**Math 220, Problem Set 5.**

**1.** Let $f : \mathbb{C} \setminus S \to \mathbb{C}$ be a bounded holomorphic function, defined away from a finite set $S$. Show that $f$ is constant.

**2.** Find the Laurent expansions around 0 for the function

$$f(z) = \frac{1}{z^2 + 3z + 2}$$

valid in three different regions of the complex plane.

**3.** Show that there is no meromorphic function $f$ on the unit disc $\Delta(0,1)$ such that $f'$ has a pole of order exactly one at $z = 0$.

**4.** Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function. Show that $f(\mathbb{C})$ is dense in $\mathbb{C}$.

**5.** Let $f : U \setminus \{a\} \to \mathbb{C}$ be a holomorphic function with an isolated singularity at $a \in U$. Let $P$ be a non-constant polynomial.

Show that $f$ has a removable singularity, or a pole, or an essential singularity at $a$ respectively then $P \circ f$ has a removable singularity, or a pole, or an essential singularity at $a$.

**6.** Solve Conway, Problem 13(a)(b), Chapter V.1.

**7.** Assume that $f : \mathbb{C} \to \mathbb{C}$ is entire and injective. Show that $f(z) = az + b$. You can solve this problem using the notions introduced in Problem 6 above.