HOMEWORK 6

DUE MAY 12-19, 2021 AT 11:59PM

Note: This is material that is part of the qual.

1. Let $R = \mathbb{Z}[\sqrt{-6}] = \{a + b\sqrt{-6}; a, b, \in \mathbb{Z}\}$. Let $I = (2, \sqrt{-6})$ be the ideal of R generated by 2 and $\sqrt{-6}$.

(a) Show that I is not a free R-module.

(b) Show that I is a projective R-module.

2. Show that

(a) $R[X] \otimes_R R[Y] \simeq R[X, Y]$ as R-algebras.

(b) $R/I \otimes_R R/J \simeq R/(I + J)$ for any two ideals I, J of R.

From Atiyah-MacDonald:

Chapter 2: 11

Chapter 3: 5, 12, 13, 14, 15