

Homework 1, Math 181A Winter 2021

Due by Saturday midnight, Jan 9 (pacific time). This change applies to students in Section B and homework 1 only, since there was a Canvas access error earlier. Starting from homework 2, the due time is Friday midnight.

Relevant section in textbook by Larsen and Marx: 3.2-3.7, 3.9, 3.11, 4.2-4.4.

Relevant lecture notes: Lecture 1 and Lecture 2.

Problem 1: Larsen and Marx question 3.6.6.

Problem 2: Larsen and Marx question 3.9.20.

Problem 3: Larsen and Marx question 4.3.33.

Problem 4: Larsen and Marx question 4.3.34.

Problem 5: Apply Chebyshev's inequality to prove the Weak Law of Large Numbers for the sample mean of i.i.d. random variables with a finite variance.

R simulation:

Let X_1, \dots, X_n be i.i.d. random variables from a uniform distribution on $[0, 2]$. Generate and plot 10 paths of sample means from $n = 1$ to $n = 40$ in one figure for each case. Describe what you can observe from each plot and relate to the theory in this specific context to empirically check the Law of Large Numbers.

(a) When n is large,

$$\frac{X_1 + \dots + X_n}{n} \text{ converges to } E[X_i].$$

(b) When n is large,

$$\frac{X_1^2 + \dots + X_n^2}{n} \text{ converges to } E[X_i^2].$$