



# 기계이상진단을 위한 인공지능 학습 기법

## 제 4강 분류 태스크를 이용한 이상진단

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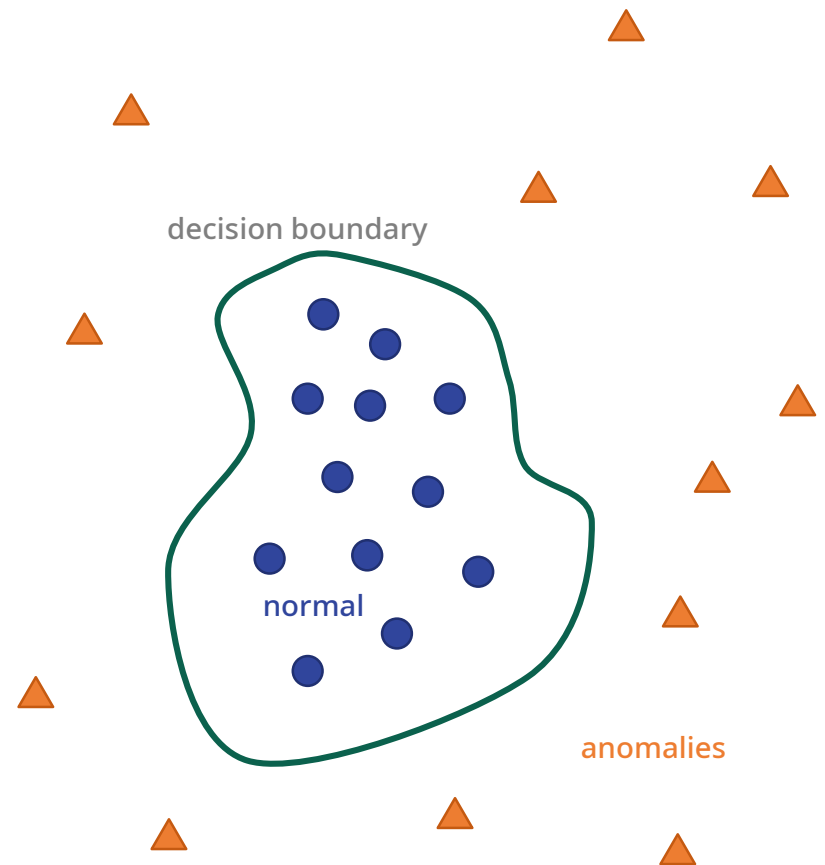
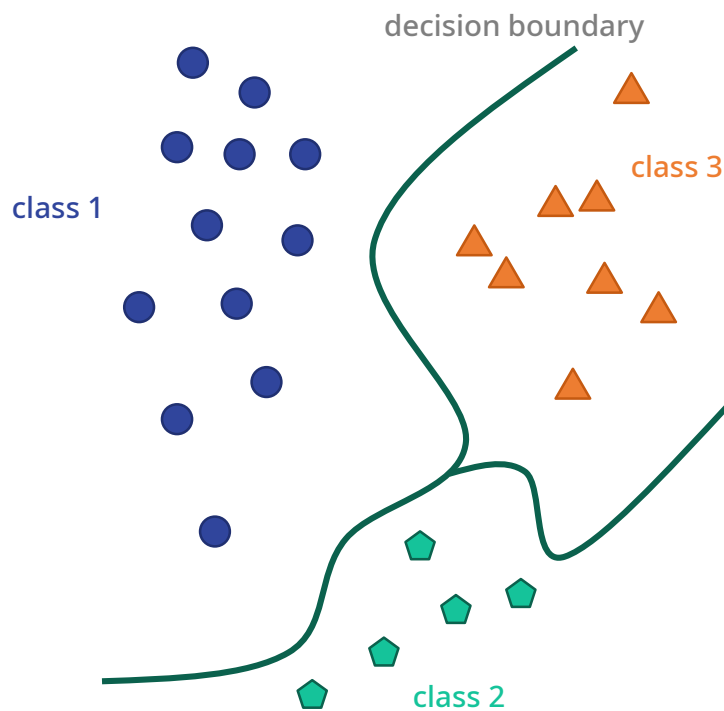
**KAIST EE**

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- 심층 신경망 분류기 조정 기법
  - Label smoothing
  - Temperature scaling

# 단일 클래스 분류

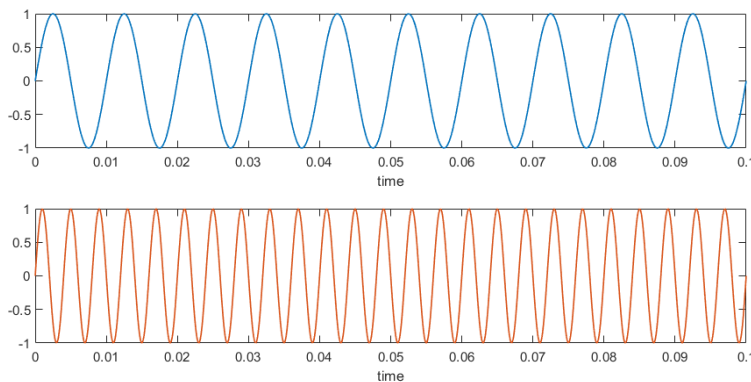
- 이상진단과의 관계



# 예제: 푸리에 변환

- Distinguishing two sinusoidal signals

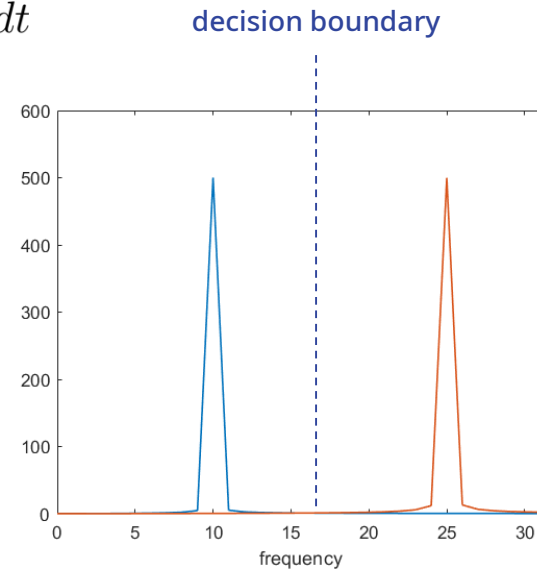
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$



Input data



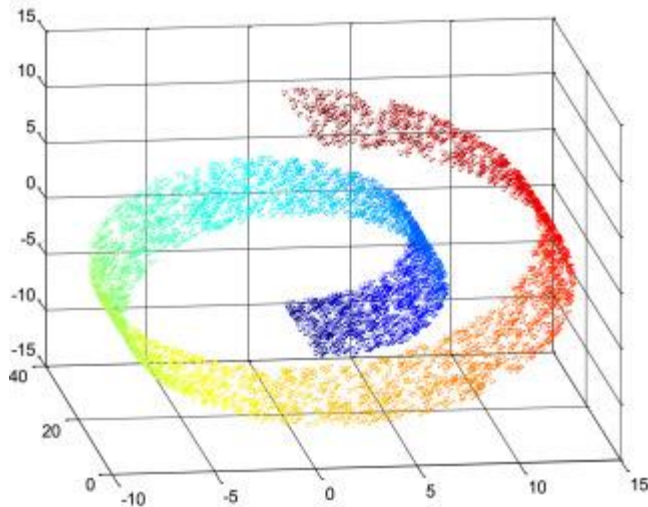
transform



feature space

# 복잡한 차원 변환

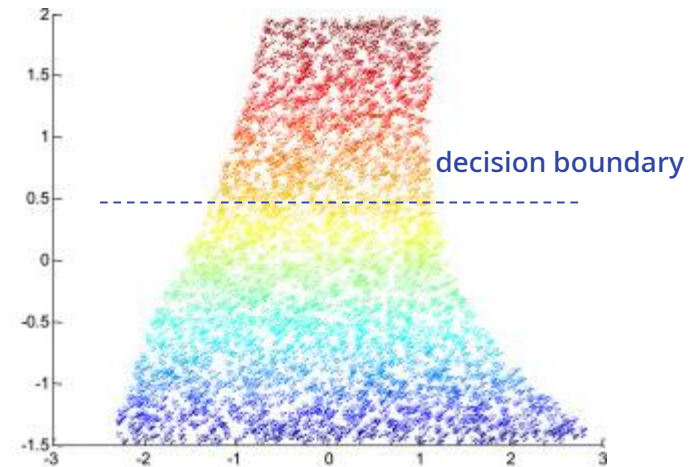
- Representation learning



Input data



transform



feature space

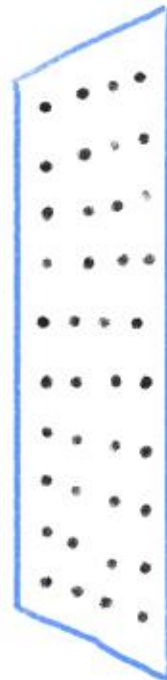
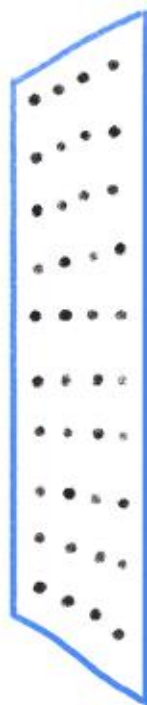
- Good representation can be more important than finding a good decision boundary

CAT



(LABELED  
PHOTOS)

DOG



OUTPUT

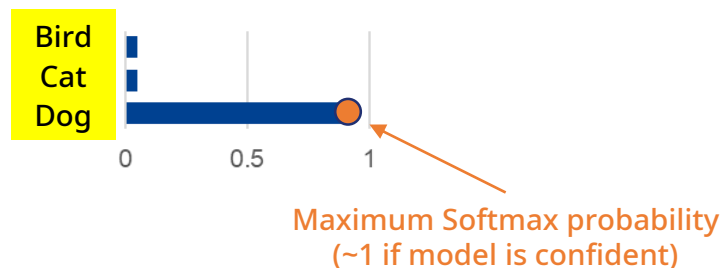
# 이상진단을 위한 분류 태스크 활용

- 확신(confidence) 기반 강아지와 고양이 데이터

Training dataset



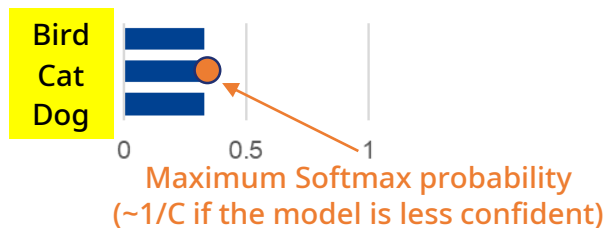
Softmax probability



Test dataset



Softmax probability



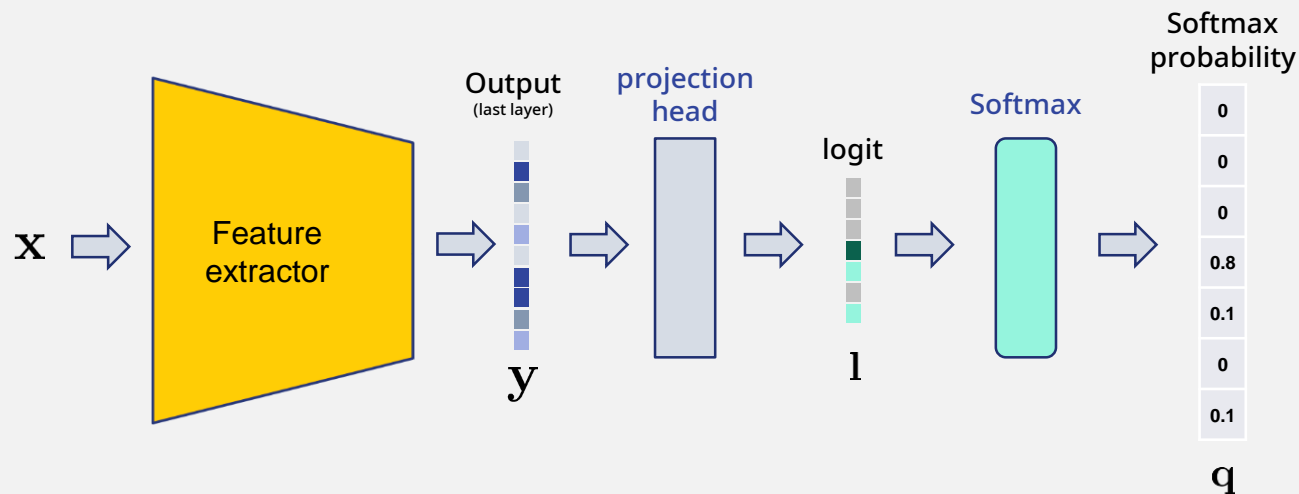
$$\text{Anomaly Score} = 1 - \text{Maximum Softmax probability}$$

# 확신 기반 이상진단

- **Model's confidence = Anomaly detection score**
  - Confidence should be similar to the model's actual performance
  - Calibration of confidence is required
- **In-depth study of model's confidence**
  - Logits
  - Softmax
  - Cross-entropy and KL divergence
- **Tips & Tricks**
  - Temperature scaling
  - Label smoothing

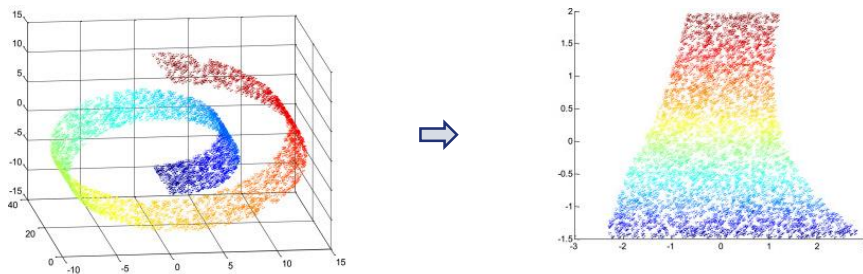


# DNN 분류기의 구조

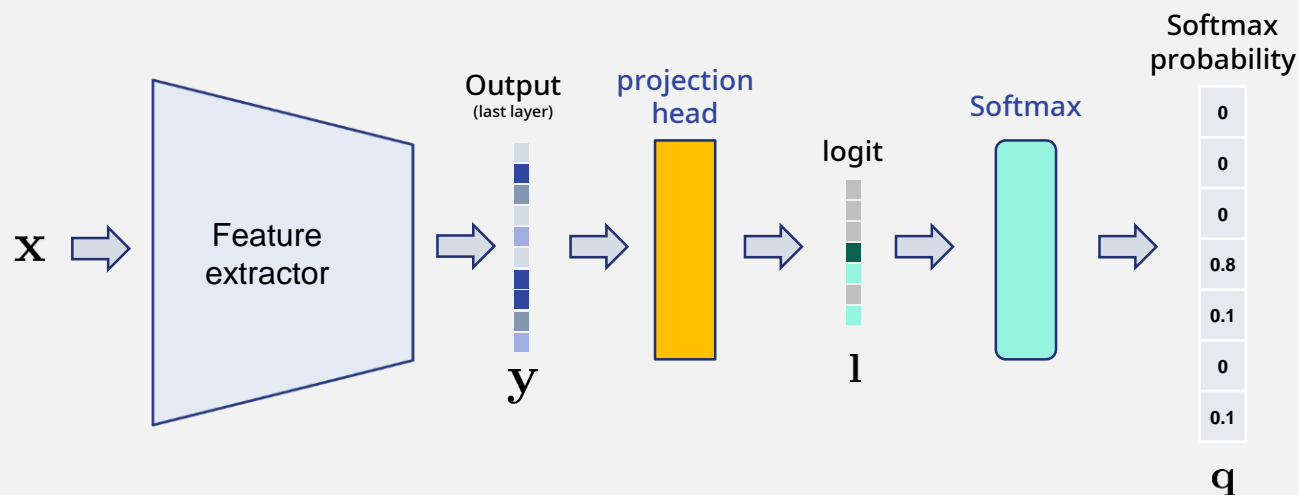


## ● 특징 추출기 (Feature extractor)

- Transform input data  $x$  to feature space data  $y$
- Trained to find a good mapping for classification



# DNN 분류기의 구조



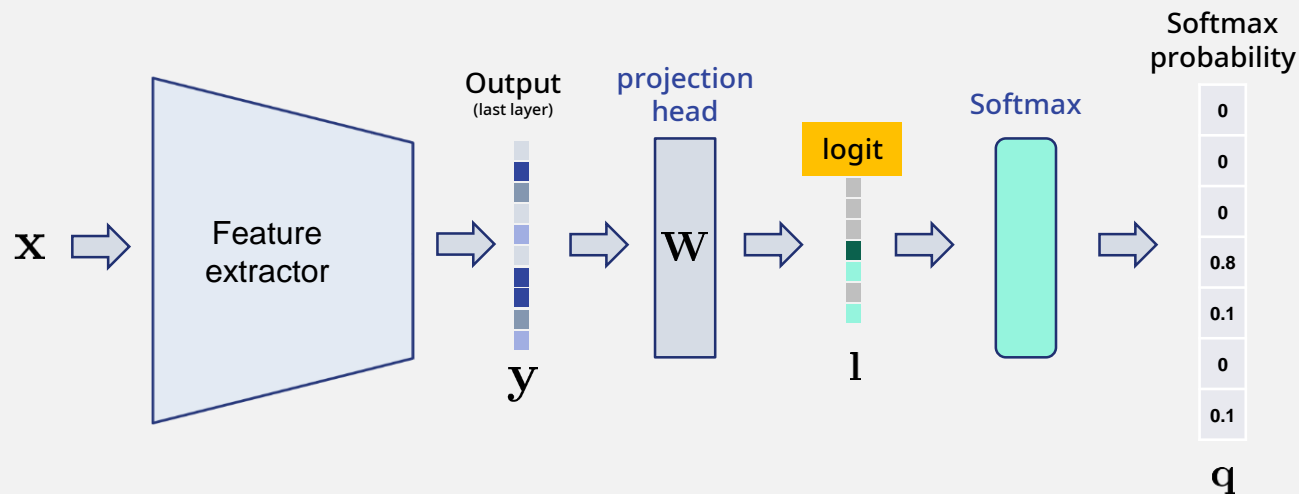
## ● 투영기 (Projection head)

- Compare output from the last layer with a template trained for each class
- Usually fully connected network
- Module for finding a decision boundary

$$\text{sigmoid} \left[ \begin{array}{c} \text{template} \\ \text{(for the class 1)} \end{array} \times \begin{array}{c} y \\ \text{Output (last layer)} \end{array} \right] = \begin{array}{c} \text{logit} \\ \text{(of class 1)} \\ l \end{array}$$

The equation shows the calculation of the logit for class 1. It involves a template matrix (labeled 'template (for the class 1)') which is a 5x5 grid with the first row highlighted in green. This matrix is multiplied (indicated by  $\times$ ) by the output vector  $y$  (a 5x1 column vector). The result of this multiplication is then passed through a sigmoid function (indicated by 'sigmoid' in a large bracket) to produce the logit  $l$  (a 1x1 scalar value, represented by a single green cell in a 5x1 column vector labeled 'logit (of class 1)').

# DNN 분류기의 구조



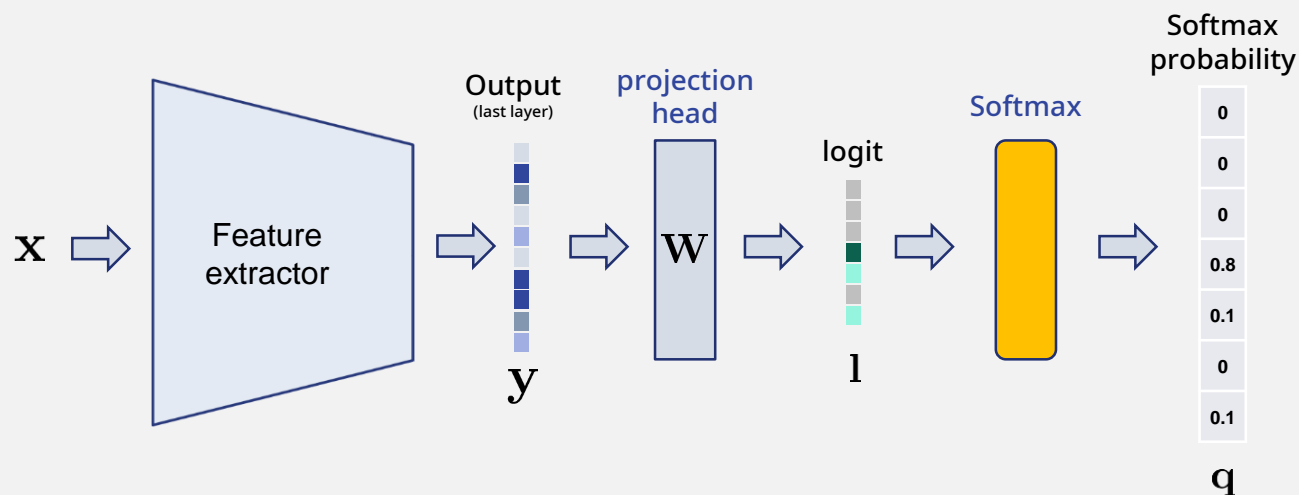
## ● logit (logistic probit)

- describes how it is likely to belong to each class
- not a probability yet

$$\text{sigmoid} \sum \left[ \begin{array}{c} y \\ \text{Feature map} \end{array} \times \begin{array}{c} w_i \\ \text{Weight vector} \end{array} \right] = l_{\text{yellow}}$$

$$\text{sigmoid} \left[ \begin{array}{c} \text{template (for the class 1)} \\ w_i \end{array} \times \begin{array}{c} y \\ \text{Feature map} \end{array} \right] = \begin{array}{c} \text{logit (of class 1)} \\ l \end{array}$$

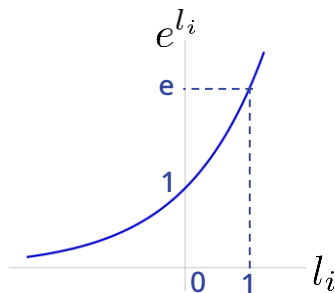
# DNN 분류기의 구조



## ● Softmax function

- Changes logit into a probability
- Sum of all elements = 1

$$\int q(x)dx = 1$$



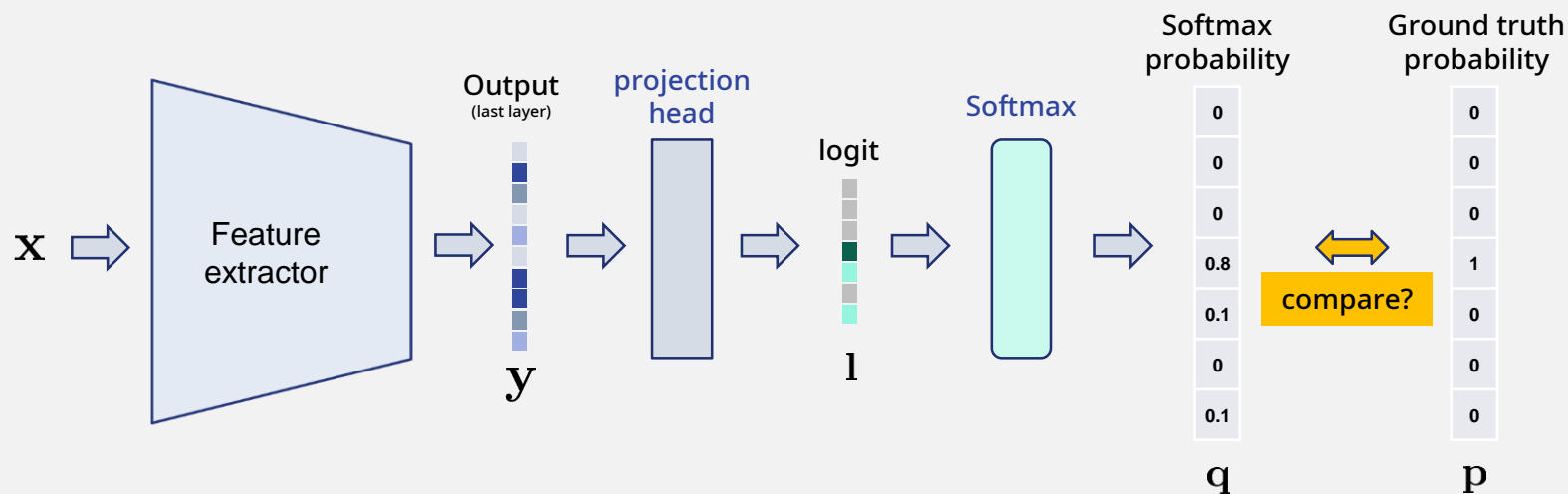
$$q = \text{softmax}(l)$$

$$q_i = \frac{e^{l_i}}{\sum_{k=1}^C e^{l_k}}$$

for two classes

$$= \frac{e^{l_1}}{e^{l_1} + e^{l_2}} = \text{sigmoid}$$

# DNN 훈련을 위한 손실 함수



- 분포간의 유사성 척도

- Kullback-Leibler (KL) divergence

$$KL(\mathbf{p}||\mathbf{q}) = \sum_{i=1}^C p_i \log \frac{p_i}{q_i}$$

- $KLD \geq 0$
- asymmetric:  $KL(\mathbf{p}||\mathbf{q}) \neq KL(\mathbf{q}||\mathbf{p})$

# Kullback-Leibler 발산

- Derivative

$$\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

one-hot vector

$$\begin{aligned} KL(\mathbf{p}||\mathbf{q}) &= \sum_{i=1}^C p_i \log \frac{p_i}{q_i} \\ &= \underbrace{\sum_{i=1}^C p_i \log p_i}_{=0} - \underbrace{\sum_{i=1}^C p_i \log q_i}_{\text{cross-entropy}} \end{aligned}$$

- For one-hot target labels, KLD = cross-entropy
- Even if  $\mathbf{p}$  is not a one-hot vector,  $\mathbf{p}$  is a static variable. So, derivative of KLD = derivative of cross-entropy
- Cross-entropy loss is used instead of KLD

# 심층 신경망 조정하기

Tuning of DNN Classifiers

# Cross-entropy loss 의 단점

- Backprop을 위한 미분시 문제점 (for one-hot target labels)

$$CE(\mathbf{q}) = - \sum_{i=1}^C p_i \log q_i$$

$$\frac{\partial CE(\mathbf{q})}{\partial q_j} = - \frac{\partial \left( p_j \log q_j + \sum_{i \neq j} p_i \log q_i \right)}{\partial q_j}$$

$$\text{if } p_j = 1, \frac{\partial CE(\mathbf{q})}{\partial q_j} = -\frac{1}{q_j} \quad \text{if } p_j = 0, \frac{\partial CE(\mathbf{q})}{\partial q_j} = 0$$

0	0	0
0	0	0
0	0	0
1	0.8	0.8
0	0.1	0.2
0	0	0
0	0.1	0
<b>p</b>	<b>q</b>	<b>q'</b>

- Ground truth label이 1인 class에 대해서만 미분치가 존재
- 다른 레이블 들에 대해서는 gradient 발생하지 않음 (no backpropagation)
- CE loss only tries to maximize Softmax probability of the **correct label**



# Label smoothing 기법

- 미분 (for one-hot target labels)

$$\frac{\partial CE(\mathbf{q})}{\partial q_j} = - \frac{\partial \left( p_j \log q_j + \sum_{i \neq j} p_i \log q_i \right)}{\partial q_j}$$

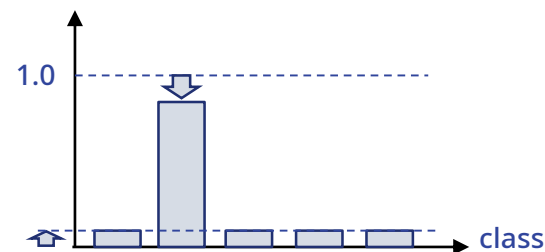
- Label smoothing

- Adds a small noise to target probability
- generates gradients for other labels

$$\text{if } p_j = \epsilon, \frac{\partial CE(\mathbf{q})}{\partial q_j} = -\epsilon \frac{1}{q_j}$$

- Label smoothing can increase the distance to other classes

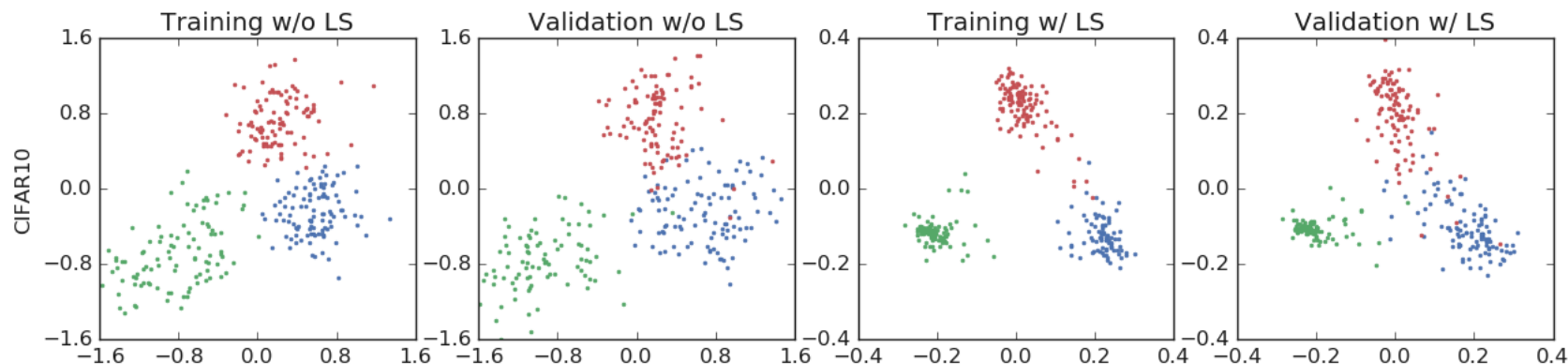
hard target		soft target
0		$\epsilon/C$
0		$\epsilon/C$
0		$\epsilon/C$
1	➔	$(1-\epsilon)+\epsilon/C$
0		$\epsilon/C$
0		$\epsilon/C$
0		$\epsilon/C$
<b>p</b>		<b>p</b>



<https://arxiv.org/pdf/1906.02629.pdf>

# Label smoothing 기법

- 예제

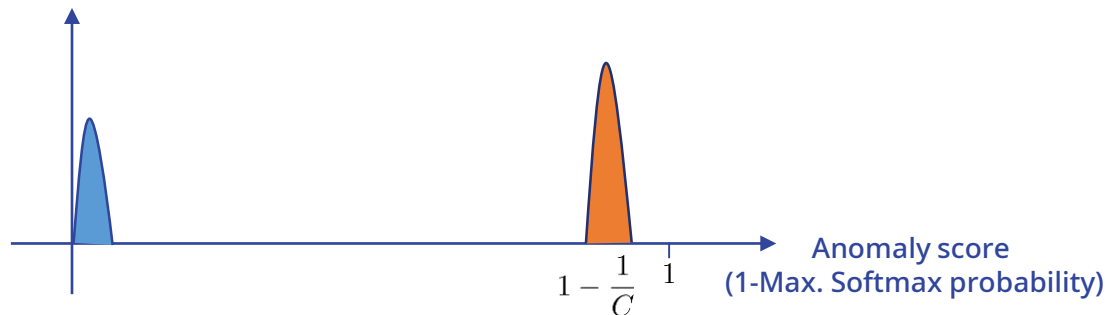
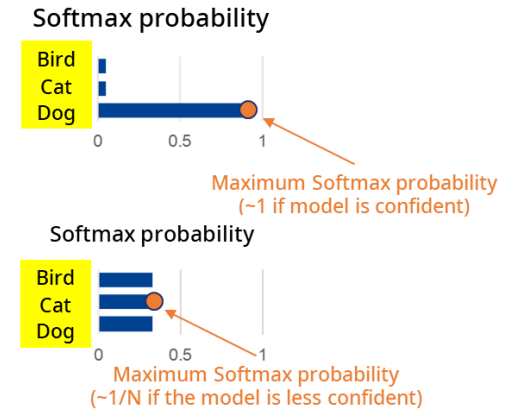


- Label smoothing 효과

- 동일한 클래스의 데이터를 보다 좁은 영역으로 바인딩
- 다른 클래스 데이터끼리 특징 차원에서의 거리를 더 분리

# Temperature scaling

- **확신 (confidence) 기반 이상진단의 문제**
  - Maximum Softmax probability = Confidence of model
- **모델의 과신 (overconfidence) 문제**
  - The confidence of a model is often overrated.
  - Calibration of Softmax probability according to true correctness likelihood



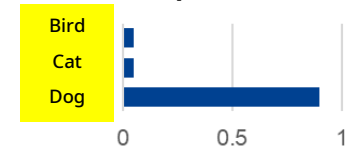
# Temperature scaling

- 아이디어

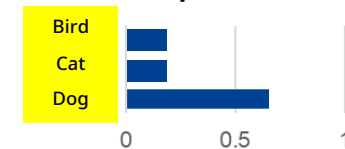
- Rescale the logit scores before applying Softmax
- Calibration without affecting the classification result over normal data

$$q_i = \frac{e^{(l_i/T)}}{\sum_{k=1}^C e^{(l_k/T)}}$$

Softmax probability



Softmax probability



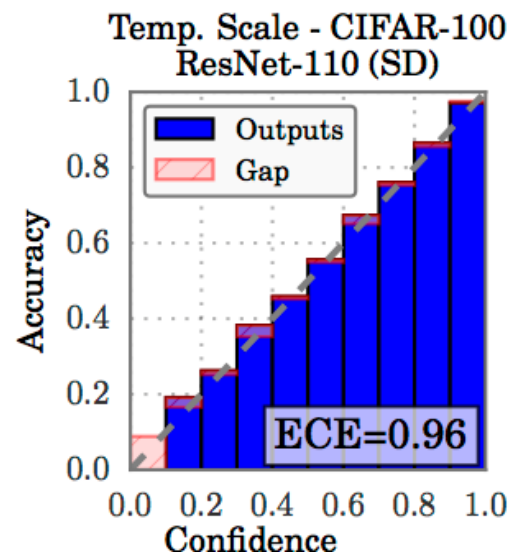
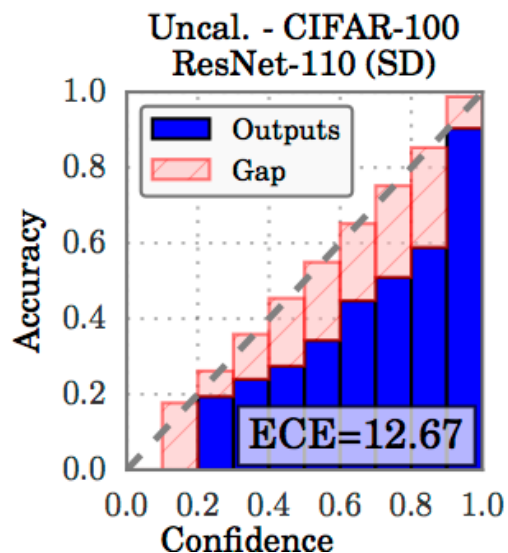
- Temperature  $T$

- Softens the Softmax (increases output entropy) with  $T > 1$
- As  $T \rightarrow \infty$ ,  $q_i$  approaches to  $1/K$

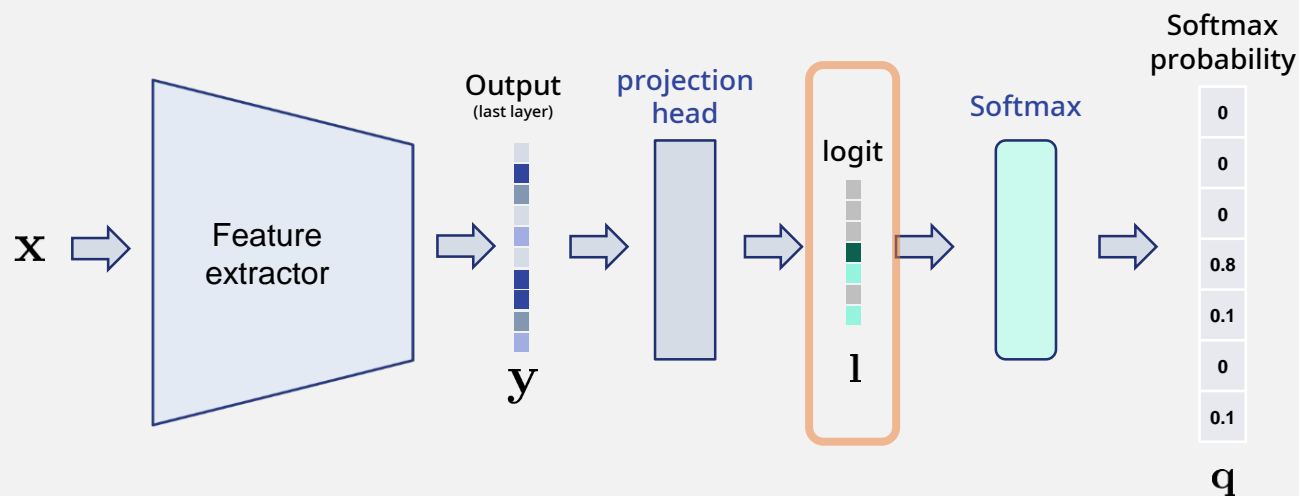
On Calibration of Modern Neural Networks, <https://arxiv.org/pdf/1706.04599.pdf>

# Temperature Scaling을 사용한 학습된 모델 보정

- Using the validation set
  - Accuracy vs. Confidence

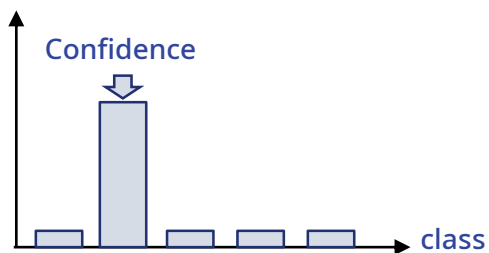


# Free-energy 기반 이상진단



## • Softmax function & Confidence

$$q_i = \frac{e^{l_i}}{\sum_{k=1}^C e^{l_k}}$$



## • Free energy

$$E_F = -\log \sum_{k=1}^C e^{l_k}$$

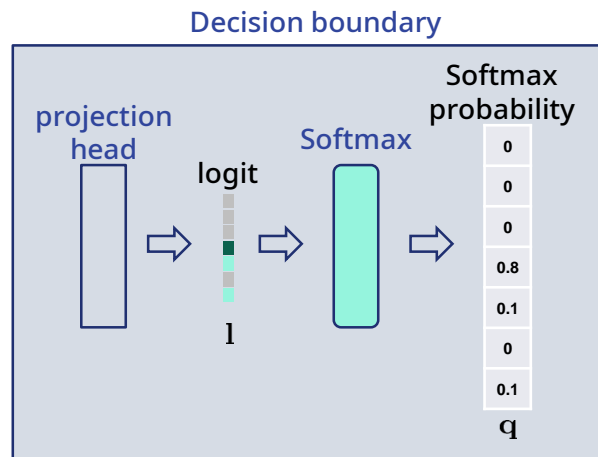
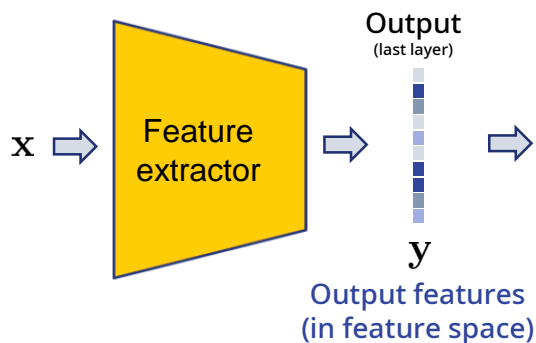
- Overall activation of logits
- Using  $-E_F$  for detecting outliers  
→ Low overall activation

# 특징 추출

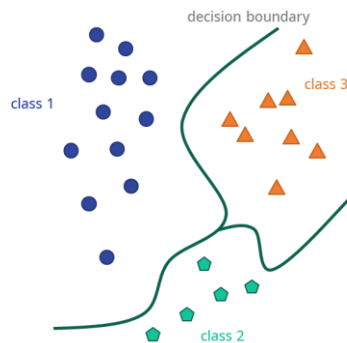
Good transformation?

# 좋은 분류기란?

- 좋은 representation이 좋은 decision boundary보다 중요
  - 어떻게 feature space 상에서 잘 분리되어 있도록 할까?



- 좋은 representation?
  - 유사한 레이블들은 가깝게
  - 다른 레이블들은 멀게

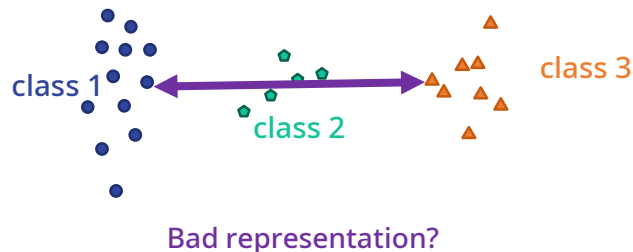
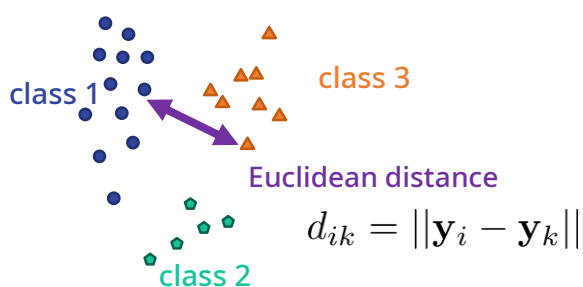




# 다른 손실 함수(loss fn.)을 사용한 훈련법

## ● 거리 기반 손실함수

- 반드시 먼 거리가 좋은 분류를 뜻하지는 않음



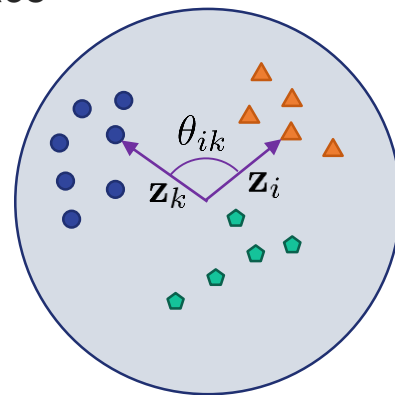
## ● 각도 기반 손실함수

- Normalize: map the output feature onto the spherical surface

$$\mathbf{z}_i = \frac{\mathbf{y}_i}{||\mathbf{y}_i||} \rightarrow ||\mathbf{z}_i|| = 1$$

- Measure the cosine angle between normalized features

$$\cos \theta_{ik} = \mathbf{z}_i^T \mathbf{z}_k$$

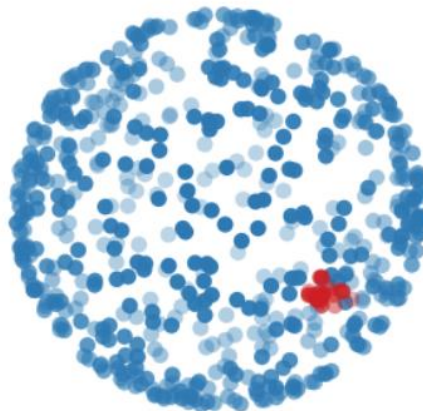
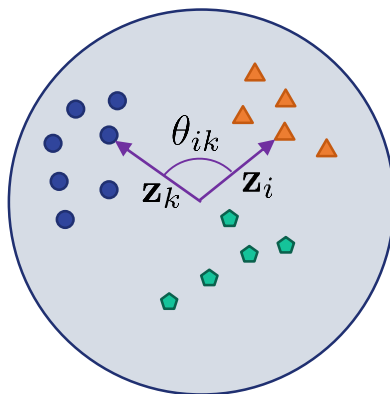


# 각도 기반 손실 함수 (Angular loss)

- Cosine similarity loss

- Cosine: 1 for similar data, -1 for dissimilar data
- For gradient descent minimization, we use the following loss function

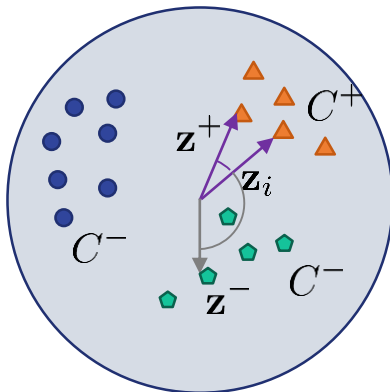
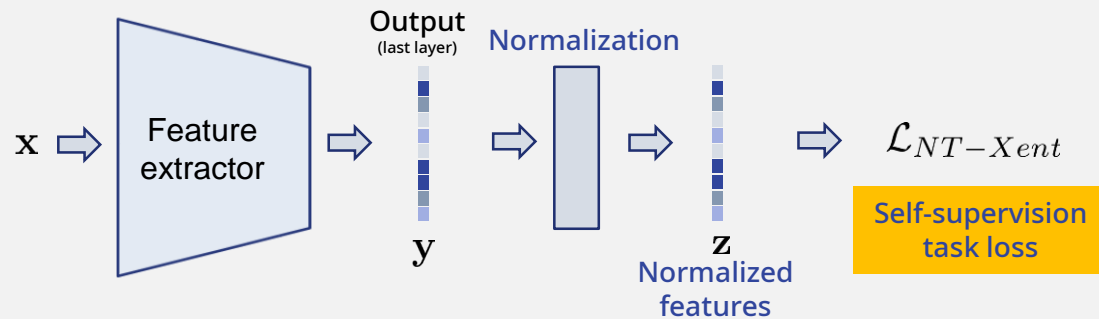
$$\mathcal{L}_{ang} = 1 - \cos \theta_{ik} \in [0, 2]$$



# NT-Xent loss

- Contrastive 훈련을 위한 동종 및 이종 레이블 손실 함수

- 같은 레이블은 가깝게 (minimize angles)
- 다른 레이블은 멀게 (maximize angles)



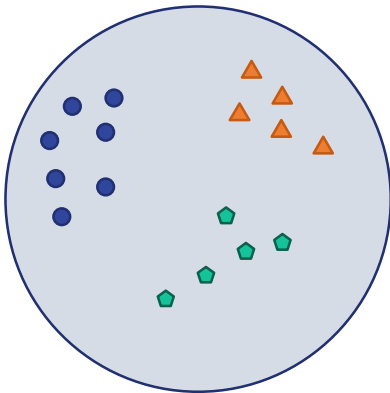
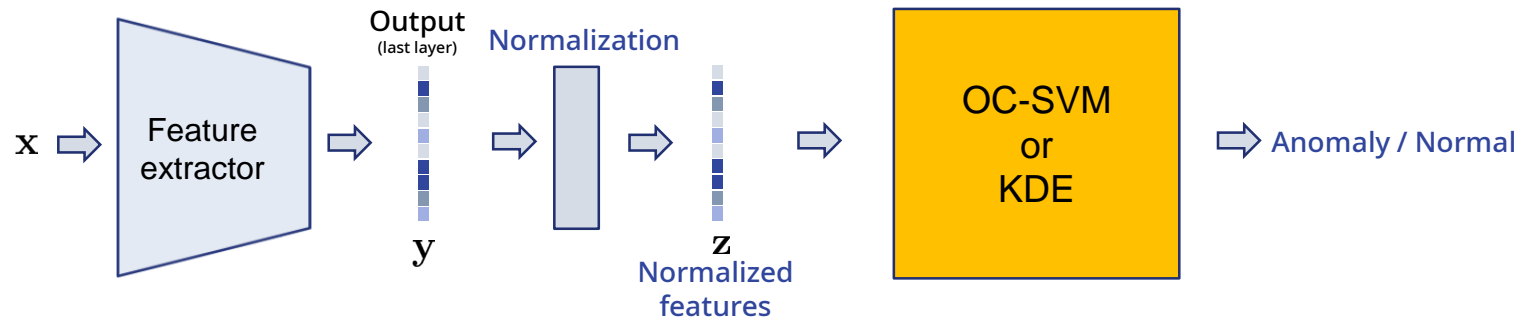
$$\mathcal{L}_{NT-Xent} = - \sum_{\mathbf{z}^+ \in C^+} \log \frac{e(\mathbf{z}_i^T \mathbf{z}^+ / T)}{\sum_{k=1}^C e(\mathbf{z}_i^T \mathbf{z}^+ / T) + \sum_{\mathbf{z}^- \in C^-} e(\mathbf{z}_i^T \mathbf{z}^- / T)}$$

c.f.) 
$$q_i = \frac{e(l_i / T)}{\sum_{k=1}^C e(l_k / T)}$$

# Downstream Task

- 자기지도학습 후 downstream task를 통한 이상치 검출

- 이상진단점수 (Anomaly Score) 설정
- 기존의 Kernel density estimation 이나 One-class Support Vector Machine (SVM) 사용

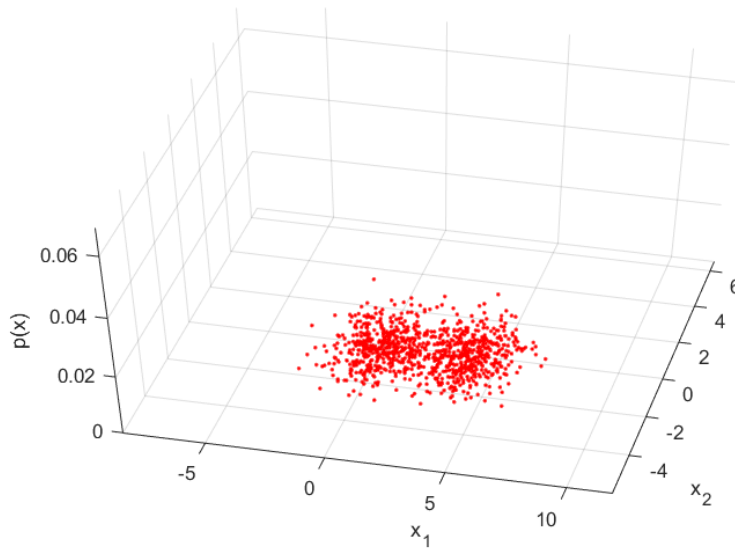


# Kernel density estimation (KDE)

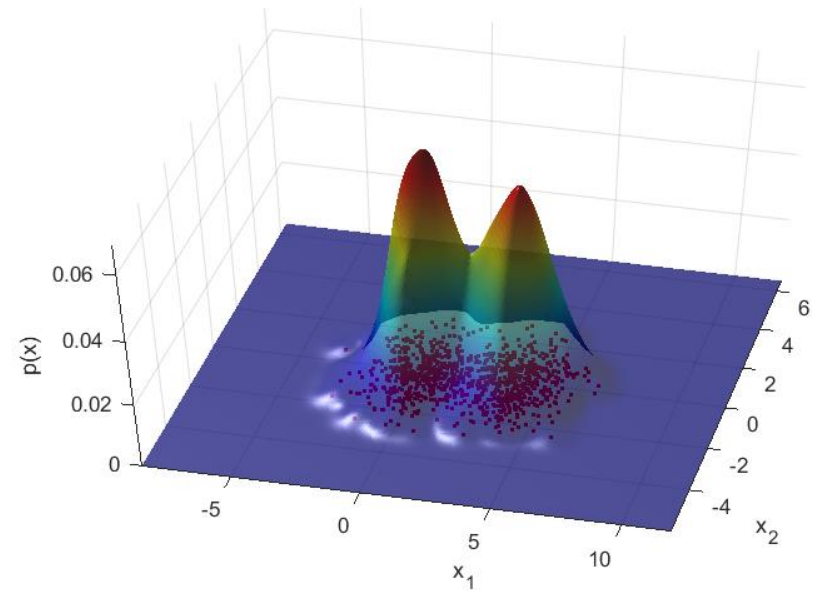
Classical classification techniques

# Feature distribution → Density function

- 정상 데이터 분포 (Distribution of normal feature data)



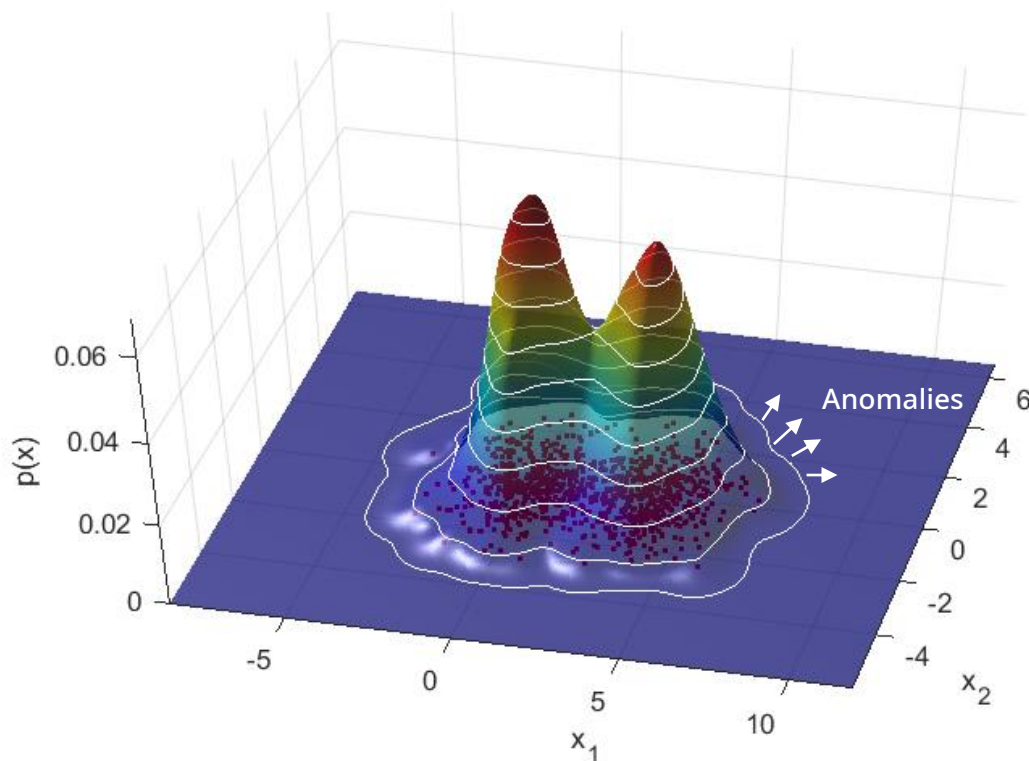
- 밀도 함수 (Density function)



# 이상 진단

- 밀도 = 이상 점수

- Data outside of a certain density level becomes anomalies
- Level of density function = decision boundary

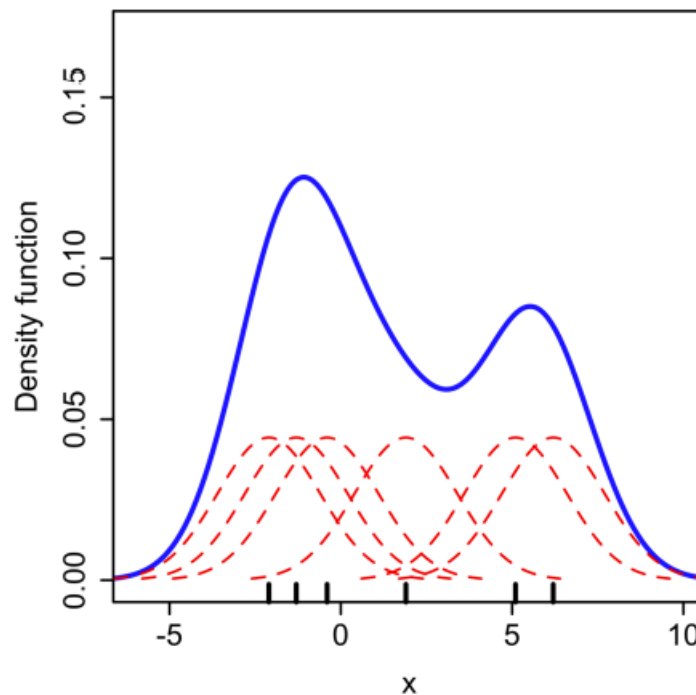


# Kernel density estimation (KDE)

- Smoothing discrete observation using a **kernel function K**

$$f(x) = \frac{1}{nh} \sum_i K\left(\frac{x - x_i}{h}\right)$$

- n: number of data
- h: bandwidth (hyperparameter)
- $x_i$ : position of  $i^{\text{th}}$  feature data



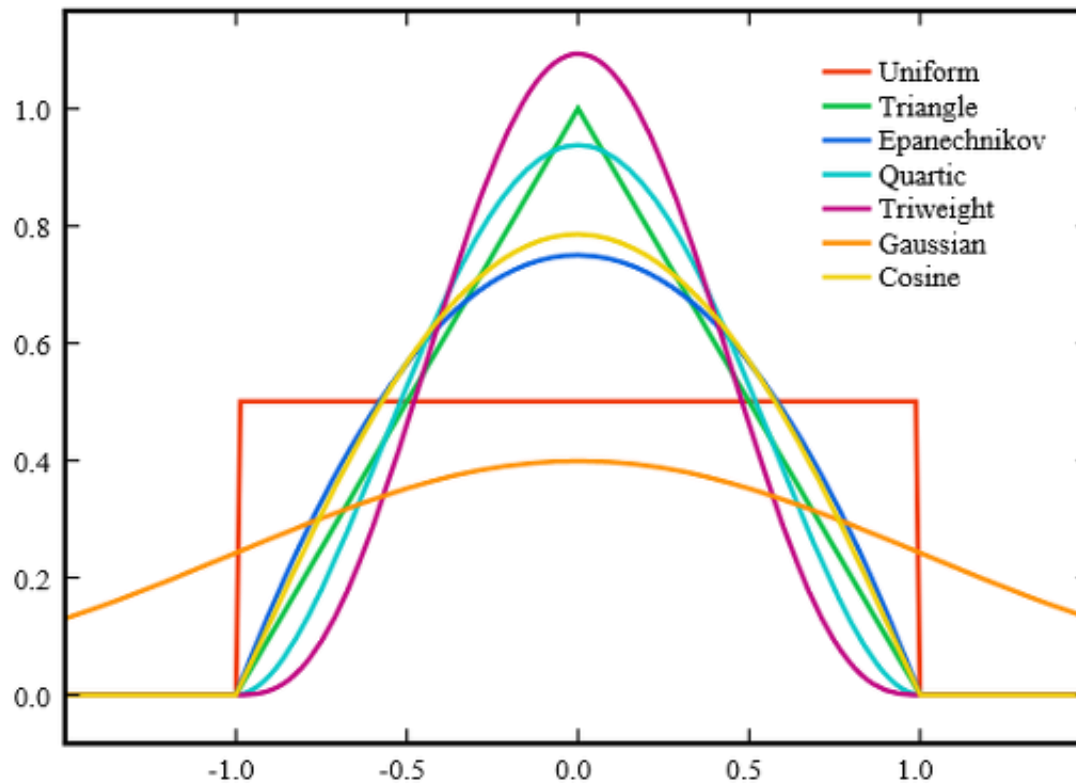
- For multivariate feature

$$f(\mathbf{x}) = \frac{1}{n} |\mathbf{H}|^{-1/2} \sum_i K\left(|\mathbf{H}|^{-1/2}(\mathbf{x} - \mathbf{x}_i)\right)$$

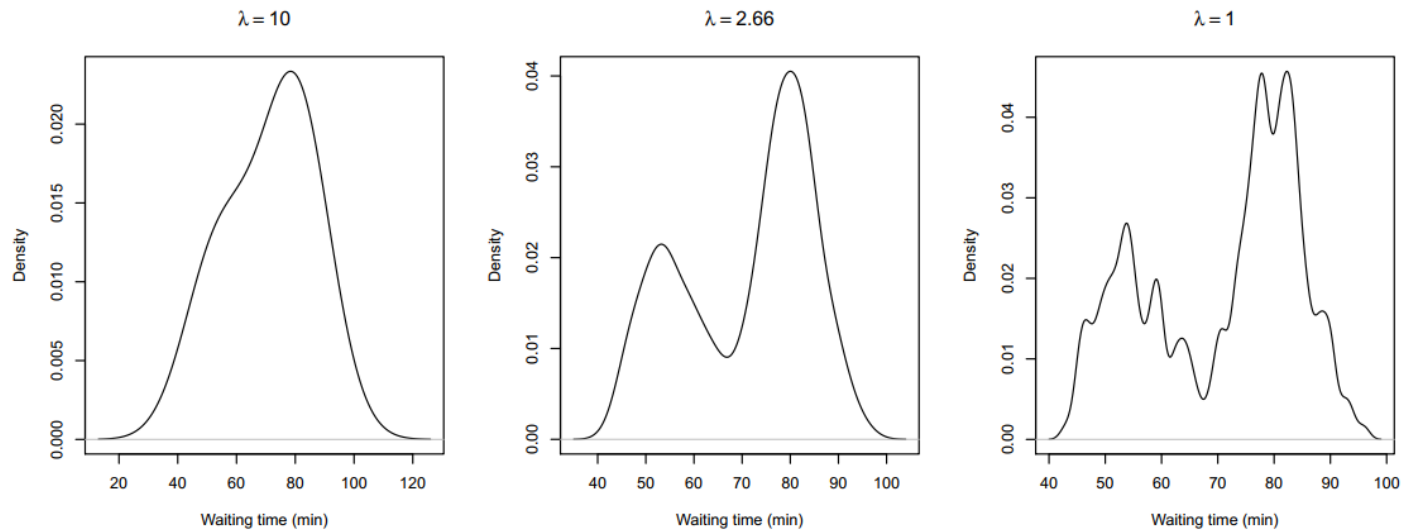


# Choice of kernel function

- Various kernel functions



# Choice of bandwidth



- Higher bandwidth  $\rightarrow$  reduces variance, increases bias
- Rule of thumb for Gaussian kernel

$$h = \left( \frac{4\sigma^5}{3n} \right)^{\frac{1}{5}} \approx 1.06\sigma n^{-1/5}$$

$$\mathbf{H} = n^{-1/(d+4)} \boldsymbol{\Sigma}^{1/2}$$

# KDE in Python

- scikit-learn library

```
class sklearn.neighbors.KernelDensity(*, bandwidth=1.0, algorithm='auto', kernel='gaussian', metric='euclidean', atol=0, rtol=0,
breadth_first=True, leaf_size=40, metric_params=None)
```

[\[source\]](#)

```
>>> from sklearn.neighbors import KernelDensity
>>> import numpy as np
>>> rng = np.random.RandomState(42)
>>> X = rng.random_sample((100, 3))
>>> kde = KernelDensity(kernel='gaussian', bandwidth=0.5).fit(X)
>>> log_density = kde.score_samples(X[:3])
>>> log_density
array([-1.52955942, -1.51462041, -1.60244657])
```

>>>

# 분류 태스크를 활용한 다양한 자기지도 학습기법

Transform-based techniques

# 클래스 레이블이 없는 경우의 자기지도 분류학습

- For classification task, internal labels are required

Training dataset

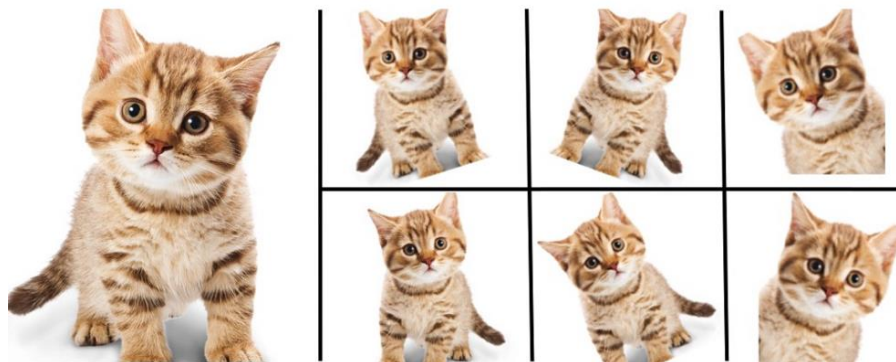


- What if we don't have such labels?
  - Prior information from humans can be utilized
  - 경험에 기반한 상식 → DNN model에 주입

# 어떤 상식을 활용할 수 있을까?

- Common senses

- Some transforms **do not change the identity** of data
- e.g., a cat image is still a cat image after the **rotation, stretching, translation, zoom, cutout, brightness/ contrast/ color** change ...

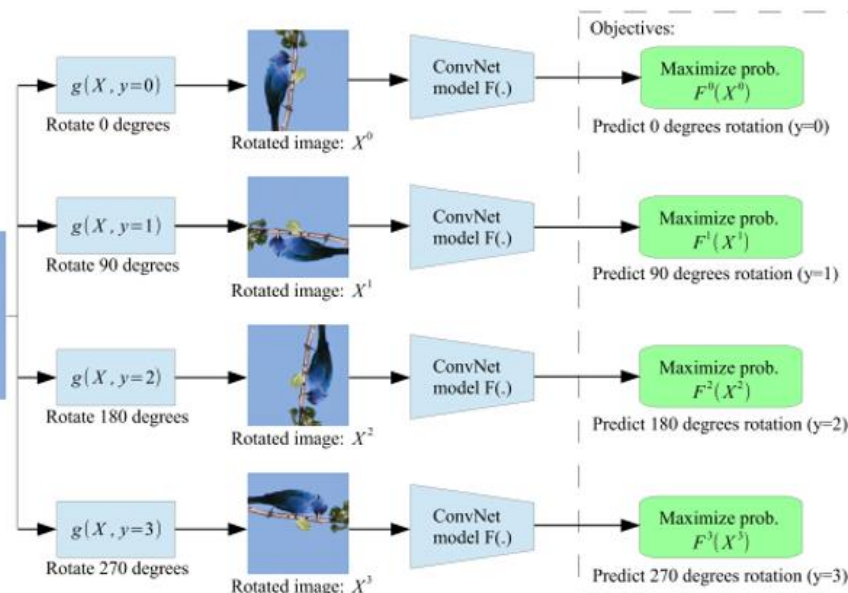


- How to utilize these common senses in DNN training?

# Let the model learn common senses

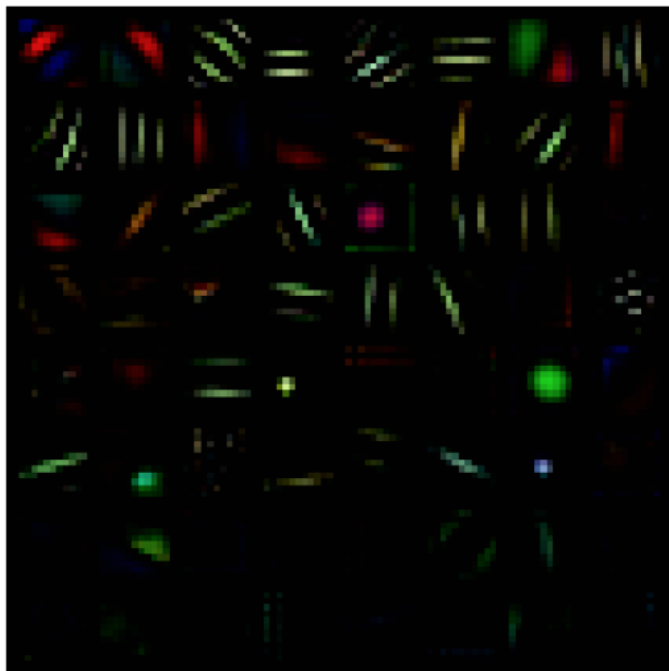
- Using transform as classification labels
- Example: RotNet (ICLR 2018)
  - Find the label of rotation angle from a given image

Training data (normal)

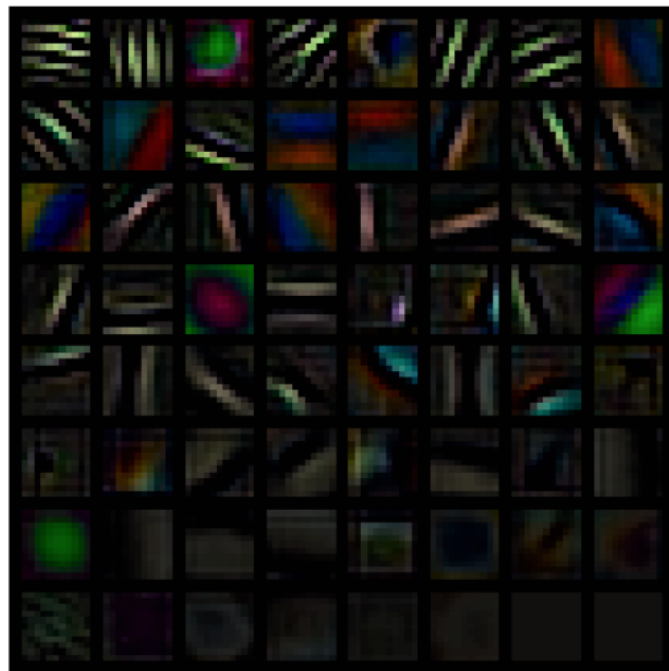


# 이미지 회전을 통한 자기지도 학습

- To find a right angle, the DNN model tries to learn various features
  - While the scale or resizing transform does not provide useful features for image classification



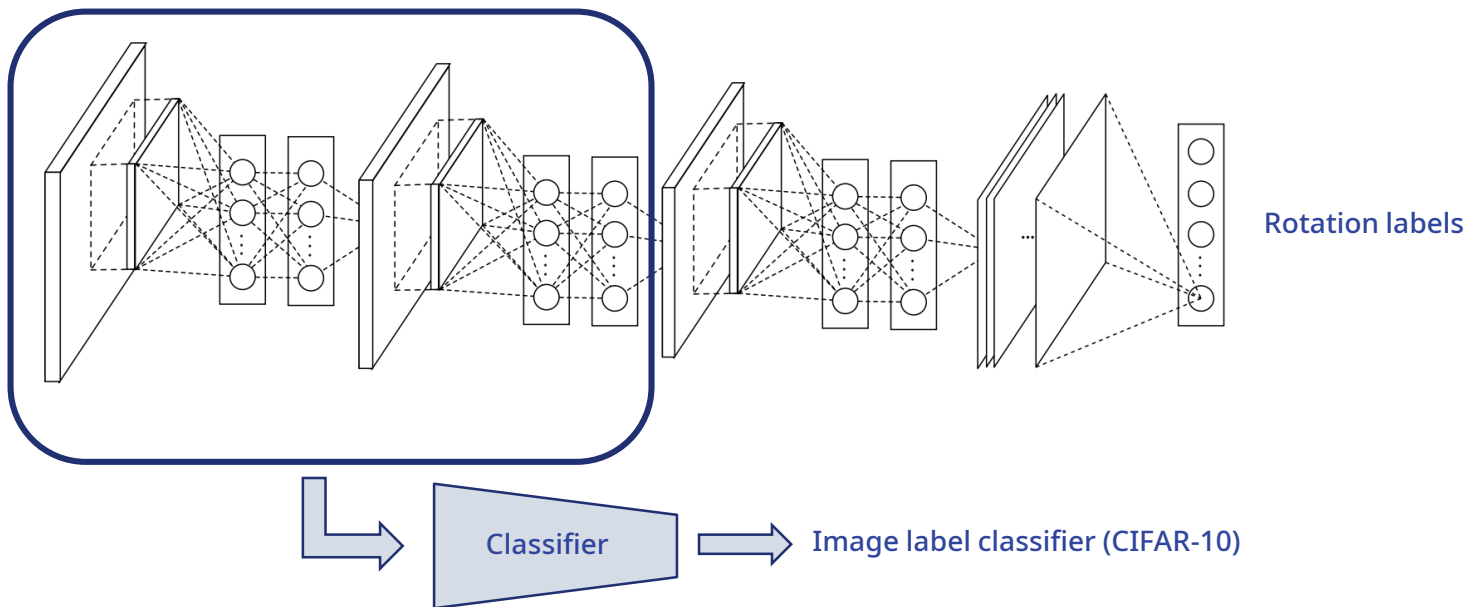
(a) Supervised



(b) Self-supervised to recognize rotations



# Transfer learning with RotNet



Model	ConvB1	ConvB2	ConvB3	ConvB4	ConvB5
RotNet with 3 conv. blocks	85.45	88.26	62.09	-	-
RotNet with 4 conv. blocks	85.07	89.06	86.21	61.73	-
RotNet with 5 conv. blocks	85.04	<b>89.76</b>	86.82	74.50	50.37

# 분류 기반 자기지도 학습의 단점

- Class labels include only small information
  - Cat, dog, tiger ... : high level context is missing
  - e.g.) cat eating a Churu (normal)  
cat eating dog food (abnormal)



Normal

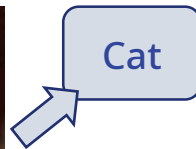


Abnormal

- 간단한 레이블로는 고차원의 학습 정보를 얻기 어려움

# Contrastive Learning

- Classification



Dog

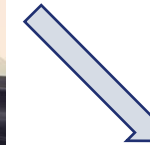
Tiger

- Contrastive learning

- 남들과 비교를 통해 더 많은 정보 습득



Different



Similar

Negative sample



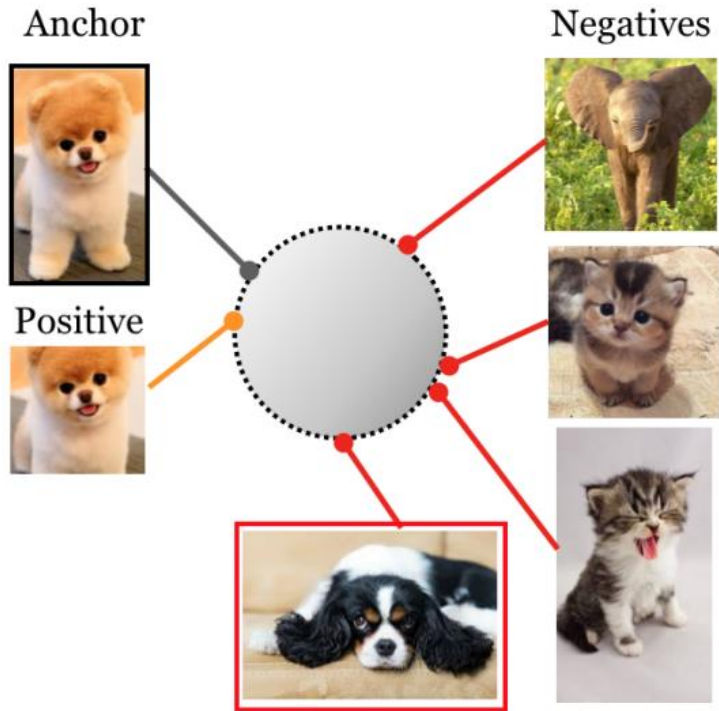
Positive sample



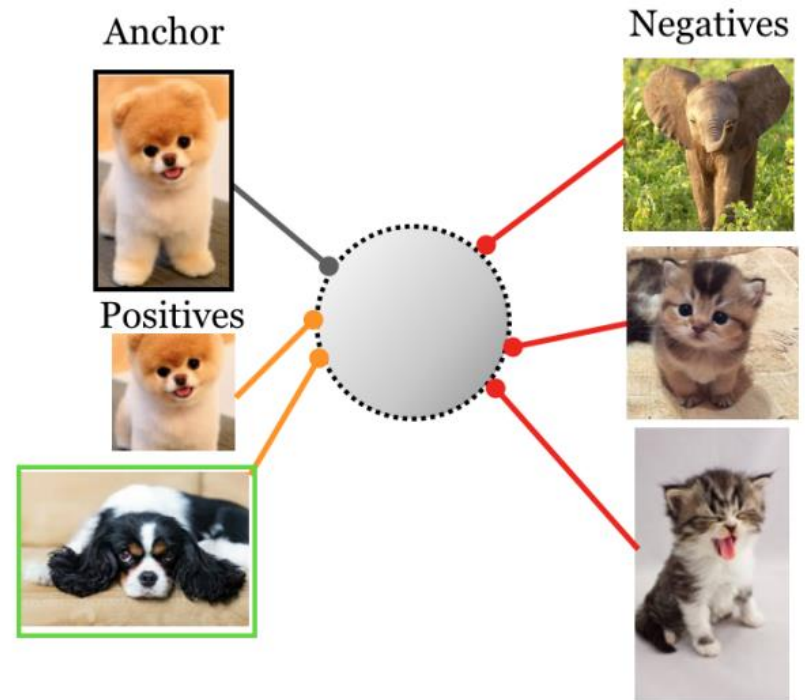
Negative sample



# Contrastive Learning



Self Supervised Contrastive



Supervised Contrastive

Supervised Contrastive Learning, NeurIPS, 2020, <https://arxiv.org/pdf/2004.11362.pdf>

# 요약

- **분류 태스크를 통한 이상 진단**

- Using maximum Softmax probability as anomaly score
- Maximum Softmax probability = Model's confidence
- Confidence needs calibration: temperature scaling

- **좋은 분류기?**

- Good representation + Good decision boundary
- Feature extractor = representation learning
- Projection head + Softmax = decision boundary

- **더 나은 특징 획득을 위한 기법들**

- Label smoothing: considering non-target labels
- NT-Xent loss: push/pull angular distances between inter- and intra-class data

- **레이블이 없는 경우의 분류 학습**

- Using known transforms to generate labels

# Appendix

## 개발 단계에서 특징 분포 확인

Feature Visualization

# t-Stochastic Neighbor Embedding

- Vector Visualization

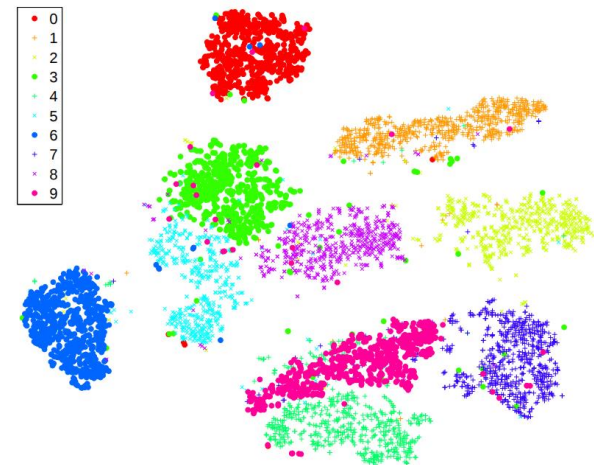
- High dimensional data  $\rightarrow$  2D vector mapping

- Objective

- Retaining both the **local** and the **global** structure of the data in a single map

- Example

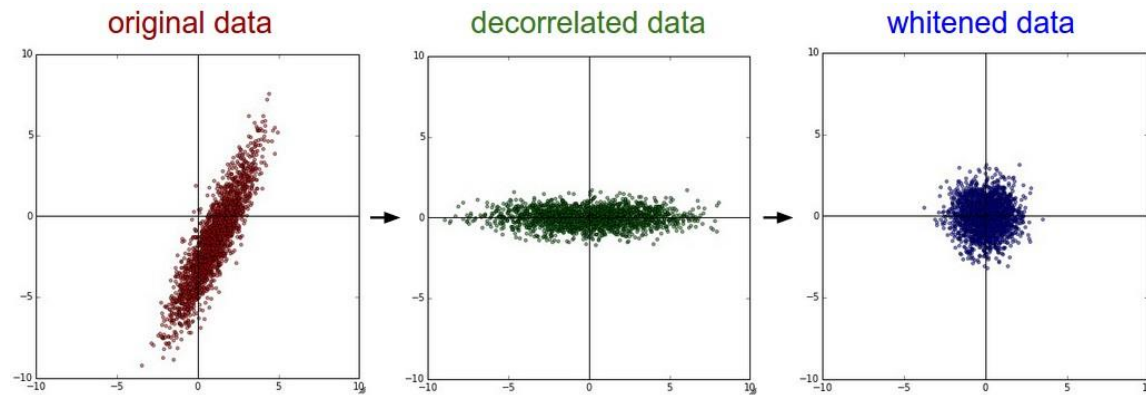
- 2D visualization of MNIST dataset
- Original data:  $28 \times 28 = 784$  dim.  $\rightarrow$  2 dim.
- 10 numerical digits (classes)



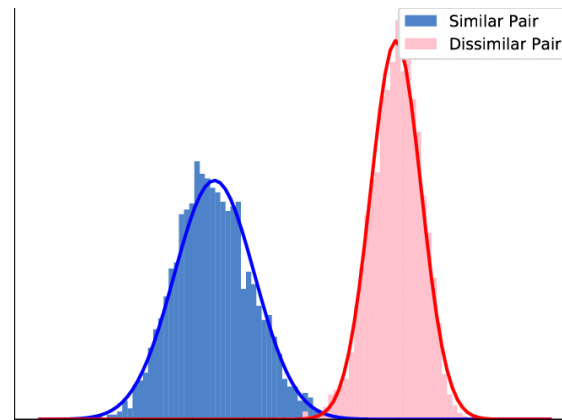
(a) Visualization by t-SNE.

# Whitening for measuring distances

- Whitening



- Normalized distances





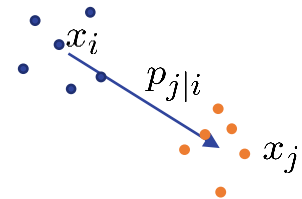
# Similarity between vectors

- Similarity in (input) **high dim. space**

- Softmax probability of Mahalanobis distance
- Gaussian assumption

$$p_{j|i} = \frac{\exp(-|x_i - x_j|^2 / 2\sigma_i^2)}{\sum_{k \neq i} \exp(-|x_i - x_k|^2 / 2\sigma_i^2)}$$
$$p_{ij} = \frac{p_{i|j} + p_{j|i}}{2n}$$

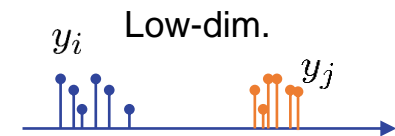
High-dim.



transform

- Similarity in (feature embedding) **low dim. space**

$$q_{ij} = \frac{\boxed{(1 + |y_i - y_j|^2)^{-1}}^{\text{t-distribution}}}{\sum_{k \neq l} (1 + |y_k - y_l|^2)^{-1}}$$



- Making two distributions similar

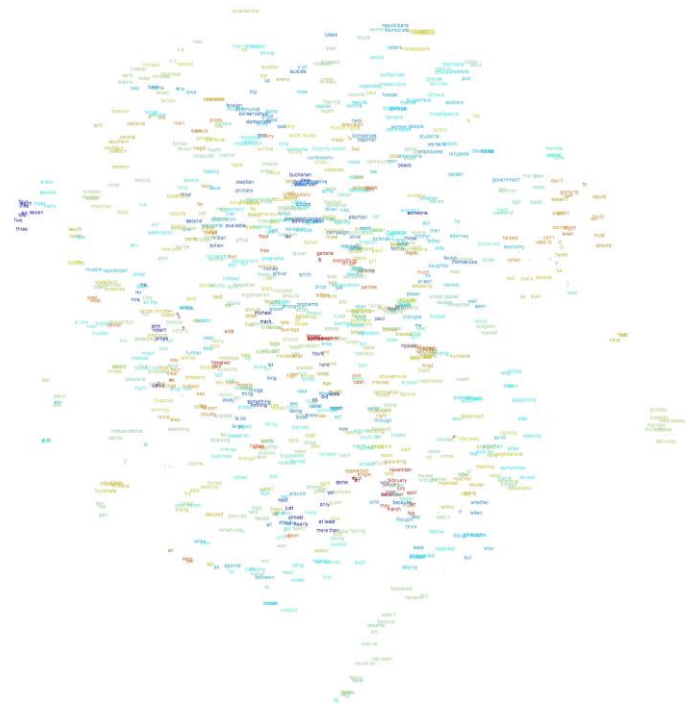
# Optimization

- KL divergence between  $p$  and  $q$

$$\begin{aligned} Cost &= \sum_i KL(P_i || Q_i) \\ &= \sum_i \sum_j p_{ij} \log \frac{p_{ij}}{q_{ij}} \end{aligned}$$

$$\text{t-SNE} \quad \frac{\delta C}{\delta y_i} = \sum_j (p_{ij} - q_{ij})(y_i - y_j) \frac{1}{1 + |y_i - y_j|^2}$$

- Update  $y_i$  to minimize the KL divergence
- Obtain 2D embedding  $y_i$



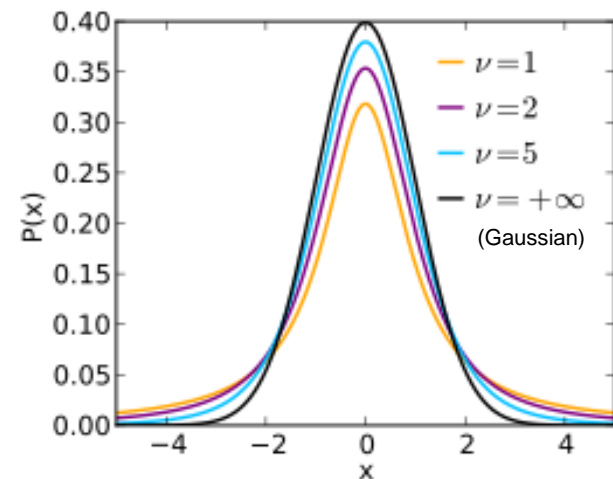
# Student t-distribution

- Definition

$$p(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

- Special case (nDOF:  $\nu = 1$ )

$$p(t) = \frac{1}{\pi (1 + x^2)}$$



# Why t-distribution?

- The Crowding problem

$$p_{j|i} = \text{softmax}[\exp(-\frac{|x_i - x_j|^2}{2\sigma_i^2})]$$

Related to perplexity

$$q_{ij} = \frac{(1 + |y_i - y_j|^2)^{-1}}{\sum_{k \neq l} (1 + |y_k - y_l|^2)^{-1}}$$

t-distribution

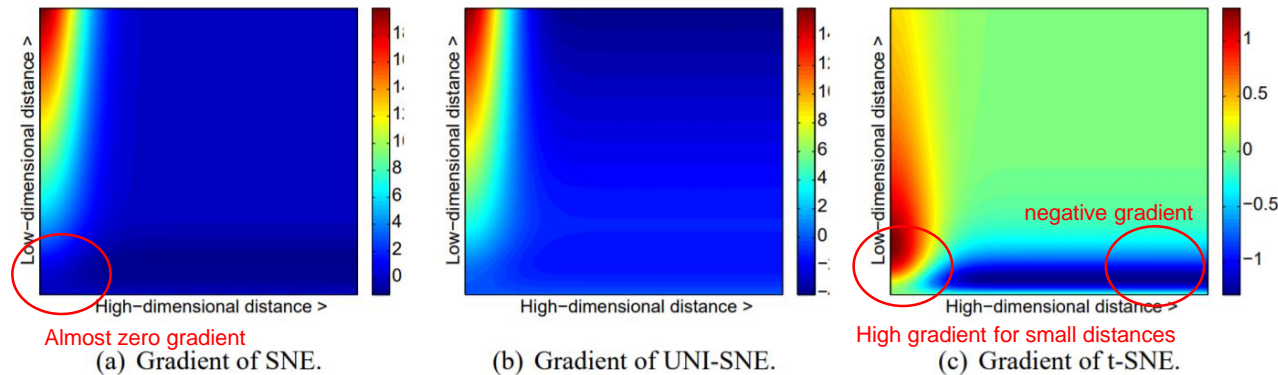
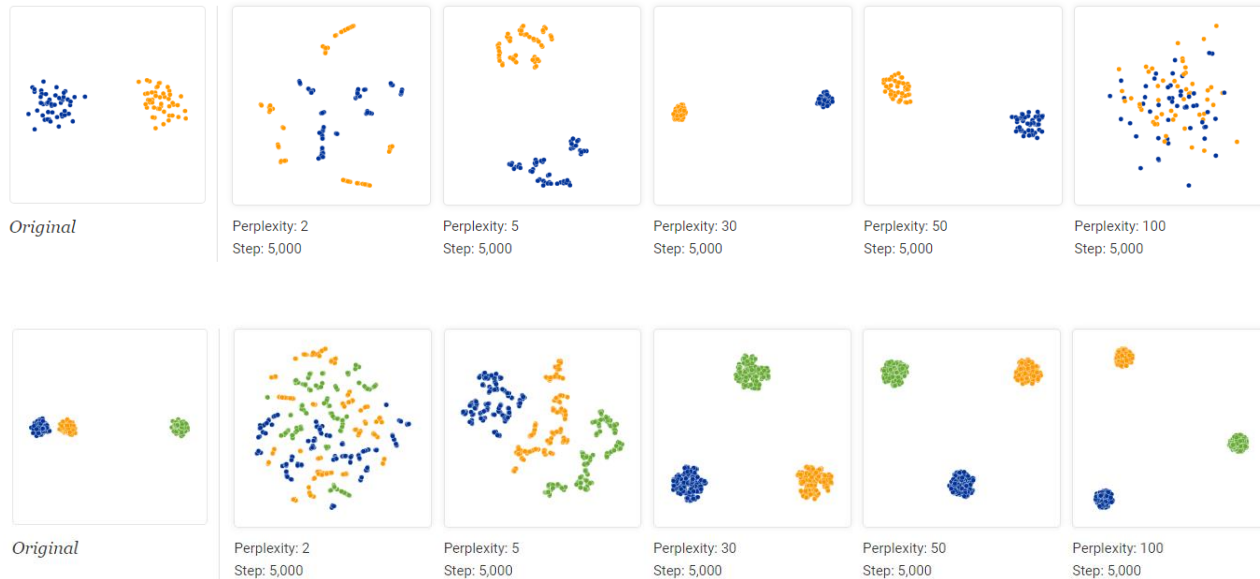


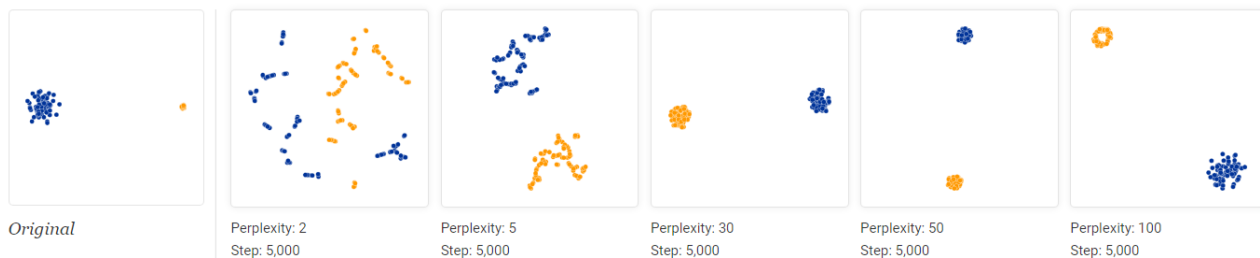
Figure 1: Gradients of three types of SNE as a function of the pairwise Euclidean distance between two points in the high-dimensional and the pairwise distance between the points in the low-dimensional data representation.

# Myth of t-SNE

- Effect of hyperparameters <https://distill.pub/2016/misread-tsne/>



distances between well-separated clusters in a t-SNE plot may mean nothing



One cannot see relative sizes of clusters in a t-SNE plot

# In Python

```
from sklearn.manifold import TSNE
```

```
TSNE(n_components=2, perplexity=30.0, early_exaggeration=12.0,  
learning_rate=200.0, n_iter=1000, n_iter_without_progress=300,  
min_grad_norm=1e-07, metric='euclidean', init='random', verbose=0,  
random_state=None, method='barnes_hut', angle=0.5)
```

```
tsne = TSNE(n_components=2)  
y = tsne.fit_transform(x)
```