Math 220A - Fall 2020 - Midterm

Name: ______________________________

Student ID: ______________________________

Instructions:

Please print your name and student ID (if you know it).

You may not use any books, notes or internet.

There are 4 questions which are worth 40 points. You have 60 minutes to complete the test. Please upload your answers in Gradescope at the end of the exam.

Question	Score	Maximum
1		10
2		10
3		10
4		10
Total		40

Problem 1. [*10 points.*]

Let

$$f(z) = \frac{z}{z^2 - 4}.$$

Expand f into Laurent series around 0 in the two regions $|z| < 2$ and $|z| > 2$ respectively.

Problem 2. [*10 points; 5, 5.*]

Let $U \subset \mathbb{C}$ be a connected open set.

(i) Show that if $h : U \to \mathbb{C}$ is nonconstant and holomorphic, then $\mathrm{Re}\, h : U \to \mathbb{R}$ is an open map.

(ii) Let $f : U \to \mathbb{C}$ be holomorphic with $f'(z) \neq 0$ for all $z \in U$. Show that

$$\{\mathrm{Re}\ f(z) \cdot \mathrm{Im}\ f(z) : z \in U\}$$

is an open subset of $\mathbb{R}$.

Problem 3. [*10 points.*]

Suppose $f : \Delta(0,1) \to \mathbb{C}$ is holomorphic such that for all $z \neq 0$, we have

$$|f(z)| \leq -\log|z|.$$

Show that $f \equiv 0$.

Problem 4. [*10 points.*]

Assume that $f : \overline{\Delta}(0,1) \to \mathbb{C}$ is continuous, and f is holomorphic in $\Delta(0,1)$. Show that if $f(z) = 0$ for all $z = e^{it}$ with $0 \leq t < \pi$ then $f \equiv 0$.

Hint: You may wish to work with a convenient auxiliary function.