

Quant Trading Interview Questions and Answers

-
1. For a 3 sets tennis game, would you bet on it finishing in 2 sets or 3 sets?
 2. I have a square, and place three dots along the 4 edges at random. What is the probability that the dots lie on distinct edges?
 3. You have 10 people in a room. How many total handshakes if they all shake hands?
 4. Two decks of cards. One deck has 52 cards, the other has 104. You pick two cards separately from a same pack. If both of two cards are red, you win. Which pack will you choose?
 5. What is 39×41 ?
 6. A group of people wants to determine their average salary on the condition that no individual would be able to find out anyone else's salary. Can they accomplish this, and, if so, how?

7. How many digits are in 99 to the 99th power?
8. A line of 100 passengers is waiting to board a plane. They each hold a ticket to one of the 100 seats on that flight. (For convenience, let's say that the n th passenger in line has a ticket for the seat number n .) Unfortunately, the first person in line is crazy, and will ignore the seat number on their ticket, picking a random seat to occupy. All of the other passengers are quite normal, and will go to their proper seat unless it is already occupied. If it is occupied, they will then find a free seat to sit in, at random. What is the probability that the last (100th) person to board the plane will sit in their proper seat (#100)?
9. What is the sum of the numbers one to 100?
10. You have a 3 gallon jug and 5 gallon jug, how do you measure out exactly 4 gallons? Is this possible?
11. You have 17 coins and I have 16 coins, we flip all coins at the same time. If you have more heads then you win, if we have the same number of heads or if you have less then I win. What's your probability of winning?
12. What is the probability you draw two cards of the same color from a standard 52-card deck? You are drawing without replacement.
13. You're in a room with three light switches, each of which controls one of three light bulbs in the next room. You need to determine which switch controls which bulb. All lights are off to begin, and you can't see into one room from the other. You can inspect the other room only once. How can you find out which switches are connected to which bulbs? Is this possible?
14. In world series, what are the odds it goes 7 games if each team equal chance of winning?
15. Given 100 coin flips, what is the probability that you get an even number of heads?
16. There are 5 balls, 3 red, and 2 black. What is the probability that a random ordering of the 5 balls does not have the 2 black balls next to each other?
17. What is the least multiple of 15 whose digits consist only of 1's and 0's?
18. Is 1027 a prime number?
19. Does the price of a call option increase when volatility increases?
20. 2 blue and 2 red balls, in a box, no replacing. Guess the color of the ball, you receive a dollar if you are correct. What is the dollar amount you would pay to play this game?
21. What is the singles digit for 2^{230} ?

22. If A, B, and C are integers between 1 and 10 (inclusive), how many different combinations of A, B, and C exist such that $A < B < C$?
23. A has 6 points and B has 4 points. They flip a coin and if it's a head, then A gets a point from B. If it's a tail, then B gets a point from A. What's the probability that A wins with 10 points?
24. Minimum number of people to guarantee at least 5 people share the same birthday month?
25. What is the sum of all the odd numbers in between 1 and 100?
26. How do you select 5 random numbers that add up to 1? Is this possible?
27. Apples cost 27 cents. How many apples can you purchase with \$10? *1 minute*
28. What is the sum of the digits of all the numbers from 1 to 1000000?
29. How many 7-digit numbers are in the form of $abcdcba$ (symmetric about the digit in the middle)?
30. Throw a fair die with 6 faces, what is the expected number of times until 2 consecutive 6 come out?
31. The surface of a $3 \times 3 \times 3$ block is painted. The block is split into 27 cubes and one is dropped on the floor. If no visible face is painted, what is the probability that it is the centre cube?
32. If you have four coins and I have four. We both throw the four and if your four sides equal to mine, I will give you 2 dollar and otherwise you give me 1. Will you do it?
33. You have 2 fair dice. What is the probability of both showing six if I have observed at least one six.
34. 27% of 115? *Time pressure -- should be done in < 15 seconds*
35. You flip 4 fair coins and get x dollars where x is the number of heads squared. What are your expected winnings?
36. Player A had a better batting average than Player B in both the first and second halves of the season. Is it possible for Player B to have a better average than A in the overall season? How?
37. A steel ball with 8cm diameter weighs 8 ounces. With the exact same density, how much does a 12cm diameter steel ball weigh?
38. If you roll a 10 sided die and you roll a 20 sided die, what is the probability that you get a bigger roll on the 10 sided die?

39. You have 3 pancakes. One has both sides burned, one has only one side burned, and the last one has no sides burned. Now, choose a random pancake. The top side is burned. what is the probability of the other side is burned?
40. What is 45^{69} ? *As quickly as possible -- maximum of 15 seconds*
41. I have a square, and place three dots along the 4 edges at random. (1) What is the probability that the dots, when connected, do not form a triangle?
42. In a country that only wants girls, every family continues to have children until they have a girl. If they have a boy, they have another child. If they have a girl, they stop. What is the proportion of boys to girls in the country?
43. How many times a day does a clock's hands overlap?
44. You have a 100 coins laying flat on a table, each with a head side and a tail side. 10 of them are heads up, 90 are tails up. You can't feel, see or in any other way find out which 10 are heads up. Your goal: split the coins into two piles so there are the same number of heads-up coins in each pile. Is This possible? How do you do it?
45. There are three boxes, one contains only apples, one contains only oranges, and one contains both apples and oranges. The boxes have been incorrectly labeled such that no label identifies the actual contents of its box. Opening just one box, and without looking in the box, you take out one piece of fruit. How many tries does it take to label all boxes?
46. You start out with 1 dollar and your friend starts out with 2 dollars. You bet 1 dollar until one of you runs out of money. You have a $\frac{2}{3}$ chance of winning each bet. What is your chance of winning?
47. There are 2 boys and unknown number of girls in a nursery . A new baby is just born inside the room. We pick randomly a baby from the room, it turns out that the baby is a boy. What is the probability that the new baby just born is a boy?
48. Chance of seeing a shooting star in 1 hour is 60%. What is the chance of seeing star in half an hour?
49. 11% of 111,111
50. Which is bigger, $2^{(1/3)}$ or $10^{(1/10)}$?
51. You have 5 friends of different ages going to dinner. What is the probability that they will sit in age-order around the table? Can be clockwise or counter-clockwise.
52. If I toss 3 pieces on a tic-tac-toe board at random, and I pay you \$9 if the pieces create tic-tac-toe, and you pay me \$1 if they do not, what's the expected value?

53. You have the option to throw a die two times. You will earn the face value of the die. You have the option to stop after the first throw and walk away with the money earned. The earnings are not additive. What is the expected payoff of this game?
54. If a submarine can fire has two missiles to fire with each one hitting with a probability of $\frac{1}{3}$, what is the probability that at least one missile will hit?
55. If you flip a coin until you decide to stop and you want to maximize the ratio of heads to total flips, what is the approximate expected ratio?
56. Square root of .16? *Time pressure; <15 seconds to answer*
57. If you flip 4 coins what is the probability that you get exactly 3 heads?
58. You can distribute 50 red and 50 blue marbles between two jars so as to maximize the chance of picking a red marble. You choose one of the two jars randomly, and then you select a marble from the jar randomly. What is your approximate probability of selecting a red marble with your optimal strategy?
59. A 100 ml drink is 20% alcohol. How many liters of pure alcohol would need to be added to the drink in order for it to be 25% alcohol?
60. If you have 8 teams playing in a simple bracket would you rather win 1 million dollars for getting all of the game outcomes correct or receive 10,000 for getting at least one game correct.
61. Solve $x^x x^x \dots = 2$
62. A piece of land has a 30% chance of being located over an oil reserve, in which case it is going to be worth 100M. If there is no oil, the land is worth 30M. You are offered an option to buy the land at 40M after inspecting it and ascertaining if there is oil. How much are you willing to pay for this option?
63. We are going to bet on a football game for \$10. Halfway through the game I can choose to raise the bet to \$20. After I raise the bet to \$20 you have a crystal ball that tells your chances of winning the game, what is the minimum probability of winning in order for you to take the bet. (note: if you decline the raise you lose your original \$10. If you accept, \$20 is on the line.)
64. Say we have a pond with lily pads. The lily pads double every minute. After 60 minutes, the pond is completely covered. How long does it take for the pond to be $\frac{1}{4}$ covered?
65. You draw six cards randomly from a deck of 52 cards. What is the approximate probability there is no pairs?

66. How many ways can 11 # signs and 8 * signs be arranged in a row so that no two * signs come together?
67. A number from 0-1 is put on a piece of paper. You and Person X are playing a game. You have to get less than the number on the paper, but above X's guess. You know X chooses to pick a number at random. What is the lowest number you can choose to max. your probability of winning?
68. If two cars are traveling in a two lap race on a track of any length, one going 60 mph and the other going 30mph, how fast will the slower car have to go to finish at the same car to finish at the same time?
69. Given a six-sided die, what's the expected value of the difference between two die rolls?
70. You have 12 black socks and 12 white socks mixed up in a drawer. You're up very early and it's too dark to tell them apart. What's the smallest number of socks you need to take out (blindly) to be sure of having a matching pair?
71. There is a car auction. The price of the car is uniform $[0, 1000]$, you do not know the actual value of the car. If you bid higher than the value of the car you get it, if you bid lower than the value of the car you don't. If you know you can sell it on afterwards for x times its worth, what should you bid when: $x=1.5$ (e.g. For $x=1.5$, you bid 100, the car is worth 80, you get it and sell it on for 120, which is a 20 profit).
72. You can buy chicken nuggets in packs of 7 or 11. What is the maximum number of chicken nuggets you can *not* buy using only packs of 7 or 11?
73. One by one, we throw three dice. What's the probability that our 3 rolls are strictly in increasing order?
74. I throw six dice. What's the approximate probability three outcomes show up exactly two times each?
75. Five boys and five girls sit on five two seated benches. One boy and one girl must be on each bench. How many different orderings are possible?
76. We have N bins, and we decide to toss 100 balls into these N bins. What is the expected number of bins with at least one ball?
77. We have three horses (A, B, C). Every dollar I wager, I get \$2, \$4, or \$6 if A, B, or C wins the race, respectively. Design me a strategy that never loses money.
78. A drunk person randomly chooses two letters from a "HAPPY HOUR" sign. His drunk friend then puts the two letters back in random order. What is the probability "HAPPY HOUR" still appears on the sign?

79. A card from a 52 card deck is lost. We then draw 2 cards from the 51 remaining cards. What is the probability they are both diamonds?
80. You are offered a contract to buy a piece of land for 300K. This land is worth 1000K 70% of time, 500K 20% of the time, and 150K 10% of the time. The contract says you can pay somebody x dollars to determine the land's value and then decide to purchase the land or not. What is the highest value of x you should be willing to pay if you're completely rational?
81. How many outcomes are possible when five dice are rolled in which at least one dice shows a three?
82. How many digits in 7^7 ?
83. I keep rolling a die until a 6 appears. What is the probability the sum of all rolls is even?
84. A germ population begins with one germ. Then, after each period, the germ can divide into 1, 2, 3, or 0 germs with equal probability, where 0 signifies death of the germ. What is the approximate probability the population of germs will eventually die out?
85. What is the last digit of 3^{33} ?
86. You have \$20 in both five and one dollar bills. What's the expected number of bills you have?
87. You have 100 dollars, and there is a dollar bill behind each door. You roll a 100 sided die 100 times, and you take the dollar behind the door on the die roll if the bill has not been taken already (e.g. you roll 16, then you take the dollar behind door 16 if you haven't already taken it). What is your expected payoff?
88. It is 10:02am. What is the angle between the minute and hour hand?
89. You have two ropes. Each takes exactly 60 minutes to burn. They are made of different material so even though they take the same amount of time to burn, they burn at separate rates. In addition, each rope burns inconsistently. How do you measure out exactly 45 minutes? Is this possible?
90. What is the probability that a random number from 1 and 100 does not contain a 7?
91. You have a deck of 97 cards and I will pay you \$10 if I draw 4 cards and they are in ascending order (not necessarily consecutive order) and you pay me \$1 if they are not. Would you play?
92. You have five pirates, ranked from 5 to 1 in descending order. The top pirate has the right to propose how 100 gold coins should be divided among them. But the others get to

- vote on his plan, and if fewer than half agree with him, he gets killed. How will the coins end up being divided, assuming all the pirates are rational and want to end up alive?
93. What is the best approximation for the derivative of x^x at $x=2$?
94. When I walk up an escalator in 7 steps, it takes me 30 seconds to reach the top. If I run up the same escalator in 13 steps, it takes me 21 seconds. How much time will it take for me to reach the top if I do not move up any steps on the escalator?
95. Expected amount of rolls of the dice to get three 6's in a row?
96. How many squares/rectangles on a chessboard?
97. There are two fresh cookies and two moldy cookies. Two kids grab two cookies each at random. What is the probability that each kid gets one fresh cookie and one moldy cookie?
98. You have a deck of 52 cards. What's the probability you draw exactly 1 heart in 2 draws with replacement?
99. Two people play a game. The first person starts with the number 1 and can multiply 1 by any number from 2-9. Then, the second person multiplies this new number by any number from 2-9. The winner is the first person to get a million or more. Who wins the game with an optimal strategy?
100. You have 75 blue balls, 25 red balls, and 1 yellow ball in an urn. You get \$1 for every red ball you draw, but you lose everything if you draw the yellow ball. What should be your strategy if you can choose to stop or re-draw after each ball?
101. What is the probability of getting dealt two cards with the same value from a standard, 52 card deck?
102. What happens to the price of a call option when interest rates increase?
103. What happens to the price of a call option when dividends unexpectedly increase?
104. What is delta (for financial options)?
105. What is the difference between American and European options?
106. What is the probability you are dealt pocket Aces in a standard poker hand?
107. What is the probability you are dealt at least one Ace in a standard poker hand?

Answers:

1. **Two sets** - Let p =prob team 1 wins and q =prob team 2 wins. $p^2 + q^2$ = probability finish in two sets. $2*p*q$ = probability finish in three sets. $p^2 + q^2$ always $\geq 2*p*q$, so the answer is two sets.
2. **3/8** - Given the edge the first dot is on, the probability the other two dots are on distinct edges is $(3/4)*(2/4)$
3. **45** - $(10 \text{ choose } 2) = 45$ -- this is the total number of ways two people can shake hands.
4. **104 card pack** - $(52/104)*(51/103) > (26/52)*(25/51)$, or $51/103 > 25/51$
5. **1599** - $39*41 = (40-1)*(40+1) = 40*40 - 1 = 1599$
6. **Yes, it's possible** - The first person thinks of a random number, say X . This person adds this number to her salary. The rest of the group simply adds their salary to the initial number. Then, the first person subtracts the random number X and divides the total salary sum by the size of the group to obtain the average.
7. **198** - $99^{99} = (100)^{(99)} * (.99)^{99} = (10)^{(198)} * (.99)^{99}$. You can convince yourself 10^{198} has 199 digits, and 0.99^{99} approaches $1/e$. Thus, $(10)^{(198)} * (.99)^{99}$ has 198 digits.
8. **0.5** - The fate of the last passenger is determined the second either the first or last seat on the plane is taken. This statement is true because the last person will either get the first seat or the last seat. All other seats will necessarily be taken by the time the last passenger gets to pick his/her seat. Since at each choice step, the first or last seat has an equal probability of being taken, the last person will get either the first or last with equal probability: 0.5.
9. **5050** - Sum of numbers from $1, 2, \dots, n = (n)*(n+1)/2$. You can also think about this problem by pairing off numbers - 1 and 100, 2 and 99, 3 and 98, 4 and 97, etc. We have 50 of these pairs, and each pair sums up to 101, so the final sum = $50*101 = 5050$.
10. **Yes, it's possible** - Fill up the 3 gallon jug. Then, pour the liquid into the 5 gallon jug. Fill the 3 gallon jug again, and then fill the 5 gallon jug until it is full. We now have 1 gallon remaining in the 3 gallon jug. We empty the five gallon jug and pour the remaining 1 gallon into our 5 gallon jug. Finally, we fill the 3 gallon jug and add this to the 5 gallon jug (which already had 1 gallon). We are left with 4 gallons in the 5 gallon jug.
11. **0.5** - Use recursion - The initial 16 flips have the same probability of everything. Thus, the game completely depends on if the last coin flip is tails or head (50/50 chance of H vs. T).

- 12. 25/51** - You either draw a black or a red card first. Then, there are 51 cards left in the deck and 25 of these cards have the same color. Thus, the probability is 25/51.
- 13. Yes, it's possible** - Leave switch 1 off. Then, turn switch 2 on for ten minutes. After the ten minutes, turn it off and quickly turn on switch 3. Now, go into the room. The currently lit up bulb connects to switch 3. The bulb that off but still warm is from switch 2, and the remaining bulb is from switch 1.
- 14. 20/64** - Out of the first three games, each team needs to win three. Thus, $(6 \text{ choose } 3) \cdot (.5^6) = 20/64$, as each team has a $1/2$ probability of winning each game.
- 15. 1/2** - Whether there is an odd or even number of heads is ultimately determined by the final flip (50/50 chance of being heads vs. tails), for any number of flips.
- 16. 0.6** - Because of repeats of black/red balls, there are 10 combinations of red/black balls: (5 choose 2) or (5 choose 3) spots to put the black or red balls, respectively. There are 4 places that 2 black balls can be next to each other, so the other 6 combinations do NOT have two black balls next to each other.
- 17. 1110** - The last digit must be zero (30, 45, 60, 75, etc.). Multiples of 15 never end in 1. Then, starting checking numbers. 10, 100, 110, 1000, 1100, 1110. You will quickly arrive at the answer if you are good with your mental math.
- 18. No** - $1027 = 1000 + 27 = 10^3 + 3^3$. We know $a^3 + b^3$ can be factored, so 1027 is NOT prime.
- 19. Yes** - sometimes a rare finance question is included in these interviews; remember that both time and volatility *increase* the prices of both calls and puts
- 20. 17/6 dollars** - You'll always get the last ball right as your sampling w/o replacement. The first ball you have a 50% chance of getting right. The second ball you have a $2/3$ chance of getting right.
- 21. 4** - Repeating patterns -- 2,4,8,6,2 -- follow the pattern.
- 22. 120** - $120 = (10 \text{ choose } 3)$. This is the number of ways to choose three different numbers. Then let A = the smallest one and C = the largest one.
- 23. 6/10** - By symmetry; if A loses the first flip, then A and B will both have 5 points and thus must have an equal chance of winning
- 24. 49** - Imagine we have four people with birthdays each month - $12 \cdot 4 = 48$ people. Then the 49th person will NEED to be the 5th birthday for some month.

- 25. 2500** - Sum of numbers between 1 and $n = n(n+1)/2$. Sum of even numbers between 1 and $n = (n/2)(n/2+1)$. Thus, the sum of odd numbers between 1 and $n = n(n+1)/2 - (n/2)(n/2+1)$. Just plug in $n=100$ to get the answer.
- 26. Yes, it's possible** - Pick 5 random numbers and divide by their sum.
- 27. 37** - Fast mental math. $37 \times 27 = 999$ cents. That's the best you can do.
- 28. 1000000** - Write down 0 to 999999 as six digit numbers (e.g. 009319). You will quickly see all digits appear an equal number of times. Thus, each digit appears $6 \times 1000000/10$ times. Thus, we see $600000 \times 45 + 1$ is the answer, as the sum of numbers from 1-9 = $9 \times 10/2$.
- 29. 9000** - b,c,d can be any number from 0-9. 'a' must be between 1-9. Thus, the answer is $9 \times 10 \times 10 \times 10$.
- 30. 42** - $x = 1/6 \times 1/6 \times 2 + 5/6 \times (x+1) + 1/6 \times 5/6 \times (x+2)$
- 31. 1/2** - Bayes rule -- $p(\text{center} \mid \text{no visible face painted}) = p(\text{center} \mid \text{no visible face painted}) / p(\text{no visible face painted}) = (1/27) / (1/27 + (1/6) \times (6/27)) = 1/2$
- 32. No** - Find the expected value of the game. $2 \times (70/256) - (1) \times (1 - 70/256) = -46/256$
- 33. 1/11** - Bayes rule: $P(\text{both six} \mid \text{at least one six}) = P(\text{both six}) / P(\text{at least one six}) = (1/36) / (1 - (5/6)^2) = (1/36) / (1 - 25/36) = 1/11$
- 34. 31.05** - keep track of your decimal places.
- 35. 5** - Let x = the number of heads. Then, we know $E(x^2) = \text{var}(x) + e(x)^2 = (1/2) \times (1/2) \times 4 + 2^2 = 5$. Remember, $\text{var}(x) = p(\text{heads}) \times p(\text{not heads}) \times (\# \text{ flips})$.
- 36. Yes** - Yep, this is Simpson's Paradox: Imagine the following -- first half B goes 1/10 and A goes 0/1. Second half, B goes 1/1 and A goes 9/10. B has a higher average in both the first half and second half, but A has a higher average overall.
- 37. 27** - $\text{weight} = 8 \times (12/8)^3 = 8 \times (3/2)^3 = 8 \times (27/8) = 27$
- 38. 9/40** - There's a 1/2 chance the 20 sided die ≤ 10 . Then, 9/20 chance the roll on the 10-sided die is bigger given the 20 sided die ≤ 10 . Thus, $(1/2) \times (9/20) = 9/40$
- 39. 2/3** - Bayes rule: $p(\text{both sides burned} \mid \text{at least one side burned}) = p(\text{both sides burned}) / p(\text{at least one side burned}) = (1/3) / (1/3 + (1/2) \times (1/3)) = (1/3) / (1/2) = 2/3$
- 40. 3105** - $50 \times 69 - 5 \times 69 = 3450 - 345 = 3105$
- 41. 1/16** - Cannot form a triangle if and only if all dots lie on the same edge. The probability of that is $4 \times (1/4)^3$
- 42. Ratio will be 1:1** -- To understand this problem, assume this country has 1000 people. Initially, they should have 500 boys and 500 girls. Then, only the family with 500 boys

will have another kid. We can expect 250 boys and 250 girls from this next batch of children. Then, from the 250 families that had two boys, they will reproduce again, and we can expect 125 girls and 125 boys. We see that the ratio b/w boys and girls will never change in this country, regardless of their desire to have all girls.

43. 22 - Here's one way to think about the question. We know the minute hand and hour hand move at different speeds. The hour hand is always moving, so it takes the minute hand a bit over an hour to overlap with the hour hand again. The minute hand moves at a speed of 360 degrees/hour. The hour hand moves at a speed of 30 degrees/hour. Thus, the difference in their speeds is 330 degrees/hour. At 330 degrees/hour, we see that the clock hands will overlap 22 times: $(24) \cdot (330/360)$.

44. Yes, it's possible - Take any 10 coins, put them in a separate pile, and flip them over. Remember - there are 90 tails, 10 heads initially, so let's say there are x heads left in the 90 coin pile. Then, before we flip the coins over, we know there are $(10-x)$ heads in the 10-coin pile. Thus, if there are $(10-x)$ heads in the 10-coin pile, we know there are x tails, as there are 10 total coins. Once we flip them over, those x tails become x heads and we have an even number in each pile.

45. 1 try - Pick a fruit from 'Apples + Oranges' You know they're all labeled wrong, so if you draw an apple from this box, the box must be 'Apples'; and otherwise 'Oranges'. Then, let's say you drew an apple from the first box. You know the 'Oranges' box is still labeled incorrectly and cannot be 'Apples' (as this is already taken), so it must be 'Apples + Oranges'. The last box then has to be 'Oranges'

46. 4/7 - Use recursion - $p = (2/3) \cdot (2/3) + (2/3) \cdot (1/3) \cdot p$

47. 3/5 - Assume there are X girls. Let the event A be that the baby born is a boy. $P(A) = 1/2$
Let the event B be that the baby picked is a boy. $P(A|B) = P(A \text{ and } B)/P(B)$ by Bayes rule.
 $P(B) = P(\text{born is a boy}) \cdot P(\text{picked is a boy}) + P(\text{born is a girl}) \cdot P(\text{picked is a girl}) = 0.5 \cdot (3/(3+X) + 2/(3+X))$
 $P(A \text{ and } B) = P(\text{born is a boy}) \cdot P(\text{picked is a boy}) = 0.5 \cdot (3/(3+X))$
Therefore, $P(A|B) = P(A \text{ and } B)/P(B) = \frac{3}{5}$

48. 0.37 - Probability of not seeing a shooting star in half an hour = $1 - 0.6 = 0.4$. Let x = probability of NOT seeing a shooting star in half an hour, then $x^2 = 0.4$, $x = 0.632$. Thus, the probability of seeing a shooting star in half an hour = $1 - x = 1 - 0.632 = 0.37$

49. 12222.21 - mental math; keep track of your decimal places

50. 2^{10} - Raise both to the 30th power. Then, we have 2^{30} vs. 10^3 and we see that 2^{30} is larger, so $2^{10} > 10^{10}$.

- 51. 1/12** - A circular table has $(n-1)!$ different permutations. $4! = 24$. Then, since the ordering can be counter-clockwise or clockwise, we have a $2/24 = 1/12$ probability that the group will be sitting in age order.
- 52. -1/21** - There are $(9 \text{ choose } 3) = 84$ ways to distribute three pieces on a tic-tac-toe board at random. There are 8 ways to form tic-tac-toe on a board. Thus, we can calculate the expected value as follows: $9*(8/84) - 1*(76/84) = -1/21$
- 53. 4.25** - The expected value of dice roll is 3.5. Thus, when playing optimally, you'll reroll if you get a 1, 2, or 3 on your first roll. Thus, the expected value with one re-roll is $p(\text{die roll} > 3.5) * (\text{EV if roll} > 3.5) + p(\text{die roll} \leq 3.5) * (\text{EV of one die roll}) = (.5)*(5) + (.5)*(3.5) = 4.25$. Now, try to figure out how to find the expected value if you get up to two re-rolls (hint - 4.67 is the answer with two re-rolls).
- 54. 5/9** - Probability miss twice = $(2/3)*(2/3)$. Thus, the final answer to this question is $1 - (2/3)^2 = 5/9$
- 55. 3/4** - On the first flip, there is a 50% chance of getting heads (1/1 heads = 1:1 ratio of heads to total flips). If the first flip is heads, you stop flipping, as you have the maximum possible ratio. If you flip tails first, you'll continue to play until the ratio approaches 1:2 (law of large numbers, it eventually will happen). $(1/2)(1/1) + (1/2)(1/2) = 3/4$.
- 56. 0.4** - $\sqrt{16}/\sqrt{100} = 4/10 = 0.4$
- 57. 1/4** - Binomial theorem - $(4 \text{ choose } 3) * (1/2)^4 = 4/16 = 1/4$
- 58. 3/4** - Let jar 1 have only one red marble, and the other jar will have the other 99 marbles. Then, our probability of selecting a red marble is $(1/2)*(1) + (1/2)*(49/99)$, or approximately 3/4.
- 59. 6.67** - $.25 = (20+x)/(100+x)$
- 60. At least one game correct** - Expected value of getting all games correct = $1,000,000 * (1/2^7) = 7812.5$. $1 - (1/2)^4 = (15/16)*10,000 = 9375 =$ expected value second option of getting at least one game correct.
- 61. sqrt(2)** - $x^2 = x^{(x^{(x^{(x^{\dots})})})}$, so $x = \sqrt{2}$
- 62. 18M** - Payoff = 60M, so value contract value = $60M * 0.3 = 18M$
- 63. 0.25** - Set up the following formula: $p*20 - (1-p)*20 \geq -10$
- 64. 58 minutes** - After 59 minutes, the pond is 1/2 covered. So after 58 minutes, the pond is 1/4 covered

- 65. 0.34** - Sample space - (52 choose 6). The number of possible choices without pairs = $(13 \text{ choose } 6) * 4^6$, as each card can be a diamond, heart, spade, or club. $(13 \text{ choose } 6) * 4^6 / (52 \text{ choose } 6) = 0.34$.
- 66. 495** - $(12 \text{ choose } 8) = 495$. We have 12 possible places to put the * signs, and we need to choose 8 of these slots.
- 67. 0.5** - $X * (1 - X)$ = probability of winning. This is maximized at $X = 0.5$.
- 68. The slow car could *never* catch up** - By the time that the slower car finishes the 1st lap, the faster car will have finished the race, so this is a trick question!
- 69. 35/18** - The sample size is small, so you can look at all possible differences. $(5*2 + 4*4 + 3*6 + 2*8 + 1*10 + 0*6)/36 = 35/18$
- 70. 3** - In the first two draws, there are only 3 possible outcomes - you draw two white socks, two black socks, or one white sock and one black sock. The maximum number of socks you need to guarantee a pair is thus 3, as you'll either have 3 black socks, 3 white socks, 2 white socks and 1 black sock, or 2 black socks and 1 white sock.
- 71. You should not bid** - You only get to purchase the car if your bid > car value. So define $y = \text{car value}$, and we can calculate our expected payoff = $(1.5)*y - x$, where $y = x/2$. Thus, for any bid, your expected pay off = $-.25*bid$.
- 72. 59** - Chicken McNugget Theorem - since 7 and 11 are coprime, the answer is $7*11 - (7+11)$
- 73. 5/54** - $P(\text{three numbers are different}) * P(\text{order is also increasing}) = 1*(5/6)*(4/6) * (1/2)*(1/3) = 5/54$
- 74. 0.0386** - $(6 \text{ choose } 3)$ ways to choose 3 outcomes. Then $(6 \text{ choose } 2)*(4 \text{ choose } 2)*(2 \text{ choose } 2)$ ways to distribute these outcomes. The sample space is 6^6 . Thus, the final probability is $(6 \text{ choose } 3)*(6 \text{ choose } 2)*(4 \text{ choose } 2)*(2 \text{ choose } 2)/(6^6) = 0.0386$
- 75. $(5!)*(5!)*2^5$** - There are $5!$ ways to place the boys on the five benches and $5!$ ways to place the girls. Then, there are 2^5 ways to order the kids on the five benches (either boy girl or girl boy on each bench).
- 76. $N*(1-((N-1)/N)^{100})$** - The probability that a random bin does not have any balls thrown into it after 100 throws is $((N-1)/N)^{100}$, as there is a $(N-1)/N$ chance that any given ball thrown does NOT go into this bin. Thus, $1-((N-1)/N)^{100}$ is the probability at least one ball is thrown into this bin after 100 throws. We have 'N' total bins, so our final answer is $N*(1-((N-1)/N)^{100})$.
- 77. Put 6 on A, 3 on B, and 2 on C.** - I will always spend \$11 and win \$12.

- 78. 38/72** - If the first drunk person chooses two different letters, there is a 50% chance the next drunk person puts the letters back correctly. However, if the first drunk person draws two of the same letter, "HAPPY HOUR" has a 100% chance of remaining intact. Thus, our final probability = $(1/2) * ((9 \text{ choose } 2) - 2) / (9 \text{ choose } 2) + 2 / (9 \text{ choose } 2) = 38/72$
- 79. 1/17** - The fact that a card is lost is irrelevant information, as we do not know which card was lost. Thus, the probability is $(13 \text{ choose } 2) / (52 \text{ choose } 2) = 1/17$
- 80. 530K** - 70% of the time the land is worth 1000K and you make a profit of 700K. 20% of the time the land is worth 500K and you make a profit of 200K. 10% of the time we should not buy. Thus, the maximum value of x we should be willing to pay is $0.7 * 700K + 0.2 * 200K = 530K$.
- 81. $6^5 - 5^5$** - With no restrictions, there are 6^5 possible outcomes when rolling five dice. There are 5^5 possible outcomes when rolling five dice if zero threes are rolled. Thus, $6^5 - 5^5$ is the number of possible outcomes when at least one dice shows a three.
- 82. 6 digits** - $7^2 = 49$, or about 50. Thus, $7^6 = \text{about } 50^3 = \text{about } 125000$. Then $7^7 = \text{about } 875000$, so 6 digits
- 83. 4/7** - Use recursion. $p = 1/6 + (1/3) * p + (1/2) * (1 - p)$
- 84. $(\sqrt{2}) - 1$** - Use recursion. The probability the population dies out can be found using the following equations, where p = the probability the population dies out. $p = 1/4 + (1/4) * p + (1/4) * p^2 + (1/4) * p^3$. When we solve for p , we see the only solution between 0 and 1 = $(\sqrt{2}) - 1$
- 85. 3** - The last digit of multiples of 3 follow the following pattern: 3, 9, 7, 1, 3. Follow the pattern and it becomes clear that the last digit of 3^{33} is 3.
- 86. 12 bills** - The expected number of bills is 12, as you can have 4 (all 5's), 8, 12, 16, or 20 (all 1's) bills.
- 87. 63 dollars** - The probability a given door is selected is $1 - (99/100)^{100}$. Thus, the number of dollars we can expect to make = $100 * (1 - (99/100)^{100}) = \text{approximately } 63$.
- 88. 71 degrees** - Remember the hour hand is not exactly at 10. It moved slightly towards 11 within the first two minutes of the hour, and we need to determine how much. Minute hand at $(2/60) * (360 \text{ degrees}) = 12 \text{ degrees}$. Hour hand at $(300 \text{ degrees} + (2/60) * 30) = 301 \text{ degrees}$. The difference in the angle is thus 71 degrees.
- 89. Yes, it is possible** - Take one rope and light it at both ends. At the exact same time, burn one end of the remaining rope. When the first rope finishes burning (which will be after 30 minutes), light the un-lit end of the other rope. This rope has already burned for

30 minutes, so after another 15 minutes (as both ends are now lit), the rope will burn out.
 30 minutes (for first rope to burn out) + 15 minutes (for second rope to burn out) = 45 minutes.

90. 81 - Remember not to count 77 twice!

91. No - Probability in increasing order = $(1) \cdot (1/2) \cdot (1/3) \cdot (1/4) = 1/24$. Thus, the expected value of the game = $(1/24) \cdot 10 - (23/24) = -13/24$. So do not play!

92. 98, 0, 1, 0, 1 - 98 coins to pirate #1, and only one coin to #3 and #5. Use recursion to solve this brain teaser.

93. 6.77 - derivative of $x^x = (x^x) \cdot (\ln x + 1)$; then plug in $x=2$

94. 40.5 seconds - The 6 additional steps took 9 seconds. That's 1.5 seconds per step.
 Thus, it will take me $7 \cdot 1.5 + 30$ seconds = 40.5 seconds.

95. 258 - Use recursion. $x = 5/6 \cdot (x+1) + (5/6) \cdot (1/6) \cdot (x+2) + (5/6) \cdot (1/6) \cdot (1/6) \cdot (x+3) + (1/6) \cdot (1/6) \cdot (1/6) \cdot (x+3)$

96. 1296 - There are 9 horizontal and 9 vertical lines on a chessboard. A rectangle/square is formed by choosing 2 horizontal lines and 2 vertical lines. Thus, the answer is $(9 \text{ choose } 2)^2 = 1296$

97. 2/3 - Kid 1 or Kid 2 will get the first moldy cookie. The probability this kid also picks the second moldy cookie = $1/3$. So our answer = $1 - 1/3 = 2/3$

98. 0.375 - Binomial theorem - $2 \cdot (39/52) \cdot (13/52)$

99. Second person - 111,112 to 999,999 = winnable numbers (as can multiply to get >1 million). Thus, 55,556 to 111,111 are losable numbers. Thus, 6,173 to 55,555 are winnable, and 3,087 to 6,172 are losable. If we keep following this pattern backwards, we see 1 is losable. Thus, the if the second person plays an optimal strategy, he or she will always win.

100. Stop after drawing 13 balls - Ignore the blue balls, as they are not relevant to the problem. Then, imagine we have drawn x red balls. We need $(25-x)/(26-x) \geq (x)/(26-x)$ to draw again. Solve the equation to see we will stop drawing after 13 balls.

101. 3/51 - This question assumes a standard 52 card deck. After the first card is dealt, there are 51 remaining cards and 3 cards left with the same value as the first card.

- 102. Price of a call increases, price of a put decreases** - Owning a call is similar to owning a share of stock, but you have to put up less capital. For example, imagine XYZ stock = \$200 and some given call option for XYZ = \$5. Thus, you save \$195 by owning the call, and you'd prefer to invest this money in a high interest rate environment.
- 103. Price of a call decreases, price of a put increases** - After a dividend, the stock price of the company will fall by the dividend amount. Thus, call options become less valuable when dividends increase.
- 104. Change in option value for a \$1 increase in the underlying security.** You can think of delta as the "odds an option ends up in the money." Call deltas range from 0-1, and put deltas range from -1 to 0.
- 105. Exercising before expiration** - American options *can* be exercised before expiration, but European options *cannot* be exercised before expiration.
- 106. 0.0045** - Use combinatorics: $(4 \text{ choose } 2)/(52 \text{ choose } 2) = 0.0045$. There are four Aces in a standard deck of 52 cards.
- 107. 0.149** - $1 - (48/52) * (47/51)$