Math 281a – Problem Set # 5
Module 6

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Due @ December 2nd, 2021

Consider Maximum Likelihood and Uniform Laws of Large Numbers (Rademacher's complexity).

1. Compute the population risk
$$R(\theta, \theta^*) = \mathbb{E}_{\theta^*}\left[\log \frac{p_{\theta^*}(X)}{p_\theta(X)}\right]$$
in the following cases:
   (a) Bernoulli:
   $$p_\theta(x) = \frac{\exp\{\theta x\}}{1+\exp\{\theta\}}$$
   for $x \in \{0, 1\}$;
   (b) Poisson:
   $$p_\theta(x) = \frac{\exp\{\theta x\}\exp\{-\exp\{\theta\}\}}{x!}$$
   for $x \in \{0, 1, 2, \cdots\}$;
   (c) Multivariate Gaussian $\mathcal{N}(\theta, \Sigma)$ where the covariance matrix Σ is known and is fixed (also invertible).
2. For each of the cases above
   (a) Let $\hat{\theta}$ denote the maximum likelihood estimate. Give an explicit expression for the excess risk
   $$E(\hat{\theta}, \theta^*) = R(\hat{\theta}, \theta^*) - \inf_{\theta\in\Theta} R(\theta, \theta^*)$$
   (b) Give an upper bound on the excess risk in terms of an appropriate Rademacher complexity.