.74	0.77
HF ₁₃₀	221,842	0.96	0.37	0.95	0.36	0.64	1.16	1.54	—	—	—	—	—	—	—	—	0.76	0.75
BV	75,548	0.99	0.24	0.88	0.68	0.88	1.19	1.46	—	—	—	—	—	—	—	—	—	0.89
BV ₃₀	73,883	1.00	0.22	0.90	0.72	0.88	1.17	1.42	—	—	—	—	—	—	—	—	—	—

Notes. This table presents summary statistics of various beta estimates used in the paper. For all HIST estimators and BV, we use a sampling window of 12 months, whereas for all HF estimators and BV₃₀, we use a 6-month window. The summary statistics are similar for alternative sampling windows. Additionally, we present averages of firm-level correlations of the different beta estimation techniques. The total number of monthly estimates for individual firms is indicated by the number of observations. Mean_{vw} is the overall value-weighted average of the estimates over the entire sample period. Standard deviation and AR(1) present further summary statistics on the average of firm-level standard deviations and first-order autocorrelations, whereas $q^{0.05}$, $q^{0.25}$, $q^{0.75}$, and $q^{0.95}$ provide the respective average quantiles. The sample spans January 1996 until December 2014.

discussion on both HF₃₀ and HF₃₀⁽¹⁾. Overall, the potential downward bias in high-frequency betas does not seem to be a big issue, but we are careful to examine the robustness of all our main results to using lag-adjusted estimators.

Table 1 also presents the average firm-level correlation coefficients for different estimators. We observe a correlation of 0.78 between the historical daily and the 30-minute high-frequency estimators. These figures indicate that the two estimators may to some extent contain different information. Because for BV and BV₃₀ only the input to obtain correlations differs, the correlation of BV and BV₃₀ is higher and amounts to 0.89.

3. The Conditional CAPM Revisited

3.1. Empirical Setup

We start our main empirical analysis by exploring the ability of the conditional CAPM to explain the main empirical asset pricing anomalies. Specifically, we set out to examine the following questions: Can the new high-frequency beta estimates allow us a second look at the conditional CAPM? Can high-frequency estimates of beta help explain why small firms generally outperform large firms? Can we explain the value effect when using high-frequency betas? Can the conditional CAPM explain why past winner stocks generally outperform past loser stocks?

To perform the analysis, we follow the approach suggested by Lewellen and Nagel (2006) and use direct estimates for conditional betas. Along with each conditional beta estimate, we obtain an estimate for the conditional alpha. We thus partition the entire sample used for unconditional tests allowing for time variation in betas. For our main empirical tests, we use quarterly nonoverlapping windows. For each of these quarterly windows, we obtain conditional alphas and betas for each of the anomaly portfolios. Using the time series of conditional alphas, we can perform a direct test of the conditional CAPM by testing whether the average alpha is significantly different from zero. We assess the statistical significance using robust Newey and West (1987) standard errors with four lags.

We test the model for different anomaly portfolios. For all anomaly portfolios, we use NYSE breakpoints and obtain the conditional alphas and betas as value-weighted averages of the respective conditional alphas and betas of the stocks in that portfolio. We construct size and value portfolios following Lewellen and Nagel (2006). To obtain the book-to-market ratio for each stock, we use the firm's most recent observation for the book value (assuming that the book equity from the previous year becomes available at the end of June in the current year) and divide it by the market capitalization at the end of December of

the previous fiscal year. Thus, the book-to-market ratio changes at the end of June each year according to new information both on the book equity and the market capitalization from the previous fiscal year. Each month, we sort the stocks independently into 25 size-book-to-market portfolios and base our tests on combinations of these portfolios: S (small) is the average of the five low-market-cap portfolios, and B (big) is the average of the five high-market-cap portfolios, whereas S–B is their difference. Similarly, G (growth) is the average of the five low-book-to-market portfolios, and V (value) is the average of the five high-book-to-market portfolios, and V–G is the difference between the value and growth portfolios. For momentum, we sort the stocks into 10 portfolios on the basis of their prior 12-month return while skipping the most recent 1 month (Jegadeesh and Titman 1993). L (losers) and W (winners) are the bottom and top deciles, and W–L is the difference between the winner and loser portfolios.

3.2. The Unconditional CAPM

We present the empirical results in Table 2. To set the stage, we first present the results of the unconditional CAPM. That is, we regress the monthly portfolio excess returns on a constant and that of the S&P 500 Index. We find that the portfolio of small stocks has a significant alpha of 0.65% per month. Thus, the

unconditional CAPM is not able to explain the return on small stocks. For big stocks, the model works substantially better: the portfolio of big stocks has an alpha of 0.01%, which is not statistically significant. We observe an unconditional size effect with an S–B alpha of 0.63%, which is significant at 5%. Thus, the unconditional CAPM fails to explain the return of the size anomaly portfolio.

For the value anomaly, we find a significant alpha for the growth portfolio of 0.46% and an insignificant alpha for the value portfolio of 0.42%. Thus, the unconditional CAPM can explain the return of the portfolio of value stocks but not that of growth stocks. In total, there is no significant difference between the value and growth alphas. For momentum, we detect an insignificant alpha of –0.57% per month for the loser portfolio and a significant alpha of 0.49% for the winner portfolio. Thus, the unconditional CAPM can explain the return of the portfolio of loser stocks but fails with the portfolio of winner stocks. Finally, there is a significant alpha of momentum anomaly portfolio of 1.06% per month.

The significant size effect is consistent with results in the previous literature (e.g., Banz 1981, Fama and French 1992). The results for the value anomaly are also consistent with the recent empirical literature, showing that the value effect is substantially attenuated for the more recent period (e.g., Hou et al. 2015, Bollerslev et al. 2016,

Table 2. Conditional CAPM Tests

	Size				Value				Momentum			
	α_S	α_B	α_{S-B}	β_{S-B}	α_G	α_V	α_{V-G}	β_{V-G}	α_L	α_W	α_{W-L}	β_{W-L}
Unconditional	0.0065** (0.0032)	0.0001 (0.0011)	0.0063** (0.0028)	0.2303*** (0.0785)	0.0046** (0.0019)	0.0042 (0.0027)	–0.0004 (0.0027)	–0.1679* (0.0970)	–0.0057 (0.0037)	0.0049* (0.0028)	0.0106** (0.0053)	–0.5550*** (0.2102)
Daily data												
HIST	0.0079** (0.0035)	0.0010 (0.0009)	0.0069** (0.0030)	0.0620 (0.0860)	0.0060** (0.0023)	0.0052** (0.0024)	–0.0007 (0.0021)	–0.1142* (0.0677)	0.0009 (0.0029)	0.0025 (0.0025)	0.0016 (0.0043)	0.0775 (0.1599)
HIST ⁽¹⁾	0.0074** (0.0036)	0.0012 (0.0009)	0.0061* (0.0031)	0.1144 (0.0806)	0.0050** (0.0023)	0.0053** (0.0025)	0.0002 (0.0021)	–0.1404* (0.0756)	0.0003 (0.0029)	0.0023 (0.0025)	0.0021 (0.0044)	0.0357 (0.1839)
HIST ⁽²⁾	0.0078* (0.0041)	0.0013 (0.0010)	0.0065* (0.0035)	0.1376 (0.0836)	0.0051** (0.0024)	0.0057** (0.0028)	0.0006 (0.0021)	–0.1231* (0.0729)	–0.0002 (0.0034)	0.0025 (0.0024)	0.0027 (0.0047)	–0.0310 (0.1918)
High-frequency data												
HF ₃₀	0.0013 (0.0031)	–0.0006 (0.0010)	0.0019 (0.0029)	–0.1609** (0.0805)	0.0001 (0.0015)	0.0013 (0.0024)	0.0012 (0.0022)	–0.1110** (0.0554)	–0.0086* (0.0046)	–0.0038 (0.0027)	0.0048 (0.0050)	0.0942 (0.1311)
HF ₃₀ ⁽¹⁾	0.0004 (0.0031)	–0.0010 (0.0011)	0.0014 (0.0029)	–0.0392 (0.0865)	–0.0007 (0.0015)	0.0008 (0.0024)	0.0015 (0.0022)	–0.1306** (0.0618)	–0.0094** (0.0046)	–0.0040 (0.0025)	0.0054 (0.0049)	0.1037 (0.1370)
HF ₃₀ ⁽⁵⁾	0.0000 (0.0034)	–0.0010 (0.0011)	0.0010 (0.0030)	0.0267 (0.0809)	–0.0008 (0.0017)	0.0009 (0.0024)	0.0017 (0.0021)	–0.1414** (0.0667)	–0.0090** (0.0042)	–0.0041* (0.0024)	0.0050 (0.0047)	0.0980 (0.1469)

Notes. This table reports average conditional alphas for different value-weighted anomaly portfolios. We present the results for size, value, and momentum portfolios. For size and value, we build 25 independently sorted portfolios. S is the average of the five low-market-cap portfolios, and B is the average of the five high-market-cap portfolios, whereas S–B is their difference. Similarly, G is the average of the five low-book-to-market portfolios, and V is the average of the five high-book-to-market portfolios. V–G is the difference between the value and growth portfolios. For momentum, we sort the stocks into 10 portfolios according to their return over the past 12 months (skipping 1 month). L and W are the bottom and top deciles, whereas W–L is their difference. Unconditional denotes the unconditional CAPM based on monthly returns for the full sample period. The remaining segments test the conditional CAPM using quarterly nonoverlapping windows. In parentheses, we report Newey and West (1987) standard errors with four lags.

*, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Hou et al., 2018). The positive momentum effect is consistent with Jegadeesh and Titman (1993) and Goyal and Jegadeesh (2018), among many others.

To further validate that the results of our sample are reasonable and representative for the full NYSE/AMEX/NASDAQ sample, we compute the correlations of the hedge portfolio returns resulting from our sorts and those that result from using the corresponding portfolio returns available from Kenneth French's web page. That is, we use the 25 Fama–French portfolios sorted on size and book-to-market as well as the 10 portfolios sorted on momentum to obtain full NYSE/AMEX/NASDAQ sample hedge portfolio returns in a similar vein as for our main results. We obtain a correlation of 71% between S–B based on the Fama–French portfolios and our S–B returns and a correlation of 87% between H–L based on the Fama–French portfolios and our H–L portfolio returns. For momentum, we obtain a correlation of 96% with the W–L hedge portfolio returns based on the Fama–French portfolios. Thus, even though we limit our sample to stocks with available high-frequency data, the constructed portfolios do not seem to be materially different.

3.3. The Conditional CAPM with Daily Data

Second, we test whether the conditional CAPM based on daily data is able to explain the size, value, and momentum anomalies. The results of this analysis are also presented in Table 2. For HIST, we detect a statistically significant alpha for the portfolio of small stocks of 0.79% per month. The conditional CAPM with daily data is thus not able to explain the return on small stocks. For the portfolio of big stocks, the alpha amounts to 0.10% per month, which is not statistically significant. Hence, small stocks seem to pose a much stronger challenge for the model than do big stocks.

For the S–B portfolio, we detect a significantly positive alpha of 0.69% per month. The results for the small, big, and S–B portfolios are broadly consistent with those of Lewellen and Nagel (2006). In table 3 of their paper, for example, the alpha of the S–B portfolio with quarterly windows amounts to 0.42% per month, with a standard error of 0.22. These numbers correspond to a *t*-statistic of 1.91, which is marginally insignificant at 5% (the criterion for boldface printing in their table), but it is clearly significant at 10%. For our sample, the size anomaly is even somewhat stronger, and the conditional CAPM cannot explain this anomaly when using daily data.

Turning to the value anomaly, we obtain a statistically significant alpha of 0.60% per month for growth stocks. For the portfolio of value stocks, the corresponding figure amounts to 0.52% per month, which is also statistically significant. Thus, the

conditional CAPM with daily data cannot explain the returns of both growth and value stocks. The V–G portfolio has an insignificant alpha, consistent with that of the unconditional CAPM. In this case, the results for our sample period differ from those of Lewellen and Nagel (2006), who find a significant V–G alpha of the conditional CAPM for the 1964–2001 period.⁷

Finally, examining the momentum anomaly, we find that for HIST, neither the loser nor the winner portfolio generates a significant alpha. Thus, the conditional CAPM can explain the returns of both past losers and past winners when using daily return data. The alpha of the W–L portfolio amounts to 0.16% per month, which is not statistically significant.

Thus, overall, we find that the conditional CAPM works quite well for the recent sample period. With daily data, the conditional CAPM can explain the returns of big, loser, and winner stocks, as well as the value and momentum anomaly portfolios, but it fails to explain the returns of small, growth, and value stocks, as well as the size anomaly portfolio.

Adding lags in the estimation of beta does not alter these conclusions. HIST⁽¹⁾ and HIST⁽²⁾ yield statistically significant alphas in exactly the same cases as HIST. With daily data and a lag-adjustment in the beta estimation, the conditional CAPM can explain the value and momentum anomalies but not the size anomaly.

To make sure that these results are not driven by the use of a subsample, which relies on stocks for which we have high-frequency data, in untabulated results we also repeat the previous analysis for the full NYSE/AMEX/NASDAQ sample of eligible stocks for our sample period. We obtain very similar results. The S–B unconditional return is slightly smaller, but the conditional CAPM based on daily data cannot explain the size anomaly. The S–B alpha of HIST amounts to 0.69% per month. However, the conditional CAPM using daily data can explain the value and size anomalies. The V–G and W–L alphas are of similar magnitude as for our sample. Similar to our main sample, the average alphas and betas change only marginally when using lag-adjusted betas.

3.4. The Conditional CAPM with High-Frequency Data

In a next step, we examine the performance of the conditional CAPM when using betas based on the potentially more informative high-frequency data. We also present the analysis in Table 2. With betas estimated from high-frequency data (HF₃₀), the alpha of the portfolio of small stocks amounts to only 0.13% per month, which is substantially smaller than the 0.79% observed for HIST. The alpha of the portfolio of big stocks amounts to –0.06% per month. Thus, using high-frequency data, the conditional CAPM can explain the returns of both small and big stocks.

The average alpha of the combined S–B portfolio is 0.19% per month, which is also not statistically significant. Hence, the conditional CAPM based on high-frequency data can explain the size anomaly. Thus, HF₃₀ seems to substantially better capture the time variation in betas of small and big stocks.

One may argue that it is not very surprising that HF₃₀ can explain the size anomaly because the betas of small and infrequently traded stocks are likely biased downward when using high-frequency data because of asynchronous trading times with the market. However, this argument does not square well with the empirical evidence. The literature, and also our tests based on HIST, typically detects an outperformance of small over big stocks. If small stocks tend to have lower betas when using high-frequency data, this, in turn, would imply that these stocks have higher conditional alphas on average. However, what we find is the exact opposite. The small stock portfolio has a substantially lower average conditional alpha for HF₃₀ compared with HIST. Thus, it is very unlikely that a potential downward bias in high-frequency betas drives these results. Moreover, the fact that the conditional CAPM can explain the size anomaly with high-frequency but not with daily data points toward a superior information content of high-frequency betas.

The alpha of the portfolio of growth stocks amounts to 0.01% per month and that of value stocks to 0.13% per month. Both alphas are smaller in magnitude for HF₃₀ than for HIST and are not statistically significant. Thus, as opposed to when using daily data, with high-frequency data, the conditional CAPM can explain the returns of growth and value stocks. We also find that the conditional CAPM with high-frequency data is able to explain the return of the value anomaly portfolio: the V–G alpha amounts to an insignificant 0.12% per month.

Finally, for momentum, we detect a monthly alpha for the loser portfolio of –0.86% per month, which is significant at 10%. Thus, the conditional CAPM with high-frequency data has problems in explaining the returns of past loser stocks. For the winner portfolio, the alpha is –0.38% per month, which is not statistically significant. Thus, in explaining the returns of past winner stocks, the conditional CAPM with high-frequency data performs as well as with daily data. The W–L portfolio yields an insignificant alpha of 0.48% per month. Hence, with high-frequency data, the conditional CAPM can overall also explain the return of the momentum anomaly portfolio.

Using a lag adjustment for the high-frequency betas does not qualitatively change our results. The conditional CAPM with HF₃₀⁽¹⁾ and HF₃₀⁽⁵⁾ can explain the returns of the size, value, and momentum anomaly portfolios.

3.5. Underconditioning Bias

Following Lewellen and Nagel (2006) and Boguth et al. (2011), we now examine the potential bias that could be introduced by underconditioning (i.e., by using the unconditional CAPM instead of the conditional CAPM when betas indeed vary over time). Boguth et al. (2011) show that if the conditional alphas are uncorrelated with the market excess return, the bias in unconditional alphas, that is, the difference between the unconditional alpha (α_j^{unc}) and the average conditional alpha ($\bar{\alpha}_j^{\text{cond}}$), can be described as

$$\alpha_j^{\text{unc}} - \bar{\alpha}_j^{\text{cond}} = \underbrace{\left(1 + \frac{\bar{R}_M^2}{\sigma_M^2}\right) \text{cov}(\beta_{j,t}, \mathbb{E}_t(R_{M,t})) - \left(\frac{\bar{R}_M}{\sigma_M^2}\right) \text{cov}(\beta_{j,t}, \mathbb{E}_t(R_{M,t}^2))}_{\text{market-timing bias}} - \underbrace{\left(\frac{\bar{R}_M}{\sigma_M^2}\right) \text{cov}(\beta_{j,t}, \sigma_{M,t}^2)}_{\text{volatility-timing bias}}, \quad (5)$$

where $R_{M,t}$ denotes the market excess return (i.e., $r_{M,t} - r_{f,t}$), \bar{R}_M is the average market excess return, σ_M^2 is the unconditional variance of the market excess return, and $\sigma_{M,t}^2$ is its conditional expectation.

The underconditioning bias thus consists of a market-timing and a volatility-timing bias. The market-timing bias results from the covariation of conditional betas with the expected market excess return or the expected squared market excess return and reflects the fact that betas likely vary with the business cycle (Jagannathan and Wang 1996). The volatility-timing bias results from the negative covariation of conditional betas with the conditional volatility of the market, which seems to be particularly pronounced for momentum, where the current market return predicts both future betas and future market volatility (Grundy and Martin 2001, Boguth et al, 2011).

Lewellen and Nagel (2006, p. 295) argue that the market-timing bias is small in magnitude. They examine this bias for different sets of parameters and obtain a bias that is “typically less than 0.20%” per month “with a maximum of 0.35%.” Boguth et al. (2011) find that the volatility-timing bias can be substantially larger than the market-timing bias. For the parameter sets used in their paper, the volatility-timing bias reaches values between 0.10% per month and 1.01% per month. For the portfolios in our sample, the alphas in absolute terms range between 0.01% per month for the portfolio of big stocks and 1.06% per month for the momentum anomaly portfolio, whereas most returns are below 0.7% per month in absolute terms. Thus, the magnitude of the unconditional returns we observe is potentially consistent with an underconditioning bias. For the most extreme parameter calibrations,

even an unconditional alpha as high as the 1.06% for the momentum anomaly portfolio is still within a possible range.⁸

3.6. Alternative Window Lengths

For the main tests of the conditional CAPM in Sections 3.3 and 3.4, we use a three-month window to estimate conditional alphas and betas. However, the window length involves a trade-off: on the one hand, a shorter window might improve the model's performance because it allows for greater variation in beta; on the other hand, longer windows could increase the precision of the conditional alpha and beta estimates and improve the model performance in this way.

Hence, we examine alternative window lengths of 1 month, 6 months, and 12 months in Table 3. We only report the results for our main estimators HIST, HF₃₀, and HF₃₀⁽¹⁾. As the table shows, these changes do not affect our main conclusions.

3.7. Alternative Frequencies

Next, we examine the impact of alternative sampling frequencies of the high-frequency estimator on our results. In Table 4, we examine estimators based on 5-, 75-, and 130-minute data with and without lags. For all these estimators, we obtain qualitatively similar results as for HF₃₀: the conditional CAPM based on high-frequency data can explain the size, value, and momentum anomalies.

3.8. Alternative Conditioning Approaches

Boguth et al. (2011) caution that one might introduce a bias in the analysis because the beta estimates used for computation of the conditional alpha are not available to investors ex ante, before they invest. To check whether our results are affected by such a bias, we repeat the previous analysis using three different alternative approaches. First, we simply use lagged betas as proxies for future betas. Second, we estimate a regression of the form

$$\beta_{j,t} = a_j + b_j \beta_{j,t-1} + \epsilon_{j,t}, \quad (6)$$

where $\beta_{j,t}$ is the beta for time t of one approach. We then use $\hat{\beta}_{j,t}^{\text{instr}} = \hat{a}_j + \hat{b}_j \beta_{j,t-1}$ instead of $\beta_{j,t}$ to compute the conditional alphas. We use two variations of this approach: one that uses the full sample as in Boguth et al. (2011) and one that would be implementable in real time and uses only data up to time t with an expanding window initialized with a 36-month window.

We present the results of these approaches in Table 5. The results are very similar to our benchmark findings. We find that the conditional CAPM based on high-frequency data still explains the size, value, and momentum anomalies in all three cases.

4. The Precision of High-Frequency Betas

Section 3 showed that the use of high-frequency data can improve the performance of the conditional CAPM. This section explores whether high-frequency betas provide more accurate forecasts for future betas than those based on daily return data. We examine this by evaluating the predictability of HIST and HF₃₀ for future realized betas. We start the analysis by picking the optimal estimation period for the estimators and then examine the estimation accuracy using root mean squared prediction errors and Mincer–Zarnowitz regressions, as well as cross-sectional tests.

4.1. Estimation Period

We perform an out-of-sample evaluation of beta estimation precision. At the end of each month, we use all observations available during the measurement period to obtain estimates for beta and evaluate these estimates in a separate evaluation period. We use a one-month evaluation period. To conduct a fair comparison, we first identify the best estimation window for each estimator. By taking this step, we ensure that the competing models are put on an equal footing.⁹ We consider various estimation windows of 1, 3, 6, and 12 months. We then proceed as follows. We examine the average RMSEs of the estimators for each measurement window. We obtain the RMSE as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^n (\beta_{t,T}^R - \beta_t)^2}, \quad (7)$$

where n is the number of out-of-sample observations, $\beta_{t,T}^R$ is the realized beta in the period ranging from t to T , and β_t denotes an estimate for beta. For HIST, we find that the lowest average RMSE is obtained for a 12-month estimation window. For HF₃₀, BV, and BV₃₀, a 6-month estimation period turns out to be best. Thus, in the following we stick to these measurement periods for our estimators.

The shorter optimal estimation window for HF₃₀ delivers a first indication that high-frequency data can help resolve the trade-off between conditionality of the estimates and a large sample, which improves the statistical properties of the estimator. For example, for HIST, we need a longer estimation window to reduce the measurement error in the estimates and thereby inevitably lose conditionality.

To ensure a fair comparison, for each of the following analyses, we only use stock-month observations that are available for all estimators considered in that analysis.

4.2. Estimation Accuracy

To perform the analysis, we follow the approach of Fama and MacBeth (1973) and Hollstein and Prokopczuk (2016). Hence, we first sort stocks into

Table 3. Conditional CAPM Tests: Different Window Lengths

	Size			Value			Momentum					
	α_S	α_B	α_{S-B}	β_{S-B}	α_G	α_V	α_{V-G}	β_{V-G}	α_L	α_W	α_{W-L}	β_{W-L}
Unconditional	0.0065** (0.0032)	0.0001 (0.0011)	0.0063** (0.0028)	0.2303*** (0.0785)	0.0046** (0.0019)	0.0042 (0.0027)	-0.0004 (0.0027)	-0.1679* (0.0970)	-0.0057 (0.0037)	0.0049* (0.0028)	0.0106** (0.0053)	-0.5550*** (0.2102)
Monthly												
HIST	0.0081*** (0.0031)	0.0009 (0.0008)	0.0073** (0.0028)	0.0678 (0.0550)	0.0065*** (0.0019)	0.0057** (0.0023)	-0.0009 (0.0023)	-0.1102** (0.0427)	0.0009 (0.0039)	0.0071*** (0.0024)	0.0062 (0.0054)	0.0300 (0.1137)
HF ₃₀	0.0017 (0.0031)	-0.0002 (0.0009)	0.0019 (0.0030)	-0.1655*** (0.0493)	0.0006 (0.0018)	0.0021 (0.0023)	0.0015 (0.0023)	-0.1105*** (0.0355)	-0.0083** (0.0038)	0.0001 (0.0029)	0.0084 (0.0052)	0.0462 (0.0965)
HF ₃₀ ⁽¹⁾	0.0010 (0.0030)	-0.0008 (0.0010)	0.0018 (0.0029)	-0.0441 (0.0535)	-0.0001 (0.0016)	0.0016 (0.0022)	0.0017 (0.0022)	-0.1317*** (0.0397)	-0.0093** (0.0037)	0.0001 (0.0029)	0.0093* (0.0051)	0.0607 (0.1009)
Semiannually												
HIST	0.0083** (0.0038)	0.0010 (0.0009)	0.0074** (0.0033)	0.0725 (0.1162)	0.0059** (0.0028)	0.0052** (0.0023)	-0.0006 (0.0022)	-0.1123 (0.0862)	0.0047 (0.0032)	0.0024 (0.0022)	-0.0023 (0.0033)	0.1037 (0.1472)
HF ₃₀	0.0016 (0.0028)	-0.0004 (0.0012)	0.0021 (0.0027)	-0.1517 (0.1109)	0.0002 (0.0014)	0.0016 (0.0021)	0.0014 (0.0022)	-0.1069 (0.0721)	-0.0043 (0.0043)	-0.0029 (0.0023)	0.0014 (0.0036)	0.1240 (0.1248)
HF ₃₀ ⁽¹⁾	0.0007 (0.0030)	-0.0009 (0.0013)	0.0016 (0.0029)	-0.0320 (0.1182)	-0.0005 (0.0016)	0.0011 (0.0022)	0.0016 (0.0022)	-0.1250 (0.0805)	-0.0046 (0.0043)	-0.0031 (0.0020)	0.0015 (0.0035)	0.1304 (0.1297)
Annually												
HIST	0.0083** (0.0036)	0.0010 (0.0007)	0.0073** (0.0031)	0.0735 (0.1430)	0.0072** (0.0030)	0.0044* (0.0022)	-0.0028 (0.0022)	-0.0871 (0.1119)	0.0057 (0.0036)	0.0027 (0.0018)	-0.0030 (0.0030)	0.0581 (0.1458)
HF ₃₀	0.0020 (0.0028)	-0.0006 (0.0010)	0.0026 (0.0029)	-0.1452 (0.1434)	0.0018 (0.0015)	0.0010 (0.0022)	-0.0009 (0.0019)	-0.0906 (0.0936)	-0.0020 (0.0027)	-0.0033 (0.0023)	-0.0013 (0.0025)	0.0866 (0.1163)
HF ₃₀ ⁽¹⁾	0.0011 (0.0031)	-0.0010 (0.0011)	0.0020 (0.0031)	-0.0243 (0.1506)	0.0012 (0.0017)	0.0005 (0.0023)	-0.0008 (0.0019)	-0.1004 (0.1033)	-0.0026 (0.0029)	-0.0036 (0.0023)	-0.0010 (0.0025)	0.0876 (0.1265)

Notes. This table reports average conditional alphas for different value-weighted anomaly portfolios. We present the results for size, value, and momentum portfolios. For size and value, we build 25 independently sorted portfolios. S is the average of the five low-market-cap portfolios, and B is the average of the five high-market-cap portfolios, whereas S-B is their difference. Similarly, G is the average of the five low-book-to-market portfolios, and V is the average of the five high-book-to-market portfolios. V-G is the difference between the value and growth portfolios. For momentum, we sort the stocks into 10 portfolios according to their return over the past 12 months (skipping 1 month). L and W are the bottom and top deciles, whereas W-L is their difference. Unconditional denotes the unconditional CAPM based on monthly returns for the full sample period. The remaining segments test the conditional CAPM using monthly, semiannual, and annual nonoverlapping windows, as indicated in the segment headings. In parentheses, we report Newey and West (1987) standard errors with four lags.

*, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 4. Conditional CAPM Tests: Different Frequencies

	Size				Value				Momentum			
	α_S	α_B	α_{S-B}	β_{S-B}	α_G	α_V	α_{V-G}	β_{V-G}	α_L	α_W	α_{W-L}	β_{W-L}
Unconditional	0.0065** (0.0032)	0.0001 (0.0011)	0.0063** (0.0028)	0.2303*** (0.0785)	0.0046** (0.0019)	0.0042 (0.0027)	−0.0004 (0.0027)	−0.1679* (0.0970)	−0.0057 (0.0037)	0.0049* (0.0028)	0.0106** (0.0053)	−0.5550*** (0.2102)
Different HF estimators												
HF ₅	0.0023 (0.0033)	−0.0005 (0.0011)	0.0028 (0.0029)	−0.3182*** (0.0707)	0.0007 (0.0016)	0.0016 (0.0025)	0.0009 (0.0023)	−0.0832 (0.0505)	−0.0087* (0.0049)	−0.0036 (0.0027)	0.0052 (0.0051)	0.0791 (0.1279)
HF ₅ ⁽¹⁰⁾	0.0001 (0.0032)	−0.0012 (0.0011)	0.0013 (0.0029)	−0.0201 (0.0861)	−0.0008 (0.0016)	0.0007 (0.0024)	0.0014 (0.0022)	−0.1370** (0.0632)	−0.0096** (0.0045)	−0.0038 (0.0025)	0.0057 (0.0048)	0.0869 (0.1400)
HF ₇₅	0.0006 (0.0031)	−0.0005 (0.0010)	0.0011 (0.0029)	−0.0840 (0.0809)	−0.0002 (0.0015)	0.0012 (0.0023)	0.0013 (0.0022)	−0.1269** (0.0593)	−0.0090* (0.0047)	−0.0037 (0.0026)	0.0053 (0.0050)	0.1026 (0.1344)
HF ₇₅ ⁽³⁾	−0.0005 (0.0033)	−0.0008 (0.0010)	0.0003 (0.0031)	0.0526 (0.0807)	−0.0009 (0.0018)	0.0007 (0.0023)	0.0016 (0.0022)	−0.1421* (0.0714)	−0.0092** (0.0039)	−0.0040 (0.0025)	0.0052 (0.0046)	0.1011 (0.1609)
HF ₁₃₀	0.0010 (0.0032)	−0.0007 (0.0011)	0.0017 (0.0029)	−0.0541 (0.0781)	−0.0001 (0.0016)	0.0012 (0.0024)	0.0013 (0.0022)	−0.1244** (0.0614)	−0.0085* (0.0046)	−0.0038 (0.0027)	0.0047 (0.0050)	0.0913 (0.1354)
HF ₁₃₀ ⁽³⁾	0.0002 (0.0035)	−0.0003 (0.0012)	0.0005 (0.0031)	0.0876 (0.0822)	−0.0006 (0.0020)	0.0014 (0.0023)	0.0020 (0.0022)	−0.1482* (0.0796)	−0.0086** (0.0037)	−0.0037 (0.0024)	0.0049 (0.0044)	0.0806 (0.1801)

Notes. This table reports average conditional alphas for different value-weighted anomaly portfolios. We present the results for size, value, and momentum portfolios. For size and value, we build 25 independently sorted portfolios. S is the average of the five low-market-cap portfolios, and B is the average of the five high-market-cap portfolios, whereas S–B is their difference. Similarly, G is the average of the five low-book-to-market portfolios, and V is the average of the five high-book-to-market portfolios. V–G is the difference between the value and growth portfolios. For momentum, we sort the stocks into 10 portfolios according to their return over the past 12 months (skipping 1 month). L and W are the bottom and top deciles, whereas W–L is their difference. Unconditional denotes the unconditional CAPM based on monthly returns for the full sample period. The remaining segments test the conditional CAPM using quarterly nonoverlapping windows. In parentheses, we report Newey and West (1987) standard errors with four lags.

*, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

five portfolios according to their estimate of HIST 12 months ago, thereby ensuring that we do not sort on the current measurement error of either of the approaches.¹⁰

To examine the out-of-sample estimation accuracy of the different approaches, we use the RMSE criterion of Equation (7), a loss function commonly applied in the literature. We rely on the RMSE criterion because it is shown that it is robust to the presence of (mean zero) noise in the evaluation proxy, whereas other commonly used loss functions are not (Patton 2011). In Section 4.6, we also examine the MAE criterion as an alternative.

We test for significance in RMSE using the modified Diebold–Mariano test, as proposed by Harvey et al. (1997), and for significance in root median squared error (RMedSE) using the nonparametric Wilcoxon signed rank test.¹¹ The results are provided in panel A of Table 6. In parentheses, we report the share of portfolios for which a difference in RMSE or RMedSE is statistically significant at 5%. In general, the results for the RMedSE and its significance are similar to those for the RMSE. Hence, in discussing our results, we mostly focus on the RMSE.

We find that HIST yields an average RMSE of 0.135. The corresponding figure for HF₃₀ is lower by almost one-third, amounting to 0.095. The RMSE of HF₃₀ is significantly lower than that of HIST for each of the five portfolios. Thus, the superior economic value of

HF₃₀ seems to be linked to its superior predictive power for future realized betas. For both HIST and HF₃₀, the lag adjustments increase the RMSE.

One might argue that the realized beta based on 30-minute data, which we use in panel A, could be downward biased without the lag adjustment. Therefore, we repeat the previous analysis using realized beta with one lag (panel B of Table 6) and with realized beta based on daily data (panel C of Table 6). In the former case, we find that HF₃₀ still yields a substantially lower RMSE than HIST. In this setup, however, the high-frequency estimator with one lag (HF₃₀⁽¹⁾) yields the overall lowest RMSE. When realized beta is estimated with daily data, we obtain the same pattern: the average RMSE of HF₃₀ is lower than that of HIST. Owing to the increased noise introduced by measurement error in the daily realized beta, however, only a few of the differences in this panel are statistically significant. Nevertheless, we believe that the results of this analysis are meaningful in that if HF₃₀ were systematically biased, it would also be systematically biased in comparison with the realized beta based on daily data used in this analysis. Thus, in this case, it would be very unlikely that we observe that HF₃₀ yields a lower average RMSE than HIST.

We find that in general the average RMSEs increase for all estimators when moving from 30-minute realized betas to those with lag adjustments or those

Table 5. Conditional CAPM Tests: Conditioning Approaches

	Size				Value				Momentum			
	α_S	α_B	α_{S-B}	β_{S-B}	α_G	α_V	α_{V-G}	β_{V-G}	α_L	α_W	α_{W-L}	β_{W-L}
Unconditional	0.0065** (0.0032)	0.0001 (0.0011)	0.0063** (0.0028)	0.2303*** (0.0785)	0.0046** (0.0019)	0.0042 (0.0027)	-0.0004 (0.0027)	-0.1679* (0.0970)	-0.0057 (0.0037)	0.0049* (0.0028)	0.0106** (0.0053)	-0.5550*** (0.2102)
Lagged proxies												
HIST	0.0077** (0.0034)	0.0004 (0.0009)	0.0073** (0.0031)	0.0515 (0.0847)	0.0059** (0.0023)	0.0044* (0.0024)	-0.0015 (0.0023)	-0.1244** (0.0717)	0.0008 (0.0030)	0.0011 (0.0026)	0.0002 (0.0046)	0.0058 (0.1499)
HF ₃₀	0.0012 (0.0032)	-0.0014 (0.0012)	0.0025 (0.0029)	-0.1704** (0.0799)	0.0000 (0.0016)	0.0007 (0.0024)	0.0007 (0.0023)	-0.1139* (0.0593)	-0.0093** (0.0045)	-0.0049 (0.0030)	0.0044 (0.0048)	0.0357 (0.1267)
HF ₃₀ ⁽¹⁾	-0.0000 (0.0032)	-0.0019 (0.0013)	0.0018 (0.0030)	-0.0466 (0.0868)	-0.0006 (0.0016)	-0.0001 (0.0024)	0.0006 (0.0023)	-0.1318* (0.0663)	-0.0096** (0.0046)	-0.0052* (0.0030)	0.0044 (0.0048)	0.0494 (0.1325)
Instruments: Full sample												
HIST	0.0078** (0.0035)	0.0005 (0.0009)	0.0073** (0.0031)	0.0580 (0.0772)	0.0060*** (0.0023)	0.0045* (0.0024)	-0.0016 (0.0023)	-0.1157* (0.0687)	0.0007 (0.0030)	0.0009 (0.0026)	0.0002 (0.0046)	0.0714 (0.1287)
HF ₃₀	0.0011 (0.0032)	-0.0012 (0.0012)	0.0023 (0.0029)	-0.1608** (0.0788)	0.0001 (0.0016)	0.0008 (0.0024)	0.0007 (0.0023)	-0.1146* (0.0580)	-0.0096** (0.0046)	-0.0051* (0.0030)	0.0045 (0.0051)	0.0964 (0.1067)
HF ₃₀ ⁽¹⁾	0.0008 (0.0032)	-0.0013 (0.0013)	0.0021 (0.0029)	-0.1600* (0.0839)	0.0000 (0.0016)	0.0004 (0.0025)	0.0004 (0.0023)	-0.1140* (0.0643)	-0.0097** (0.0048)	-0.0051* (0.0030)	0.0047 (0.0051)	0.0916 (0.1070)
Instruments: Expanding window												
HIST	0.0091** (0.0038)	0.0006 (0.0010)	0.0085** (0.0034)	0.1256* (0.0743)	0.0070*** (0.0025)	0.0055** (0.0027)	-0.0015 (0.0024)	-0.0450 (0.0708)	0.0008 (0.0035)	-0.0001 (0.0029)	-0.0009 (0.0053)	0.0333 (0.1371)
HF ₃₀	0.0012 (0.0033)	-0.0014 (0.0013)	0.0026 (0.0032)	-0.0633 (0.0757)	0.0005 (0.0018)	0.0008 (0.0026)	0.0003 (0.0024)	-0.0577 (0.0618)	-0.0096* (0.0054)	-0.0066** (0.0033)	0.0031 (0.0058)	0.0488 (0.1236)
HF ₃₀ ⁽¹⁾	0.0011 (0.0034)	-0.0013 (0.0014)	0.0024 (0.0032)	-0.0658 (0.0773)	0.0006 (0.0018)	0.0006 (0.0028)	-0.0000 (0.0025)	-0.0516 (0.0642)	-0.0098* (0.0057)	-0.0067* (0.0033)	0.0032 (0.0057)	0.0440 (0.1228)

Notes. This table reports average conditional alphas for different value-weighted anomaly portfolios. We present the results for size, value, and momentum portfolios. For size and value, we build 25 independently sorted portfolios. S is the average of the five low-market-cap portfolios, and B is the average of the five high-market-cap portfolios, whereas S-B is their difference. Similarly, G is the average of the five low-book-to-market portfolios, and V is the average of the five high-book-to-market portfolios. V-G is the difference between the value and growth portfolios. For momentum, we sort the stocks into 10 portfolios according to their return over the past 12 months (skipping 1 month). L and W are the bottom and top deciles, whereas W-L is their difference. Unconditional denotes the unconditional CAPM based on monthly returns for the full sample period. The remaining segments test the conditional CAPM using quarterly nonoverlapping windows. In parentheses, we report Newey and West (1987) standard errors with four lags.

*, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

based on daily data, whereas the general ranking among the estimators largely prevails. The higher RMSEs of panels B and C of Table 6 indicate that the corresponding realized betas are contaminated by higher measurement error. Therefore, in the following, we will focus the main discussion on the 30-minute realized beta while examining the robustness to this choice for all the main tests.

To shed further light on the predictability of the five portfolios, Figure 1 shows scatterplots of the predicted and realized betas for each estimation approach. We find that HIST and HIST⁽¹⁾ spread relatively widely from the 45° line. Especially for portfolios with high and low predicted betas, the approaches produce large forecast errors. For HF₃₀ and HF₃₀⁽¹⁾, in contrast, the dots are much closer to the 45° line. The predictions and realizations match very well, independently of the magnitude of the predicted betas.

However, HIST might not be the optimal alternative estimator for beta. Taken together, the studies of Buss and Vilkov (2012) and Hollstein and Prokopczuk (2016) show that the BV estimator outperforms all other

approaches based on daily data both statistically and economically. Therefore, we also add this estimator as a second benchmark. At each point in time, the BV estimator is only available for the stocks that are currently in the S&P 500. Thus, to be able to compare the results and assess their statistical significance, we repeat the previous analysis only including the constituents of the S&P 500 at each point in time.

We present these results in Table 7. Using high-frequency realized beta in panel A, we find that HF₃₀ clearly outperforms all other models, including BV and BV₃₀. When using high-frequency realized beta with lag adjustment in panel B, we find once more that HF₃₀⁽¹⁾ yields the lowest average RMSE. However, that of HF₃₀ and BV is only marginally and not statistically significantly higher. Using realized beta based on daily data (panel C), HF₃₀ again yields a lower average RMSE than HIST. Interestingly, for the S&P 500 stocks, the outperformance of HF₃₀ over HIST is even more pronounced, indicating that high-frequency data are particularly useful for the large stocks, which are traded most frequently among those in our total sample.

Table 6. Prediction Errors

	HIST	HIST ⁽¹⁾	HF ₃₀	HF ₃₀ ⁽¹⁾
Panel A: Realized beta				
Average RMSE	0.1352	0.1542	<i>0.0949</i>	0.1180
HIST	—	−0.0190 (−1.00)	0.0403 (1.00)	0.0172 (0.40)
HIST ⁽¹⁾	0.0118 (1.00)	—	0.0593 (1.00)	0.0362 (1.00)
HF ₃₀	−0.0277 (−1.00)	−0.0395 (−1.00)	—	−0.0231 (−1.00)
HF ₃₀ ⁽¹⁾	−0.0064 (−0.40)	−0.0181 (−0.80)	0.0214 (1.00)	—
Panel B: Realized beta with lag-adjustment				
Average RMSE	0.1491	0.1587	0.1344	<i>0.1276</i>
HIST	—	−0.0095 (−0.40)	0.0147 (0.20)	0.0216 (0.80)
HIST ⁽¹⁾	0.0060 (0.40)	—	0.0242 (0.40)	0.0311 (1.00)
HF ₃₀	−0.0011 (−0.20)	−0.0070 (−0.20)	—	0.0069 (0.60)
HF ₃₀ ⁽¹⁾	−0.0111 (−0.60)	−0.0170 (−0.80)	−0.0100 (−0.60)	—
Panel C: Daily realized beta				
Average RMSE	0.1510	0.1561	0.1493	<i>0.1466</i>
HIST	—	−0.0052 (−0.20)	0.0017 (0.00)	0.0044 (0.00)
HIST ⁽¹⁾	0.0061 (0.20)	—	0.0069 (0.20)	0.0096 (0.00)
HF ₃₀	0.0058 (0.40)	−0.0003 (0.20)	—	0.0027 (0.20)
HF ₃₀ ⁽¹⁾	0.0023 (0.00)	−0.0039 (−0.20)	−0.0035 (0.00)	—

Notes. This table presents the out-of-sample prediction errors of different approaches. Each month, we sort the stocks into five portfolios according to their estimate for HIST 12 months before. We obtain value-weighted average betas for each portfolio and approach and measure the realized beta of each portfolio using 30-minute returns over the subsequent month. The first row reports the average (Avg.) RMSE across all portfolios. The lowest average error is indicated by *italic* font. The remainder of the table reports the differences in prediction errors. The upper triangular matrices report the differences in RMSE, averaged over all portfolios. Similarly, the lower triangular matrices report the average differences in RMedSE. We report the error loss differential between the model [*name in row*] and the model [*name in column*]. The absolute values of the numbers in parentheses indicate the share of portfolios for which the difference is significant at 5% (e.g., 0.4 indicates that the difference is significant for 40% of the portfolios). Figures printed in **bold** indicate that the differences are significant for *all* portfolios. We test significance with the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign of the number in parentheses indicates the direction of the significant differences.

Comparing HF₃₀ and BV, we find that HF₃₀ performs at least as well as BV. Finally, another interesting observation relates to the comparison of BV and BV₃₀. We find that BV yields lower

average RMSEs than BV₃₀ for every specification. Thus, the use of high-frequency data does not seem to be beneficial for this estimator. This finding is likely caused by the need to estimate the full correlation matrix and not only one bivariate correlation as for HF₃₀, where the issue of nonsynchronous trading is severely amplified.

We have also generated scatterplots, similar to that in Figure 1 for BV and BV₃₀ (not shown). We find that both approaches tend to underestimate the betas of low-beta portfolios and overestimate those of high-beta portfolios. For beta estimates around 1, the approach works reasonably well. The scatterplots for HIST and BV are comparable to those in Buss and Vilkov (2012).

4.3. Alternative Frequencies

In Table 8, we present the results for high-frequency beta estimators based on alternative sampling frequencies of 5, 75, and 130 minutes, both with and without lag adjustment. All results are very similar to before for the 30-minute frequency. The high-frequency estimators for coarser frequencies outperform HIST throughout. For our entire sample, the 5-minute frequency seems to be a little too high. Especially for realized beta with lag adjustment or daily data, the 5-minute frequency leads to high average RMSEs.

4.4. Information Content

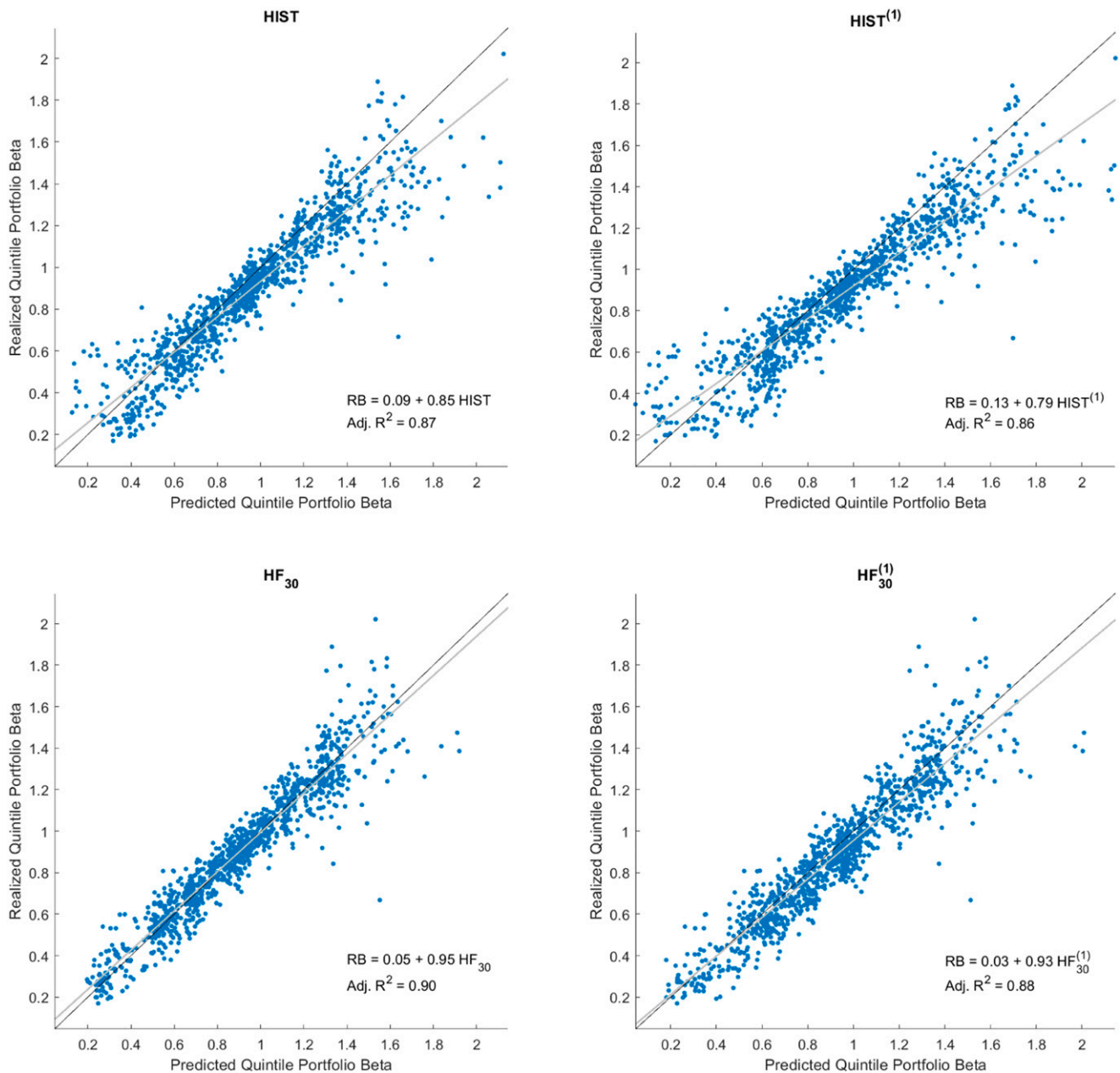
An alternative way to evaluate the performance of ex ante estimates is to use Mincer and Zarnowitz (1969) regressions. We regress the one-month (ex post) realized beta on the different predictions for beta:

$$\beta_{i,T}^R = a + b\beta_i + \epsilon_T, \quad (8)$$

where β_i denotes one beta estimate in univariate regressions or a vector of several beta estimates in encompassing regressions, and $\beta_{i,T}^R$ is as previously defined.

The regression model in Equation (8) is designed to test for the informational efficiency and unbiasedness of different estimators. We stick to level Mincer–Zarnowitz regressions instead of logarithmically transforming our variables because beta is theoretically unbounded and can also take negative values. Hansen and Lunde (2006) show that level Mincer–Zarnowitz regressions are robust to (mean zero) errors in the evaluation proxy.

We test for unbiasedness in univariate regressions using a Wald test, which imposes the joint hypothesis that $a = 0$ and $b = 1$. If the model is unbiased (i.e., it adequately describes the data), we will not be able to reject this hypothesis. We test for informational efficiency in encompassing regressions by restricting the slope parameters of alternative estimators to zero. This test determines whether an estimator contains information beyond that of a baseline model.

Figure 1. (Color online) Predicted Beta vs. Realized Beta: Scatterplots

Notes. This figure shows the scatterplots of predicted and realized beta. Each month, we sort the stocks into five portfolios according to their estimate for HIST 12 months before. For each of the portfolios, we compute the value-weighted average predicted beta as well as the realized beta over the next month, using 30-minute returns. We plot the realized quintile betas versus the predicted portfolio betas for each quintile, month, and estimation methodology. Each figure includes a 45° line and a fitted regression line, as well as the adjusted R^2 of the regression.

If an estimator adds value, its encompassing slope estimate must be significant (and positive), and the explanatory power must rise compared with the restricted model. We perform a Wald test to assess whether an estimator fully subsumes all the information contained in another estimator. To do this, we test the null hypothesis that adding an estimator does not increase the explanatory power compared with the (restricted) univariate models.

Table 9 presents the results of the Mincer–Zarnowitz regressions. In panel A, we present the univariate regressions. Note that for univariate regressions, the t -statistics of the slope coefficients, as part of the unbiasedness hypothesis, test the hypothesis of $b = 1$ and not, as is usually done, $b = 0$. Because we expect a positive relationship between the expected and realized beta, in multivariate regressions, we consider a one-sided test of $b = 0$ against $b > 0$. All t -statistics

Table 7. Prediction Errors: S&P 500 Stocks

	HIST	HIST ⁽¹⁾	HF ₃₀	HF ₃₀ ⁽¹⁾	BV	BV ₃₀
Panel A: Realized beta						
Average RMSE	0.1320	0.1421	<i>0.0945</i>	0.1163	0.1072	0.1145
HIST	—	−0.0101 (−0.60)	0.0375 (1.00)	0.0157 (0.40)	0.0247 (0.60)	0.0174 (0.40)
HIST ⁽¹⁾	0.0107 (0.80)	—	0.0476 (1.00)	0.0258 (0.80)	0.0349 (0.60)	0.0275 (0.40)
HF ₃₀	−0.0215 (−1.00)	−0.0322 (−1.00)	—	−0.0218 (−1.00)	−0.0128 (−0.40)	−0.0201 (−0.60)
HF ₃₀ ⁽¹⁾	−0.0053 (−0.20)	−0.0161 (−0.80)	0.0162 (1.00)	—	0.0090 (0.20)	0.0017 (0.00)
BV	−0.0114 (−0.40)	−0.0221 (−0.80)	0.0101 (0.60)	−0.0061 (−0.20)	—	−0.0073 (−0.60)
BV ₃₀	−0.0061 (0.00)	−0.0169 (−0.20)	0.0154 (0.60)	−0.0008 (−0.20)	0.0053 (0.40)	—
Panel B: Realized beta with lag-adjustment						
Average RMSE	0.1511	0.1571	0.1301	<i>0.1292</i>	0.1294	0.1364
HIST	—	−0.0060 (−0.40)	0.0211 (0.60)	0.0220 (0.60)	0.0218 (1.00)	0.0148 (0.60)
HIST ⁽¹⁾	0.0040 (0.40)	—	0.0271 (0.80)	0.0280 (1.00)	0.0278 (0.80)	0.0208 (0.60)
HF ₃₀	−0.0089 (−0.60)	−0.0129 (−0.40)	—	0.0009 (0.00)	0.0007 (0.00)	−0.0063 (0.00)
HF ₃₀ ⁽¹⁾	−0.0116 (−0.20)	−0.0156 (−0.80)	−0.0027 (0.00)	—	−0.0002 (0.00)	−0.0072 (−0.20)
BV	−0.0126 (−0.60)	−0.0166 (−0.60)	−0.0037 (−0.20)	−0.0010 (0.20)	—	−0.0070 (−0.60)
BV ₃₀	−0.0044 (−0.40)	−0.0085 (−0.60)	0.0044 (0.00)	0.0071 (0.20)	0.0082 (0.60)	—
Panel C: Daily realized beta						
Average RMSE	0.1550	0.1579	0.1431	0.1476	<i>0.1416</i>	0.1486
HIST	—	−0.0029 (−0.20)	0.0119 (0.40)	0.0074 (0.20)	0.0135 (0.60)	0.0065 (0.40)
HIST ⁽¹⁾	0.0000 (0.20)	—	0.0148 (0.40)	0.0103 (0.20)	0.0164 (0.60)	0.0093 (0.60)
HF ₃₀	−0.0038 (−0.40)	−0.0039 (−0.40)	—	−0.0045 (0.00)	0.0015 (0.20)	−0.0055 (0.00)
HF ₃₀ ⁽¹⁾	−0.0027 (0.20)	−0.0027 (0.00)	0.0011 (0.20)	—	0.0061 (0.20)	−0.0010 (0.00)
BV	−0.0116 (−0.40)	−0.0116 (−0.40)	−0.0077 (−0.40)	−0.0089 (−0.60)	—	−0.0070 (−0.40)
BV ₃₀	−0.0065 (0.00)	−0.0065 (−0.40)	−0.0026 (−0.20)	−0.0038 (−0.20)	0.0051 (0.80)	—

Notes. This table presents the out-of-sample prediction errors of different approaches. Each month, we sort the stocks in the S&P 500 into five portfolios according to their estimate for HIST 12 months before. We obtain value-weighted average betas for each portfolio and approach and measure the realized beta of each portfolio using 30-minute returns over the subsequent month. The first row reports the average (Avg.) RMSE across all portfolios. The lowest average error is indicated by *italic* font. The remainder of the table reports the differences in prediction errors. The upper triangular matrices report the differences in RMSE, averaged over all portfolios. Similarly, the lower triangular matrices report the average differences in RMSE. We report the error loss differential between the model [*name in row*] and the model [*name in column*]. The absolute values of the numbers in parentheses indicate the share of portfolios for which the difference is significant at 5% (e.g., 0.4 indicates that the difference is significant for 40% of the portfolios). Figures printed in **bold** indicate that the differences are significant for *all* portfolios. We test significance with the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign of the number in parentheses indicates the direction of the significant differences.

Table 8. Prediction Errors: Different Frequencies

	HIST	HIST ⁽¹⁾	HF ₃₀	HF ₃₀ ⁽¹⁾	HF ₅	HF ₅ ⁽¹⁰⁾	HF ₇₅	HF ₇₅ ⁽¹⁾	HF ₁₃₀	HF ₁₃₀ ⁽¹⁾
Panel A: Realized beta										
Average RMSE	0.1353	0.1543	0.0949	0.1181	0.1012	0.1420	0.0980	0.1320	0.0995	0.1371
HIST	—	−0.0190	0.0404	0.0172	0.0341	−0.0067	0.0373	0.0033	0.0358	−0.0018
HIST ⁽¹⁾	—	(−1.00)	(1.00)	(0.40)	(1.00)	(−0.40)	(1.00)	(0.00)	(1.00)	(−0.20)
	0.0120	—	0.0594	0.0362	0.0531	0.0122	0.0563	0.0223	0.0548	0.0172
	(1.00)	—	(1.00)	(1.00)	(1.00)	(0.20)	(1.00)	(0.60)	(1.00)	(0.40)
HF ₃₀	−0.0273	−0.0393	—	−0.0232	−0.0063	−0.0472	−0.0031	−0.0371	−0.0046	−0.0422
HF ₃₀ ⁽¹⁾	(−1.00)	(−1.00)	—	(−1.00)	(−0.80)	(−1.00)	(−0.60)	(−1.00)	(−0.60)	(−1.00)
	−0.0060	−0.0180	0.0213	—	0.0169	−0.0240	0.0201	−0.0139	0.0186	−0.0190
	(−0.40)	(−0.80)	(1.00)	—	(0.80)	(−1.00)	(1.00)	(−1.00)	(1.00)	(−1.00)
HF ₅	−0.0205	−0.0326	0.0067	−0.0145	—	−0.0409	0.0032	−0.0308	0.0017	−0.0359
HF ₅ ⁽¹⁰⁾	(−0.80)	(−1.00)	(0.80)	(−0.80)	—	(−1.00)	(0.40)	(−1.00)	(0.00)	(−1.00)
	0.0053	−0.0067	0.0326	0.0113	0.0259	—	0.0440	0.0100	0.0425	0.0050
	(0.60)	(−0.40)	(1.00)	(1.00)	(1.00)	—	(1.00)	(0.60)	(1.00)	(0.40)
HF ₇₅	−0.0276	−0.0396	−0.0003	−0.0216	−0.0071	−0.0329	—	−0.0340	−0.0015	−0.0391
HF ₇₅ ⁽¹⁾	(−1.00)	(−1.00)	(0.20)	(−1.00)	(−0.20)	(−1.00)	—	(−1.00)	(−0.20)	(−1.00)
	0.0036	−0.0084	0.0309	0.0096	0.0241	−0.0017	0.0312	—	0.0325	−0.0051
	(0.40)	(−0.40)	(1.00)	(1.00)	(1.00)	(−0.20)	(1.00)	—	(1.00)	(0.00)
HF ₁₃₀	−0.0262	−0.0382	0.0011	−0.0202	−0.0056	−0.0315	0.0015	−0.0297	—	−0.0376
HF ₁₃₀ ⁽¹⁾	(−1.00)	(−1.00)	(0.20)	(−1.00)	(−0.20)	(−1.00)	(0.00)	(−1.00)	—	(−1.00)
	0.0021	−0.0099	0.0294	0.0081	0.0226	−0.0032	0.0297	−0.0015	0.0283	—
	(0.60)	(−0.40)	(1.00)	(0.40)	(1.00)	(−0.20)	(1.00)	(0.00)	(1.00)	—
Panel B: Realized beta with lag-adjustment										
Average RMSE	0.1492	0.1587	0.1343	0.1275	0.1512	0.1377	0.1299	0.1343	0.1307	0.1411
HIST	—	−0.0095	0.0149	0.0217	−0.0020	0.0114	0.0193	0.0149	0.0185	0.0081
	—	(−0.40)	(0.20)	(0.80)	(0.00)	(0.40)	(0.80)	(0.40)	(0.60)	(0.20)
HIST ⁽¹⁾	0.0060	—	0.0244	0.0312	0.0075	0.0210	0.0288	0.0244	0.0280	0.0176
	(0.40)	—	(0.40)	(1.00)	(0.00)	(0.40)	(0.80)	(0.80)	(0.60)	(0.40)
HF ₃₀	−0.0001	−0.0061	—	0.0068	−0.0169	−0.0035	0.0044	−0.0000	0.0036	−0.0068
	(−0.20)	(−0.20)	—	(0.60)	(−1.00)	(0.00)	(0.60)	(0.00)	(0.40)	(0.00)
HF ₃₀ ⁽¹⁾	−0.0114	−0.0174	−0.0113	—	−0.0238	−0.0103	−0.0024	−0.0069	−0.0032	−0.0137
	(−1.00)	(−0.80)	(−0.60)	—	(−0.80)	(−0.60)	(−0.20)	(−0.40)	(−0.40)	(−0.80)
HF ₅	0.0098	0.0038	0.0099	0.0212	—	0.0135	0.0214	0.0169	0.0205	0.0101
	(0.60)	(0.20)	(1.00)	(0.80)	—	(0.40)	(0.80)	(0.60)	(0.80)	(0.40)
HF ₅ ⁽¹⁰⁾	−0.0028	−0.0089	−0.0027	0.0085	−0.0126	—	0.0079	0.0034	0.0071	−0.0034
	(−0.40)	(−0.40)	(0.00)	(0.60)	(−0.40)	—	(0.20)	(0.40)	(0.20)	(−0.20)
HF ₇₅	−0.0042	−0.0102	−0.0041	0.0072	−0.0140	−0.0013	—	−0.0044	−0.0008	−0.0112
	(−0.20)	(−0.60)	(−0.60)	(0.00)	(−0.60)	(0.00)	—	(−0.20)	(−0.20)	(−0.40)
HF ₇₅ ⁽¹⁾	−0.0045	−0.0105	−0.0044	0.0068	−0.0143	−0.0017	−0.0004	—	0.0036	−0.0068
	(−0.20)	(−0.60)	(0.20)	(0.20)	(−0.80)	(−0.20)	(−0.20)	—	(0.20)	(−0.80)
HF ₁₃₀	−0.0034	−0.0094	−0.0033	0.0080	−0.0132	−0.0005	0.0008	0.0011	—	−0.0104
	(−0.40)	(−0.60)	(−0.80)	(0.60)	(−1.00)	(0.00)	(0.20)	(0.00)	—	(−0.40)
HF ₁₃₀ ⁽¹⁾	−0.0025	−0.0085	−0.0024	0.0089	−0.0123	0.0003	0.0017	0.0020	0.0009	—
	(−0.40)	(−0.40)	(−0.20)	(0.60)	(−0.60)	(0.00)	(0.20)	(0.20)	(0.20)	—
Panel C: Daily realized beta										
Average RMSE	0.1509	0.1561	0.1491	0.1464	0.1681	0.1588	0.1433	0.1432	0.1432	0.1495
HIST	—	−0.0052	0.0018	0.0045	−0.0172	−0.0079	0.0076	0.0077	0.0078	0.0014
	—	(−0.20)	(0.00)	(0.00)	(−0.80)	(−0.40)	(0.20)	(0.00)	(0.20)	(0.00)
HIST ⁽¹⁾	0.0058	—	0.0070	0.0097	−0.0120	−0.0027	0.0128	0.0129	0.0130	0.0066
	(0.20)	—	(0.20)	(0.00)	(−0.40)	(−0.40)	(0.20)	(0.20)	(0.20)	(0.00)
HF ₃₀	0.0061	0.0002	—	0.0027	−0.0190	−0.0097	0.0058	0.0059	0.0060	−0.0004
	(0.40)	(0.00)	—	(0.20)	(−1.00)	(−0.40)	(1.00)	(0.20)	(0.80)	(0.00)

Table 8. (Continued)

Panel C: Daily realized beta										
Average RMSE	0.1509	0.1561	0.1491	0.1464	0.1681	0.1588	0.1433	0.1432	<i>0.1432</i>	0.1495
HF ₃₀ ⁽¹⁾	0.0015 (0.00)	−0.0043 (−0.20)	−0.0045 (0.00)	— (0.00)	−0.0217 (−1.00)	−0.0124 (−0.60)	0.0031 (0.20)	0.0032 (0.00)	0.0033 (0.40)	−0.0031 (0.00)
HF ₅	0.0171 (0.80)	0.0113 (0.80)	0.0111 (1.00)	0.0156 (1.00)	— (0.20)	0.0093 (1.00)	0.0248 (1.00)	0.0249 (1.00)	0.0250 (1.00)	0.0186 (0.60)
HF ₅ ⁽¹⁰⁾	0.0187 (0.60)	0.0129 (0.60)	0.0126 (0.40)	0.0172 (0.80)	0.0016 (−0.60)	— (0.80)	0.0155 (0.60)	0.0156 (0.60)	0.0157 (0.80)	0.0093 (0.60)
HF ₇₅	0.0043 (−0.20)	−0.0015 (−0.60)	−0.0017 (−0.60)	0.0028 (0.00)	−0.0128 (−0.60)	−0.0144 (−0.60)	— (0.20)	0.0001 (0.20)	0.0002 (0.20)	−0.0062 (−0.20)
HF ₇₅ ⁽¹⁾	0.0033 (0.00)	−0.0025 (0.00)	−0.0028 (−0.20)	0.0018 (0.00)	−0.0138 (−0.80)	−0.0154 (−0.60)	−0.0010 (0.00)	— (0.00)	0.0000 (0.00)	−0.0063 (−0.40)
HF ₁₃₀	0.0023 (−0.20)	−0.0036 (−0.20)	−0.0038 (−0.80)	0.0007 (−0.20)	−0.0148 (−1.00)	−0.0164 (−0.60)	−0.0021 (−0.20)	−0.0010 (0.00)	— (0.00)	−0.0064 (−0.20)
HF ₁₃₀ ⁽¹⁾	0.0072 (0.20)	0.0014 (0.00)	0.0012 (−0.40)	0.0057 (0.00)	−0.0099 (−0.80)	−0.0115 (−0.80)	0.0029 (0.20)	0.0039 (0.20)	0.0049 (0.00)	— (0.00)

Notes. This table presents the out-of-sample prediction errors of different approaches. Each month, we sort the stocks into five portfolios according to their estimate for HIST 12 months before. We obtain value-weighted average betas for each portfolio and approach and measure the realized beta of each portfolio using 30-minute returns over the subsequent month. The first row reports the average (Avg.) RMSE across all portfolios. The lowest average error is indicated by *italic* font. The remainder of the table reports the differences in prediction errors. The upper triangular matrices report the differences in RMSE, averaged over all portfolios. Similarly, the lower triangular matrices report the average differences in RMedSE. We report the error loss differential between the model [name in row] and the model [name in column]. The absolute values of the numbers in parentheses indicate the share of portfolios for which the difference is significant at 5% (e.g., 0.4 indicates that the difference is significant for 40% of the portfolios). Figures printed in **bold** indicate that the differences are significant for *all* portfolios. We test significance with the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign of the number in parentheses indicates the direction of the significant differences.

and Wald tests use robust Newey and West (1987) standard errors with four lags.

If an approach yields noisy estimates of beta, the slope coefficient of the Mincer–Zarnowitz regression will be biased downward. As a result, the intercept estimate is biased upward. Comparing the average slope coefficients of HIST and HF₃₀, we find them to be substantially closer to 1 (0.77 versus 0.88) for the high-frequency estimator. The intercept is substantially smaller for HF₃₀, whereas the average adjusted R^2 increases from 0.64 to 0.73 when moving from HIST to HF₃₀. These results are consistent with the notion that using high-frequency data reduces the measurement error in historical beta. For HIST, we can reject the unbiasedness hypothesis for each portfolio, whereas for HF₃₀, we cannot reject this hypothesis for three out of five portfolios. For the approaches that include lag adjustments, we find qualitatively similar results with somewhat smaller adjusted R^2 values.

We next examine the results of the encompassing regressions in panel B of Table 9. In a joint regression with HIST and HF₃₀, we find that the slope coefficient on HIST is −0.01 on average, whereas that on HF₃₀ is

0.88. Thus, HF₃₀ seems to subsume most of the information contained in HIST.

4.5. Cross-Sectional Predictability

Up to this point, we have focused on the time-series dimension of prediction errors. However, a good approach for estimating beta should do well both in the time series and in the cross section (i.e., the stocks should be properly ranked, and the expected cross-sectional differences in beta should line up well with the realized cross-sectional differences). This insight motivates two additional tests. First, we examine the average of the cross-sectional Spearman rank correlations of the estimates of each approach with the realizations of beta. For an approach to work well, the rank correlation should be close to 1. Second, to capture the magnitude of cross-sectional predictability, we use Fama and MacBeth (1973) cross-sectional regressions of realized beta on the estimates of each approach. That is, each month, we fit the following cross-sectional regression model:

$$\beta_{j,t,T}^R = a_T + b_T \beta_{j,t} + \epsilon_{j,t,T}, \quad (9)$$

Table 9. Univariate and Encompassing Regressions for Realized Beta

Panel A: Univariate regressions													
	a	sig	b_{HIST}	sig	$b_{\text{HIST}^{(1)}}$	sig	$b_{\text{HF}_{30}}$	sig	$b_{\text{HF}_{30}^{(1)}}$	sig	R^2_{adj}	sig(Wald ₁)	
HIST	0.19	(0.20)	0.77	(0.20)	—	—	—	—	—	—	0.64	(1.00)	
HIST ⁽¹⁾	0.24	(0.80)	—	—	0.70	(1.00)	—	—	—	—	0.62	(1.00)	
HF ₃₀	0.13	(0.40)	—	—	—	—	0.88	(0.40)	—	—	0.73	(0.40)	
HF ₃₀ ⁽¹⁾	0.11	(0.20)	—	—	—	—	—	—	0.88	(0.20)	0.65	(0.80)	
Panel B: Encompassing regressions													
	a	sig	b_{HIST}	sig	$b_{\text{HIST}^{(1)}}$	sig	$b_{\text{HF}_{30}}$	sig	$b_{\text{HF}_{30}^{(1)}}$	sig	R^2_{adj}	sig(Wald ₁)	sig(Wald ₂)
HIST & HIST ⁽¹⁾	0.20	(0.40)	0.38	(0.40)	0.37	(0.40)	—	—	—	—	0.65	(0.40)	(0.40)
HIST & HF ₃₀	0.13	(0.40)	−0.01	(0.00)	—	—	0.88	(0.80)	—	—	0.73	(0.00)	(0.80)
HIST & HF ₃₀ ⁽¹⁾	0.11	(0.20)	0.37	(0.80)	—	—	—	—	0.50	(0.80)	0.69	(0.80)	(0.80)
HIST ⁽¹⁾ & HF ₃₀	0.14	(0.40)	—	—	0.08	(0.00)	0.78	(0.80)	—	—	0.73	(0.00)	(0.80)
HIST ⁽¹⁾ & HF ₃₀ ⁽¹⁾	0.13	(0.20)	—	—	0.33	(0.60)	—	—	0.52	(0.80)	0.69	(0.60)	(0.80)
HF ₃₀ & HF ₃₀ ⁽¹⁾	0.15	(0.40)	—	—	—	—	0.98	(1.00)	−0.11	(0.00)	0.73	(1.00)	(0.00)

Notes. This table reports the results of univariate (panel A) and encompassing regressions (panel B). Each month, we sort the stocks into five portfolios according to their estimate for HIST 12 months before. We obtain value-weighted average betas for each portfolio and approach and measure the realized beta of each portfolio using 30-minute returns over the subsequent month. Finally, we regress realized beta ($\beta_{i,T}^R$) on a constant and one or several ex ante estimates, denoted by β_i . The regression equation is $\beta_{i,T}^R = a + b\beta_i + \epsilon_i$, where a denotes the regression intercept and b is a vector of the slope coefficients for the different beta estimates. For each regression coefficient we report in parentheses (sig) the share of portfolios for which the coefficient is significant at 5% using Newey and West (1987) standard errors with four lags. We separately test the null hypotheses $a = 0$ and $b = 1$ in univariate regressions, whereas in multivariate regressions we use the one-sided hypothesis of $b = 0$ against $b > 0$. R^2_{adj} indicates the adjusted R^2 of the regressions. The columns sig(Wald₁) and sig(Wald₂) present the results of the Wald test statistic as a percentage for which the test statistic is significant at 5%. In univariate regressions, the Wald test (reported in Wald₁) imposes the joint hypothesis that $a = 0$ and $b = 1$. In multivariate regressions, the hypothesis of Wald₁ is that the explanatory power of the multivariate model is equal to that of the univariate model just using the explanatory variable mentioned second (first for Wald₂).

where a_T and b_T denote the time T regression intercept and slope, respectively. All other variables are as previously defined. To estimate Equation (9), we follow the generalized method of moments instrumental variables (IV-GMM) estimator approach of Kim and Skoulakis (2016). A good model should have an intercept close to 0, a slope coefficient close to 1, and a high R^2 .

We present the results in Table 10. In panel A, we examine the rank correlations. We observe that high-frequency data are valuable for estimating beta in terms of ranking the stocks. The average rank correlation of HF₃₀ is 0.80, whereas for HIST, the rank correlation amounts to only 0.69. For HF₃₀⁽¹⁾ and HIST⁽¹⁾, the rank coefficients are somewhat lower than those for HF₃₀ and HIST, respectively. For all approaches, the average p -value is close to 0. Hence, all approaches seem to be able to significantly predict the cross-sectional ranks of beta. Overall, the highest average rank correlation is obtained for HF₃₀.

Panel B of Table 10 shows the results of cross-sectional regressions. We find that using high-frequency data improves the cross-sectional predictability. Looking at the parameter estimates for HF₃₀, we can see that the slope coefficient is of a similar magnitude as that of HIST. However, HF₃₀ clearly yields the highest cross-sectional R^2 , with 0.66, which indicates its superior cross-sectional predictability compared with all other approaches.

4.6. Mean Absolute Error

Lastly, we test the robustness of our results to an alternative loss function (i.e., the MAE):

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^n \left| \beta_{i,T}^R - \beta_i \right|, \quad (10)$$

where all variables are as previously defined. The results for five portfolios are presented in Table 11. These are qualitatively similar to those using the RMSE criterion. Overall, HF₃₀ also yields the lowest average MAE.

5. Conclusion

This paper tests the conditional CAPM with betas based on daily and high-frequency data. Whereas with daily data the model cannot explain the size anomaly and three of the six component returns of the three main anomalies, the conditional CAPM can explain the size, value, and momentum anomalies, as well as five out of six of the component portfolio returns when using high-frequency data.

We find that the superior economic value of high-frequency betas is associated with a superior predictability for future realized beta. Betas estimated from high-frequency data are significantly more accurate than those estimated from daily return data. We find that betas estimated from high-frequency data also outperform those based on daily data in

Table 10. Cross-Sectional Predictability

	HIST	HIST ⁽¹⁾	HF ₃₀	HF ₃₀ ⁽¹⁾
Panel A: Rank correlations				
ρ	0.6937***	0.6308***	0.8034***	0.7700***
(<i>p</i> -value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Panel B: Cross-sectional regressions				
Const.	−0.0871***	−0.0264	0.0411***	−0.0602***
(s.e.)	(0.0221)	(0.0251)	(0.0125)	(0.0161)
Slope	0.9547***	0.8352***	0.9670***	0.9662***
(s.e.)	(0.1654)	(0.1377)	(0.1106)	(0.1371)
R^2_{adj}	0.4903	0.3900	0.6613	0.5984

Notes. This table presents results for the cross-sectional predictability of different approaches for 30 minute realized beta over the horizon of one month. Panel A reports averages of cross-sectional rank correlations of the approaches with ex-post realized beta in each period. ρ is the average rank correlation coefficient, whereas *p*-value indicates the corresponding average *p*-value for the null hypothesis of zero correlation. Panel B shows the results of cross-sectional Fama and MacBeth (1973) regressions, regressing realized beta on ex ante estimates. We estimate the coefficients and perform the inference following the IV-GMM approach suggested by Kim and Skoulakis (2016). Const. and Slope denote the time-series averages of the regression intercept and slope coefficients, respectively, whereas s.e. indicates the corresponding standard errors. The row R^2_{adj} presents the average adjusted R^2 s of the regressions.

*, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 11. Prediction Errors: MAE

	HIST	HIST ⁽¹⁾	HF ₃₀	HF ₃₀ ⁽¹⁾
Avg. MAE	0.1020	0.1183	0.0703	0.0919
HIST	—	−0.0162	0.0318	0.0101
	—	(−1.00)	(1.00)	(0.40)
HIST ⁽¹⁾	0.0118	—	0.0480	0.0264
	(1.00)	—	(1.00)	(1.00)
HF ₃₀	−0.0277	−0.0395	—	−0.0216
	(−1.00)	(−1.00)	—	(−1.00)
HF ₃₀ ⁽¹⁾	−0.0064	−0.0181	0.0214	—
	(−0.40)	(−0.80)	(1.00)	—

Notes. This table presents the out-of-sample prediction errors of different approaches. Each month, we sort the stocks into five portfolios according to their estimate for HIST 12 months before. We obtain value-weighted average betas for each portfolio and approach and measure the realized beta of each portfolio using 30-minute returns over the subsequent month. The first row reports the average MAE across all portfolios. The lowest average error is indicated by *italic* font. The remainder of the table reports the differences in prediction errors. The upper triangular matrices report the differences in MAE, averaged over all portfolios. Similarly, the lower triangular matrices report the average differences in MedAE. We report the error loss differential between the model [name in row] and the model [name in column]. The absolute values of the numbers in parentheses indicate the share of portfolios for which the difference is significant at 5% (e.g., 0.4 indicates that the difference is significant for 40% of the portfolios). Figures printed in **bold** indicate that the differences are significant for *all* portfolios. We test significance with the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign of the number in parentheses indicates the direction of the significant differences.

Mincer–Zarnowitz regressions and have a superior cross-sectional predictability.

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Appendix. Model-Free Option-Implied Volatility

The hybrid beta estimation approach is based on option-implied moments. Therefore, we build on the work of Bakshi et al. (2003) and compute the model-free option-implied volatility. Similar to Chang et al. (2012), we first compute ex-dividend stock prices. Next, we interpolate implied volatilities on a grid of 1,000 moneyness levels (K/S , strike-to-spot), equally spaced between 0.3% and 300%, for any given stock and trading day. We extrapolate implied volatilities outside the range of available strike prices using the value for the smallest, respectively largest, available moneyness level (as in Jiang and Tian 2005 and Chang et al. 2012). We use the interpolated volatilities to compute Black and Scholes (1973) option prices for calls $C(\cdot)$ if $K/S > 1$ and puts $P(\cdot)$ if $K/S < 1$. We use these prices to obtain the prices of the volatility (QUAD), the CUBIC, and the quartic (QUART) contracts (Jiang and Tian 2005):

$$\text{QUAD} = \int_S^\infty \frac{2(1 - \ln[\frac{K}{S}])}{K^2} C(T - t, K) dK + \int_0^S \frac{2(1 + \ln[\frac{S}{K}])}{K^2} P(T - t, K) dK, \quad (\text{A.1})$$

$$\text{CUBIC} = \int_S^\infty \frac{6 \ln[\frac{K}{S}] - 3(\ln[\frac{K}{S}])^2}{K^2} C(T - t, K) dK + \int_0^S \frac{6 \ln[\frac{S}{K}] + 3(\ln[\frac{S}{K}])^2}{K^2} P(T - t, K) dK, \quad (\text{A.2})$$

$$\text{QUART} = \int_S^\infty \frac{12(\ln[\frac{K}{S}])^2 - 4(\ln[\frac{K}{S}])^3}{K^2} C(T - t, K) dK + \int_0^S \frac{12(\ln[\frac{S}{K}])^2 + 4(\ln[\frac{S}{K}])^3}{K^2} P(T - t, K) dK. \quad (\text{A.3})$$

We approximate the integrals using a trapezoidal rule (Dennis and Mayhew, 2002). The option-implied moments can be computed as

$$\mu_{t,T}^Q = e^{r_{f,t}(T-t)} - 1 - \frac{e^{r_{f,t}(T-t)}}{2} \text{QUAD} - \frac{e^{r_{f,t}(T-t)}}{6} \text{CUBIC} - \frac{e^{r_{f,t}(T-t)}}{24} \text{QUART}, \quad (\text{A.4})$$

$$(\sigma_{t,T}^Q)^2 = e^{r_{f,t}(T-t)} \text{QUAD} - (\mu^Q)^2, \quad (\text{A.5})$$

where $r_{f,t}$ denotes the risk-free rate, $T - t$ is the time to maturity of the contract, and $(\sigma^Q)^2$ is the option-implied variance.

Endnotes

¹ The starting date of our study is determined by the need to have high-frequency data. The TRTH database does not start until the beginning of 1996.

² IvyDB uses a kernel smoothing algorithm that generates standardized options only “if there exists enough option price data on that date to accurately interpolate the required values” (https://wrds-www.wharton.upenn.edu/documents/755/IvyDB_US_Reference_Manual.pdf, p. 28). For more details, refer to the IvyDB technical document.

³ Note that the formula for realized beta makes use of the expanded formula for the variance, neglecting both the drift term and the risk-free rate. Andersen et al. (2006) note that the effect of the drift term vanishes as the sampling frequency is reduced, which effectively “annihilates” the mean. This view is supported by empirical facts (e.g., the average 30-minute return of the S&P 500 Index amounts to only 0.0017%). Similarly, the average daily riskless interest rate during our sample period amounts to 0.01%, which is equivalent to an average risk-free rate as low as 0.0007% over 30-minute intervals, indicating that it can indeed be neglected.

⁴ In this case, we compute the realized beta with respect to several lagged market returns and simply sum up the contemporaneous and lagged realized betas.

⁵ The technical condition is motivated by several points. First, the correlation needs to be between -1 and 1 . Second, the correlation matrix must be positive (semi-) definite. Third, the correlation risk premium must be consistent with empirical observations, namely that implied correlations are higher than realized correlations and that the correlation risk premium $\rho_{ijt}^{\mathbb{P}} - \rho_{ijt}^{\mathbb{Q}}$ is higher for stocks with low correlations under the physical measure. For more details, refer to Buss and Vilkov (2012).

⁶ Note that to keep the presentation manageable, we select only one estimation window length for the estimators. That is, according to the optimization that becomes relevant in Section 4.1, we use 12 months for all HIST estimators and 6 months for the HF and BV estimators. The summary statistics for alternative sampling windows are very similar.

⁷ Using the same stocks and sample period as in Lewellen and Nagel (2006), we obtain similar results as the original authors.

⁸ Unfortunately, it is not possible to pin down the bias for one portfolio to exact numbers because the factors that drive these biases are latent and not observable.

⁹ We thank an anonymous referee for suggesting this.

¹⁰ For the stocks for which we do not have an estimate of HIST 12 months ago, we set the quantity to 1 (Hollstein and Prokopczuk 2016).

¹¹ Strictly speaking, the Wilcoxon signed rank test incorporates the joint null hypothesis of zero median in the loss differentials as well as a symmetric distribution. We stick to this test instead of an alternative only testing on zero median, such as the simple sign test, because the Wilcoxon signed rank test turns out to be more powerful in many applications (Conover 1999).

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