Homework due Tuesday, June 9, 11:00 am, on Gradescope.
Throughout, A is a ring (commutative with 1).

(1) Problem 2, page 67 of Atiyah-MacDonald.

(2) Problem 10, page 68 of Atiyah-MacDonald.

(3) Problem 16, page 69 of Atiyah-MacDonald.

(4) Problem 31, page 72 of Atiyah-MacDonald.

(5) Let D be an integral domain, and let $0 \neq d \in D$. Then

$$S^{-1}D \simeq D[x]/\langle dx - 1\rangle$$

where $S = \{d^n : n = 0, 1, 2 \dots\}$.

(6) A ring A is called a *Jacobson ring* if for every $\mathfrak{p} \in \mathrm{Spec}(A)$ we have

$$\mathfrak{p} = \bigcap_{\substack{\mathfrak{m} \in \mathrm{Max}(A), \\ \mathfrak{p} \subset \mathfrak{m}}} \mathfrak{m}.$$

Let F be an algebraically closed field. Let A be a finitely generated F-algebra. Then

(a) A is a Jacobson ring.
(Hint: Reduce to $A = F[x_1, \dots, x_n]$. Apply 4th version of Hilbert's Nullstellensatz, and then 1st version of Hilbert's Nullstellensatz.)

(b) There exists some $n \in \mathbb{N}$ so that $\mathrm{J}(A)^n = 0$.
(Hint: Use part (a), and the fact that by Hilbert's basis theorem A is Noetherian.)