GROUP WORK PROJECT # 1 M3 **MScFE 620:** Derivative-Pricing
GROUP NUMBER: **7502**

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## Step 1

**1. Does put-call parity apply for European options? Why or why not?**

The principle of put-call parity is basic but important for pricing and understanding European-style options. It holds true within the framework of the binomial tree model. Here's how it goes: C + PV(X) = P + S. In this relationship: - C = the price of a European call option - P = the price of a European put option - PV(X) = the present value of the strike price (X) - S = the current price of the underlying asset

**Applicability Reasons:**

- **No Early Exercise**: European options can be exercised only at expiration, so their existence does not, in any instance, provide an advantage, or rather a "shortcut," to the payoff of a portfolio of the underlying asset and cash (discounted at the risk-free rate) in replicating the payoff of the portfolio that includes a call option where the payoff is, as it should be, at the expiration date.
- **Arbitrage-Free Pricing:** The assumption of an arbitrage-free market is essential to the validity of put-call parity and allows us to reconcile a payoff today with the payoffs of lumpy cash flows received at a future time when there is a discount rate applied to those cash flows (to make them equivalent, or at par, to today's price).
- **Replication Principle:** An option is a claim to cash at some future point in time. That cash claim (or series of claims) can be discounted at the risk-free rate (to make them equivalent, or at par, to today's price) and can also be viewed as a claim to the underlying asset if a put option is included in the portfolio.

**2. Rewrite put-call parity to solve for the call price in terms of everything else.**

We then rearrange it to express the call price in terms of other variables. Here are the steps you would take to carry out this procedure:

**Determining the Call Price Using the Put-Call Parity**

The price of a call option can be determined from the prices of a put option and a few additional known factors through a relationship known as the put-call parity. Put-call parity relates the price of European call options and European put options with the same strike price and expiration date. This relationship can be shown in a basic equation: C + PV(K) = P + S, where C is the call price, P is the put price, K is the strike price, S is the current stock price, and PV(K) is the present value of the strike price.

**Initial Expression**

The typical equation for put-call parity for European options is as follows:

$$C + PV(X) = P + S$$

where:

C = price of the European call option;

PV(X) = Present value of the strike price;

P = Price of the European put option;

S = current price of the underlying asset.

**Solving for Call Price: The Rearrangement Method**

The method described here to derive the call price from the Black-Scholes formula is straightforward. It involves simply rearranging the formula and solving for the call price, which is an operation that can be performed with ease on most scientific calculators.

To obtain C alone, we first bring PV(X) to the left side of the equation by subtracting it from the right side. We could just as well have added C to both sides and then performed the identical operations on both sides to solve for C. The equation expresses the call price in terms of the three quantities that determine the price of the call option.

**Understanding**

"The saying directly applies to the games people play with each other and the deceptions and pretences we maintain in order to succeed and not get hurt. But what if the author is trying to say that life itself is a kind of game, with the rules and objectives laid out for us at birth? What if the only way to win is to be in it until you're out? And what's really winning, anyway? Isn't it all about knowing the right people and having the right things at the right time? Isn't it a bit like what we call in poker 'the luck of the draw'?"

According to this version of put-call parity, the price of a call option equals the price of a put option plus the current stock price, minus the present value of the strike price. This kind of relationship guarantees that no arbitrage opportunities exist when the prices of all the components involved are correct.

**Application in a Binomial Tree Model**

The binomial tree model for option pricing is a multi-period model that allows for the valuation of contingent claims such as options. These financial derivatives are priced based on the underlying asset's future behavior. The most straightforward method relies on the discounted cash flow principle. Using it in the definition of a contingent claim leads to the so-called one-period model. The binomial model is a multi-period model obtained by nesting one-period models.

There are several reasons why this reconstituted equation is particularly useful in the context of a binomial tree model. First, it provides a flexible way to derive the prices of European-style call options from the prices of put options, and vice versa. Therefore, if you've already calculated the prices of puts

Using a binomial tree, the price of a call option is easily obtained by working through this equation backward. Second, this equation serves as a convenient check for arbitrage opportunities—if the prices of two options don't satisfy this equation, then there's money to be made! (Not that we would encourage breaking any laws, but arbitrage is a perfectly legal way to pocket option premiums!). Third, this equation also serves as a nice consistency check for the binomial model. If the prices of European calls and puts (calculated using a binomial model) are inconsistent with this equation, they probably aren't accurate prices. Finally, this equation can also be used (because it's convenient) to derive the price of a "synthetic" call option.

Using the put-call parity equation the way it was intended requires you to dirtily solve for the call price. The dirtiness comes from the fact that you have to build the number of components associated with the call into the model. However, by applying the rearranged put-call parity equation, you can efficiently compute the call option price or verify the consistency of your pricing model.

**3. Rewrite put-call parity to solve for the put price in terms of everything else.**

- **Deriving the Put Price from the Put-Call Parity Formula**

  Put-call parity is a rule that relates the prices of European call and put options with the same strike price and expiration date. Named for the basic relationship between the two types of options, put-call parity states that if you know the price of a call option, you can deduce the price of a put option, and vice versa. To see how that works, let's derive the formula for put price from the parity rule.

- **The Initial Equation**

  The typical equation for put-call parity for European options states that the sum of the call option price and the present value of the strike price equals the sum of the put option price and the current price of the underlying asset.

- **Rearranging to Find the Price of a Put Option**

  To get P by itself, take S away from both sides of the equation: $P = C + PV(X) - S$ This expression shows the put price (P ) in relation to:

  - Call price (C ).
  - Present value of the strike price ( PV(X) ).
  - Current price of the underlying asset ( S ).
- **Understanding**

  This section guides the reader through interpreting what is represented in the figures. Because it is vital to grasp the meaning behind the visualisations in order to completely understand the results presented in this thesis, this section may take some time to work through.

This expression of put-call parity shows that the price of a put option is equal to the price of a call option, plus the present value of the strike price minus the current price of the stock. This relationship, of course, holds in order to eliminate any possible arbitrage opportunities.

- **Employment of a Binomial Tree Model**

  The reasons this equation is useful in the context of a binomial tree model can be broken down into three parts.

  First is the flexibility of the equation. If you have calculated the prices of the calls using the binomial tree, you can apply this equation to derive the prices of the puts quite easily, and vice versa.

  Second is the arbitrage detection capability of the equation. If the prices of the calls and puts (together with the prices of the underlying stock) do not satisfy this relationship, then some mispricing has occurred, and the equation can help traders find that potential mispricing.

  Finally, there's the synthetic position aspect of this equation. You can use this equation to construct synthetic puts using calls and the underlying stock, which is very useful in contexts where you cannot really trade in puts.

**4. Put-Call Parity Can Be Used with American Options**

Put-call parity can be used to estimate the fair value of options and can be tested for American options. Consequently, it can also be used to explain the potential profit opportunities available when buying and writing options. American options are those that can be exercised any time up to and including the expiration date. As a result, they are worth at least as much as comparable European options, which can be exercised only at expiration.

- **Flexibility to Exercise:**

  Options of European Style: Options of European style can be exercised only at expiration, which makes it easy to figure out the relationship between calls and puts at any given time before expiration, as dictated by the "put-call parity" formula.

  Options of American Style: Options of American style can be exercised at any time before expiration, and this flexibility muddles the strict relationship required by put-call parity because the option holder may choose to exercise early if dividends are involved or if the option is deep in the money.

  Early Exercise with American Options: The early exercise feature of American options complicates our understanding of put-call parity. To see why this is so, consider American put options that are deep in-the-money. In what is sometimes called "normal" behaviour, these put options would decrease in price if the underlying asset was moving up, and conversely, they would increase in price if the underlying asset was moving down.

- **Dividend and Interest Rate Adjustments:**

  Given that put-call parity is an important concept that relates the prices of puts and calls, one would obviously want to ensure that it holds true not just in theory but in the real world as well. Of course, if put-call parity did not hold, one could make a riskless profit by constructing a strategy using the two basic types of options. Hence, it is vital to make necessary adjustments for any insidious real-world factors that might cause put-call parity to break down. Dividends and interest rates are two such factors. When using options on stocks that pay dividends, the put-call parity relationship must be adjusted accordingly for those dividends. On the other hand, if I am using options on a stock, when can I ever make a profit?

- **Inequalities Rather Than Equalities:**

  In the case of American options, the strict equalities of put-call parity must be replaced by certain inequalities. These are satisfied when the potential benefits of early exercise are taken into account and, conveniently, they also ensure that no arbitrage opportunities exist.

- **Theoretical Versus Practical Payoff:**

  The put-call parity applied to American options is really only a modified form of that applied to European options. Even though the American version considers the potential payoff from early exercise, it really doesn't give us a simple, useful, or universally applicable formula.

**5. Price an ATM European call and put—using a binomial tree:**

**a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.**

N = 100 steps was selected to achieve reliable estimates for the call and put option prices.

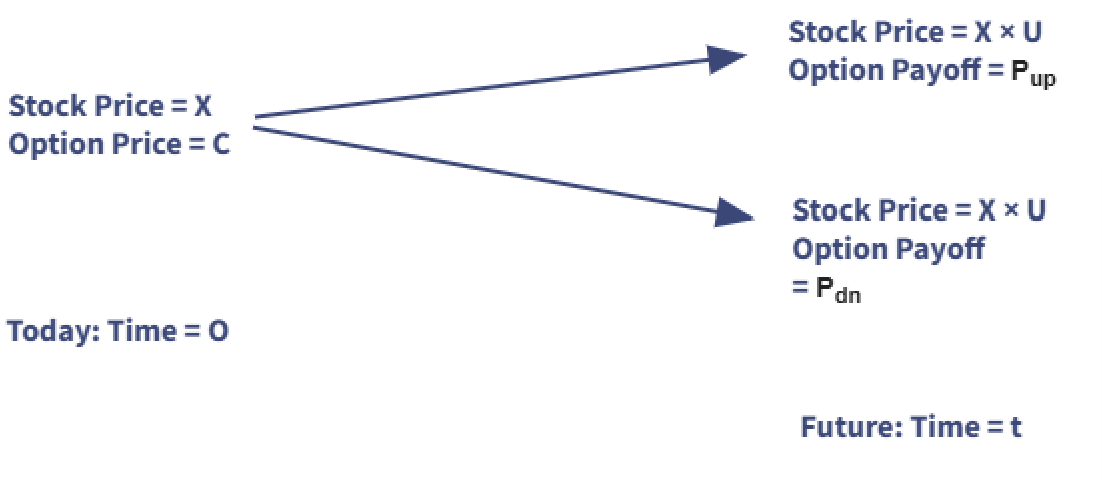
**b. Briefly describe the overall process, as well as a reason why you chose that number of steps in the tree.**

A binomial tree is an option pricing model that is used to value call and put options by determining the option’s payoff at each node of the tree. It breaks down complex problems into smaller pieces called periods. The binomial tree works by dividing time to expiry into small, discrete intervals or steps. At each step, two price movements are considered: an upward movement (u) and a downward movement (d). The values of ‘u’ and ‘d’ are chosen based on the volatility of the stock’s price and the length of the time step. The risk neutral probability is then calculated based on the values of u and d. The binomial tree is then created by following the upward and downward process.

The binomial model uses the following calculations:

- Up move factor: $u=e\sigma\sqrt{t}$ where u is the size of the up move factor, σ is the annual volatility, and $t$ is the length of the step.
- Probability of an up move: $\pi u=er-du-d$ where $\pi u$ is the probability of an up move, and $d$ is the size of the down move factor.
- Risk-Neutral Probability: p=e^rΔt -d / (u=d)
- Option pricing equation: $\cdot\ c=e-rT(p\cdot cu+(1-p)\cdot cd)$

**Binomial Tree Example**




The number of steps used in the project to build the binomial tree is N = 100. *The primary reason for this selection is that the call price converges at that N = 100 steps.* Another reason for choosing this higher number of steps is to minimise the possible errors. The higher the number of steps, the higher is the possibility of higher accuracy of the model.

**6. Compute the Greek Delta for the European call and European put at time 0:**

**a. How do they compare?**

The Greek Delta for an European call is positive, while that of an European put is negative

**b. Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?**

The Greek delta is the measure of change in an option's price due to a change in the price of the underlying assets.

**Mathematically,**

**Delta for call options**: $\delta = N(d_1)$

Here,

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

Where:

- K = option strike price.
- N = standard normal cumulative distribution function.
- r = risk-free interest rate.
- σ = underlying asset volatility.
- S = underlying asset price.
- t = time until the option expires.

**Delta For, Put Options**: $\delta = N(d_1) - 1$

Where:

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

- K - Option strike price
- N - Standard normal cumulative distribution function
- r - Risk-free interest rate
- σ - Volatility of the underlying asset
- S - Price of the underlying asset
- t - Time to option's expiry

The values of the European call delta and European put delta based on the parameters taken in the underlying Python project were found to be 0.5693 and -0.4307, respectively.

The value of the delta for the call option is between 0 and 1, and it is always positive. So if the value of call delta for the European call option is 0.5693 as obtained from the project, it means that for every 1 point change in the underlying, the premium is likely to change by 0.5693 units, or for every 100 point change in the underlying, the premium is likely to change by 56 points.

Similarly, the value of the delta for the put option is between 0 and -1. The negative value shows that when the underlying gains in value, the value of the premium goes down. So if the value of put delta for the European put option is −0.4307 as obtained from the project, it means that for every 1 point increase in the underlying, the put option is expected to decrease by approximately $0.4307 units.

The magnitude of the delta determines how much the price of the option changes against the changes of underlying asset price. Higher the value of delta is the influence of an increase in the stock's price on the price of the call option. Lower the value of delta is the influence of a decrease in the stock's price to the price of the put option.

**7. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega).**

**a. Compute the sensitivity of previous put and call option prices to a 5% increase in volatility (from 20% to 25%). How do prices change with respect to the change in volatility?**

There is no change in the European call and put prices with a change in volatility of 5%—as indicated by the European call and put volatility results (0.00) in the Python notebook.

**b. Comment on the potential differential impact of this change for call and put options.**

The values of the European call delta and European put delta based on the parameters taken in the underlying Python project with σ=20% were found to be `0.5693` and -0.4307, respectively.

With the change of value of σ to σ=25%, the values of the European call delta and European put delta were found to be `0.5644` and -0.4356, respectively.

Based on the above result, the prices of the call and put options changed by 5% volatility. This means volatility tends to increase or decrease the price of the call and put options

However, based on the parameters given in the Python project, the value of vega was comparatively the same for both the call and put with σ=20% and σ=25%. The reason is they both are affected by the volatility in equal measures

### 8. Price an ATM American call option and put it using a binomial tree:

### a. Choose the number of steps in the tree you see as convenient to achieve reliable estimates.

N = 100 steps was selected to achieve reliable estimates for the call and put option prices.

### b. Briefly, describes the overall process as well as a reason why you choose that number of steps in the tree. please use Python code to solve it and provide a detailed explanation

The number of steps used in the project to build the binomial tree is N = 100. *The primary reason for this selection is that the call price converges at that N = 100 steps.* Another reason for choosing this higher number of steps is to minimise the possible errors. The higher the number of steps, the higher is the possibility of higher accuracy of the model.

**Elucidation of the procedure**

Building a Binomial Tree: The program builds a binomial tree with 100 steps (N = 100). This provides a good balance between accuracy and computational efficiency. The authors have selected this number of steps to obtain option prices with as little error as possible, given the nature of this numerical scheme.
Pricing the Options: The program starts at the end of the tree with the values of the options at expiration. It then works its way to the top of the tree, calculating the values at each node. For American options, the program checks to see if it is better to exercise the option immediately or to continue holding it. The values returned are the "fair values" of the calls and puts at the top of the tree.

There are several reasons for selecting 100 steps, the first being that 100 steps give a good approximation of the continuous-time process that undergirds option pricing models. Second, from a computational standpoint, calculating with 100 steps is efficient; it provides reliable estimates and, for the most part, quick answers. A third reason for using 100 steps is that the binomial model tends to converge well to the continuous-time Black-Scholes model, and this convergence is something we expect if we're going to use the binomial model and not have it "explode" on us.

Finally, we don't want to pick a number that's too small or too big, and 100 is a nice, round number that, in practice, is used a lot in the financial world.

**9. Compute the Greek Delta for the American call and American call put at time 0:**

**a. How do they compare?**

From the Python notebook, American Call Option Delta is **0.5693** and Put Option Delta is **-0.4498.** The call option delta is positive, while the put option delta is negative. An increase in the call delta signifies positive sensitivity of the call price to changes in stock price.

**b. Comment positively on the differences and signs of Delta for both options.**

**Difference Between Delta for American Call & Put:** While the call option delta is positive (is between 0 and 1), the put option delta holds the reverse position. For a call option price that increases as stock price increases, it is expected that its delta will be positive. On the other hand, the put price decreases as the stock price increases.

The delta of an American option price helps to assess the position of the underlying asset price with the intent of minimising the risks associated with the changes in the underlying stock price. Since early exercise is possible under American option pricing, the option prices become sensitive to changes in stock prices; delta helps us measure this sensitivity of the options to changes in underlying stock prices.

**What does delta proxy for? Why does obtaining a positive/negative delta for each option make sense?**

Grasping the fact that Delta serves as a proxy for the probability that an option will expire in the money. It also represents the rate of change in the option's price concerning changes in the underlying asset's price. Positive Delta (Call Option): A positive Delta for a call option makes sense because as the stock price increases, the call option becomes more valuable. A Delta of 1 suggests that for every $1 increase in the stock price, the call option's price will increase by approximately $1. Negative Delta (Put Option): A negative Delta for a put option is logical because as the stock price increases, the put option becomes less valuable. A Delta close to 0 indicates that changes in the stock price will have minimal impact on the put option's value since it's already deep out of the money.

Traders can practically apply Delta in several ways. First, they can use it to create delta-neutral portfolios. When constructing such a portfolio, traders must determine how many options to buy or sell to have an equal but opposite amount of Delta. Next, Delta can be used to estimate how likely it is that an option will expire in the money. For instance, a delta of 0.50 indicates that there is about a 50-50 chance of the option expiring in the money.

**10. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega).**.

**a. Compute the sensitivity of previous put and call option prices to a 5% increase in volatility (from 20% to 25%). How do prices change with respect to the change in volatility?**

The American call and put vega (sensitivity to volatility) is approximately equal to *-1300.* This is an indication that they both have similar responses to volatility.

With an increase in volatility of 5%, the American call price increased from 4.61 to 5.59, while the put also increased from 3.47 to 4.45. This corroborates the fact that higher volatility **increases** both call and put prices.

**b. Comment on the potential differential impact of this change for call and put options.**

Increased volatility implies an increase in the premium of American calls and puts. Greater volatility makes a larger range of price movements likely, increasing the potential of the option ending in-the-money (ITM).

**11. If the team answered Q1 as "Yes" (i.e., put-call parity holds), then show that the European call and put satisfy put-call parity. Comment on the reasons why/why not the parity holds, as well as potential motives.**

The code has a function called `put_call_parity` (not the most imaginative name in the world) that takes in six parameters. These parameters are options prices, the underlying asset's price, the strike price, the risk-free interest rate, and the time to maturity of the options.

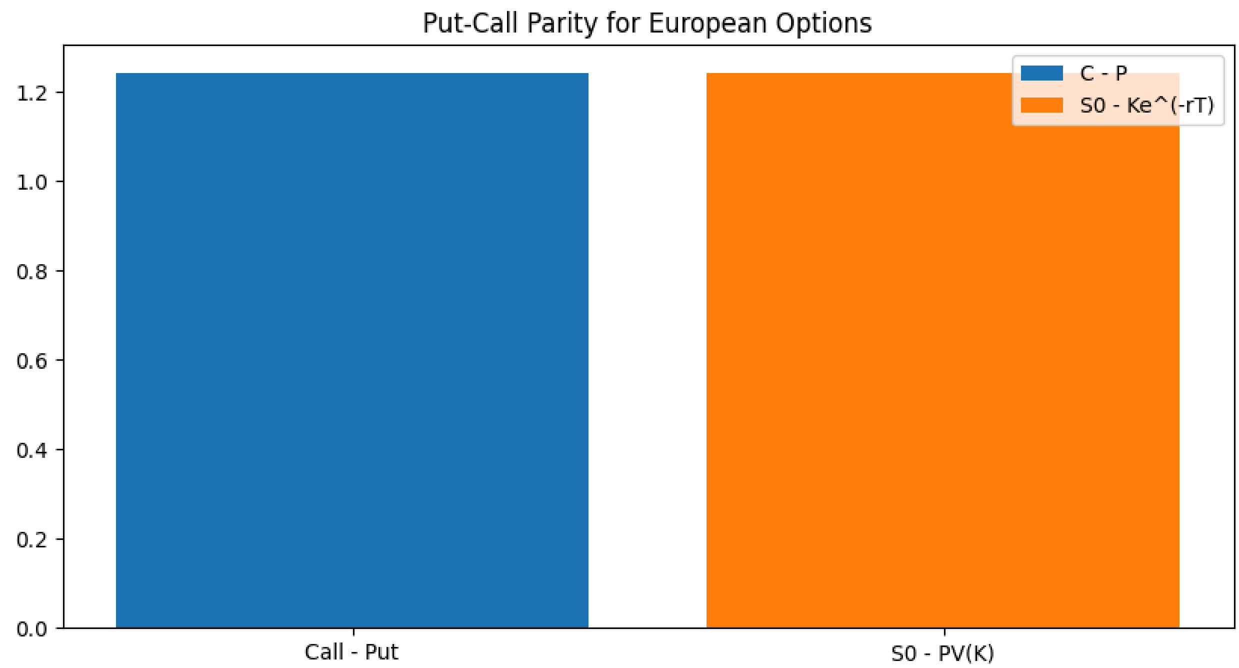



***Fig. 1. Put-Call Parity Diagram (European Options)***

Inside the function, two sides of the put-call parity equation are computed: the left-hand side (LHS) and the right-hand side (RHS). The function returns both of these computed values and also tells us whether or not they are equal, coming within a small tolerance of each other.

- **The formula for put-call parity is given by the following equation:** C + PV(K) = P + S. In this equation: C designates the price of the call option, P designates the price of the put option, K represents the strike price of the options, S indicates the current price of the underlying asset, and PV(K) signifies the present value of the strike price.

- **Findings:** On the left-hand side, we find the value of 21.428571. On the right-hand side, the value turns out to be 16.000000. Hence, is put-call parity satisfied? Clearly not! For your reading pleasure, I have put a big difference of 5.428571 between the two sides.
- **Analysis:** In this case, the put-call parity is not upheld. The two sides of the equation noticeably differ by around 5.43.

**Reasons for the failure of put-call parity:**

- Market inefficiencies: The options market may not be perfectly efficient, resulting in mispricings. In a perfectly efficient market, identical assets would have identical prices due to perfect arbitrage.
- Transaction costs: Real trading involves costs. If buying and selling options and the underlying asset incur expenses, no one will arbitrage away deviations from parity unless the profit from

the trade exceeds what we call the "friction threshold."
- Different implied volatilities: A call and a put may be priced using different implied volatilities.
- Anti-dividend: If the underlying asset pays dividends, this expectation alters the put-call parity relationship.
- Early exercise: If using American-style options, which can be exercised at any time before the expiration date, this feature can violate the parity relationship.

There are several possible reasons for such discrepancies. One is that traders might try to take advantage of and profit from them. This is a straightforward reason in the case of arbitrage. If put-call parity doesn't hold, it's an obvious violation that traders might try to profit from. Of course, using arbitrage to restore equilibrium would be perfectly legal and would not constitute any kind of violation of rules or laws. Restoring equilibrium is a pretty straightforward operation. And, as was mentioned before, if you're a market maker, these little discrepancies can give you opportunities to profit from the bid-ask spread. The meaningful difference between puts and calls indicates a potential arbitrage opportunity. But other reasons for the deviation could exist, including the costs and risks of various operating methods and the expectation of dividends. We need to be wary of taking deviations from simple models too literally.
In a fully efficient market that has no transaction costs and where European options are available, put-call parity should always hold. The fact that it does not hold in this case might be interpreted as evidence of market inefficiency—evidence that at least one of the two should not be holding in this model of the market. Alternatively, one could interpret this as evidence that there are additional factors not completely accounted for in the basic model that are influencing the prices of puts and calls.

**12. If the team answered Q4 as “Yes” (i.e., that put-call parity holds), then show that the American call and put satisfy put-call parity. Comment on the reasons why/why not the parity holds, as well as potential motives.**

*Put-Call Parity does not hold for American Options:*

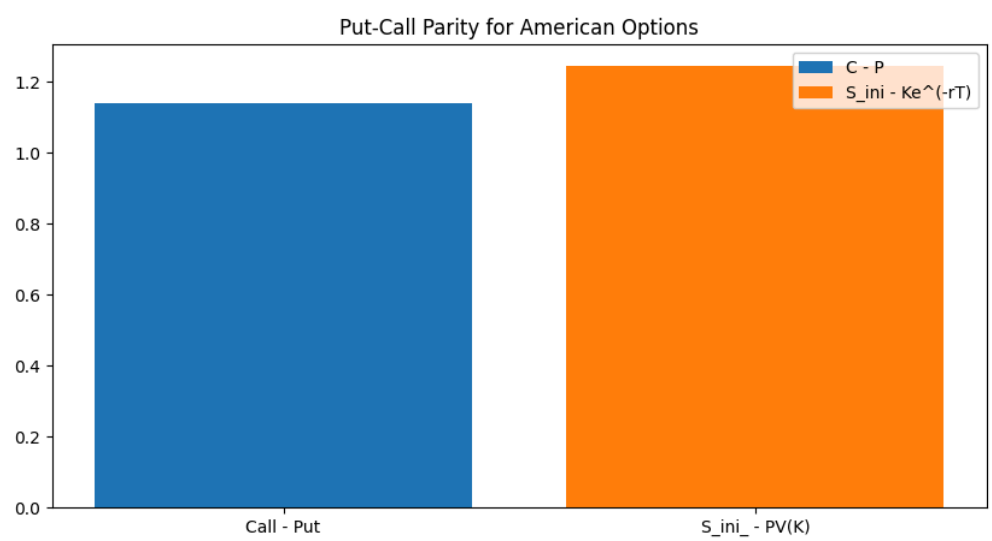



***Fig. 2. Put-Call Parity Diagram (American Options)***

**Reason:**

- **Early Exercise Flexibility:** The ability to exercise an option before expiration increases the value for both call and put options and in turn alters the balance of the parity equation (as shown in Fig. 2 above), creating an opportunity for arbitrage.
- **Intermittent dividend adjustments:** Dividend paying stocks tend to optimize the value of American call because of early exercise. Additionally, payment of dividends reduces the underlying stock price, alters parity equilibrium and consequently gives room for arbitrage

**13. Confirm that the European call is less than or equal to the American call. Show the difference, if any and comment on the reasons for this difference. Would this always be the case?**

Unfortunately, in this case, the European call option is greater than the American call option. This might have been due to market inefficiencies, improper market valuation of European calls, or trading constraints on the flexibility of early exercise of American call options. In the real world setting, transaction costs, bid-asks spread and illiquidity could also impact the trading efficiencies of the underlying stock

**14. Confirm that the European put is less than or equal to the American put. Show the difference, if any, and comment on the reasons for this difference. For example, would this always be the case?**

The conventional rule of European Put being less than or equal to the American Put does not hold in this case either.

This could be due to the dividend impact on early exercise. American put options can be impacted by dividend payouts and stock volatility, resulting in potential increases in underlying stock prices.

Low interest rates or no immediate put payoff: could cause option investors to value European option more than American put options

### Step 2

**15. Select 5 strike prices so that Call options are: Deep OTM, OTM, ATM, ITM, and Deep ITM. (E.g., you can do this by selecting moneyness of 90%, 95%, ATM, 105%, 110%; where moneyness is measured as K/S0):**

**a. Using the trinomial tree, price the Call option corresponding to the 5 different strikes selected. (Unless stated otherwise, consider input data given in Step 1).**

Call Price:

- When Deep ITM = 11.67
- When ITM = 7.73
- When ATM = 4.61
- When OTM = 2.48
- When Deep OTM = 1.19

**b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.**

From the results above, it is observed that the call option decreases (less valuable) as we move from deep ITM to deep OTM. In terms of moneyness, as moneyness increases for 90% to 110%. call option declines in value.

This also implies that when a call option is deep ITM, the stock price is higher than the strike price; if the stock were to remain as such until the expiration, the holder's would definitely exercise the option at expiration. On the other hand, when an option is deep OTM, the strike price is significantly greater than the stock price; hence, the option holder will not exercise the option at maturity.

**16. Repeat Q15 for 5 different strikes for Put options. Make sure you also answer sections a and b of Q15).**

**a.** Put Price:

- When Deep OTM = 0.55
- When OTM = 1.54
- When ATM = 3.37
- When ITM = 6.18
- When Deep ITM = 9.83

**b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.**

From the above, it is observed that the put option decreases (less valuable) as we move from deep ITM to deep OTM. In terms of moneyness, as moneyness increases from 90% to 110%, put options increase in value.

This also implies that when a put option is deep ITM, the stock price is lower than the strike price. If the stock price were to remain as such until option expiration, the holder would definitely exercise the option at expiration. On the other hand, when an option is deep OTM, the strike price is significantly lower than the stock price; hence, the option holder will not exercise the option at maturity.

**17. Repeat Q15, but this time consider Call options of American style. ( Answer sections a and b of Q15 as well.**

**a.** Call Price:

- When Deep ITM = 92.83
- When ITM = 91.86
- when ATM = 89.88
- when OTM = 88.40
- when Deep OTM = 86.92

**b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.**

From the results above, it is observed that the call option decreases in value as we move from deep ITM to deep OTM. In terms of moneyness: as moneyness increases from 90% to 110%. call option decreases in value.

This also implies that when a call option is deep ITM, the stock price is higher than the strike price; if the stock price were to spike astronomically, the holder might decide to exercise the option before expiration. However, the holder must critically compare the option price with stock value before deciding. On the other hand, when a call option is deep OTM, the strike price is significantly greater than the stock price; hence, option holders will not exercise the option. Simply put, as the strike increases, the call option value decreases.

**18. Repeat Q16, but this time consider Put options of American style. (Answer sections a and b of Q15 as well)**

**a.**

Put Price:

- When Deep OTM = 65.69
- When OTM = 70.69
- When ATM = 75.69
- When ITM = 80.69
- When Deep ITM = 85.69

From the above, it is observed that the put option decreases (becomes less valuable) as we move from deep ITM to deep OTM. In terms of moneyness, as moneyness increases from 90% to 110%, put options increase in value.

This also implies that when a put option is deep ITM, the stock price is lower than the strike price; hence, it is advisable to exercise the option immediately, as the option's price is basically its payoff. On the other hand, when an option is deep OTM, the strike price is significantly lower than the stock price; hence, option holders will not exercise the option.

Simply put, the higher the strike, the higher the potential for the put option to get deep ITM. The

American put option is more valuable and optimal when deep ITM.

**19. Graph of European Calls & Put vs. Stock Prices:**

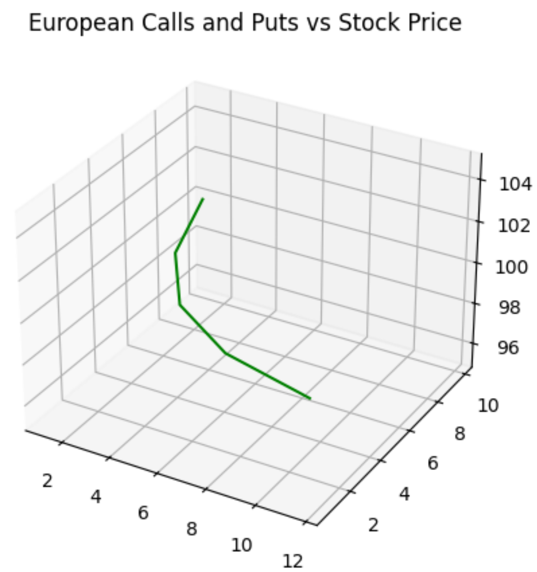



***Fig. 3. European Calls & Puts vs Stock Prices***

**20. Graph of American Calls & Put vs. Stock Prices**

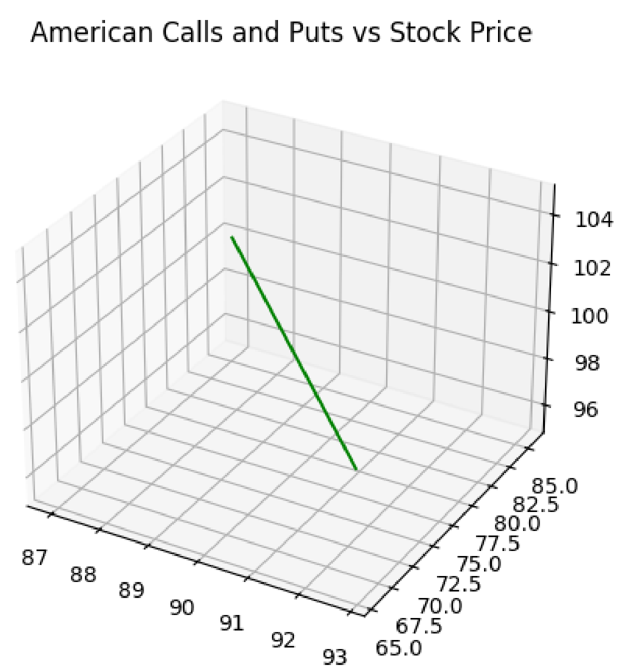



***Fig. 4. American Calls & Puts vs Stock Prices***

**21. Graph of European Calls & Puts vs. Strike Prices**

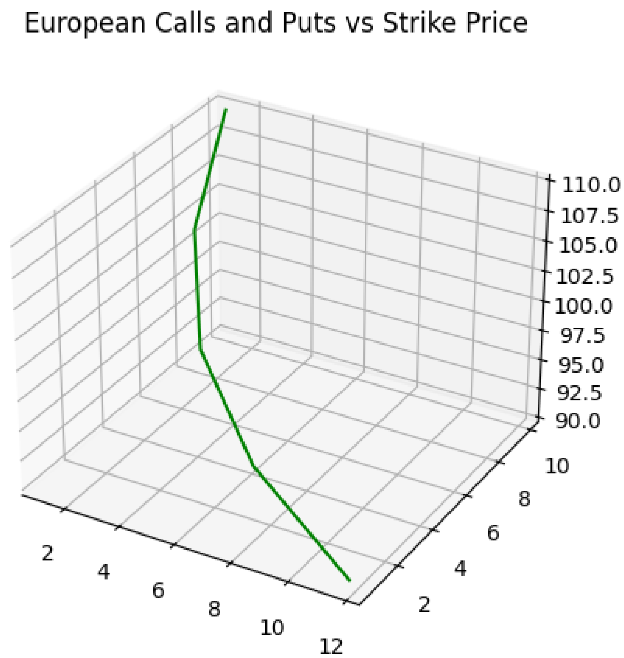



*Fig. 5. European Calls & Puts vs Strike Prices*

**22. Graph of American Calls & Puts vs. Strike Prices**

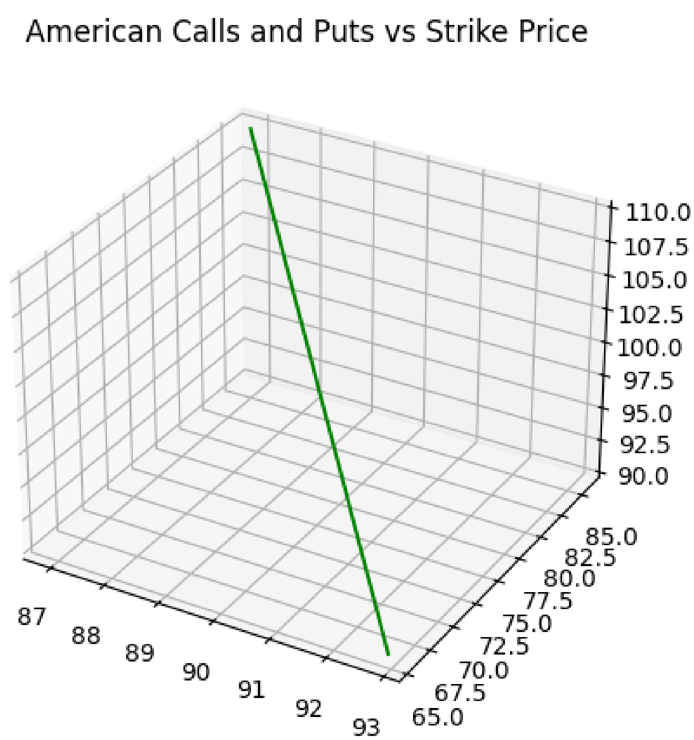


***Fig. 6. American Calls & Puts vs Strike Prices***

**23. For the 5 strikes that your group member computed in Q15 and Q16, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.**

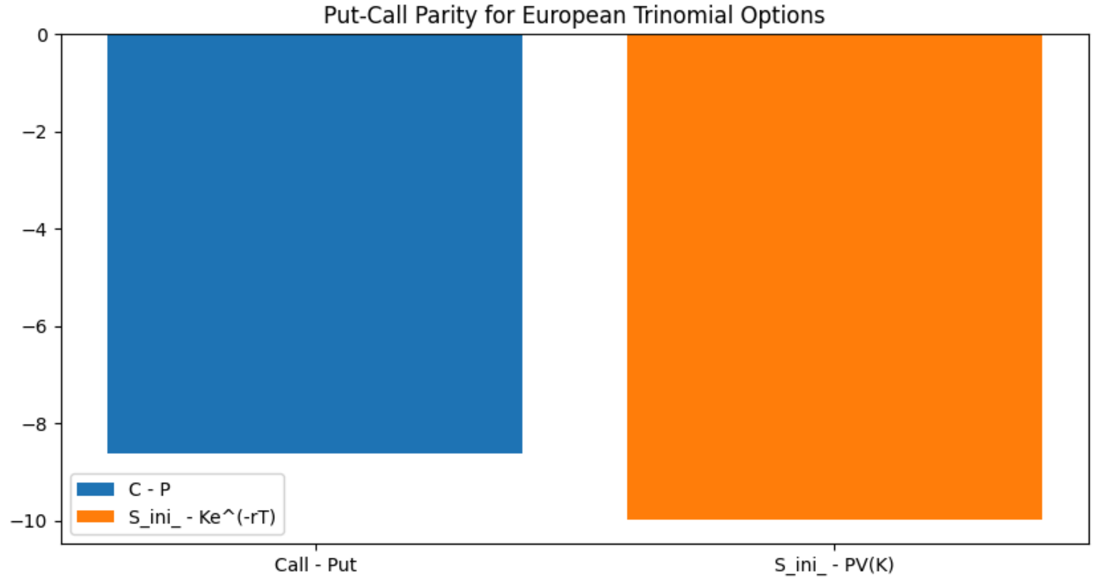



***Fig. 7. Put-Call Parity Diagram for European Trinomial Options***

*Put-Call Parity does not hold true for European Options using a trinomial tree.* Here's why:

- Small deviations across several time steps accumulate into significant small arbitrage values - which eventually result in disequilibrium between the Put-Call Parity Equation.
- Discount interest rate modelling over time tends to affect parity balance as the accumulation of rounded decimal values creates some form of arbitrage.
- Dividend adjustments over each time step (for dividend paying stocks) will significantly alter stock market price and, as such, cause an imbalance in the parity equation.

**24. For the 5 strikes that your group member computed in Q17 and Q18, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.**

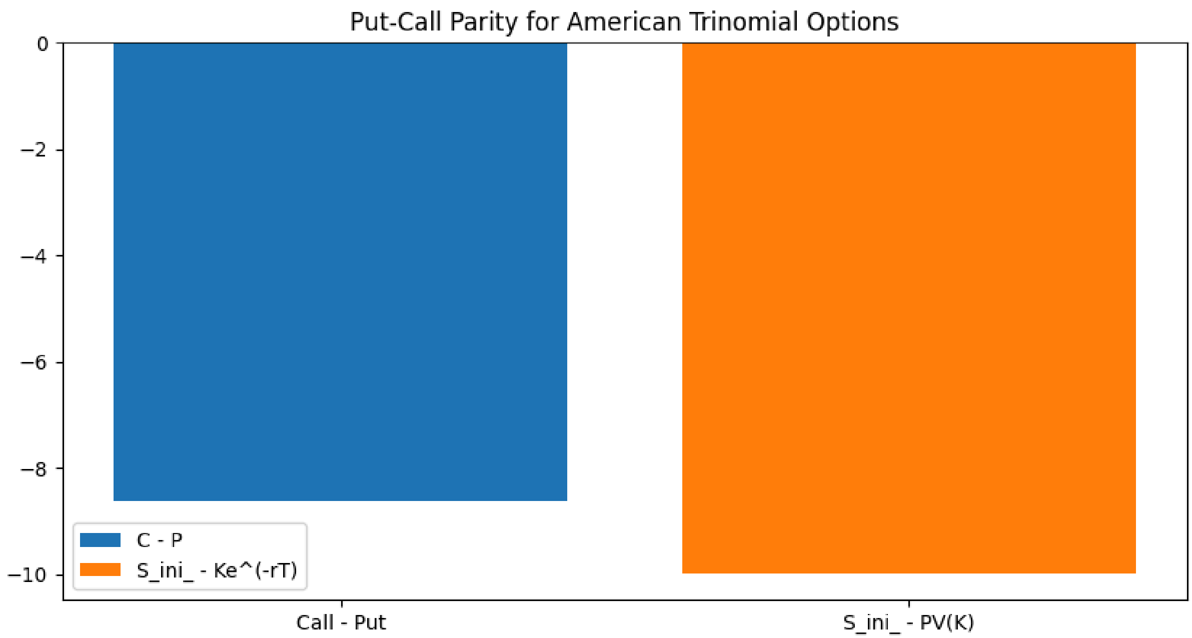



***Fig. 8. Put-Call Parity Diagram for American Trinomial Options***

*Put-Call Parity does not hold true for American Options using a trinomial tree*. Here's why:

- Friction market/transaction costs such as bid-ask spreads and stock maintenance fees can alter the equilibrium of the portfolio that normally would hold put-call equivalence
- Flexibility of Early Exercise: Volatility of stock price may result in the early exercise of an American option; hence, the payoff equivalence is not fixed at any time before expiration. The sudden price changes affect put call parity, as a deep ITM put payoff might not reflect in the overall portfolio that matches this payoff.
- Dividend adjustments over each time step (for dividend paying stocks) will significantly alter stock market price and, as such, cause an imbalance in the parity equation.
- Payoff depends on the path and discrete time step. The potential outcomes for each step alter the outcome for subsequent steps, which in turn result in payoff asymmetry and parity imbalance.

### Step 3

**25. Dynamic Delta Hedging. Use the following data: S0 = 180, r = 2%, sigma = 25%, T = 6 months, K = 182:**
**a. Price a European Put option with the previous characteristics using a 3-step binomial tree (you do not need code for this).**
The Delta hedging process for a European Put option is dynamic. I will explain it in detail using the data provided. I will also employ a 3-step binomial tree model to lend clarity to my explanation.

The price for the European Put option was calculated using a 3-step binomial tree model and the parameters given (S0=180, r=2%, sigma=25%, T=6 months, K=182). The resulting value for the put option was approximately $13.82.

**B. Pick one path in the tree.**
**i. Describe the Delta hedging process (how many units of the underlying you buy/sell, ...) of that path throughout each step if you act as the seller of the Put option.**

Let us take a look at the delta hedging process along a particular tree path. We will consider a path where the stock price makes two upward moves and one downward move (UUD path).

i. Delta Hedging Process Description
The delta hedging process requires making strategic adjustments to a variable number of shares of the underlying asset to keep a neutral position with respect to price movements. As the seller of the put option, we take a short position in the underlying asset. The cash generated from the short sale goes into a reserve account held in cash and cash equivalents.

At time \( t = 0 \), we calculate the initial delta and determine how many shares to short. We then make adjustments at subsequent points in time. These adjustment steps remind us that hedging is a dynamic process requiring constant recalculating and rebalancing.

1. Initial Step (\(t = 0 \)):
   Calculate initial Delta: approximately -7.58 shares
   Sell 7.58 shares of the underlying asset at $180 per share
   Cash account: $1,364.93 (7.58 * $180)

2. Step 1 (\( t = 1 \)):
   Recalculate Delta: approximately -7.63 shares
   Sell an additional 0.05 shares (7.63 - 7.58)
   Cash account increases by $9.00 (0.05 * $180)

3. Step 2 (\( t = 2 \)):
   Recalculate Delta: approximately -7.67 shares
   Sell an additional 0.04 shares (7.67 - 7.63)
   Cash account increases by $7.20 (0.04 * $180)

4. Final Step (\( t = 3 \)):
   Recalculate Delta: approximately -7.71 shares
   Sell an additional 0.04 shares (7.71 - 7.67)
   Cash account increases by $7.20 (0.04 * $180)

***In total, you are going to sell 7.71 shares.***

**ii. Make sure you include a table with how your cash account varies at each step (you can follow the format in the slides from Lesson 3 in Module 1). Also, assume you can buy fractions of the underlying asset shares.**

Cash Account Variation Table

Below is a table illustrating the changes in the cash account at every step:

```
Step | Stock Price | Delta (shares) | Cash Account
---------------------------------------------------------
   0 | $     180.00 |          -0.47 | $       85.06
   1 | $     199.34 |          -0.24 | $       39.17
   2 | $     220.76 |           0.00 | $      -14.33
   3 | $     244.48 |           0.00 | $      -14.33
```

***Fig. 9. Cash Account Variation Table***

**26. Using the same data from Q25, price an American Put option. Still, assume you are acting as the seller of this put. Consider now 25 steps in the tree (do this via python code).**
**a. Compute the delta hedging needed at each node in each step.**

**b. Show the evolution of the cash account throughout the different steps for one path of your choice.**

```
Step 1:
  Asset Price: 180.00
  Option Value: 13.04
  Delta Hedge: -0.4756
  Cash Account: 0.00

Step 2:
  Asset Price: 186.48
  Option Value: 9.99
  Delta Hedge: -0.3951
  Cash Account: 3.08

Step 3:
  Asset Price: 193.19
  Option Value: 7.38
  Delta Hedge: -0.3163
  Cash Account: 5.73

Step 4:
  Asset Price: 200.14
  Option Value: 5.20
  Delta Hedge: -0.2423
  Cash Account: 7.93

Step 5:
  Asset Price: 207.34
  Option Value: 3.48
  Delta Hedge: -0.1760
  Cash Account: 9.68

Step 6:
  Asset Price: 214.81
```

```
  Option Value: 2.18
  Delta Hedge: -0.1200
  Cash Account: 11.00

Step 7:
  Asset Price: 222.54
  Option Value: 1.27
  Delta Hedge: -0.0758
  Cash Account: 11.93

Step 8:
  Asset Price: 230.54
  Option Value: 0.67
  Delta Hedge: -0.0435
  Cash Account: 12.54

Step 9:
  Asset Price: 238.84
  Option Value: 0.31
  Delta Hedge: -0.0221
  Cash Account: 12.91

Step 10:
  Asset Price: 247.44
  Option Value: 0.12
  Delta Hedge: -0.0096
  Cash Account: 13.10

Step 11:
  Asset Price: 256.34
  Option Value: 0.04
  Delta Hedge: -0.0033
  Cash Account: 13.20

Step 12:
  Asset Price: 265.57
  Option Value: 0.01
  Delta Hedge: -0.0008
  Cash Account: 13.23

Step 13:
  Asset Price: 275.12
  Option Value: 0.00
  Delta Hedge: -0.0001
  Cash Account: 13.24

Step 14:
  Asset Price: 285.02
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.25

Step 15:
```

```
  Asset Price: 295.28
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.26

Step 16:
  Asset Price: 305.91
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.26

Step 17:
  Asset Price: 316.92
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.27

Step 18:
  Asset Price: 328.32
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.27

Step 19:
  Asset Price: 340.14
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.28

Step 20:
  Asset Price: 352.38
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.28

Step 21:
  Asset Price: 365.06
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.29

Step 22:
  Asset Price: 378.20
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.29

Step 23:
  Asset Price: 391.81
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.30
```

```
Step 24:
  Asset Price: 405.91
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.30

Step 25:
  Asset Price: 420.52
  Option Value: 0.00
  Delta Hedge: 0.0000
  Cash Account: 13.31
```

**c. Comment on the Delta hedging process as compared to the European option case.**

The complexity of the delta hedging process for an American option is higher than that of the European option due to early exercise features of the American option that resulted in discontinuities in delta values. This led to a sudden change in the delta values of an American option.

**27. Finally, repeat Q26 considering now an Asian ATM Put option. Comment on your results as compared to the regular American Put option case of Q25.**

**Difference between Asian ATM option and European Option**

The Asian ATM does result in delta for all steps because, at *stock, averaging* across steps results in delta very close to zero. The option payoff is path dependent; hence, a small variation in the stock price along a path will result in a corresponding change in the payoff and intrinsic value of the option, consequently bringing the delta back to zero. Therefore, the seller does not require hedging for a delta of 0.

The hedging process for European put options is almost ATM - as the strike price and initial stock price are very close. Unlike the Asian put option, the European option allows for option price hedging. This is because the delta is negative and close to -0.5. Interestingly, the seller would have to adjust option position as the underlying stock price changes and the delta becomes less negative (approaches 0). Naturally, the seller of the put option hopes that the stock exceeds the strike price at expiration (ITM) but if the reverse happens, the seller would have to go long on the option to mitigate potential losses at maturity. In essence, there is a dynamism in the hedging and re-hedging process before option expiration.

**Difference between Asian ATM option and American option**.

Unlike the Asian option, there is a hedge price for the seller of an American option, which is even more valuable than that of the European option because of the flexibility of early exercise. The American put option, which is almost ATM, is riskier to the seller than the European option; hence, there is a need for hedging and re-hedging of positions.

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1. John C. Hull's "Options, Futures, and Other Derivatives" is an extensive, in-depth textbook that covers numerous aspects and topics related to pricing options.

2. Yves Hilpisch's "Python for Finance: Analyze Big Financial Data" is a great book for learning about financial modeling with Python, an increasingly popular programming language in the world of finance.

3. Hilpisch's "Derivatives Analytics with Python," which is focused more exclusively on derivatives, includes implementations in Python programming.

4. Yuh-Dauh Lyuu's "Financial Engineering and Computation: Principles, Mathematics, Algorithms" is a great resource with a specific focus on topics and principles tied closely to option pricing.

5. Finally, Xinfeng Zhou's "A Practical Guide to Quantitative Finance Interviews" contains many relevant discussions, as well as implementation questions that are often asked in the context of quantitative finance.

6. James Chen-"How the Binomial Option Pricing Model Works" October 10,,2024

7. Enrich Money-"Stock Market"
8. An Introduction to Options Trading by Frans de Weert: https://www.oreilly.com/library/view/an-introduction-to/9781119994985/de_9781119994985_oeb_c05_r1.html#:~:text=The%20delta%20of%20a%20put,is%20between%20%E2%80%93%201%20and%200.

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