Math 281a – Problem Set # 2
Module 3

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Due @ October 30th, 2021

Problem 1

Let $X_1, X_2, \cdots, X_n$ be i.i.d. from Poisson distribution with parameter $1/\theta$. Derive the MLE estimator for θ and show that it is asymptotically normal (under the right scaling and assumptions).

Problem 2

Let $X_1, X_2, \cdots, X_n$ be i.i.d. from Normal distribution with parameters (θ, θ). Derive the MLE estimator for θ and show that it is asymptotically normal (under the right scaling and assumptions).

Problem 3

Let $X_1, X_2, \cdots, X_n$ be i.i.d. from Uniform distribution on $[0, \theta]$. Derive the MLE estimator for θ. Show that it is asymptotically consistent. Show that it is not Normally distributed.

Problem 4

Show asymptotic normality (with details) of the least absolute deviation estimator, $\hat{\theta}$ that minimizes

$$\sum_i |Y_i - \theta X_i|$$

Problem 5

Let $X_1, X_2, \cdots, X_n$ be i.i.d. with density $f_{\lambda,a}(x) = \lambda \exp\{-\lambda(x-a)\}\mathbb{1}\{x \geq a\}$, where the parameter $\lambda > 0$ and $a \in \mathbb{R}$ are unknown. Derive the MLE estimator and derive its asymptotic properties.

Problem 6

Let $X_1, X_2, \cdots, X_n$ be i.i.d. from Cauchy distribution. Show that sample mean converges in distribution to a non-trivial random variables (Hint: find the right scaling). Show that MLE estimator has asymptotic distribution. What about one-step estimator?