MATH 200A FALL 2020 FINAL EXAM

Instructions: You may quote major theorems proved in class or the textbook, but not if the whole point of the problem is to reproduce the proof of a theorem proved in class or the textbook. Similarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and reproducing the result of the exercise is not the main point of the problem, or if the problem says you are allowed to. You may use the result of one part of the problem in the proof of a later part, even you do not complete the earlier part.

This is an open book exam— you may use Isaacs, Dummit and Foote, and any class materials if you wish (lectures, homework writeups, course notes). You may not refer to any other textbooks or online sources, or consult anyone else during your exam.

This is a 3 hour minute exam plus 10 minutes for downloading and 15 minutes for uploading. For students taking it at the regular time, the exam will be available Tuesday December 15 at 11:20am and you must upload by 2:45pm PST. Since there may be other students taking this exam at different times, you may not share this exam or discuss it with anyone else until Wednesday December 16. All problems are worth 15 points.

1. Let $Q = Q_8$ be the quaternion group. Show that $\mathrm{Aut}(Q)$ is isomorphic to S_4. You may assume that Q is given by some standard presentation, as in HW2#3, if you wish, but do not quote other homework problems without proof.

2. Classify all groups of order 44 up to isomorphism. Write down a presentation for each of the non-abelian examples you find. You may use HW6 #2 without reproving it.

3. A group G is *supersolvable* if there are subgroups $1 = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$, where $H_i \lhd G$ for all i, and with H_{i+1}/H_i cyclic for all i.

(a). Show that any finite nilpotent group is supersolvable.

(b). Find the smallest n such that all groups of finite order n are solvable but some group of order n is not supersolvable. Justify your answer.

4. Let R be an integral domain. Let X be a multiplicative system in R with $0 \notin X$.

(a). Show that the collection of ideals I of R such that $I \cap X = \emptyset$ has a maximal element. Fix such a maximal element I for parts (b) and (c).

(b). Prove that I is a prime ideal in R.

(c). Consider the localization RX^{-1}. Let $J = \{r/x | r \in I, x \in X\}$. Prove that J is a maximal ideal of RX^{-1}.

Date: December 15, 2020.