

Course Project Report: Advanced Math Analysis with Matlab

KeZheng Xiong
22920202204622

December 20, 2021

Abstract

The report for the end-of-term project of Advanced Math Analysis with MATLAB fall 2021 course. All the source code is open-sourced on the Github repository <https://github.com/SmartPolarBear/matlab-math-analysis-csxmu-2021> under **GPLv3** license

Contents

1	Problem 1	2
1.1	Problem Description	2
1.2	Solution	2
1.2.1	The gradient of the function	2
1.2.2	Find the extreme values	2
1.3	Acknowledgment	2

1 Problem 1

1.1 Problem Description

Given function $F(x, y) = 0.2x^2 + 0.1y^2 + \sin(x + y)$, please work out its gradient. Based on the gradient, please find out the local extreme of function $F(x, y)$ when both x and y are in the range of $[-2 * \pi, 2 * \pi]$. The 2D and 3D views of the function is given in Fig. 1.

1.2 Solution

1.2.1 The gradient of the function

I get the gradient of the function using the following code

```
1 syms x y;  
2 f=0.2*x^2+0.1*y^2+sin(x+y);  
3 diff(f,x)  
4 diff(f,y)
```

Based on the result, the gradient is

$$\nabla \cdot f(x, y) = (\frac{2 * x}{5} + \cos(x + y), \frac{y}{5} + \cos(x + y)) \quad (1)$$

1.2.2 Find the extreme values

To find the extreme values of $F(x, y)$ with gradient decent method, we walk little steps towards the direction of the gradient. To formalize this idea, the algorithm is shown as follows.

Algorithm 1 Gradient Descent

Input: Initial point x_0 , a constant α , $k = 0$

while termination condition does not hold **do**

$k = k + 1$

$x_{k+1} = x_k - \alpha \nabla \cdot f(x_k)$

Various problems occurs if this brute-force algorithm is implemented directly. The speed of convergence is annoyingly slow if parameters are not chosen right. In fact, I never succeeded finding a set of parameters that works. A well-known optimization is called Stochastic gradient descent, or SGD, improve it significantly.

The given parameter α in the brute-force algorithm, which is referred as learning rate, will change each round according to the situation. To be more exact, SGD tries to find a learning rate m , so that it minimize the function

$$\nabla \cdot f(x, y) = (\frac{2 * x}{5} + \cos(x + y), \frac{y}{5} + \cos(x + y)) \quad (2)$$

1.3 Acknowledgment

Thanks to (TODO)