GROUP WORK PROJECT # 3 M7 MScFE 620: Derivative-Pricing
GROUP NUMBER: 7502

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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

Step 1.

Stochastic Volatility Modelling.

5. Determine the prices of at-the-money European call and put options for the Heston model.

The Heston model, a model of stochastic volatility, is utilized for the pricing of options. It takes the Black-Scholes model and extends it by allowing not just the underlying asset but also its variance to evolve in a more sophisticated, time-dependent manner. Variance, of course, is directly related to volatility, but the Heston model has a much richer structure, which means that it can match the observed option pricing behaviour much more closely. For our particular exercise, we implement the Heston model using the following parameters and a straightforward Monte Carlo method.

The starting variance (v0) = 3.2%.

The speed of mean reversion (κ) = 1.85.

The variance over the long term = 0.045.

The correlation is negative, ρ = -0.30.

Simulating many paths for the asset price and variance is necessary. When you have done this, you'll compute the payoffs for the option. You've got to average them, discount them, and that's your price.

The European call and put prices obtained from the Heston model are **$3.49** & **$2.42** respectively.

6. Restating the European pricing but with a different correlation value (ρ = -0.70).

You'll perform this task much like you did for question 5, but this time, set the correlation parameter to -0.70. This will give you practice with the computation of option prices and help you understand how the correlation between asset price and volatility affects those prices.

The results obtained for European call and put option prices from this model are **$3.49** & **$2.42,** which is the same as the result obtained in question 5. With these results. It can be concluded that European call and option put prices for ρ = -0.30 and ρ = -0.70 are the same. The slight difference is a result of the numerous Monte Carlo simulations.

7. Calculating Heston's delta and gamma:

Delta gauges how much and how quickly an option's price changes when the price of the underlying asset changes. If you think in terms of a graph and a curve, the delta is the first derivative of the option price concerning the price of the underlying asset.

Gamma, on the other hand, is a bit more complicated: it tells us not just how much and how quickly the price of the option changes when the underlying asset changes, but also it can be understood to have this role: how much and how quickly delta changes when the underlying asset changes.

Delta:

Evaluate the option using the present stock price (S).

Value the option using a stock price that is slightly higher (S + h).

Determine delta by calculating (Price(S + h) - Price(S)) / h.

Gamma:

Evaluate the option using the present value of the stock (S).

Value the option using a slightly higher stock price (S + h).

Value the option with a stock price that is somewhat lower (S - h).

To compute gamma, use the formula: (Price(S + h) + Price(S - h) - 2 * Price(S)) / h^2.

Since volatility fluctuates in upward and downward directions, a perturbation variance (h) of *0.01* was assumed (in the Python project) to depict upward or downward movement of the underlying variable—in this case, stock price.

The results obtained for ***Heston's Call and Put Delta and Gamma*** are listed in table 1. below:

Table 1: Heston's Call and Put Delta and Gamma

Parameter	**Correlation (ρ)**	**Results**
Call Delta	-0.30	1.6282
Call Gamma	-0.30	-519.4442
Put Delta	-0.30	-0.0080
Put Gamma	-0.30	-12.6955
Call Delta	-0.70	1.2880
Call Gamma	-0.70	-500.0661
Put Delta	-0.70	0.3710
Put Gamma	-0.70	-396.9052

8. Using the Merton Model to price an ATM European call and put with a jump intensity, λ= 0.75.

The Merton Model is the structural credit risk model which basically helps to understand whether the company or a financial organization is capable of meeting the financial obligations. This means it helps to identify the valuation of the company. This model determines the theoretical pricing of the European call and put options. The Merton model is used by the stock analyst, loan officers, and others. Mathematically, the Merton model is given by:

$$E = V_t N(d_1) - Ke^{-r\Delta T} N(d_2)$$

Where:

$$d_1 = [ln(S_0/K) + (r + \sigma^2/2)T]/[\sigma T^(1/2)]$$
$$d_2 = d_1 - \sigma T^(1/2)$$

E = Theoretical value of a company's equity

Vt=Value of the company's assets in period t

K = Value of the company's debt

t = Current time period

T = Future time period

r = Risk-free interest rate

N = Cumulative standard normal distribution

e = Exponential term (i.e., 2.7183...)

σ = Standard deviation of stock returns

The Merton model makes several basic assumptions, such as European options are exercised only at the time of expiration, no dividends are paid out, no commissions are paid, and there is a single risk-free interest rate.

In the Python project, the "merton_call_mc" function is used to simulate the path of the option price using the Merton model, introducing the jump parameter. The number of time steps used in the Monte Carlo simulation is 90 for 3 months (30 per month).

With the jump intensity parameter equal to 0.75, the price of the European call option was found to be Call Price: $**8.34**, and the price of the European put options was found to be Put Price: $**7.20.**

9. Using the Merton Model to price an ATM European call and put with a jump intensity (λ= 0.25).

Using the formula for the Merton model given in question no. 8 and changing the jump intensity parameter to equal 0.25 and keeping the number of time steps used in the Monte Carlo simulation fixed to 252, the price of the European call option was found to be Call Price: **$6.83,** and the price of the European put options are found to be Put Price: **$5.74.**

10. Calculating Merton's delta and gamma:

Greek delta is the measure of change in an option's price due to a change in the price of the underlying assets.

Mathematically,

Delta for call options: $\delta = N(d_1)$

Here,

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

Where:

- K = option strike price.
- N = standard normal cumulative distribution function.
- r = risk-free interest rate.
- σ = underlying asset volatility.
- S = underlying asset price.
- t = time until the option expires.

Delta For, Put Options: $\delta = N(d_1) - 1$

Where:

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

- K: Option strike price
- N: Standard normal cumulative distribution function
- r: Risk-free interest rate
- σ: volatility of the underlying asset
- S: Price of the underlying asset
- t: Time to option's expiry.

Gamma is a metric that represents the rate of change between an option's Delta and the underlying asset's price. It is also known as the delta of delta, as it is considered a second-order risk factor. The call and put gamma values for the Merton model are always positive.

The results obtained for ***Merton's Call and Put Delta and Gamma*** are listed in Table 2 below:

Table 2: Merton's Call and Put Delta and Gamma

Parameter	**Jump Intensity (λ)**	**Results**
Call Delta	0.75	0.5433
Call Gamma	0.75	0.0195
Put Delta	0.75	-0.4421
Put Gamma	0.75	0.0222
Call Delta	0.25	0.5142
Call Gamma	0.25	0.0300
Put Delta	0.25	-0.4754
Put Gamma	0.25	0.0261

11. Put-Call Parity Difference (Heston-Merton Comparison)

From the Python project, the results obtained for Put-Parity are listed in the table 3 below:

Table 3: Put-Call Parity Difference (Heston-Merton Model Comparison)

Model	Put-Call Parity Results
Heston (ρ = -0.30)	-0.0239
Heston (ρ = -0.70)	-0.0253
Merton ($\lambda = 0.75$)	0.0542
Merton ($\lambda = 0.25$)	0.0004

Put-Call Parity is satisfied under both the Heston and Merton models. While the parity differences for both models are within a tolerance range very close to zero, it suffices to say that the averaging of option prices across a large number of simulations tends to give varying values but significantly low deviation from 0.

12. Heston & Merton Option Prices with Different Strikes

The two different Heston model scenarios (ρ = -0.30 & = -0.70) and the Merton Model scenarios ($\lambda = 0.75$ & $\lambda = 0.25$) were used to determine the European call and put prices for different strikes. Moneyness (K/S0) values of 0.85, 0.90, 0.95, 1, 1.05, 1.10, & 1.15 were used to derive equally spaced strike values of 68, 72, 76, 80, 84, 88, & 92, respectively (with S0 = 80). The values obtained for both models are represented in Table 4 below:

Table 4: Heston Option Prices vs. Different Strikes

	Strike_prices	Heston_Call_30	Heston_Put_30	Heston_Call_70	Heston_Put_70
0	68.0	12.94	0.03	12.94	0.03
1	72.0	9.20	0.25	9.20	0.24
2	76.0	5.96	0.94	5.96	0.95
3	80.0	3.49	2.42	3.49	2.42
4	84.0	1.86	4.74	1.86	4.74
5	88.0	0.92	7.74	0.91	7.73
6	92.0	0.42	11.19	0.42	11.18

Where:

Heston_Call_30 = Heston's European Call Price at ρ = -0.30

Heston_Put_30 = Heston's European Put Price at ρ = -0.30

Heston_Call_70 = Heston's European Call Price at ρ = -0.70

Heston_Put_70 = Heston's European Call Price at ρ = -0.70

Table 5: Merton's Option Prices vs. Different Strikes

	Strike_prices	Merton_Call_75	Merton_Put_75	Merton_Call_25	Merton_Put_25
0	68.0	16.35	3.37	14.81	1.88
1	72.0	13.39	4.35	11.77	2.78
2	76.0	10.71	5.61	9.09	4.06
3	80.0	8.34	7.20	6.83	5.74
4	84.0	6.34	9.14	4.99	7.84
5	88.0	4.70	11.44	3.54	10.34
6	92.0	3.40	14.09	2.46	13.20

Where:

Merton_Call_75 = Merton's European Call Price at $\lambda = 0.75$

Merton_Put_75 = Merton's European Put Price at $\lambda = 0.75$

Merton_Call_25 = Merton's European Call Price at $\lambda = 0.25$

Merton_Call_25 = Merton's European Put Price at $\lambda = 0.25$

Step 2

13. Pricing American Call Options Using Heston & Merton Models

American call options are priced with Monte Carlo simulation in both implementations.

The primary distinction between European and American options is the early exercise feature. This feature is implemented using the Least Squares Monte Carlo method.

We generate stock price trajectories in both models and then reverse-engineer from them to ascertain the best exercise strategy. We carry out this exercise first in a deterministic setting and then in a stochastic one, using both the risk-neutral and the real-world probabilities.

Whereas the Heston model comprises stochastic volatility, the Merton model comprises jump diffusion.

At every time step, the option values are updated to compare the immediate exercise value with the expected continuation value.

The option's final price equals the present value of the average of all simulated paths.

You might notice some fundamental differences between American and European call options.

The early exercise feature of American options makes them typically cost more than other options.

Usually, the price variance is most apparent for options with longer lifespans or on assets that distribute dividends (which we haven't modelled here).

American options require a more complex computation because of the backward induction method.

For the Python project, the Heston model was used to calculate the American call option price (at $\rho = -0.30$), while the Merton model was used to calculate the American call option price (at $\lambda = 0.75$). The Heston's American Call price result obtained is ***$1.96,*** while that of the Merton's American Call price is ***$8.32.***

Differences between Heston American Call Option Price and Merton American Call Price

The results show that a Merton American call price is significantly larger than that of the Heston American call option price. This is as a result of the following:

- The Merton's call is a reflection of jumps in the volatility, unlike the volatility of the Heston's model, which is stochastic and mean reverting. Hence, call option prices for the former tend to be higher for an early-exercise-american-option.
- Option premiums for Heston calls are sensitive to low-value parameters like kappa, theta, and sigma, whereas the option premium for Merton's call is sensitive to extreme jump intensity value (lambda). The impacts the call option price.

- Merton's call option price is skew-driven, while Heston's call prices are a reflection of the volatility smile.

14. Pricing a European Up-and-In Call option using the Heston model

Procedure description:

There are two primary functions we specify:

I. The Heston model *(heston_model_mc*) is used to simulate paths that the stock price may take. Using Monte Carlo simulation, we compute the price of the up-and-in call option.

Ii. Up-and-Call function (*price_uai_call_heston*) computes the price of a call option under the Heston stochastic volatility model. In this function, we:

- Mimic pathways of stock prices
- Verify which trajectories hit the barrier.
- Determine the payoffs exclusively for the trajectories that contact the barrier.
- To obtain the option price, calculate the present value of the average payoff.
- We established the parameters specified in the question prompt. These include a barrier level set at H = $95 and a strike price of K = $95.

For the Python project, the Up-and-in model was used to calculate the European call option price and then compared to simple Heston's European call price results in Question 6. The Up-and-In Call price result obtained was ***$0.03,*** while that of the simple Heston's European call price results in Question 6 was ***$3.49.***

Comparison between UAI Heston Call Option Price and Simple Heston European Call Price

The results show that a simple European Heston call is significantly larger than the up-and-in-barrier call option. This is as a result of the following:

- The call option is only active when the stock exceeds the barrier price, resulting in a lower (cheaper) call option price at expiration.
- The probability of exercise is higher for a simple European call since there is no activation barrier.
- A simple European call is easier to hedge because of the nonexistence of a barrier price.
- A simple European call has a vanilla payoff, whereas the UAI option only pays off when the stock price hits the barrier price.

15. Pricing a European Down-and-In Put option using the Merton model

Procedure description:

We outline two primary functions:

I. Merton jump diffusion model (Merton_jump_path): Uses the Merton jump-diffusion model to simulate not just stock prices but also real stock price paths. Monte Carlo simulation is used to determine the price of a down-and-in put option.

II. Down-and-Input Model: Within the function price_dai_put_merton, we:

- Mimic the trajectory of stock prices
- Examine which routes strike the barrier.
- Only payoffs for paths that collide with the barrier should be calculated.
- Take the average payoff and discount it to find the price of the option.
- We established the parameters as they were stated in the problem, with a barrier level of H = $65 and a strike price of K = $65.

For the Python project, the Down-and-in model was used to calculate the European put option price and then compared to simple Heston's European put price results in Question 8. The Up-and-In Call price result obtained was ***$2.78,*** while that of the simple Merton's European put price results in Question 8 was ***$7.20.***

Comparison between DAI Merton Put Option Price and Simple Merton European Put Price

The results show that a simple Merton European put is significantly larger than the down-and-in-barrier put option. This is as a result of the following:

- The put option is only active when the stock falls below the barrier price, resulting in a lower (cheaper) put option price at expiration.
- The probability of exercise is higher for a simple European put since there is no activation barrier.
- A simple European call is easier to hedge because of the nonexistence of a barrier price.
- A simple European call has a vanilla payoff, whereas the UAI option only pays off when the stock price falls below the barrier price.
- The DAI put option is mainly used for special hedging cases where cost is a major concern.

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