Homework due Thursday, May 7, 9:00 pm, on Gradescope.
Throughout, A is a ring (commutative with 1).

(1) Problem 16 page 45 of Atiyah-MacDonald.

(2) Let $\mathfrak{a}$ be decomposable ideal, and let $\mathfrak{a} = \bigcap_{i=1}^{n} \mathfrak{q}_i$ be a minimal decomposition. Then
   (a) $V(\mathfrak{a}) = V(\mathfrak{p}_1) \cup \cdots V(\mathfrak{p}_n)$ where $\mathfrak{p}_i = \sqrt{\mathfrak{q}_i}$.
   (b) Let $\{\mathfrak{p}_{i_1}, \ldots, \mathfrak{p}_{i_m}\} \subset \{\mathfrak{p}_1, \ldots, \mathfrak{p}_n\}$ be the set of isolated primes associated to $\mathfrak{a}$. Then $V(\mathfrak{a}) = V(\mathfrak{p}_{i_1}) \cup \cdots V(\mathfrak{p}_{i_m})$.
   (c) Show that $X = \mathrm{Spec}(A)$ is irreducible if and only if $\mathrm{Nil}(A)$ is a prime ideal.
   (d) With the notation as above, show that $V(\mathfrak{p}_{i_j})$ for $j = 1, \ldots, m$ are irreducible components of $V(\mathfrak{a})$.

   [Recall that a topological space X is called irreducible if $X \neq \emptyset$ and every two open subset of X intersect. In other words if any open subset of X is dense in X. The maximal irreducible subspaces of X are called irreducible components of X.]

(3) Problem 10, page 55 of Atiyah-MacDonald.

(4) Problem 22, page 58 of Atiyah-MacDonald.

(5) Problem 1, page 84 of Atiyah-MacDonald.

(6) Problem 9, page 85 of Atiyah-MacDonald.