Math 240B: Real Analysis, Winter 2020

Homework Assignment 7

Due Wednesday, March 4, 2020

1. Prove that a Hausdorff space is normal if and only if the conclusion of Urysohn's Lemma is true.

2. Prove that the class of open half-lines generate the topology of $\mathbb{R}$.

3. Prove that a topological space X is Hausdorff if and only if every net in X converges to at most one point.

4. Assume X has the weak topology generated by a family $\mathcal{F}$ of functions. Prove that $\langle x_\alpha \rangle$ converges to $x \in X$ if and only if $\langle f(x_\alpha) \rangle$ converges to $f(x)$ for all $f \in \mathcal{F}$.

5. Prove that any closed subset of a compact topological space is compact.

6. Prove that any sequentially compact topological space is countably compact.

7. Let X be a countably compact topological space. Prove the following:
   (1) Any sequence in X has a cluster point;
   (2) If in addition X is also first countable, then X is sequentially compact.

8. Let X and Y be two topological spaces and $f : X \to Y$ continuous. Assume X is countably compact. Prove that $f(X)$ is also countably compact.