MATH 287C HomeWork

1. Use some statistical software (e.g., R) to generate data of sample size n from an AR(1) and an MA(1) with different parameter values. Plot the estimated ACF for lag up to n and note the regions where the estimate is reliable. Try n=100, 500 and 1000.

2. Generate your own time series of length $n = 200$ from a Gaussian AR(1) model with AR coefficient 0.75 and error variance one.

   (a) Compute and plot estimates of the autocorrelations up to lag 25.

   (b) Compute and plot estimates of the partial autocorrelations up to lag 25. Do you observe a cut-off point?

   (c) Fit AR(p) models for $p = 1, 2, 3$; use three different methods: Yule-Walker, MLE and LS. Are they similar? Give the estimates of the AR coefficients and those of the error variance; are the estimates of error variance decreasing as p increases? Is this expected?

   (d) Also test the statistical significance of each estimated AR coefficient (at level 0.05) using the limiting normal distribution.

3. Generate your own time series of length $n = 200$ from a Gaussian MA(1) model with $\theta_1 = -0.95$ and error variance one. Fit an AR(p) model to your data using the AIC to pick the order p (do not assume you know it). Repeat using the BIC, and compare. Which of the two model selection procedures would you prefer in practice?

4. From Ch. 8 of Brockwell and Davis, do problems: 7, 8, 10, 12.

5. Let $\Phi_X(w)$ be the DFT of data $X_1, ..., X_n$, where w is one of the Fourier frequencies, i.e., let $\Phi_X(w) = (2\pi n)^{-1/2} \sum_{t=1}^{n} X_t e^{itw}$, and let $I_X(w)$ be the periodogram, i.e., $I_X(w) = |\Phi_X(w)|^2$.

   Assume for simplicity the MA(1) model: $X_t = Z_t + \theta_1 Z_{t-1}$ where $Z_t \sim$ i.i.d. $(0,\sigma^2)$. [A similar argument holds for MA(q) and *linear* time series.]

   a. Let $\theta(z) = 1 + \theta_1 z$, and show that $\Phi_X(w) = \theta(e^{iw})\Phi_Z(w) + R_n(w)$ where, as $n \to \infty$, $R_n(w)$ converges to zero in probability uniformly in w, i.e., $R_n(w) = o_P(1)$ where the $o_P(1)$ does not depend on w.

   b. Assume $Z_t^4 < \infty$ and use part (a) to show $I_X(w) = |\theta(e^{iw})|^2 I_Z(w) + r_n(w)$ where $r_n(w) = o_P(1)$, and $I_Z(w)$ is the periodogram associated with the inputs $Z_1, \ldots, Z_n$.

   c. Assuming $Z_t^4 < \infty$, the assertion of part (b) can be strengthened to $Er_n^2(w) = o(1)$, i.e., $r_n(w)$ converges to zero in Mean Square. Use the strengthened result to show that $I_X(w_j)$ and $I_X(w_k)$ are asymptotically uncorrelated if $j \neq k$ and $w_k = 2\pi k/n$. [Hint: use the decorrelation properties of the DFT—see Proposition 10.1.1 of Brockwell and Davis.]