Derivatives/Options Studying Material

Terminology:

Risk-free interest rate (r_f) - The risk-free interest rate, denoted r_f, is vital when pricing financial derivatives. This interest rate is the rate of return on an investment with zero risk of loss. The U.S. treasury yield is typically as the risk-free rate.

Arbitrage - Riskless trades that profit by taking advantage of price differences of similar or identical assets.

Present value - The current value of future cash flows with a given rate of return.

Option - An option is the right, but not the obligation, to buy/sell a security at a given price. Each option contract typically controls 100 shares of the underlying security.

Call - The right, but not the obligation, to buy the underlying security at a given strike price.

Put - The right, but not the obligation, to sell the underlying security at a given strike price.

Combo (Synthetic Long Stock) - Short a put option and long a call option on the same underlying asset with the same strike price.

NBB - Best (highest) available bid

NBO - Best (lowest) available offer

Intrinsic Value - Intrinsic value is defined as the value of exercising an option now. For example, imagine we purchase a call on XYZ with K = \$100. If the current price of S_{XYZ} = \$106, the intrinsic value of this call option is \$6. On the other hand, if S_{XYZ} = \$99, the intrinsic value of this call option is \$0, yet the intrinsic value of a put option on XYZ with K = \$100 and S_{XYZ} = \$99 is \$1.

Extrinsic Value - The difference between the price of an option and the intrinsic value of the option. American options always have an extrinsic value ≥ 0. Extrinsic value is often referred to as the time value of the option.

At the Money Option (ATM) - An option with K = the current price of the underlying asset. In other words, if K = \$100, and S = \$100, the option is considered ATM.

In the Money Option (ITM) - An option with a positive intrinsic value.

Out of the Money Option (OTM) - An option with no intrinsic value.

Delta - Delta is defined as the rate of change of an option's price for a \$1 move upwards in the underlying asset. The delta of an ATM option = 0.5

Gamma - Gamma is defined as the rate of change of an option's delta for a \$1 move upwards in the underlying asset. Gamma is *always* ≥ 0 for long puts and long calls.

Theta - Theta refers to the time value of an option and represents the amount by which a long option will decrease every day.

Vega - Vega is defined as the change in the value of an option for a 1 point move in the implied volatility (IV) of the underlying asset.

Rho - Rho is defined as the change in the value of an option for a 1% increase in interest rates. Rho is positive for long call options and negative for long put options.

Risk Reversal - Long an OTM put option and short an OTM call option

Straddle - Purchase a call and put option with the same strike strike price.

Reversal - Long synthetic stock (combo), short the underlying stock.

Conversion - Short synthetic stock (combo), Long the underlying stock.

PnL - Profit & Loss of a trader's position

Counterparty Risk - The risk that the counterparty of a trade will default on the trade agreement.

Carry - Defined as ($K\text{-}Ke^{-rt} - PV_{div}$). Carry is an important term when pricing calls, puts, combos, reversals & conversions, etc.

Futures contract - An exchange traded agreement to buy/sell a specific quantity of an asset at a specified price & time in the future.

Forward contract - An over-the-counter agreement to buy/sell a specific quantity of an asset at a specified price & time in the future. Forwards are similar to futures contracts, but futures contracts are marked-to-market daily and also have lower counterparty risk.

Assigned - Options traders are assigned on their short positions when the counterparty exercises a call or put option. Being assigned on a short put option means that the counterparty will sell us the underlying asset at $K, leaving us long stock, and being assigned on a short call position means that the counterparty will buy the underlying asset from us at $K, leaving us short stock.

London Interbank Offered Rate (LIBOR) - The benchmark interest rate that global banks lend to each other for short-term loans.

Forwards/Futures

Introduction

Futures contracts and forward contracts are two financial derivatives that are similar in many ways. Forwards and futures both allow investors to purchase a given asset at a specific price and time in the future. The main difference, however, is that futures are traded on an exchange, yet forwards are privately negotiated and traded over-the-counter. This means that forward contracts have higher *counterparty risk*. A loss resulting from default is consequently a much greater risk in forwards. In a futures contract, counterparty risk is much lower as an exchange clearing house acts as the counterparty for both the buyer and the seller of the contract. Additionally, futures contracts are *marked-to-market* each day, reducing credit risk even further.

Pricing Forwards/Futures

Many new traders assume that the price of forward contracts and futures contracts depends on directional assumptions of the underlying asset. However, this is not true. Forwards and futures can be priced using only the risk-free interest rate, denoted r_f, and the expected dividends from the underlying asset.

As an example, imagine r_f = 5% and we are trying to price a one-year forward on Apple stock (*NASDAQ: AAPL*). Let's assume for a moment that AAPL is *not* expected to pay a dividend in the upcoming year. Using just the risk-free interest rate, we can calculate the price of the one-year forward using the following formula, where S_0 = the current price of AAPL.

$F_{AAPL} = S_0 e^{rt}$

Thus, the one-year forward on Apple stock, where S_0 = \$198.78 (Closing price, 06/21/2019), is equal to $F_{AAPL} = 198.78e^{(0.05*1)}$ = \$208.97. Thus, ignoring transaction costs, the buyer of this forward contract will receive one share of stock for \$208.97 in one year.

Now, imagine if this relationship did not hold. For example, what if the price of F_{AAPL} = \$200? In this scenario, F_{AAPL} is underpriced, so could execute the following trade.

Purchase F_{AAPL}, Short one share of AAPL at \$198.78 and invest this money at r_f.
The following trade would guarantee us risk-free profit, or *arbitrage*, as we would be guaranteed a riskless profit of \$8.97 at expiration, regardless of the price of AAPL in one year. At expiration, our position in AAPL would become neutral, with the forward contract canceling out our short position.

Cash Flow Today	**Cash Flow in One Year**
+\$198.78 (from shorted stock)	-\$200.00 (From AAPL purchase)
	+\$208.97 (from the \$198.78 investment at r_f)

Of course, these arbitrage scenarios occur *very rarely* in financial markets, but the concept is important. Futures and forwards are priced to prevent arbitrage.

We must note, however, that in the example above, we ignored transaction costs and assumed AAPL did *not* pay a dividend. We also ignored counterparty risk, which is often relevant when trading forward contracts.

As another example, imagine F_{AAPL} = \$220. Could we profit in this scenario? Again, the answer is yes.

Sell F_{AAPL}, Borrow \$198.78 and purchase one share of AAPL.

Cash Flow in One Year
+\$220.00 (From AAPL purchase)
-\$208.97 (from borrowing \$198.78 at r_f)

Again, assuming we can borrow money at r_f, we are guaranteed a riskless profit of \$11.03 (220-208.97), regardless of the price of AAPL in one year.

The situation becomes slightly more complex if AAPL plans to pay a dividend. Imagine for a moment that we know AAPL will pay a dividend of \$2.00 in six months. Assuming the current price of AAPL is still \$198.78, will F_{AAPL} still equal \$208.97? The answer is **no**, as we will *not* receive the dividend if we purchase F_{AAPL}. Thus, the dividend benefits owners of AAPL stock, but not buyers of F_{AAPL}.

Thus, to calculate the proper price of F_{AAPL}, we must incorporate the dividend into our calculations using the following formula, $F_{AAPL} = (S_0 - Div_{PV})e^{rt}$. In other words, we must subtract the *present value* of the dividend in our calculations.

Since the dividend is paid out in six months, we know the following:

$Div_{PV} = 2e^{(-0.05*0.5)}$ = \$1.95.

Thus, in this new scenario, $F_{AAPL} = (S_0 - Div_{PV})e^{rt} = (198.78 - 1.95)e^{(0.05*1)}$ = \$206.92.

Imagine, however, that F_{AAPL} is currently priced at $204. How could we profit in this scenario?

Buy F_{AAPL}, Short one share of AAPL

Cash Flow Today	Cash Flow in Six Months	Cash Flow at Expiration
+198.78 (Short AAPL and invest at r_f)	+$5.03 (Interest in six months from investing $198.78)	-$204 (From F_{AAPL})
	-$2 (Dividend Paid to AAPL shareholder)	+206.92 from Investment

Note, when we are short AAPL stock, we must pay out the dividend. Still, we are guaranteed a risk-free profit of $2.92 in this scenario.

Futures and forwards are priced similarly, using the risk-free interest rate and the expected dividend payouts. However, even though arbitrage situations can exist, these situations are not *always* riskless.

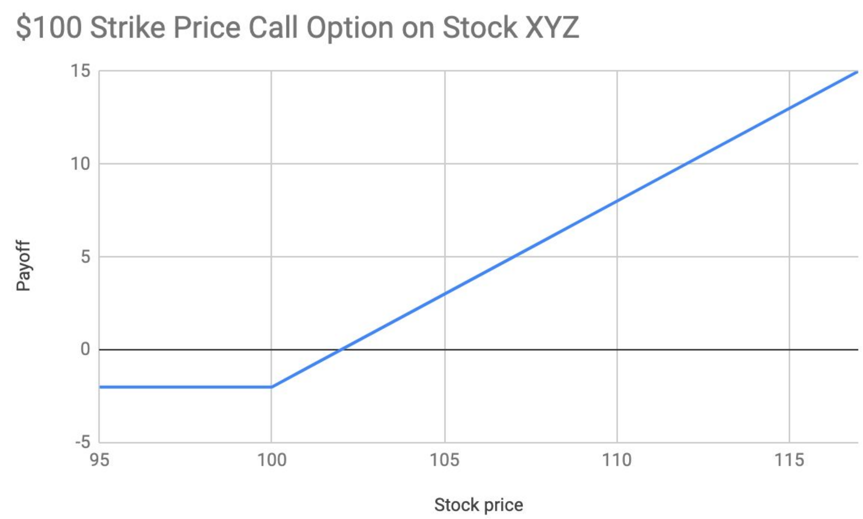
For example, imagine if, in the situation above, AAPL unexpectedly increased the dividend from $2.00 to $2.25. Clearly, this would change (more specifically, decrease) the price of F_{AAPL}. Additionally, the risk-free interest rate can change. If r_f increases, then F_{AAPL} becomes more valuable.

Financial Derivatives - Options

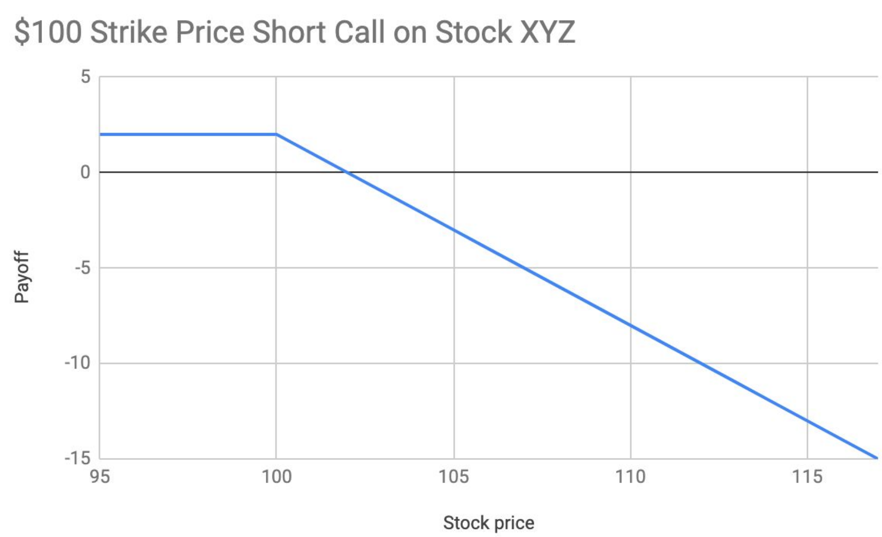
An option is the right, but not the obligation, to buy or sell an underlying security at a given “strike” price (denoted ‘K’). Calls and puts, the options discussed in this section, come in two main forms: *American* options or *European* options. The difference between American and European options is that American options can be exercised at any time, whereas European options can only be exercised at expiration date. Since American options can be exercised at any time, they are never less expensive than European options. A call option is the right to buy a given security at the strike price, and a put option is the right to sell a given security at the strike price.

For example, imagine purchasing a call option on a hypothetical stock, stock XYZ, with K = $100. We pay $2 for this option initially, so the most we can lose on this trade is $2. At expiration, if the stock price, denoted S_e, finishes ≤ $100, we will lose $2 on this trade. Why would we want to purchase XYZ for $100 if the price of XYZ in the market ≤ $100?

However, if S_e > $100, then we would exercise our option to purchase XYZ for $100. For example, if S_e = $105 at expiration, we could exercise our option, purchase XYZ for $100, and immediately sell XYZ in the market for $105, realizing a profit of $3 ($5-$2). The payoff for a long call diagram is shown below.



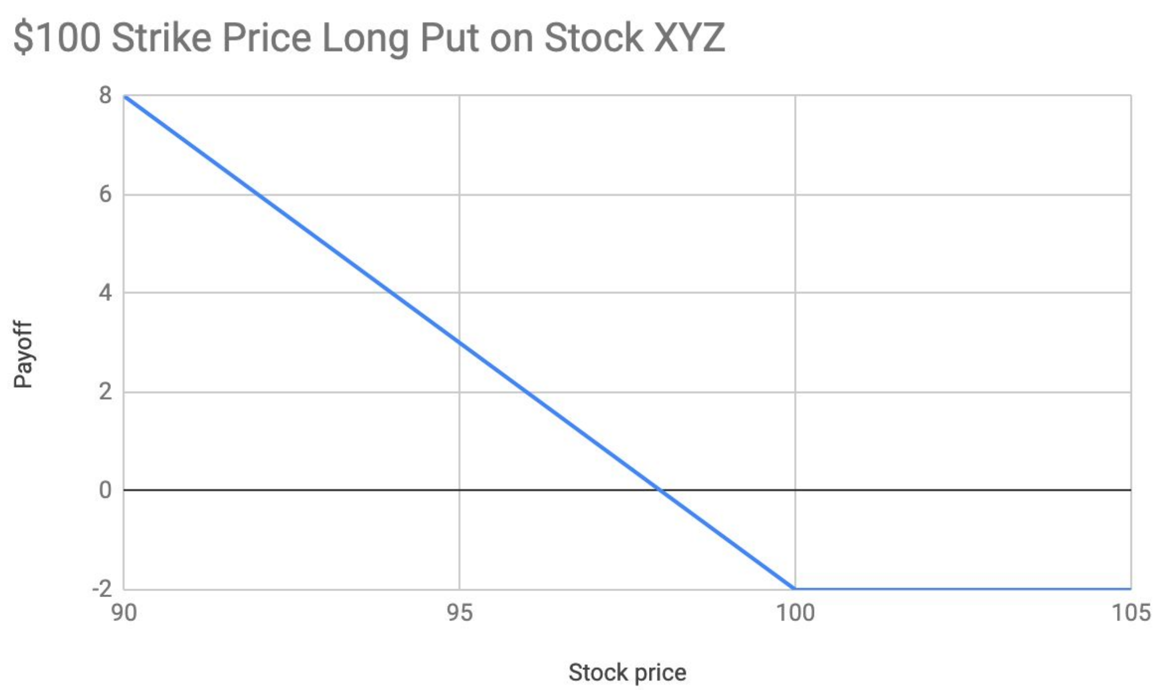
On the other hand, if we sold this call option on XYZ, we would collect $2 in premium for selling the option, and we would keep this full amount if S_e ≤ $100. Thus, our payoff diagram for a short call option with K = $100 is shown below.

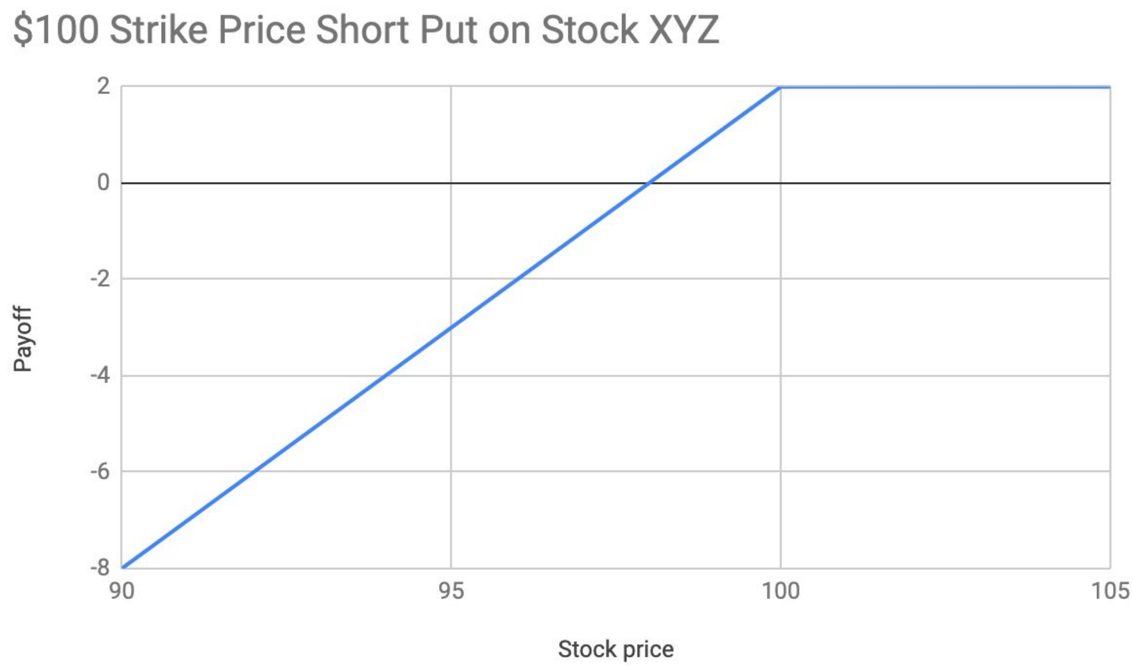


With a long call option, we clearly want the price of XYZ to increase. Thus, buying a call option is a *bullish* strategy, meaning we want the price of the underlying asset to move higher. On the other hand, a short call option is a *bearish* strategy, as we want the price of the underlying asset to decrease or sit still.

When we are long a put option, we have the right to sell XYZ at a given price. Thus, imagine that we purchase a put option on Stock XYZ with K = $100. In this scenario, we want the price of XYZ to decrease as much as possible, so a long put is a bearish strategy. Similarly, a short put is a bullish strategy, as we want the price of the underlying asset to increase.

The payoff diagrams for both a long put and a short put with K = $100 are shown below. Again, we assume the cost of this option is $2.





As you may be able to see in the diagrams above, selling options is a risky strategy. If we sell a call option, we have unlimited upside risk. If the price of stock XYZ rises to $900 before expiration, we will have lost $898 on *each* options contract, assuming each options contract controls one share of stock XYZ.

Before we continue, we need to outline a few terms that are vital to know when trading options.

1. **Intrinsic Value -** Intrinsic value is defined as the value of exercising an option now. For example, imagine we purchase a call on XYZ with K = $100. If the current price of S_{XYZ} = $106, the intrinsic value of this call option is $6. On the other hand, if S_{XYZ} = $99, the intrinsic value of this call option is $0, yet the intrinsic value of a put option on XYZ with K = $100 and S_{XYZ} = $99 is $1.
2. **Extrinsic Value -** The difference between the price of an option and the intrinsic value of the option. American options always have an extrinsic value ≥ 0. Extrinsic value is often referred to as the time value of the option.
3. **At the Money Option (ATM)** - An option with K = the current price of the underlying asset. In other words, if K = $100, and S = $100, the option is considered ATM.
4. **In the Money Option (ITM)** - An option with a positive intrinsic value.
5. **Out of the Money Option (OTM)** - An option with no intrinsic value.

Options Pricing

What factors are important when pricing an option? According to the Black Scholes formula, the current stock price (S), the strike price of the option (K), the short-term risk-free interest rate (r_f), the time to expiration (t), and the implied volatility (σ) of the underlying asset are the only factors that we need to know to price a European option. The Black-Scholes model is typically adjusted to price American options. Although the Black Scholes formula has flaws, such as assuming constant volatility and zero transaction costs, this model is still widely used today to help determine the fair price of an option.

As we can see, four of the five variables used in the Black Scholes formula are easily determined. The current stock price, the strike price, the risk-free interest rate, and the time to expiration are all assumed to be known. Thus, volatility is the only “tricky” variable that is used in the Black Scholes model. This is because we do not know the actual implied volatility of the underlying asset - it is impossible to predict the exact variability of the underlying asset.

Before continuing, we need to consider how dividends, which are ignored in the Black Scholes formula, impact the price of calls and puts. Since stocks typically decrease by the dividend payment on ex-dividend date, dividends increase the price of put options and decrease the price of call options. Thus, if a company announces an unexpected dividend, owners of calls will watch their options decrease in value, and owners of puts will watch their options increase in value. Although a special dividend is uncommon, traders need to be aware of this possibility. Microsoft announced an unexpected dividend of $3.08 in 2004, and this special dividend crushed some traders long call and short put positions, as the dividend was >10% of the price of Microsoft stock at the time. Traders need to be wary of companies with ample cash.

Similarly, the risk-free interest rate impacts both the price of call options and put options. Higher interest rates increase the price of call options and decrease the price of put options. We can think about this intuitively to see why this relationship holds. Buying a put is similar, in many ways, to shorting the underlying asset - we make money when the underlying asset decreases in price. However, when we short a stock, we receive cash (the price of the stock), which can be reinvested in the market. Thus, higher interest rates benefit traders with large short positions. However, when we purchase a put option, we do not receive cash that can be reinvested in the market, so higher rates decrease the price of puts. Similarly, when we purchase a call, we are hoping the stock will increase in price, just like when we purchase the underlying stock outright in the market. However, a call is *always* cheaper than purchasing the underlying asset, so we need to spend less money up front to get stock exposure. Thus, higher rates increase the price of call options.

Put and Call Lower and Upper Bounds

As we saw earlier, the Black Scholes formula is typically used as a baseline for option pricing. The lower and upper bounds for American and European options are slightly different and outlined below. These lower & upper bounds *must* hold, otherwise an arbitrage situation will arise. We will denote American options with capital letters (*C* = American call, *P* = American put), and European options with lowercase letters (*c* = American call, *p* = American put). Please note that these bounds assume no unexpected dividends arise, as seen in the previous example with Microsoft stock in 2004.

1. **European Put Upper Bound ($p \leq Ke^{-rt}$)**
2. **European Put Lower Bound ($p \geq Ke^{-rt} - S$)**
3. **American Put Upper Bound ($P \leq K$)**
4. **American Put Lower Bound ($p \geq K - S$)**
5. **European Call Upper Bound ($c \leq S$)**
6. **European Call Lower Bound ($c \geq S - Ke^{-rt}$)**
7. **American Call Upper Bound ($C \leq S$)**
8. **American Call Lower Bound ($C \geq S - K$)**

Imagine that the price of an American call is trading at $60 with K = $20, yet the price of the underlying stock is trading at $59. How could we take advantage of this situation? We could simply short the call and purchase the underlying security at $59. At this point, our PnL for trade is +$1, and this is a riskless trade. If the stock finishes below $20 at expiration, our profit will be ($1+S_e$), where S_e = the price of the stock at expiration. If S_e > $20, we will be assigned on our short call, and our PnL will be +$21.

Similarly, imagine an American call with K = $100 priced at $5 when S = $106. Ignoring transaction costs, we could short the stock and then purchase & exercise the call for an immediate profit of +$1. Similarly, imagine a European put with K = $100 priced at $5 when S =

$94. In this case, we could buy the stock, buy & immediately exercise the put option for a riskless profit of $1.

Derivations of each of these upper and lower bounds can be found online. Again, note that the price of American options is always *at least* that of their European counterparts. Although these arbitrage situations are rare, traders should always make sure that the upper and lower bounds hold.

Put-Call Parity

Put-Call Parity defines the relationship between European call options and European put options.

The formula is as follows, where c = European call, p = European put, S = the current price of the underlying asset, and carry = (K-Ke^{-rt} - PV_{div}). Note, both the call option and put option *must* have the same strike price (K).

$$\mathbf{c - p = S - K + carry}$$

Derivations of the formula above can be found online. However, the important part to note is that the price of call options and put options *are related*. If put-call parity does not hold, then an arbitrage situation exists.

For example, ignoring transaction costs, imagine S = $100, K = $95, carry = $0.5, c = $6, and p = $1. How could we take advantage of this situation? In this case, we would short the stock, short the put option, and long the call option. Thus, we are long the combo (synthetic stock) and short the underlying asset. This trade is called a *reversal*, and we would be guaranteed a riskless profit of $0.50, due to the carry, regardless of the price of the underlying asset at expiration. Again, we must assume that *no* special dividends are announced. How does this occur? Since we are long the call, short the put, and short the stock, when we initiate the trade, we are +$95 that can be invested in the market at r_f. Thus, on expiration date, we gain $0.50 in carry from the interest on our $95 investment, and the long synthetic stock and short underlying stock positions cancel out, leaving us with a riskless profit of $0.50. If S_e > $95, we will exercise our long call option, purchasing the underlying stock at $95. Since we were short stock, our position will be neutral, and we will be left with the $0.50 of carry. On the other hand, if S_e < $95, we will be assigned on our short put, again leaving our stock position neutral. In all scenarios, we will only be left with the carry.

Put-call parity is interesting for other reasons, too. As we have seen, a long call and short position is called a *combo*, and a combo is essentially equivalent to holding the underlying asset. When interest rates and dividends do not exist, purchasing the underlying asset and purchasing the combo will have the *exact* same payoffs at expiration.

Imagine we purchase a combo with K = \$100 and S_0 = \$105. At expiration, we can see that the payoffs of holding the underlying position and a combo are equal, assuming r_f = 0% and no dividends.

	Stock Value	**Combo Value**	**Reason:**
S_e > \$100	$S_e - S_0$	$S_e - S_0$	We exercise our long call option, leaving us long one share of the underlying asset.
S_e < \$100	$S_e - S_0$	$S_e - S_0$	We are assigned on our short put option, leaving us long one share of the underlying asset.

Note that if S_e = \$100, we *may* or *may not* be assigned on our short put. This scenario is referred to as *pin risk*.

Another interesting aspect of put-call parity comes when we rearrange the formula.

$$\mathbf{c - p = S - K + carry}$$
$$\mathbf{c = S - K + carry + p}$$

Here, we see that we can replicate a long call position by purchasing both a put option and the underlying asset. If carry = 0, we can see that a long call option (with strike price \$K) has the exact same payoff at expiration as a long stock position + a long put position (with strike price \$K). Thus, we can create a synthetic call option by purchasing a put & the underlying asset.

$$\mathbf{c - p = S - K + carry}$$

p = K - S - carry + c

Similarly, we can replicate a long put position by shorting the underlying security and purchasing a call option. Assuming the carry = $0, a long put will have the exact same payoff at expiration as a long call and short stock position.

Volatility

As we saw earlier, the only “unknown” variable in the Black Scholes model is the implied volatility of the underlying asset. Thus, options traders are often referred to as volatility traders. If an options trader is *excellent* at predicting the volatility of the underlying assets that he/she trades, then he/she will be successful.

Increased implied volatility always benefits option holders. Again, we know that an option is the right, but not the obligation, to buy/sell the underlying asset at the given strike price. Thus, higher volatility means that there is a higher probability that our option will finish ITM and be more valuable.

Additionally, since implied volatility is the only “unknown” variable in Black Scholes, if we know the price of a call option, we can solve for the implied volatility of the underlying asset.

One important thing to note is that not all options with the same underlying security have the same implied volatility. Intuitively, this does ***not*** make sense. According to the Black Scholes formula, all options with the same underlying asset should have the same implied volatility. However, due to a variety of factors, such as supply and demand, ATM options typically have the lowest IV.

The Option “Greeks”

All options traders need to understand the five main greeks - delta, gamma, theta, vega, and rho. These greeks are a way to measure the sensitivity of an option price to changes in the market. Again, this book is merely an introduction to trading, and much more information can be found online discussing the greeks and their importance in trading.

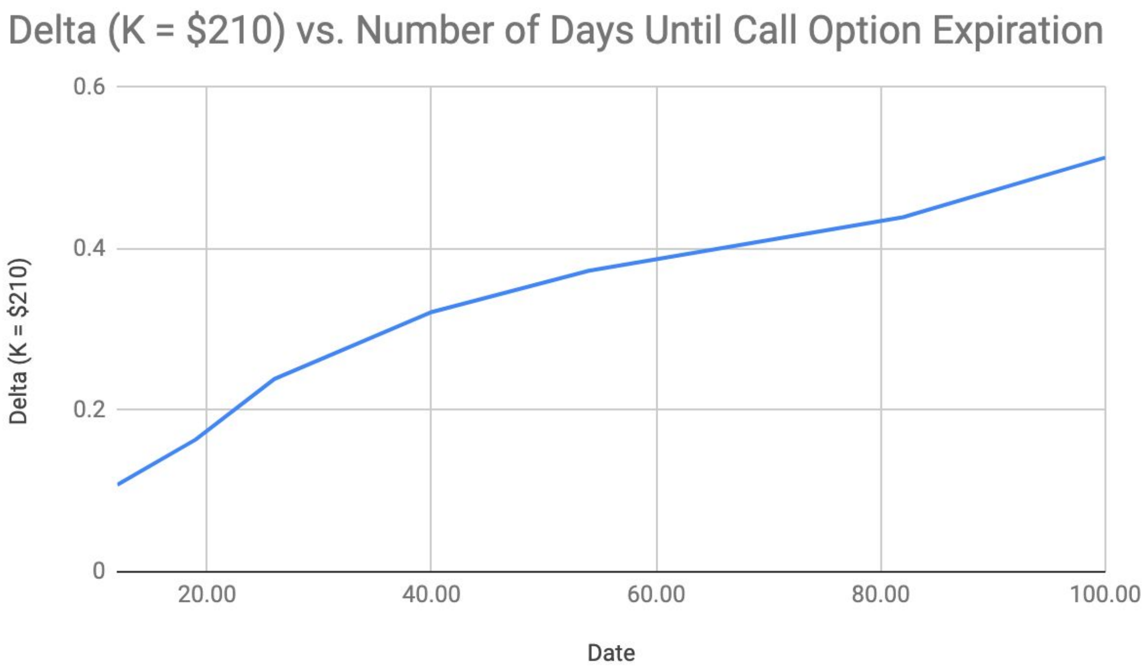
Delta

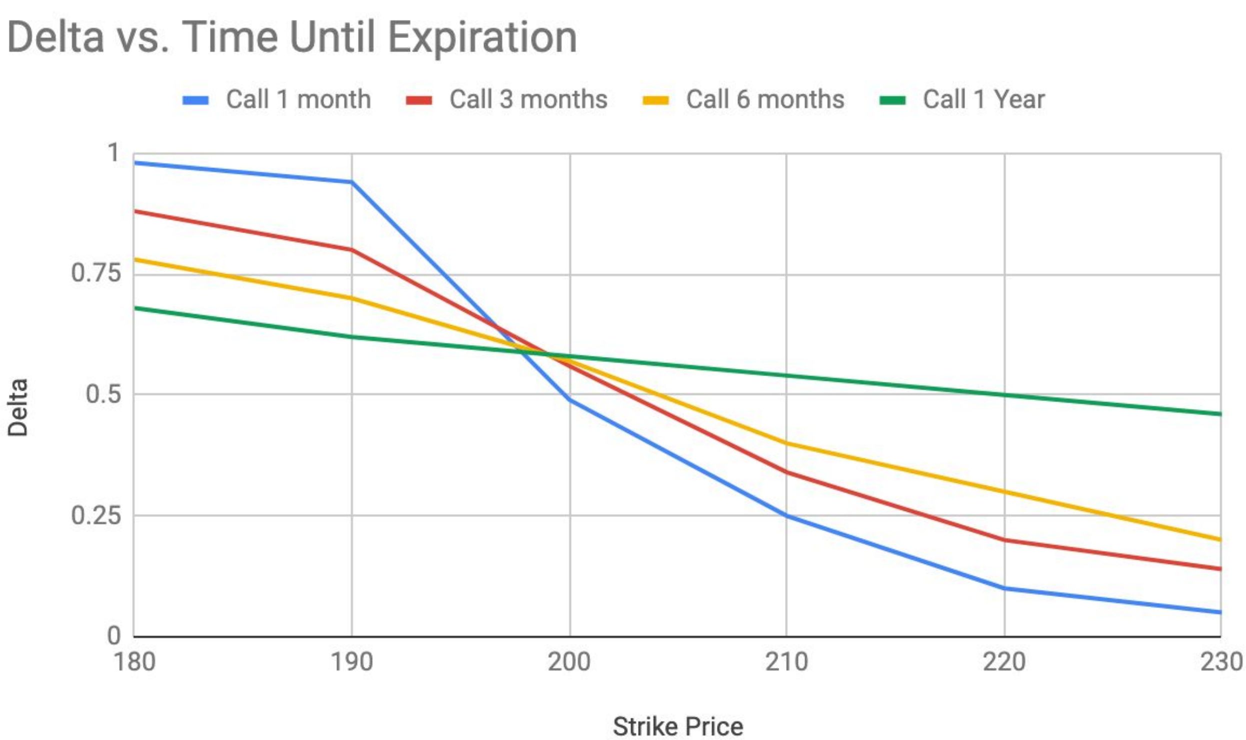
Delta is defined as the rate of change of an option’s price for a $1 move upwards in the underlying asset. For example, if a call option has a delta of 0.6, then a $1 increase in the underlying security will increase the price of this call option by $0.60. Delta is *always*

≥ 0 for call options and ≤ 0 for put options. Intuitively, this should make sense. Call options increase in value as the underlying price increases, whereas put options decrease in value as the underlying price decreases. The absolute value of delta is always between 0 and 1. A call option with a delta close to 1 is much more sensitive to the price of the underlying asset than a call option with a delta close to 0. As the option grows becomes more and more in-the-money, delta moves towards +1, and increases in the price of the option begin to mirror increases in the price of the underlying security.

Many traders think of delta as the probability that the option finishes in-the-money. Although this is not mathematically correct, it is a rough estimate. For example, an option with a delta of 0.5 has approximately a 50% chance of finishing in-the-money at expiration.

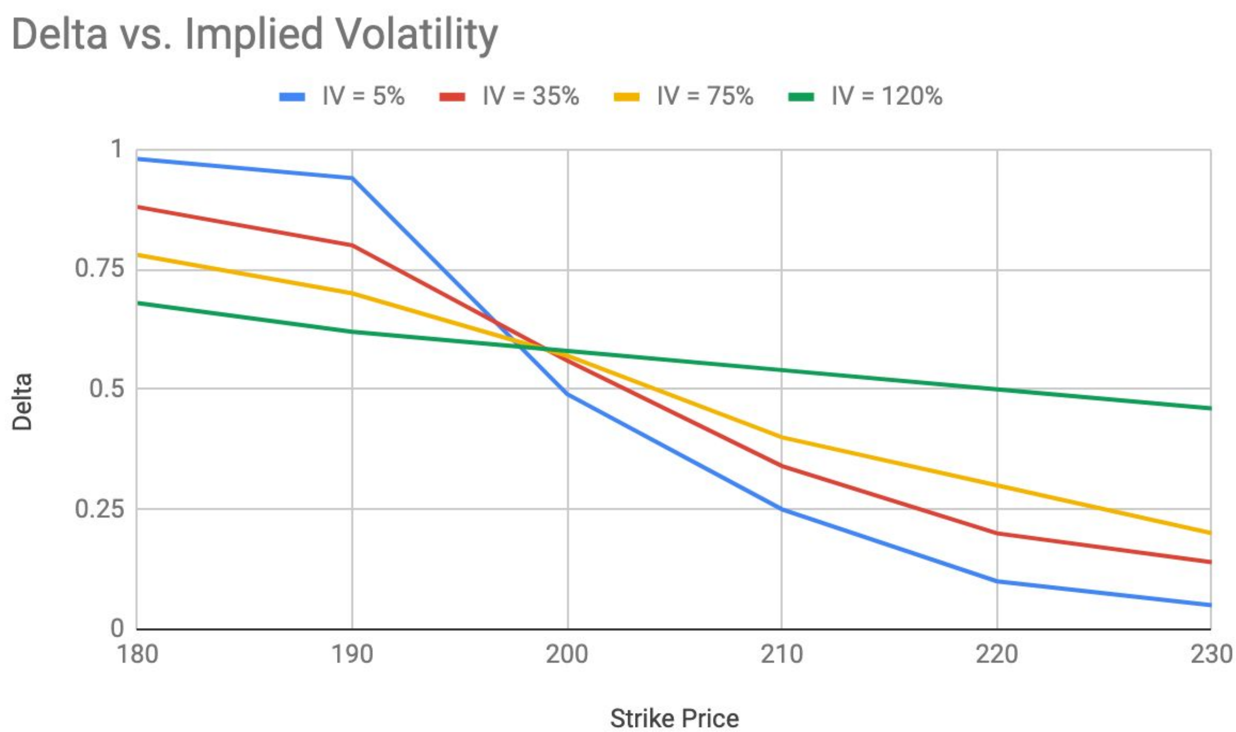
Delta is affected by time to expiration and implied volatility. Below, we can see how the number of days until expiration impacts delta. This chart was made using the option series chain for AAPL. AAPL is currently priced at $198.78 as of 06/21/2019. As we can see, for a call option, as time until expiration increases, the delta of further-dated options moves towards 0.50. Intuitively, this should make sense. If we think about delta as the approximate probability that an option finishes in the money, then more time until expiration means that delta will be approximately 50%. On the other hand, imagine we are 5 minutes until expiration. The delta of a call option with 5 minutes until expiration (with S_{AAPL} = $198.78 and K = $210) will be essentially zero, as the probability that this option finishes ITM is about 0%.





To test the charts above (the relationship between delta and time to expiration), you can visit nasdaq.com to find option greeks for any stock traded on NASDAQ.

Delta also has an interesting relationship with the implied volatility of the underlying asset. Increased implied volatility (IV) has a similar effect to increasing time until expiration. As an example, imagine a stock with 1% IV. Since this stock's price is very stable, if the option is even just a few dollars below the strike price, there is very low probability that the option will finish in-the-money, so the delta will be very low. On the other hand, if a stock has an IV of 100%, then the stock is expected to move much more, pushing the delta of a call towards 0.50. Again, we assume that AAPL is currently priced at $198.78.



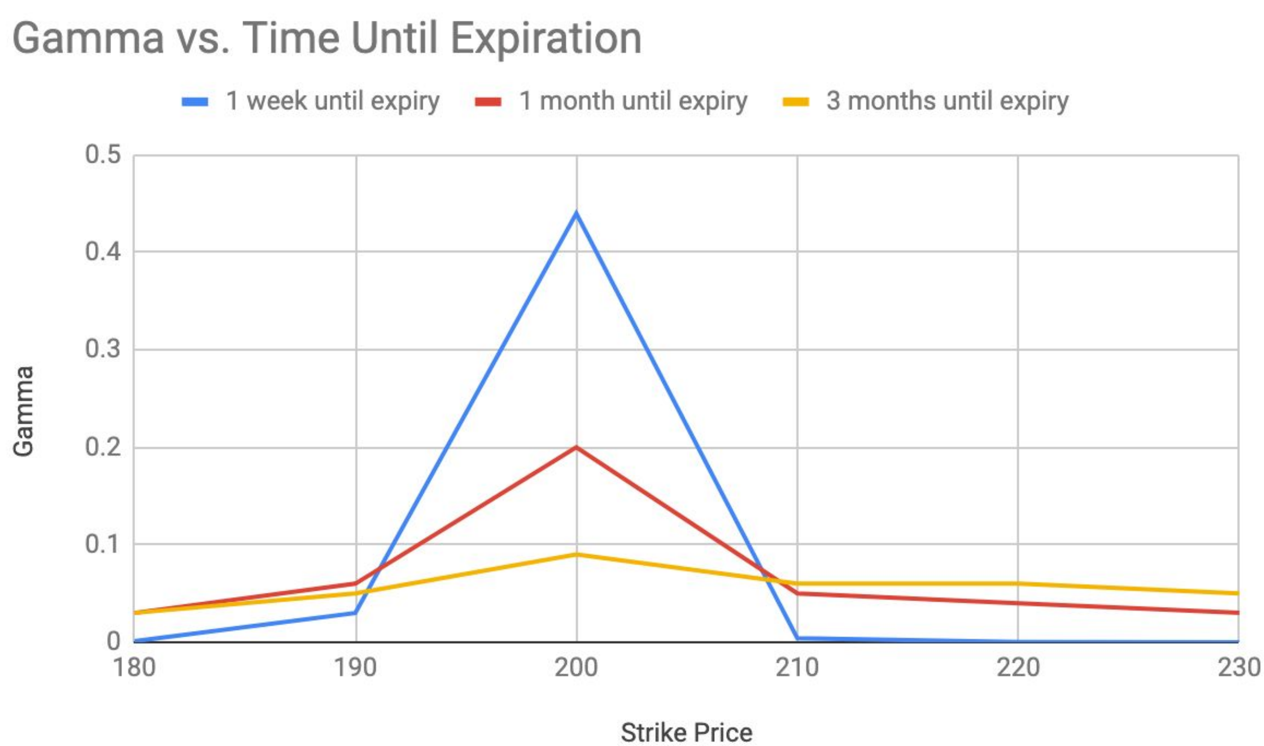
The same situation holds for put options, yet the delta scale ranges from 0 to -1.

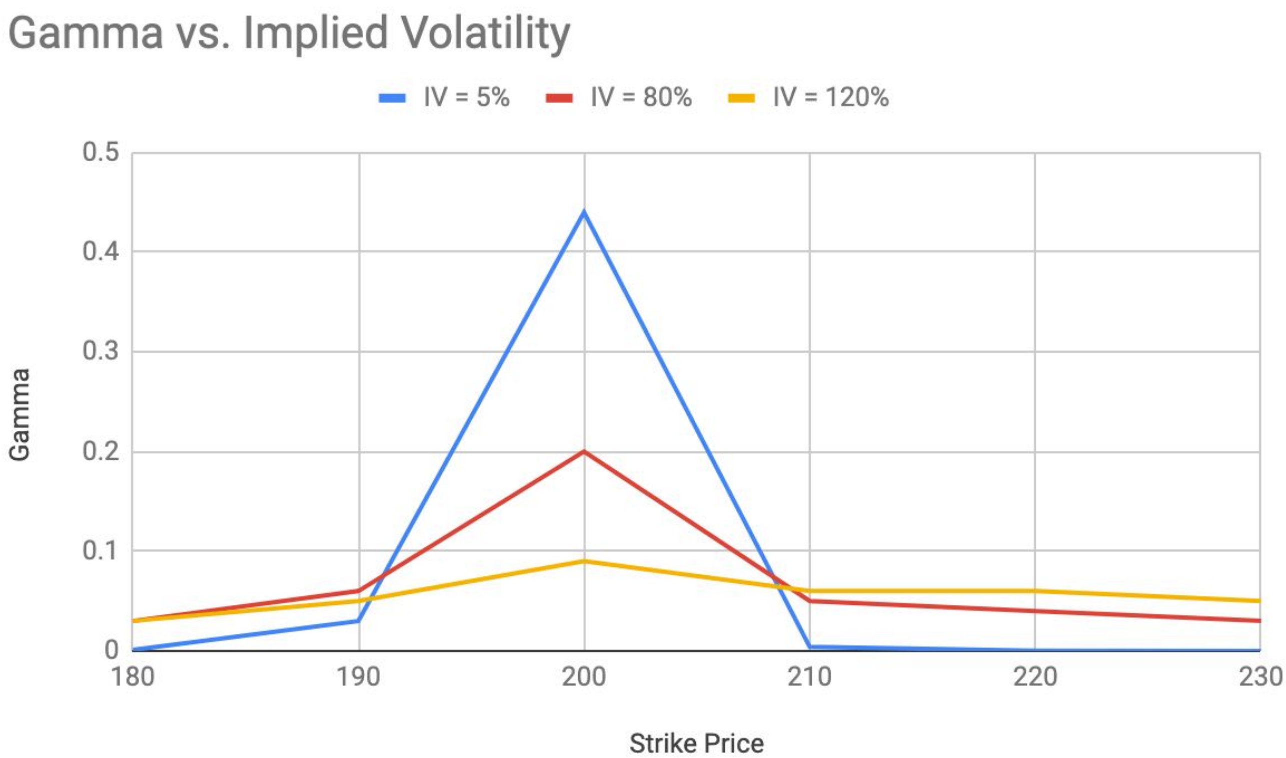
Gamma

Gamma is defined as the rate of change of an option's delta for a $1 move upwards in the underlying asset. Long option positions always have positive gamma, and short option positions always have negative gamma. Gamma is a vital measure of convexity of the option's value with respect to the underlying security.

As an example, imagine an option has a delta of 0.60 and a gamma of 0.06. Assume that the option currently has a value of $5.60. If the price of the underlying increases by $1, then the option is expected to have a value of $6.20. If the underlying increases by another dollar, we expect the option's price to rise by $0.66, as delta will now be 0.66.

Gamma is highest for ATM options. Additionally, as we move closer to expiration, the gamma of ATM options will skyrocket, as even a slight change in the underlying's price can move the option into or out of the money. The relationship between gamma and time until expiration is shown below (assuming S_0 = $198.78). Note that OTM options have a gamma close to zero as expiration nears. This is because, if the option is far OTM, then a change in stock price will not significantly impact delta. Gamma also has an interesting relationship with the implied volatility of the underlying asset. Increased implied volatility (IV) has a similar effect to increasing time until expiration. ATM options with low IV will have the highest gamma.



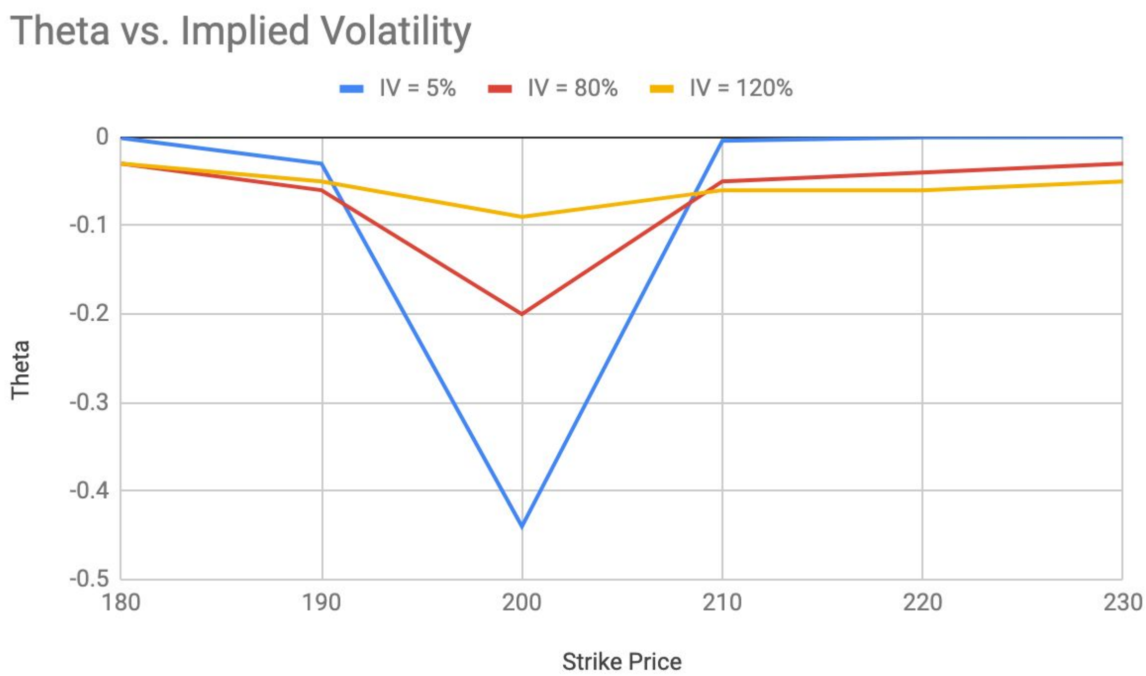


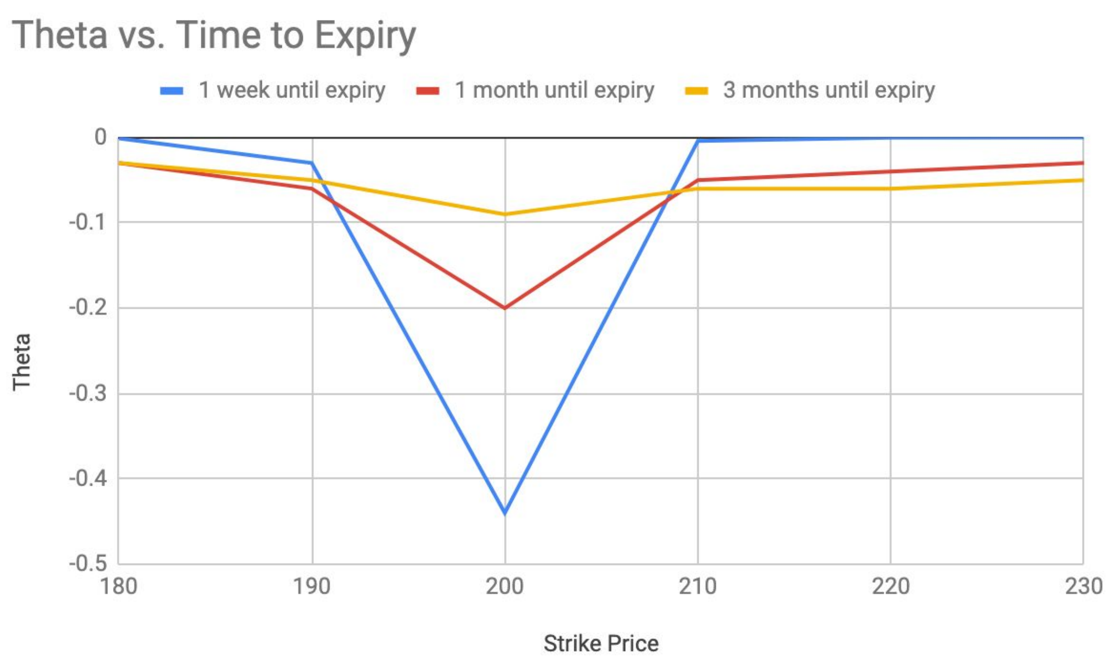
Theta

Theta refers to the time value of an option and represents the amount by which a long option will decrease every day. Increased time always increases the value of an American option. Theta is negative for long puts and long calls, and positive for short option positions. Thus, if we are short an option, we are *collecting* on theta. In other words, each day that passes and the underlying stock does not move, we make money due to positive theta.

Another way to think about theta is that, by expiration, the value of the option must decrease to *only* the intrinsic value of the option. The extrinsic value, or time value, of the option must decrease to zero.

Theta is lowest (or *most negative*) for options that are ATM. The relationship between theta and time until expiration, as well as between theta and implied volatility, is shown in the charts below. Again, we assume S_0 = $198.78. Theta increases for ATM options as time to expiration decreases, and theta also decreases as implied volatility increases.

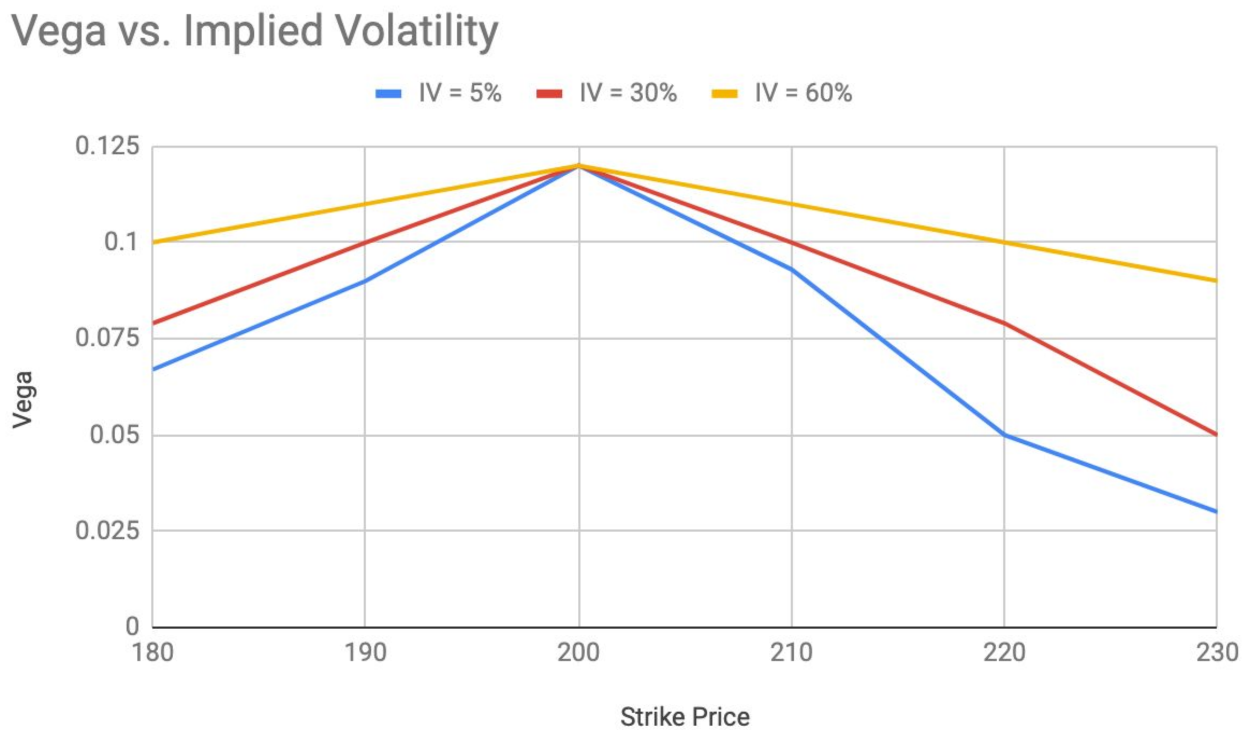




Vega

Vega is defined as the change in the price of an option for a 1 point move in the implied volatility of the underlying asset. Options always become more valuable when implied volatility increases. Vega is positive for long option positions and negative for short option positions. Vega is highest for ATM options, just like gamma. Intuitively, this should make sense. If we have purchased an OTM option that is near worthless, then a change in the underlying security's IV will not impact the price of the option, as there is still a very low probability that the option becomes in-the-money.

The relationship between vega and time to expiration is shown below. Again, we assume the current price of the asset, S_0, is \$198.78. As time to expiration increases, vega increases. Intuitively, this should make sense. Again, we can look at an extreme situation to get a better understanding of the relationship between vega and time to expiration. Imagine we have an option with five minutes until expiration. If implied volatility increases by 1%, the value of the option should not change by much, as the underlying security has very little time to move around. However, if we have a year until expiration, then a 1% increase in implied volatility will increase the price of this option by much more. The relationship between vega and implied volatility is also shown below. As we can see, the maximum value of vega is ATM, yet this value is ***not*** affected by implied volatility.



Overall, vega is a vital greek for traders to understand. If options appear relatively cheap (IV seems low), then traders should look for strategies with positive vega. On the other hand, if IV seems high, traders should look for strategies with negative vega.

Rho

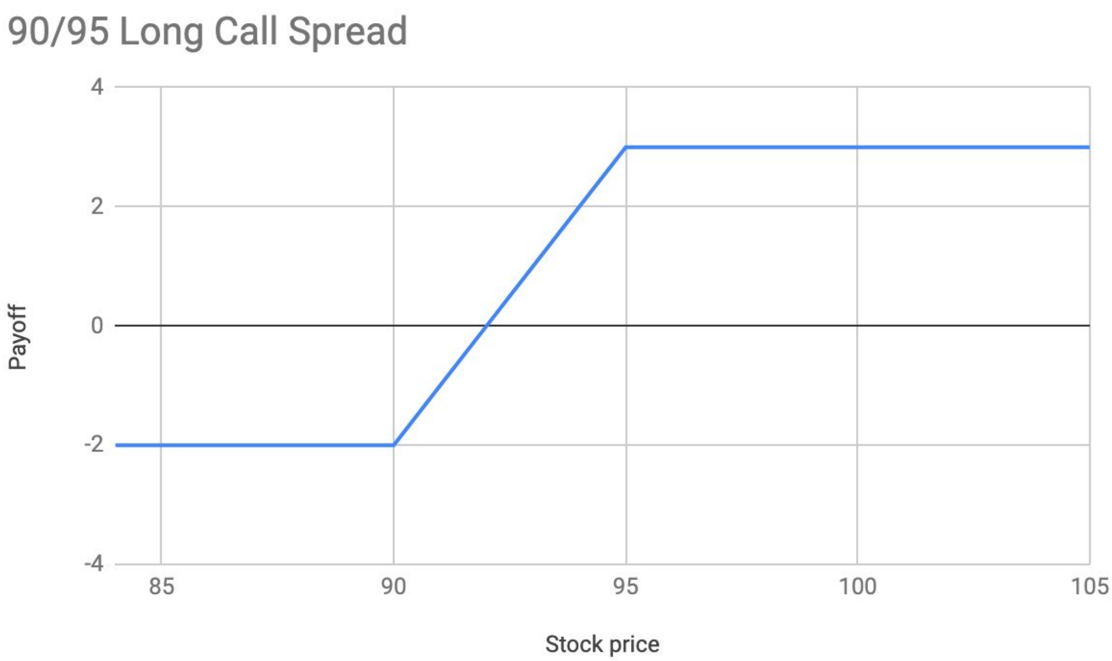
Rho is defined as the change in the price of an option for a 1% change in the risk-free interest rate. Although rho is arguably the least important greek, it is still important to know, especially in a volatilite interest rate environment. For long call options and short put options, rho is positive, as higher interest rates increase the price of call options and decrease the price of put options.

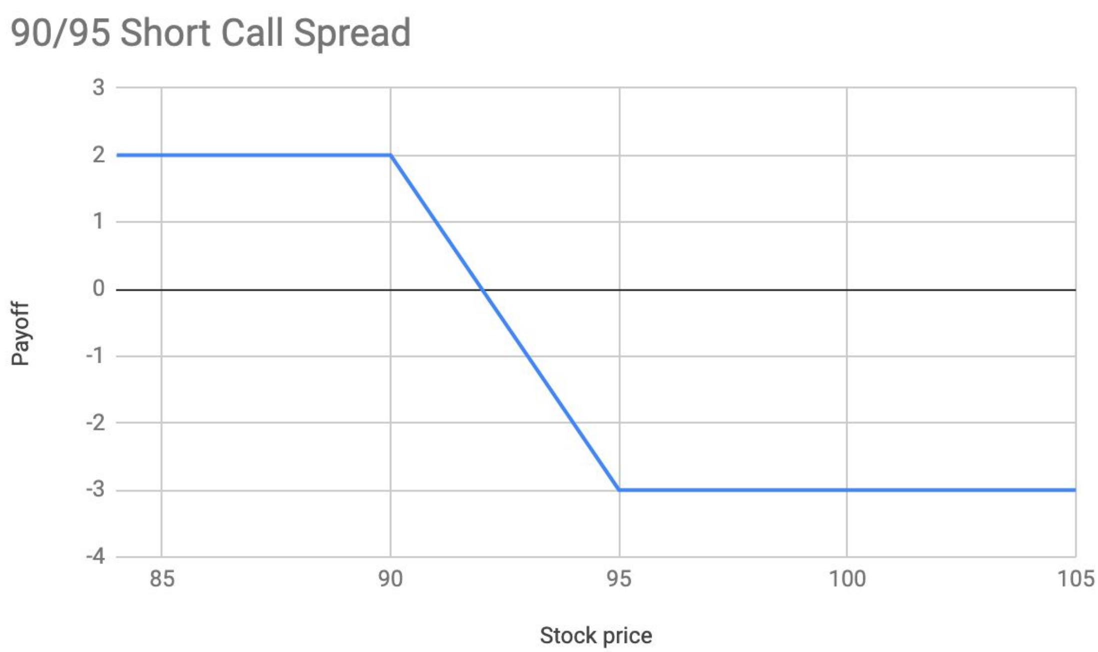
Vertical Spreads

A long call vertical spread is a trade with one long call option at K_1 and one short call option at K_2, where $K_2 > K_1$. The price of a long call vertical spread is always greater than or equal to zero. Similarly, a long put vertical spread is a trade with one long put option at K_1 and one short put option at K_2, where $K_2 < K_1$. The price of a long put vertical spread is also always greater than or equal to zero.

Vertical spreads are *very common* in financial markets. Long & short vertical spreads allow a trader to take an opinion on the direction of the underlying security, yet these trades are less risky than purchasing or selling an option outright, as there is limited profit & loss. For example, if we sell a call vertical spread, the maximum amount of money we can lose is $K_2 - K_1$, whereas if we sell a call option outright, we have unlimited risk, as the price of the underlying asset could theoretically increase to infinity.

The payoff for a 90/95 long call spread is shown below. Again, we are long the call option with K = $90, and short the call option with K = $95. In this payoff diagram, we assume we paid $2 to enter into this call spread. The payoff diagram for a short call vertical spread is also shown below.





We can also create the payoff diagrams for a long put vertical spread. Again, we assume we pay $2 to enter this trade.

Although vertical spreads seem like relatively simple trades, a few factors need to be taken into account. When we enter a vertical spread, we should have an opinion on the direction and implied volatility of the underlying asset. Long call verticals *always* have positive delta. This is because the option we buy with the lower strike price will *always* have a delta greater than the option we sell with the higher strike price. Similarly, long put verticals always have negative delta. Thus, if we have a bullish view on the underlying asset, we could either short a put vertical or purchase a long call vertical. Both of these strategies have positive delta, meaning we make money when the price of the underlying asset increases. If we are low on cash, then selling a put vertical may be the better trade, as we will receive money, or a *credit*, for entering the trade.

Note, the delta for a call vertical can still be close to zero. For example, imagine a stock is currently trading at S = \$200. If we purchase a deep ITM call vertical, buying the K = \$150 call (say, with delta = 0.99) and selling the K = \$160 call (say, with delta = 0.98), our position delta for this trade may be only 0.01. On the other hand, if we purchase the 190/210 call vertical, the delta of this trade could be close to 1 if we are close to expiration.

Interestingly, the gamma, vega, and theta of a long call vertical can be either positive or negative. Remember, gamma and vega are highest for ATM options, and theta is lowest (*most negative*) for ATM options. Thus, if S = \$200, the 200/205 call spread (long

$C_{K=\$200}$, short $C_{K=\$205}$) would be positive vega, positive gamma, and negative theta. These greeks indicate that we *want* the price of the underlying security to move. Each day that the stock stays put at S = $200, we are losing on our trade due to theta. On the other hand, if S = $200, and we purchase the 195/200 call spread, then vega and gamma are both negative, yet theta is positive, indicating that we want IV to drop. Each day that passes with the stock price staying close to $200, we are winning on our trade due to positive theta.

Remember, if our opinion is that IV is too low, we want to focus on trades with positive vega. Thus, we could buy vertical spreads that purchase the ATM option, as the ATM option will have the highest vega. On the other hand, if our opinion is that IV is too high, we should focus on selling the ATM option in our vertical spreads.

Thus, when purchasing or selling a vertical spread, options traders must consider a variety of factors. For example, imagine a scenario where we want to buy/sell a vertical spread on a stock trading at S = $200. If we think a stock is due to increase in price, yet IV is due to decrease, then we want to enter a trade with positive delta and negative vega. Thus, we may purchase the 195/200 call spread or sell the 195/200 put spread. Both of these trades have positive delta and negative vega.

Calendar Spreads (Time Spreads)
Calendar spreads are another common trade. In a long calendar spread, one option is bought with a later expiration date, and another option is sold with a shorter expiration date. Both options will have the same underlying security and strike price.

Long time spreads *always* have positive vega, and short time spreads have negative vega. Thus, long time spreads typically are used to express an opinion on the IV of the underlying asset. Long time spreads, however, can have positive or negative gamma, delta and theta.

Long time spreads are *always* most valuable when the current price of the underlying asset is equal to the strike price of the options. Thus, if S = $100, *both* a long call time spread and a long put time spread will have positive delta if K = $110. Similarly, *both* a long call time spread and a long put time spread will have negative delta if K = $90, as seen in the table below (October = closer expiry, December = further expiry).

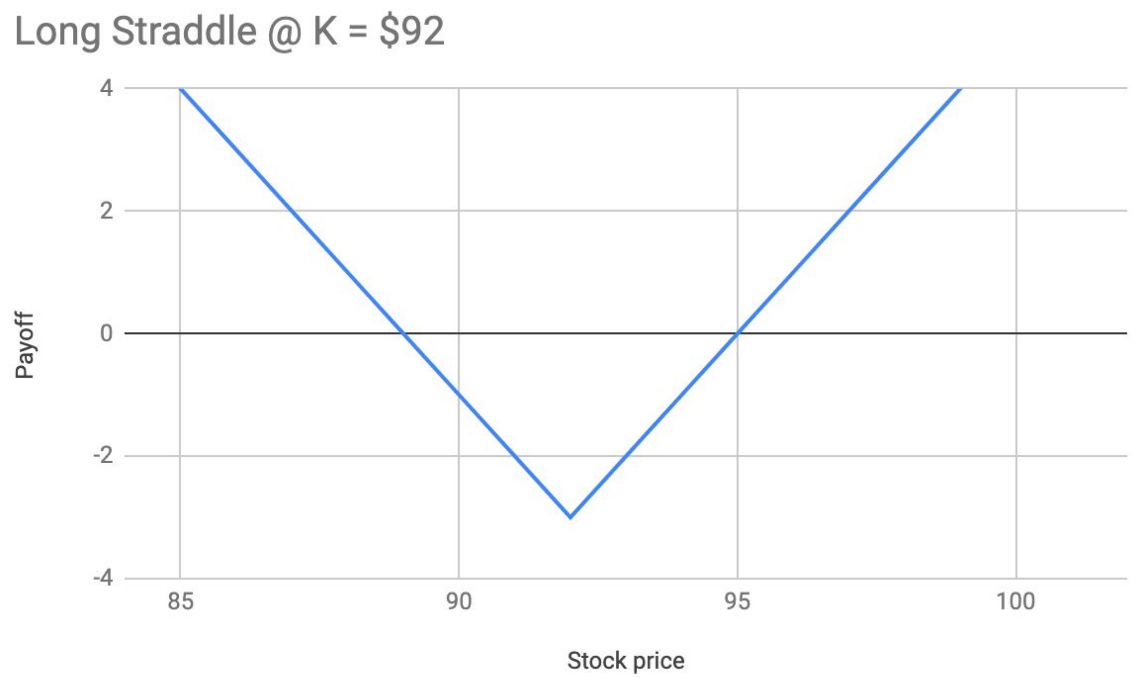
	$C_{October}$	$C_{December}$	$P_{October}$	$P_{December}$	Long call calendar	Long put calendar
Delta @ K = $110	0.23	0.44	-0.77	-0.56	0.44-0.23 = 0.21	-0.56-(-0.77) = 0.21
Delta @ K = $90	0.78	0.55	-0.22	-0.45	0.55-0.78 = -0.23	-0.45-(-0.22)= -0.23

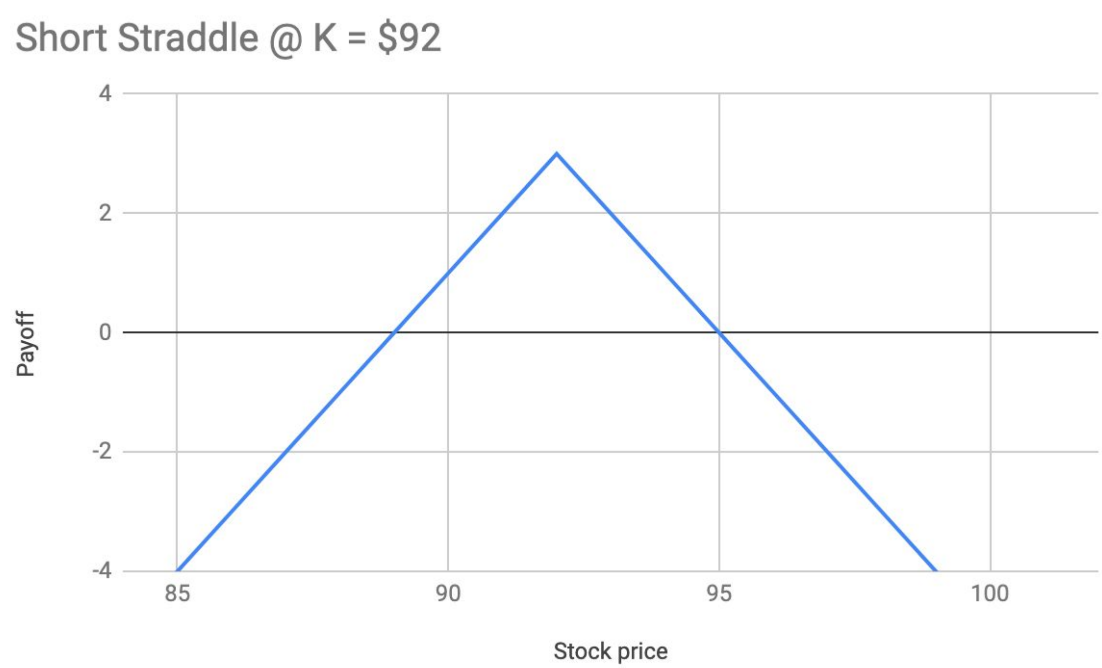
Thus, if we think the price of the underlying security is going to decrease, we should purchase an OTM call time spread (with $K < S$) or an ITM put time spread (with $K < S$). Similarly, if we think the price of the underlying security is going to increase, we should purchase an ITM call time spread (with $K > S$) or an OTM put time spread (with $K > S$).

Straddles

To create straddle, a trader purchases both a call and put option on the same underlying security with the same strike price. Straddles are positive vega, positive gamma, negative theta trades. The delta of a straddle can be either negative or positive, depending on which strike price is chosen. Typically, straddles are purchased ATM, so the delta of the trade is close to zero (ATM put = -0.5 delta, ATM call = +0.5 delta).

The payoff diagram for a long ATM (both K & S = $92) straddle is shown below. We assume that we paid $3 to enter this position. Thus, to make money on this trade, we need $S_e > 95$ or $S_e < \$89$ at expiration. In other words, we need a move greater than $3 in either direction by expiration. Note that a short straddle is a *very* risky trade. We are exposed to unlimited risk if either the stock price plummets or skyrockets. Thus, beginners should *never* trade short straddles.



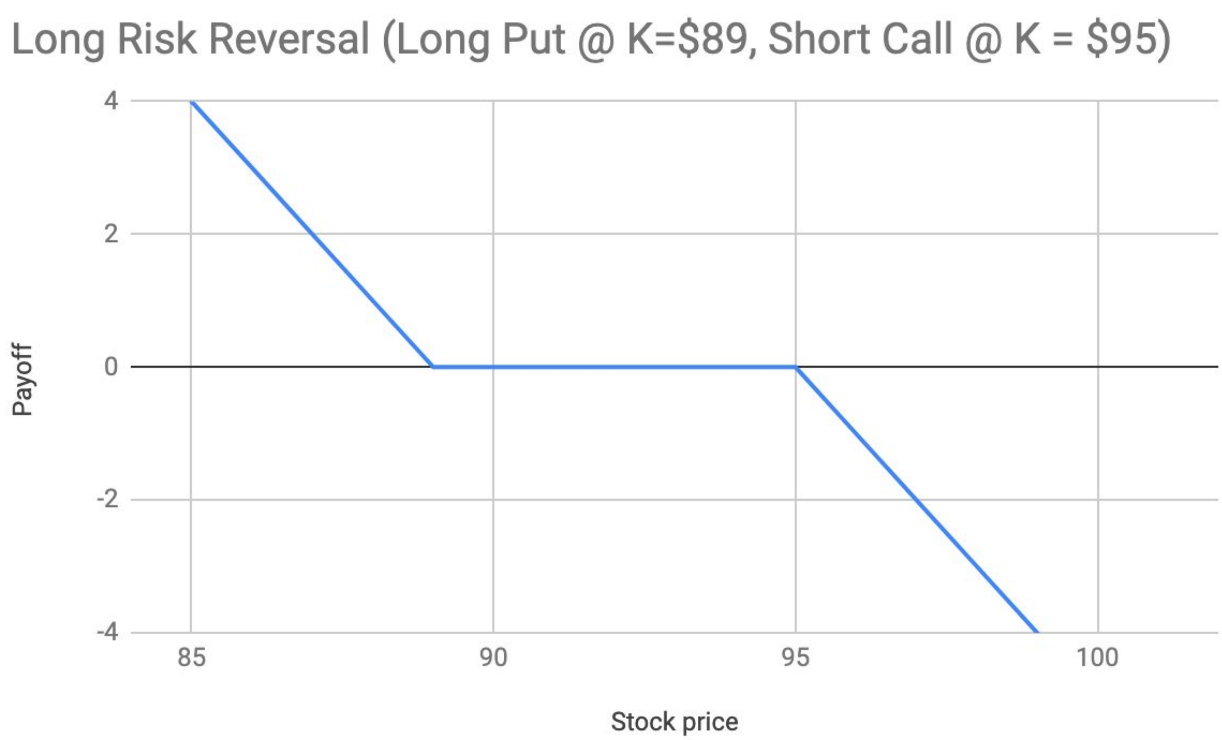


Imagine that a stock is currently trading at $100 currently. Imagine that we also know the following straddle prices. Is there any way we can profit?

Price Straddle w/ K = $95	Price Straddle w/ K = $100	Price Straddle w/ K = $105
$5.60	$4.90	$4.80

The answer is *yes*. The ATM straddle should *never* be worth more than an OTM straddle. Thus, we should sell the straddle at K = $100 and purchase the straddle at K = $105. We could also simply purchase the K = $105 straddle outright. Since S = $100, we could exercise our put for an immediate profit of $5 and hold onto our long call option.

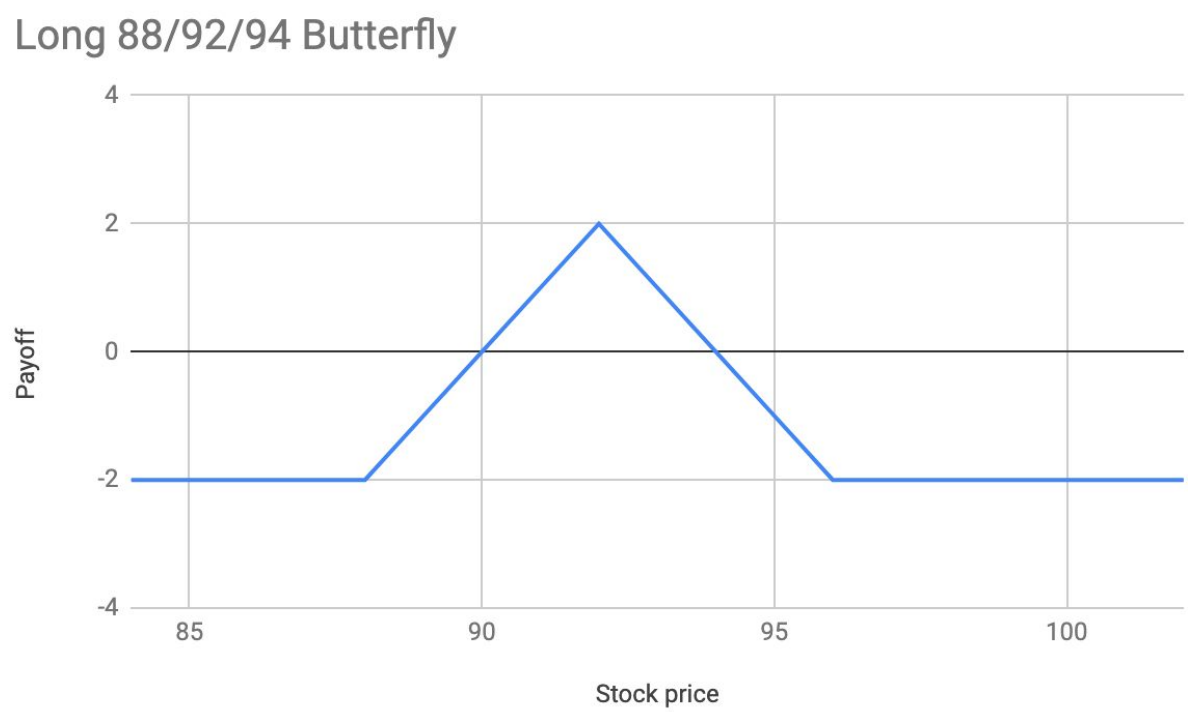
Risk Reversal
A risk reversal involves buying an OTM put and selling an OTM call. Risk reversals are a negative delta trade. The payoff for the 89/95 risk reversal is shown below. We can assume that this trade cost us $0 to enter, or $C_{K=\$95} = P_{K = \$89}$.



An interesting aspect to note is the relationship between combos, risk reversals, and call spreads. For example, if we purchase the combo (short put, long call) with K = $89 and the 89/95 risk reversal, our position would be equivalent to a 89/95 long call vertical spread. I recommend trying the payoff diagrams as an exercise, and you will see that these positions are equivalent.

Butterflies

A butterfly is another common type of trade. This trade has limited risk and limited reward, so many new traders start by trading butterflies. Typically, in a butterfly, one ITM call is purchased, two ATM calls are sold, and one OTM call is purchased. Assuming S_0 = $92, the payoff diagram for a long 88/92/94 call butterfly is shown below, and we assume this trade costs us $2 to enter.



In a long butterfly trade where our short options are ATM, our position has negative vega, positive theta, and negative gamma. Thus, each day that the underlying stock does not move, we collect on theta. Increased IV will hurt our long butterfly, as we need the underlying stock to stay within a specific price range to profit on the trade.

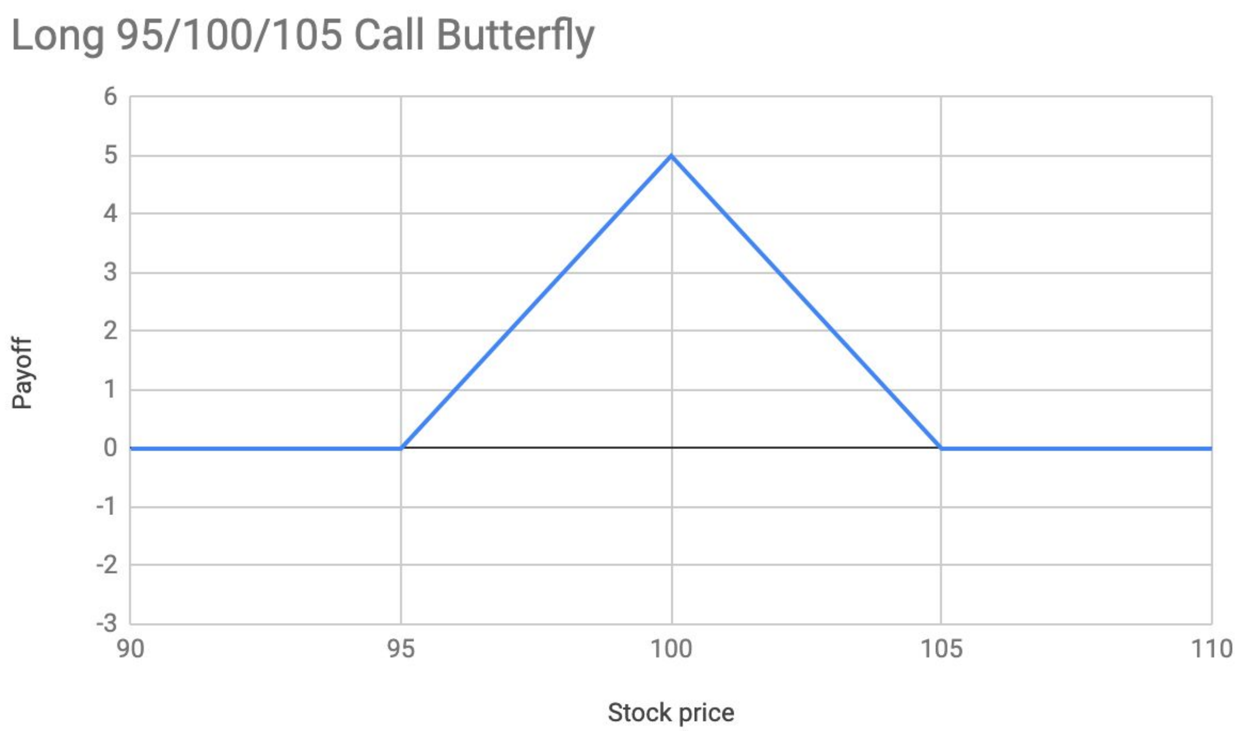
If the price of the underlying stock moves towards the outer strikes in our butterfly, gamma becomes positive and theta becomes negative, indicating the desire for the price of the underlying asset to move back to the middle strike price.

An interesting point is that a butterfly is equivalent to long a call vertical spread (purchase call @ K_1, sell call @ K_2), and short another call vertical spread (purchase call

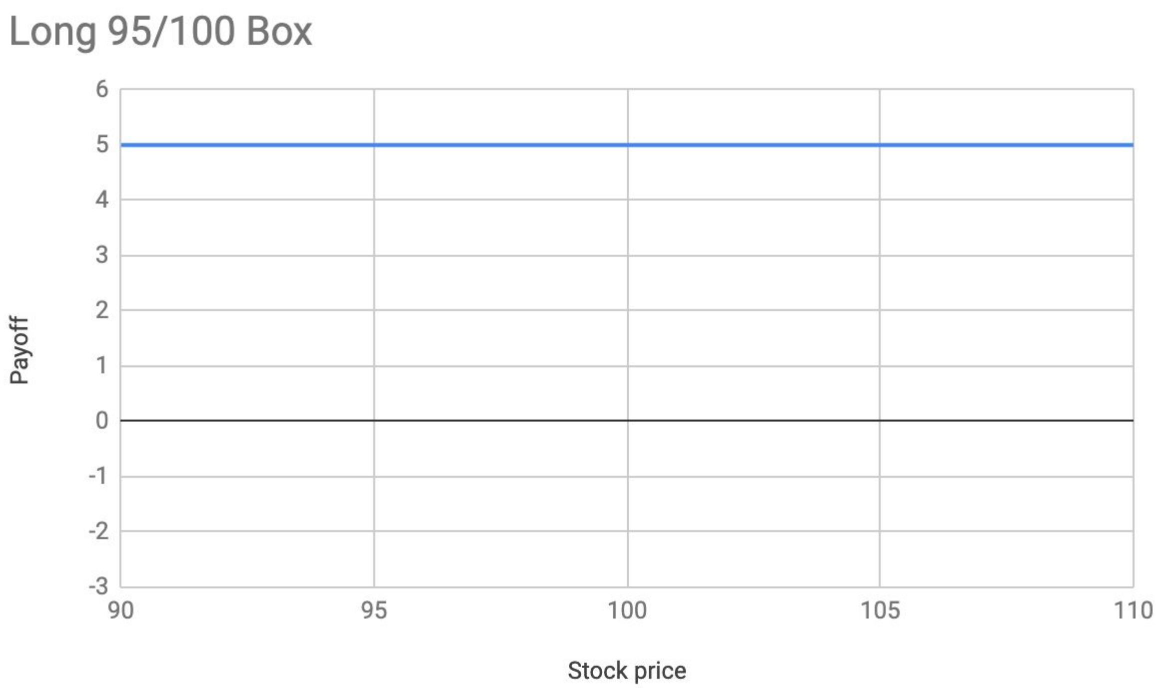
@ K_3, sell call @ K_2) where $K_1 < K_2 < K_3$. Knowing this information, is there any riskless trade we could enter using the information in the following table (assume S_0 = $100)?

$C_{K = \$95}$	$C_{K = \$100}$	$C_{K = \$105}$
Price = $5.80	Price = $3.80	Price = $1.80

The answer is *yes*. As we can see, the 95/100 call spread is worth *the same* as the 100/105 call spread. Thus, we can enter the 95/100/105 butterfly for $0, leading to the following payoff diagram.



Boxes
A long box consists of a long combo at K_1 and a short combo at K_2 where $K_2 > K_1$. Both combos have the same expiration date and the same underlying security. Since we are both long and short a combo with two different strike prices, the value of a box is $PV(K_2\text{-}K_1)$. As an exercise, I recommend that you write the payoff of a box to see why this payoff structure exists.



An interesting point is that a box is equivalent to buying a long call vertical spread and a long put vertical spread with the same strike prices.

Jelly Rolls
A long jelly roll is short a combo in a closer expiration date, and long a combo in a further expiration date. Both combos have the same strike price and the same underlying security. The value of a jelly roll is equal to the carry between the shorter expiration and further expiration. This is because, at the shorter expiration date, we will receive +$K in exchange for the underlying security. We will invest +$K at r_f, and our short position in the underlying security will cancel at the further expiration date, as we will either exercise our long call option or be assigned on our short put option. An interesting point is that a jelly roll is equal to a long call time spread combined with a short put spread.

Other types of trades
In this e-book, we've already discussed a few of the main trades that are used in financial markets: vertical spreads, time (calendar) spreads, straddles, etc. There are *hundreds* of types of trades not covered in this e-book, however, including strangles, iron condors, diagonal spreads, ratio spreads, etc. A wide variety of trades can be formed using combinations of calls & puts, so feel free to think about other types of trades that could be implemented in your own trading account!

Early Exercise of American Options
As we stated previously, American options are *always* worth at least as much as an equivalent European option. In fact, American options are typically worth more because these options are subject to early exercise.

The only time it makes sense to exercise a call option early is the day before the stock goes ex-dividend. Remember, dividends are benefits to those owning the stock, not call options. Thus, if the dividend is worth more than the combined value of the extrinsic value of the option and the carry, then the option should be exercised early to receive the dividend.

On the other hand, American put options are often exercised after the ex-dividend date so the trader does ***not*** need to pay out the dividend. Remember, if we are short stock, we need to pay out the dividend. American put options are typically exercised early in high interest rate environments, as traders can interest on their short stock positions. A trader should exercise an American put option early when the carry cost of an ITM put is greater than the extrinsic value of the put option.