Student full name:

MATH 281B – Midterm 1 – Winter 2019

Use the notation defined in the summary sheet; otherwise, define any symbol you use. If you use a result from the summary sheet, specify it (e.g., "by M1"). Be concise and clear.

Problem 1.

A. Suppose that δ is minimax for θ under absolute error loss (meaning $L(\theta, d) = |\theta - d|$). For $a, b \in \mathbb{R}$, is $a\delta + b$ minimax for $a\theta + b$?

B. Prove A1 in the summary sheet.

C. Prove M2 in the summary sheet.

D. Derive the Bayes estimator for θ under the loss $L(\theta, d) = 1_{|d-\theta|>c}$, where $c > 0$ is given.

E. Suppose we want to estimate $\theta \in \Omega = [a, b]$ based on $X \sim P_\theta$ under a loss function $L(\theta, d)$ which, for each θ fixed, is strictly decreasing on $(-\infty, \theta]$ and strictly increasing on $[\theta, \infty)$. Show that any estimator taking values outside $[a, b]$ with positive probability under some P_{θ_0} is inadmissible.

F. Consider a setting where $X \sim \text{Bin}(n, \theta)$ and the goal is to estimate $\theta \in [0, 1]$ under squared error loss. The estimation problem is invariant with respect to the group G induced by the transformation $gx = n - x$. Provide a sufficient condition on a prior Λ, with density λ with respect to the Lebesgue measure on $[0, 1]$, such that the corresponding Bayes estimator is equivariant with respect to the action of G.

Problem 2. Consider a normal sample $X_1, \ldots, X_n \sim \mathcal{N}(\theta, \sigma^2)$, where σ^2 is known. Let $\bar{X}$ denote the sample mean, i.e., $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$. We want to estimate θ with squared error loss knowing that $\theta \in [0, 1]$.

A. Is $\bar{X}$ minimax? Explain.

B. Derive the MLE for θ. Compare its maximum risk with that of $\bar{X}$.

C. For simplicity, we now restrict ourselves to linear estimators, meaning, estimators of the form $a\bar{X} + b$. Derive the estimator with minimum maximum risk among these estimators.