

**Math 220C, Problem Set 6. Due Friday, May 7.**

*For this problem set, you may assume both Little and Great Picard.*

- 1.** If  $h$  is meromorphic in  $\mathbb{C}$ , and omits three values then  $h$  is constant.

*Hint: If  $h$  omits  $a, b, c$ , what values does  $\frac{1}{h-a}$  omit?*

- 2.** Let  $n \geq 3$ . If  $f, g$  are entire such that  $f^n + g^n = 1$ , show that  $f, g$  are constant.

*Hint: Find  $n$  values that  $f/g$  omits.*

- 3.** Let  $f, g$  be two nonconstant entire functions,  $P, Q$  two nonconstant polynomials such that

$$e^f + P = e^g + Q.$$

Show that  $P = Q$ .

*Hint: Consider  $P - Q = e^g(1 - e^{f-g})$  and examine  $1 - e^{f-g}$ .*

- 4.** If  $h$  is a nonconstant polynomial and  $f$  is a nonconstant entire function, show that  $he^f$  does not omit any values.

- 5.** Let  $f$  be entire such that  $f \circ f$  has no fixed points. Show that  $f(z) = z + a$  for some  $a$ .

*Hint: Let  $g(z) = \frac{f(f(z)) - z}{f(z) - z}$ . Show that  $g$  omits the values 0 and 1, hence it is constant. Taking derivatives in  $f(f(z)) - z = c(f(z) - z)$ , show that  $f' \circ f$  omits two values. Show  $f'$  is constant and conclude.*