MATH 287A Winter 2018 Final exam —due Monday March 19 by 2pm. Leave it in a sealed envelope in Prof. Politis' mailbox but KEEP A COPY FOR YOUR RECORS!

You may use your textbook, notes and calculator but do not collaborate with anybody on this exam. All eight problems have equal weight.

1. Give an example of a dependent, white noise sequence. Be sure to prove that your example is white (uncorrelated) but not independent.

2. Do problem 5.20 of Brockwell and Davis.

3. Do problem 5.25 of Brockwell and Davis. For simplicity, assume that the autocovariance function is absolutely summable and strictly positive definite; hence, the spectral density $f(w)$ exists, and it is continuous and strictly positive for all w.

4. Let $|c| < 1$ and consider the matrix identity:

$$\begin{bmatrix} 1 & c & c^2 & \dots & c^{n-2} & c^{n-1} \\ c & 1 & c & \dots & c^{n-3} & c^{n-2} \\ c^2 & c & 1 & \dots & c^{n-4} & c^{n-3} \\ & \vdots & & \ddots & & \vdots \\ c^{n-2} & c^{n-3} & c^{n-4} & \dots & 1 & c \\ c^{n-1} & c^{n-2} & c^{n-3} & \dots & c & 1 \end{bmatrix}^{-1} =$$

$$= \frac{1}{1-c^2} \begin{bmatrix} 1 & -c & 0 & \dots & 0 & 0 & 0 \\ -c & 1+c^2 & -c & \dots & 0 & 0 & 0 \\ 0 & -c & 1+c^2 & \dots & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & \dots & 1+c^2 & -c & 0 \\ 0 & 0 & 0 & \dots & -c & 1+c^2 & -c \\ 0 & 0 & 0 & \dots & 0 & -c & 1 \end{bmatrix}$$

   How can you explain this matrix identity in light of what we know about the autocovariance and inverse autocovariance of a stationary sequence? Can you tell what the stationary sequence in question should be just by looking at this matrix formula? (Hint: you may treat the 2nd matrix as approximately Toeplitz).

5. In view of the above matrix identity, compute the BLUE (best linear unbiased estimator) of $\mu = EX_t$ based on data $X_1, \dots, X_n$ satisfying the AR(1) model: $X_t - \mu = \rho(X_{t-1} - \mu) + Z_t$ where $Z_t \sim$iid (0,1) and $|\rho| < 1$. (Hint: the form of BLUE is given in problem 7.2 of the book).

6. Do problem 7.1 of Brockwell and Davis.

7. Do problem 7.4 of Brockwell and Davis. [Note: eq. (7.2.5) in the book (and the equation for W appearing in Theorem 7.2.1) are often called 'Bartlett's formula'; these expressions only hold true for *linear* time series!]

8. Do problem 7.11 of Brockwell and Davis. [Hint: verify the identity $\sum_{k=-\infty}^{\infty} \hat{\gamma}(k) e^{ikw} = n^{-1} |\sum_{t=1}^{n} (X_t - \bar{X}) e^{itw}|^2$ which can also be used to prove problem 7.3 in the book.]