13th Jan

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If $M \cong N$ (free mods).

X, Y be basis of M & N.

Then $|X| = |Y|$.

$M \xrightarrow{\varphi} N$.

$\underbrace{M \xrightarrow{\varphi} N \xrightarrow{\text{quotient}} N / IN}$

$\ker(\text{quo} \circ \varphi) = IM$.

$IM \subseteq \ker$

$\ker \subseteq I \cdot M$ follows from the fact that φ is an iso.

Hence, $M / I \cdot M \underset{\overline{\varphi}}{\xrightarrow{\simeq}} N / I \cdot N$

$\{ m_i \}_i$ is a set of basis of M.

then : $\{ \overline{m_i} \}_i$ is a set of basis $M / I \cdot M$.

① They generate all elements in M / IM

② Independent.

t

(2). Independence.

$$\sum_{k=1}^{t} \overline{a_{i_k}}\,\overline{m_{i_k}} = 0 \quad \text{in } M/IM.$$

$$\Downarrow$$

$$\sum_{k=1}^{t} a_{i_k} m_{i_k} \in \underset{\sim}{IM} \quad \text{in } M.$$

But every elem. in $I \cdot M$ can be written as

$$\sum_{j=1}^{S} r_{i_j} \cdot m_{i_j} \quad \text{for } r_{i_j} \in I.$$

Then this forces $a_{i_k} \in I \Rightarrow \overline{a_{i_k}} = 0$

F-reps of G $\longleftrightarrow$ FG-modules.

V : F-rep of G. (V an F-v.s.).

$\phi : G \to GL(V)$.

Define a FG-mod structure on V.

$x \in F$ $\quad xv$

$g.v = \phi(g)v.$

This makes V a (left) FG-module.

FG-mod. V. ! V is an F-v.s

for any g the action of g on V. is a linear transformation

We have $G \to \mathrm{End}_F(V)$. $(F \subseteq Z(FG))$.

In fact. $G \to \mathrm{Aut}_F(V) = GL_F(V)$ (g is invertible).

This is a group homomorphism

Induced modules.

$$\underbrace{\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}[G], \mathbb{Z})}_{}$$

G-module?

ϕ^g $g \cdot \phi$.

$g \cdot \phi(x)$
$= \phi(x \cdot g)$.

This def coincides with Problem 3.

V is a k-v.s.

$R = \mathrm{End}_k(V)$. $\qquad \{v_i\}$ basis

$R \cong R \oplus R$.

$\phi \in R \qquad \phi(v_i) = \begin{cases} v_{\frac{1}{2}i} & \text{if } 2 \mid i \\ 0 & 2 \nmid i \end{cases}$.

$V = V_1 \oplus V_2$

$V_1 = \mathrm{span}\{ v_2, v_4, v_6 \cdots v \cdot \}$.

$V_2 = \mathrm{span}\{ v_1, v_3 \cdots \}$

$\phi : V_1 \xrightarrow{\sim} V$

isomorphisms.

$\psi : V_2 \xrightarrow{\sim} V$.

$R = \mathrm{Hom}_k(V, V) = \mathrm{End}_k(V)$

$= \mathrm{Hom}_k(V_1 \oplus V_2, V)$.

$= \mathrm{Hom}_k(V_1, V) \oplus \mathrm{Hom}_k(V_2, V)$. (property of Hom).

via ϕ and ψ $=$

$\mathrm{Hom}_k(V, V) \oplus \mathrm{Hom}_k(V, V)$

$$= R \oplus R.$$

$R\phi \cong R$ means. $\mathrm{Hom}(V_1, V) \cong \mathrm{Hom}(V, V)$ via ϕ.

$$\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}/5, \mathbb{Z}/5)$$

$$\mathrm{End}(\mathbb{Z}/5)$$

$$\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Q}/\mathbb{Z}, \mathbb{Q}/\mathbb{Z}) \cong \mathbb{Z}.$$

"Pontryagin dual"

Let $T = \{$ set of roots of unity $\} \subseteq \mathbb{C}$.

T is a torsion $\mathbb{Z}$-module.

Consider $\mathrm{Hom}_{\mathbb{Z}}(T, T)$.

Since T is dense in S^1,

a gp homo $T \to T$ gives a continuous homormorphism $S^1 \to S^1$

Thus $\mathrm{Hom}_{\mathbb{Z}}(T, T) \cong \mathrm{Hom}_{cts}(S^1, S^1)$

The latter is classified by widing # (homotopy).

You will have $\mathrm{Hom}_{cts}(S^1, S^1) \cong \mathbb{Z}$.