MATH 200A FALL 2020 MIDTERM

Instructions: You may quote major theorems proved in class or the textbook, but not if the whole point of the problem is to reproduce the proof of a theorem proved in class or the textbook. Similarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact, and reproducing the result of the exercise is not the main point of the problem. You may use the result of one part of the problem in the proof of a later part, even you do not complete the earlier part.

As previously announced, this is an open book exam— you may use Isaacs and any class materials if you wish (lectures, homework writeups, notes you have taken). You may not refer to any other textbooks or online sources.

This is a 50 minute exam plus 10 minutes for downloading and 15 minutes for uploading. At this point you should have a good idea of how long it takes to upload to gradescope, so your exam must be uploaded by 12:05pm Pacific time (or if you are taking the exam at another time, at the analogous time). Since there may be other students taking this exam at different times, you may not share this exam or discuss it with anyone else until Wednesday November 11.

1 (10 pts).

Suppose that G is a group with $|G| = 12$ such that G does not have a normal Sylow 3-subgroup.

Show that $G \cong A_4$.

2 (10 pts).

Suppose that G is a finite group which has normal subgroups H and K such that $HK = G$, H and K are cyclic, and $H \cap K > 1$.

Show that there does not exist another finite group L whose group of inner automorphisms $\mathrm{Inn}(L)$ is isomorphic to G.

3 (10 pts).

Let G be a group with $|G| = 2020$, and note that 2020 has prime factorization $2020 = (2^2)(5)(101)$.

Show that for every positive integer n dividing 2020, G has a subgroup of order n.

Date: November 9, 2020.