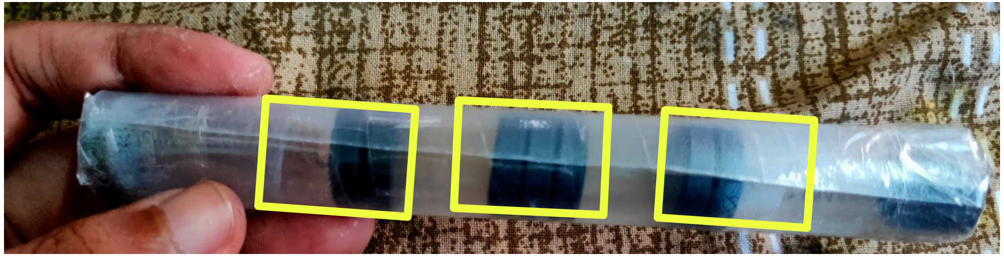
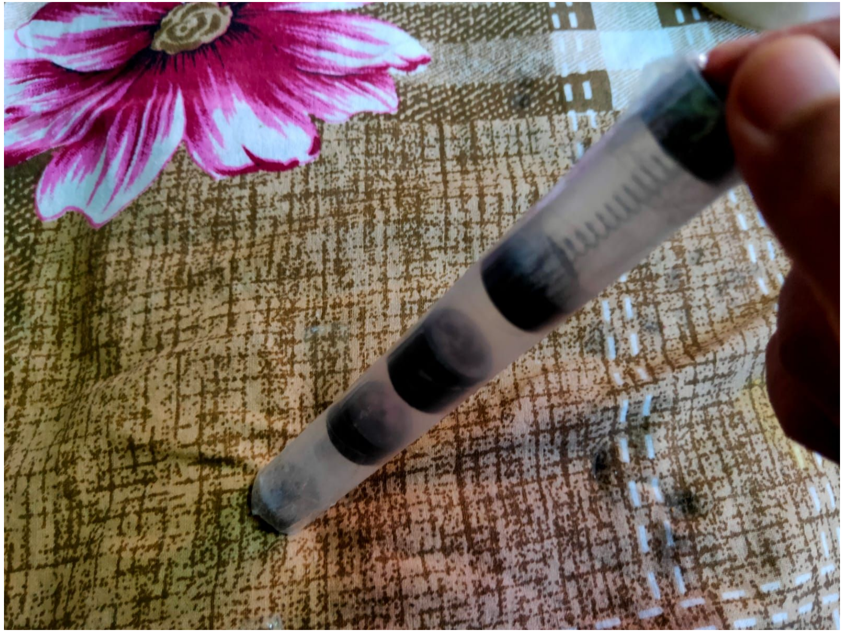
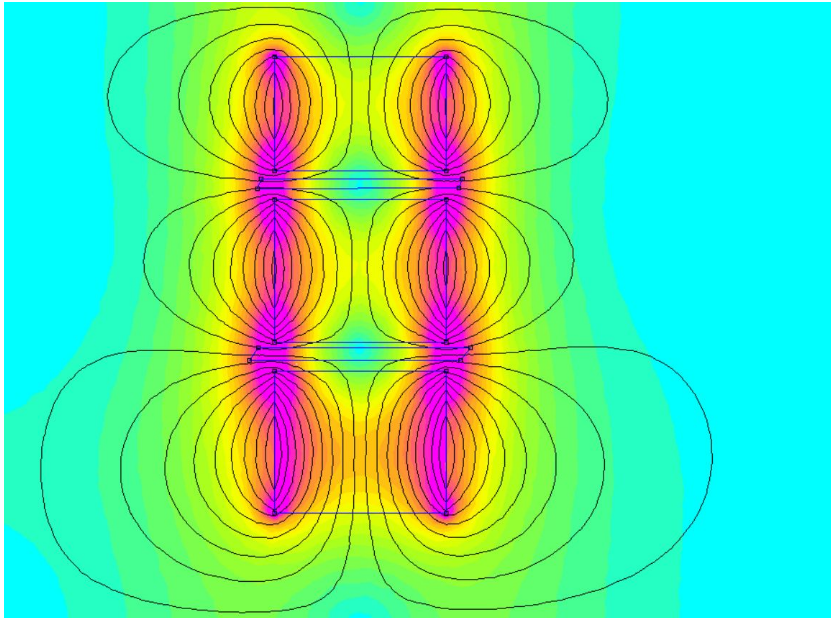
Equation Derivation - Mayukhmali Das

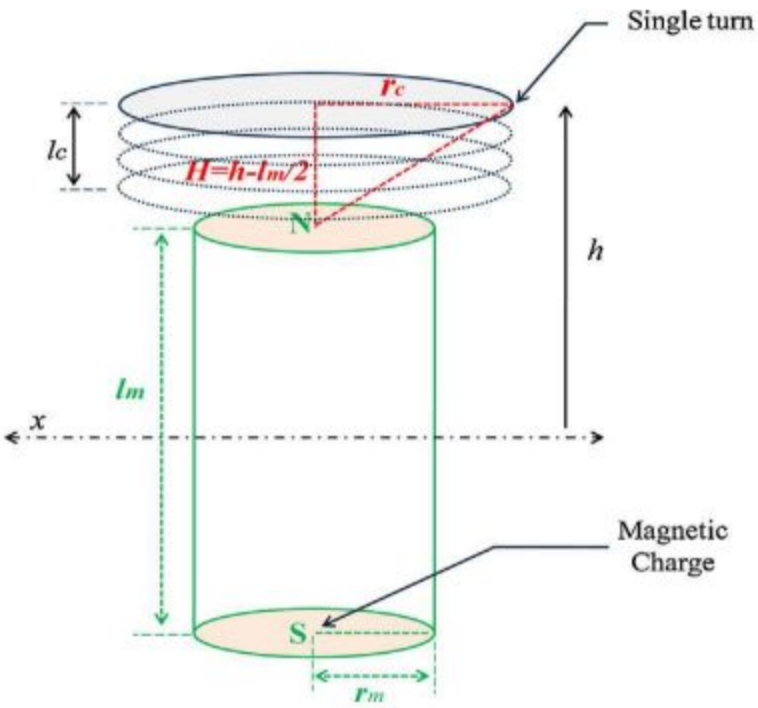


Equations we will use for describing this model referenced from paper “https://www.researchgate.net/publication/270970817_A_study_of_an_electromagnetic_energy_harvester_using_multi-pole_magnet”





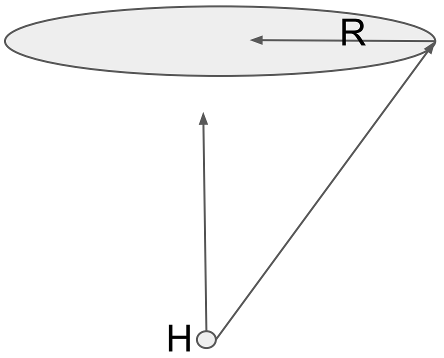
This was the final setup we saw



Let us first focus on the freely moving part of the design.

The distance from the centre of the magnet to the bottom of the coil is h. The thickness of the coil is lc and the radius is rc.

The length of the magnet stack is taken to be lm. Now we have to find the flux due to both the north and the south pole.

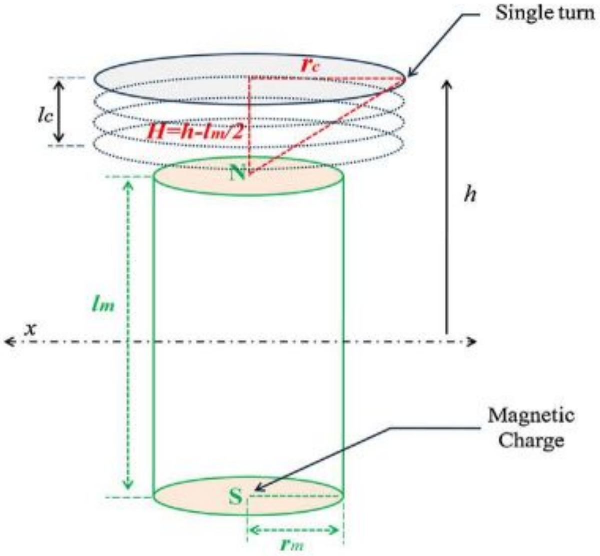


The grey part is the wire. Magnetic flux through the wire will be:

$$q_m = B\pi r_m^2$$

$$\phi = q_m \frac{2\pi R(R-H)}{4\pi R^2} = q_m \frac{(R-H)}{2R}$$

Now we will just substitute the corresponding values to the top equation to find the flux



$$\phi_1 = \mathrm{sign}(h - l_m/2)\frac{-q_m(\sqrt{r_c^2 + (h - l_m/2)^2} - |h - l_m/2|)}{2\sqrt{r_c^2 + (h - l_m/2)^2}}$$

$$\phi_2 = \mathrm{sign}(h + l_m/2)\frac{q_m(\sqrt{r_c^2 + (h + l_m/2)^2} - |h + l_m/2|)}{2\sqrt{r_c^2 + (h + l_m/2)^2}}$$

Here phi1 and phi2 are the flux on the coil due to the north and south pole respectively

Flux total will be:

$$\phi_{total} = \int_{h}^{h+l_c} \frac{N(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \cdots + \phi_n)}{l_c} dh$$

Here lc is the thickness of the coil. We are integrating over the entire thickness of the coil to find out the total flux linkage. In our case only phi1 and phi2 are applicable.

Now the change in magnetic flux with height will be

$$\frac{d\phi_{total}}{dh} = \frac{d}{dh} \int_{h}^{h+l_c} \frac{N(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \cdots + \phi_n)}{l_c} dh$$
$$= \left[\frac{N(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \cdots + \phi_n)}{l_c}\right]_{h}^{h+l_c}$$

The change in height with time is actually the velocity of the magnet which will again be sinusoidal

$$\frac{dh}{dt} = v = v_m \sin(wt) = v_m \sin(2\pi f t)$$

Now we will apply Faraday's law to calculate the voltage generated

$$V = \int \frac{d\phi_{total}}{dt} dA = \int \frac{d\phi_{total}}{dh} \frac{dh}{dt} dA \qquad V = \int v \left[\frac{N(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \cdots + \phi_n)}{l_c}\right]_{h}^{h+l_c} dA$$

For north pole

Date:

$$\phi_1 = \text{sign}(h - l_m/2)\,\frac{-q_m\sqrt{r_c^2+(h-l_m/2)^2} - |h - l_m/2|}{2\sqrt{r_c^2+(h-l_m/2)^2}}$$

$$[\phi_1]_0^{l_c}$$

$$\phi_1 = \text{sign}(l_c - l_m/2)\,\frac{-q_m\sqrt{r_c^2+(l_c-l_m/2)^2} - |l_c - l_m/2|}{2\sqrt{r_c^2+(l_c-l_m/2)^2}}$$

$$- \text{sign}(-l_m/2)\,\frac{-q_m\sqrt{r_c^2+(l_m/2)^2} - l_m/2}{2\sqrt{r_c^2+(l_m/2)^2}}$$

For south pole

Mo Tu We Th Fr Sa Su Date:

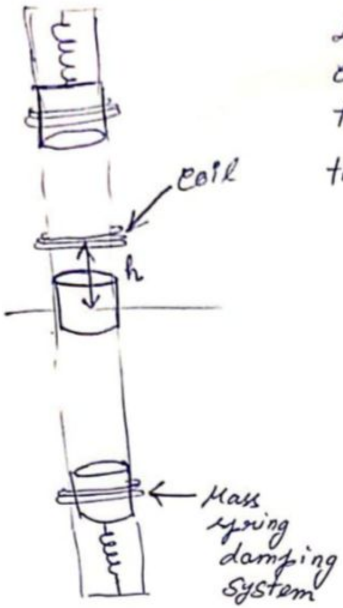
$$\phi_2 = \text{sign}(h + l_m/2)\,\frac{q_m\sqrt{r_c^2+(h+l_m/2)^2} - (h + l_m/2)}{2\sqrt{r_c^2+(h+l_m/2)^2}}$$

$$[\phi_2]_0^{l_c}$$

$$\phi_2 = \text{sign}(l_c + l_m/2)\,\frac{q_m\sqrt{r_c^2+(l_c+l_m/2)^2} - (l_c + l_m/2)}{2\sqrt{r_c^2+(l_c+l_m/2)^2}}$$

$$- \text{sign}(l_m/2)\,\frac{q_m\sqrt{r_c^2+(l_m/2)^2} - l_m/2}{2\sqrt{r_c^2+(l_m/2)^2}}$$

Now let us deal with the vibrating magnets. We will consider an ideal case where the spring damping is only due to the parasitic damping of the spring



Let us consider that the closest distance between the moving magnet and the vibrating one is p.

Max repulsive force:

$$\frac{\mu_0 m^2}{4\pi p^2}$$

Now we use the mass spring damping equation;

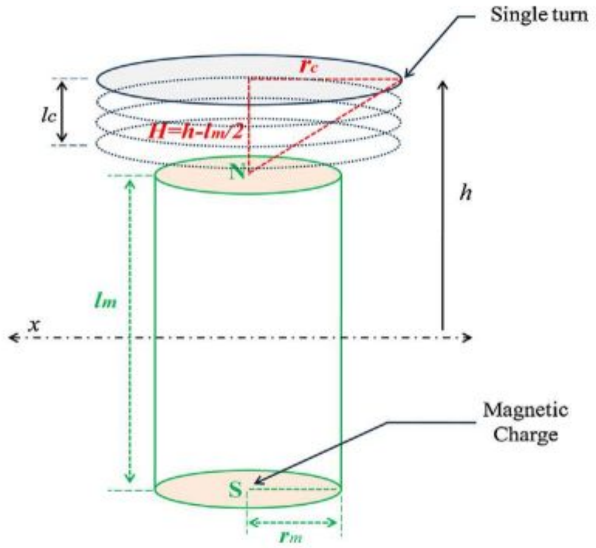
$$m\frac{d^2x}{dt^2} + D_p\frac{dx}{dt} + kx = F_0 \sin \omega t$$

Parasitic Damping

The F external is a sinusoid as the magnet is moving and its repulsive force is varying. We should be taking its absolute value as the force cannot be negative We will consider that the damping causes the vibrating magnet to stop vibrating before the repulsive force again becomes high. For solving we are taking resonant conditions.

solving for x for resonant frequency of $\sqrt{k/m}$ we get;

$$x = -F_o \cos(\omega t) / D_p \omega$$



$$x = \frac{-F_0 \cos(\omega t)}{D_p \omega}$$

$$\left\{ v = \frac{-F_0 (\sin(\omega t))}{D_p} \right\}$$

Final Voltage Generated

Net voltage

$$V = \int v \, \frac{[\phi_1 + \phi_2]_0^{l_c}}{l_c} \, dA$$

$$\therefore V = \int v \, \frac{[\phi_1 + \phi_2]_0^{l_c}}{l_c} \, dA$$

where

$$v = \Big(\underbrace{v_m \sin \omega t}_{\text{by the vibration tool}} + \underbrace{2 \frac{\mu_0 m^2}{4 \pi p^2} \frac{\sin(\sqrt{K/m}\, t}{D_p}}_{\text{for the oscillating magnets}}$$

by the vibration tool

for the oscillating magnets.