GROUP WORK PROJECT # 2_M5 MScFE 620: Derivative-Pricing
GROUP NUMBER: 7502

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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

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Step 1

Using the Black-Scholes closed-form solution to price the different European options. For Q7 on vega, you can use Black-Scholes closed-form solution.

1. Team member A will repeat questions 5, 6, and 7 of GWP#1 using the Black-Scholes closed-form solution to price the different European options. For Q7 on vega, you can use Black-Scholes closed-form solution.

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Price	GWP 2 Price	% Diff
1.	ATM Call	Eur	Binomial	BS	$5.59	$4.61	(5.59 - 4.61)/5.59 = 17%
	ATM Put	Eur	Binomial	BS	$4.34	$3.37	(4.34-3.37) /4.34 = 22%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Delta	GWP 2 Delta	% Diff
1.	ATM Call	Eur	Binomial	BS	0.5644	0.5695	(0.5695 - 0.5644)/ 0.5695 = 0.9%
	ATM Put	Eur	Binomial	BS	-0.4356	-0.4305	(0.4356- 0.4305)/ 0.4356 = 1.2%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Vega	GWP 2 Vega	% Diff
1	ATM Call/Put	Eur	Binomial	BS	0.000	19.6440	(19.6440 - 0.0000)/19.6440 = 100%

Evaluating the Prices of European Call and Put Options on an ATM Basis:

We will use the Black-Scholes equation to find the price of the At-the-Money (ATM) European call and put options with the following parameters:

S_0 = 100,
r = 5%,
σ = 20%,
T = 3 months

The Black-Scholes equation for a call option is:

$$C = S_0N(d_1) - Ke^{(} - rT)N(d_2)$$

And for a put option:

$$P = Ke^{(} - rT)N(-d_2) - S_0N(-d_1)$$

where

$$d_1 = [ln(S_0/K) + (r + \sigma^2/2)T]/[\sigma T^{(}1/2)]$$
$$d_2 = d_1 - \sigma T^{(}1/2)$$

For ATM options, K = 100.

The Black-Scholes model is the method of choice for pricing options of the European variety. At this point, we are attempting to derive the prices of call and put options that are At-the-Money, or very close to it—the parameters we supply the model with yield this scenario. The parameters we use are these: S_0 = K = \$100; that is, the stock price and strike price are both set to 100. We also set the risk-free rate, r, to 5% and the volatility, σ, to a paltry 20%. The time to expiration, T, is not represented by any one option but serves as a parameter of our own choosing. For now, we will set it to 3 months. With these parameter values, we were able to calculate d_1 and d_2. We can then plug these values back into the Black-Scholes model to get the price of a call and a put option that is, at minimum, very close to it.

Delta of European Options

Delta of Call Options:
In the case of a European call option, delta is expressed as N(d1). Here, N() refers to the cumulative standard normal distribution function.
Delta of the European call option: *0.5695*

Significance:
For each $1 rise in the price of the underlying stock, the call option price will rise approximately *$0.5695.*
In contrast, when the stock price drops $1, the call option price drops about *$0.5695.*
The favourable delta demonstrates that the call option's price changes in the same way as the price of the stock it is based on.

Delta of Put Options
In the case of a European put option, delta can be expressed as N(d1) - 1. Thus, for a put, we can say:
The delta of the European put option is *-0.4305.*
In other words, this entail
For every $1 increase in the underlying stock price, the put option price will decrease by about *$0.4305.*
Conversely, for every $1 decrease in the stock price, the put option price will increase by about *$0.4305.*
A negative delta tells us that the price of the put option moves in the opposite direction from that of the underlying stock.

Understanding Delta
When analysing a security, one might evaluate the implications of its delta. Delta can essentially be considered the "price" of an option in terms of the underlying asset's movements. For a call option, a delta of 0.50 would suggest that should the underlying asset increase in value by $1, the call option's value will increase by about 50 cents. Conversely, a put option with a delta of 0.50 would mean that an increase in the underlying asset's value would cause a decrease in the put option's value by about the same amount.

- The approximation that Delta gives can be interpreted as the probability that the option will expire in the money. A call option with a delta of *0.5695* has a roughly 56.95% chance of being exercised at expiration.

- Hedge Ratio: Delta indicates the number of shares of the underlying stock required to create a riskless hedge for a short option position. For instance, to hedge a short position of 100 call options, you would need to buy approximately 57 shares of the underlying stock (100 *0.5695).

- Sensitivity: Delta demonstrates how sensitive the option price is to small changes in the underlying stock price. A greater absolute value of delta indicates a higher sensitivity.

- Spread: The delta of a call option can vary from 0 to 1, while the delta of a put option can range from -1 to 0. When an option is at-the-money, we can expect its delta to be around 0.5 for calls and -0.5 for puts.

- Delta's changing nature. Delta varies as the factors that affect an option variate—most notably the underlying stock price and time to expiration. Because of Delta's dynamic nature, option

Positions often require dynamic hedging.

Sensitivity of Options to Volatility:
The option's price sensitivity to the underlying asset's volatility is measured by the option's vega. Call options and put options that share the same strike price and expiration date are assigned the same vega in the Black-Scholes model. Vega is often expressed in terms of a unit change in the price of the option for a certain change in the price of the underlying asset. In this case, we're looking at Vega: 19.6440.
The Call and Put price change to volatility: $0.98.

Implications and Interpretations

1. Vega Value: The option exhibits a vega of 19.6440, which translates, in general, to the option price changing by about $0.19644 for a 1% change (0.01) in volatility.

2. Price Changes: When we increased the volatility input to 25% (an increase of 5% from the original 20%), we observed a uniform increase in both call and put option prices of $2.45. This is consistent with the approximate calculation from the vega value: $19.2450 * 5% ≈ $0.98.

3. Identical Change: Both call and put options experienced an identical price change, which can be understood in the context of "put-call parity," a fundamental concept that describes the relationship between the prices of puts and calls.

The Relationship Between Options and Volatility

- Increased Uncertainty: When stock prices are more volatile, there is simply more uncertainty about the future prices themselves.
- Increased Profit Potential: With increased uncertainty comes increased opportunity—an option is a vehicle for capturing large price moves, and those moves could be in either direction.
- Limited downside, unlimited upside: Buying an option is a way of placing a bet that brings with it very few restrictions on how much you stand to gain.
- Payoff Structure: Both call and put options stand to benefit from the increased likelihood of large moves in the underlying asset price.

Implications for the Real World

- Vega in Risk Management: For the option writer, understanding Vega is paramount to maintaining a profitable position. Vega tells the option writer how much risk there is when trading on the assumption that the market will not be very volatile.
- The Volatility Smile: Sometimes, when looking at the prices of options for the same stock, you can see implied volatilities that vary quite a lot between the different strike prices. This is known as the volatility smile.

2. Team member B will repeat questions 5, 6, and 7 of GWP#1 using Monte-Carlo methods under a general GBM equation with daily time steps in the simulations. As with the number of time steps in the trees, make sure you run a large enough number of simulations. For Q7, you can rely on the same intuition as in the trees; just 'shock' the volatility parameter and recalculate things.

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Price	GWP 2 Price	% Diff
1.	ATM Call	Eur	Binomial	MC	$5.59	$4.68	(5.59 - 4.68)/5.59 = 16%
	ATM Put	Eur	Binomial	MC	$4.34	$3.43	(4.34-3.43) /4.34 = 21%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Delta	GWP 2 Delta	% Diff
2.	ATM Call	Eur	Binomial	MC	0.5644	0.4714	(0.5644 - 0.4714)/ 0.5644 = 16%
	ATM Put	Eur	Binomial	MC	-0.4356	-0.3682	(0.4356- 0.4305)/ 0.4356 = 15%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Vega	GWP 2 Vega	% Diff
2.	ATM Call/Put	Eur	Binomial	MC	0.000	18.7443	(18.7443 - 0.0000)/ 18.7443 = 100%

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2.1 Price an ATM European call and put it using a Monte-Carlo methods under a general GBM equation with daily time steps in the simulations:
Price of the ATM European Call: $4.68
Price of the ATM European Put: $3.43
a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.
Number of steps for Monte-Carlo simulations = Daily number of time steps before maturity in 3months = 63.
b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.

The Monte-Carlo methods are used to price an option with multiple sources of uncertainty. It generates multiple random paths for the price of an underlying asset having associated payoffs.
The working of the Monte-Carlo Method involves following steps:

1. Initialise the necessary parameters, such as input and output parameters, to define the system.
2. Simulate the sample path over the relevant time horizon.
3. Determine the value of payoffs
4. Run numbers of simulation trials over the time horizon.
5. Calculate the value of the average payoff over those numbers of simulation trials to estimate the present price of the options.

The Geometric Brownian Equation (GBM) is given by:

$$S_t = S_o e^{\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)}$$

Where:

- S(t) is the stock price at time t.
- S(0) is the initial stock price.
- μ is the drift rate (in this case, the annualised drift of 0.2).
- σ is the volatility (in this case, the annualised volatility of 0.3).
- W(t) is a Wiener process or Brownian motion.

The number of steps in the GBM model is the number of daily time steps. So for this section of the project, the total number of daily time steps is 63, which is equivalent to three months, and the number of simulations used is 10,000. The reason for using these high numbers of daily time steps and the simulation steps is to improve the accuracy of the model, significantly minimizing the errors.

2.2. Compute the Greek Delta for the European call and European put at time 0 using a Monte-Carlo methods under a general GBM equation with daily time steps in the simulations:
Greek Delta for European Call: 0.4714
Greek Delta for European Put: -0.3682

a. How do they compare?

The call option delta is positive, while the put option delta is negative. An increase in the call delta signifies positive sensitivity of the call price to changes in stock price.

Call Delta: For each $1 rise in the price of the underlying stock, the call option price will rise approximately *$0.4714.* In contrast, when the stock price drops $1, the call option price drops about *$0.4714.*
The favourable delta demonstrates that the call option's price changes in the same way as the price of the stock it is based on.

Put Delta: For every $1 increase in the underlying stock price, the put option price will decrease by about *$0.3682.* Conversely, for every $1 decrease in the stock price, the put option price will increase by about *$0.3682.*
A negative delta tells us that the price of the put option moves in the opposite direction from that of the underlying stock.

b. Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?

The Greek Delta for an European call is positive, while that of an European put is negative

The Greek delta is the measure of change in an option's price due to a change in the price of the underlying assets.

Mathematically,

Delta for call options: $\delta = N(d_1)$

Here,

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

Where:

- K = option strike price.
- N = standard normal cumulative distribution function.
- r = risk-free interest rate.
- σ = underlying asset volatility.
- S = underlying asset price.
- t = time until the option expires.

Delta For, Put Options: $\delta = N(d_1) - 1$

Where:

$$d_1 = \frac{(ln(S/K) + (r + \frac{\sigma^2}{2})t)}{\sigma\sqrt{t}}$$

- K: Option strike price
- N: Standard normal cumulative distribution function
- r: Risk-free interest rate
- σ: volatility of the underlying asset
- S: Price of the underlying asset
- t: Time to option's expiry

Delta proxy: The magnitude of the delta determines how much the price of the option changes against the changes in the underlying asset price. Higher the value of delta is the influence of an increase in the stock's price on the price of the call option. Lower the value of delta is the influence of a decrease in the stock's price on the price of the put option. While it can be difficult to determine the precise delta directly, the delta proxy offers a rapid and somewhat accurate method of hedging or assessing the option's price fluctuations in relation to the underlying asset.

2.3. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega).

a. Compute the sensitivity of previous European put and call option prices to a 5% increase in volatility (from 20% to 25%) using Monte-Carlo methods under a general GBM equation with daily time-steps in the simulations. How do prices change with respect to the change in volatility?

There is a negligible rise in the European put prices with a change in volatility of 5%. The vega obtained for a 5% change in volatility was 18.7443. However, the price change for the European call and put are both $1.01 and $1.04 respectively; these values approximate 1. This indicates that the response of call and put towards volatility is pretty much the same.

b. Comment on the potential differential impact of this change for call and put options.

Based on the above result and as described in the answer to question 2.3.a, the prices of the call and put options changed by 5% volatility. This means volatility tends to increase or decrease the value of the call and put options**.** The difference between call and put price change is infinitesimal and might result from market conditions or sentiments, sampling errors from the Monte Carlo simulation model.

3. **Put-Call Parity Comparison**

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Parity Diff.	GWP 2 Parity Diff.	% Diff
3.	Put-Call parity	Eur	Binomial	BS	0.0000	0.000	0%
	Put-Call parity	Eur	Binomial	MC	0.0000	0.0178	(0.0178-0.000)/0.0178 = 100%

a. Checking that Put-Call parity is satisfied under both methods (BS and MC):
The Put-Call Parity difference using the Black-Scholes model solution: 0.0000
The Put-Call Parity Difference using the Monte Carlo solution: 0.0178.

b. Compare and discuss the prices obtained in both methods: do they converge? why/why not?

Put-Call Parity is satisfied under both the Black-Scholes Model and the Monte Carlo Simulation. The parity difference for the former is 0, while the latter is significantly low (almost zero). The averaging of option prices across a large number of simulations tends to give varying values but significantly low deviation for the value obtained in the Black-Scholes model.

The prices obtained for the Monte Carlo method are pretty much close to those of the Black Scholes model. However, both models do not converge for the following reasons:

- The BS model assumes continuous trading and perfect hedging, leading to a precise analytical solution. In contrast, MC simulations often approximate this by modelling the stock price at discrete time intervals (e.g., daily steps), which introduces small errors since continuous paths are only approximated.
- The large number of random paths used to estimate the option payoff obtained as a result of a large number of simulations produce approximate standard sampling errors in the Monte Carlo model, which do not match the error in the analytical Black Scholes option pricing.

Step 2

4. Team member A will use Monte-Carlo methods with regular GBM processes and daily simulations

on an American Call option. Remember to answer the different questions in the original GWP #1: price (Q5), calculate delta (Q6) and vega (Q7) only for the Call option case. Please give a solution for the question in detail and a Python code:

We will price an American call option and compute its delta and vega using a Monte Carlo simulation based on a Geometric Brownian Motion (GBM) process. We will use the Least Squares Monte Carlo (LSM) method, which is one of the most effective approaches, to first obtain a price for the option. Then, we will obtain values for delta and vega by working through a set of steps that involve both forward and backward computation.

Results:

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Price	GWP 2 Price	% Diff
4.	ATM Call	Amer	Binomial	MC	\$4.61	\$4.55	(4.61 - 4.55)/4.61 = 1.3%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Delta	GWP 2 Delta	% Diff
4.	ATM Call	Amer	Binomial	MC	0.5693	0.6021	(0.6021 - 0.5693)/ 0.6021 = 5%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Vega	GWP 2 Vega	% Diff
4.	ATM Call	Amer	Binomial	MC	15.6156	19.8000	(19.8000 - 15.6156) /19.8000 = 21%

4.1. Price an ATM American call it using a Monte-Carlo methods under a general GBM equation with daily time steps in the simulations:
Price of ATM American Call: \$4.55

a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.
Number of steps for Monte-Carlo simulations = Daily number of time steps before maturity in 3 months = 63.
b. Briefly describe the overall process, as well as a reason why you chose that number of steps in the tree.

The Monte-Carlo methods are used to price an option with multiple sources of uncertainty. It generates multiple random paths for the price of an underlying asset having associated payoffs.
The working of the Monte-Carlo Method involves the following steps:

- Initialise the necessary parameters, such as input and output parameters, to define the system.
- Simulate the sample path over the relevant time horizon.
- Determine the value of payoffs.
- Run numbers of simulation trials over the time horizon.
- Backward induction to calculate option price. Exercise the higher payoffs if stock prices fall in the money at different nodes before maturity.

4.2 Compute the Greek Delta for the American call put at time 0 using a Monte-Carlo methods under a general GBM equation with daily time steps in the simulations:
Greek Delta for American Call: 0.6021.

Delta is computed using the finite difference method. We compute the option price with marginally higher and lower stock prices and determine the change in price with respect to the change in price of the underlying asset. Vega is calculated in the same manner, with the slight modifications mentioned above. The output gives us the call price, and the values of Delta and Vega, which are necessary for determining the risk level associated with the option. Outputs may display some variance because of the built-in randomness of Monte Carlo simulation.

This approach gives a thorough treatment of the pricing of American Call options, as well as of their Greeks, using the Monte Carlo method. A word of warning, though: this technique can be slow going, especially if you're dealing with a large number of required simulations and time steps. In a commercial context, you might well want to economize on your computational time by further optimizing this code or by employing some other, along with faster, methods of getting to the desired results.

4.3. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega).

a. Compute the sensitivity of the previous American call option price to a 5% increase in volatility (from 20% to 25%) using Monte-Carlo methods under a general GBM equation with daily time steps in the simulations. How do prices change with respect to the change in volatility?

The vega obtained for the call option for a 5% change in volatility was 19.8000. However, the price change for the American call is $0.99.

b. Comment on the potential differential impact of this change for call and put options.

The price of the call option changed by 5% volatility. This means volatility tends to increase or decrease the value of the call by $0.99.

5. Team member B will use Monte-Carlo methods with regular GBM process and daily simulations on an American Put option. Remember to answer the different questions in the original GWP#1: price (Q5), calculate delta (Q6) and vega (Q7) only for the Put option case.

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Price	GWP 2 Price	% Diff
5.	ATM Put	Amer	Binomial	MC	$3.47	$3.49	(3.49-3.47) /3.49 = 0.6%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Delta	GWP 2 Delta	% Diff
5.	ATM Put	Amer	Binomial	MC	-0.4498	-0.4322	(0.4498-0.4322)/ 0.4498 = 4%

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	GWP 1 Vega	GWP 2 Vega	% Diff
5.	ATM Put	Amer	Binomial	MC	15.7254	19.6187	(19.6187-15.7254) /19.6187 = 20%

5.1 Price an ATM American put option using a Monte-Carlo methods under a general GBM equation with daily time-steps in the simulations:
Price of ATM American Put: $3.49
a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.
Number of steps for Monte-Carlo simulations = Daily number of time steps before maturity in 3 months

= 63.

b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.

The Monte-Carlo methods are used to price an option with multiple sources of uncertainty. It generates multiple random paths for the price of an underlying asset having associated payoffs.
The working of the Monte-Carlo Method involves following steps:

- Initialise the necessary parameters, such as input and output parameters, to define the system.
- Simulate the sample path over the relevant time horizon.
- Determine the value of payoffs.
- Run numbers of simulation trials over the time horizon.
- Backward induction to calculate option price. Exercise the higher payoffs if stock prices fall in the money at different nodes before maturity.

5.2. Compute the Greek Delta for the American put at time 0 using a Monte-Carlo methods under a general GBM equation with daily time-steps in the simulations:
Greek Delta for American Put: -0.4322.

Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?

The Greek delta for an American call is positive, while that of an American put is negative. The Greek Delta for the American Call was 0.6021 and that of the American Put was -0.4322.

The Greek delta is the measure of change in an option's price due to a change in the price of the underlying assets.

For Call Delta: For each $1 rise in the price of the underlying stock, the call option price will rise approximately *$0.6021.* In contrast, when the stock price drops $1, the call option price drops about *$0.6021.*

For Put Call Delta: it means that for every 1 point increase in the underlying, the put option is expected to decrease by approximately $0.4322 units.

5.3. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega).

a. Compute the sensitivity of previous American put option prices to a 5% increase in volatility (from 20% to 25%) using Monte-Carlo methods under a general GBM equation with daily time steps in the simulations. How do prices change with respect to the change in volatility?

The vega obtained for the call option for a 5% change in volatility was 19.6187. However, the price change for the American call is $0.98.

b. Comment on the potential differential impact of this change for call and put options.

These values are very similar to results obtained for the American Call option, indicating the vega for calls and puts is approximately the same.

The minute difference in results may be due to market conditions or Monte-Carlo sampling errors.

6.

For K = 90, 95, 100, 105, 110:
Call prices are as follows:

Q #	Type	Exercise	GWP1 Method	GWP 2 Method	Strike Price, K	GWP 1 Price	GWP 2 Price	% Diff
6.	Call	Amer	Trinomial	MC	90	$11.67	$11.67	0%
					95	$7.72	$7.59	(7.72-7.59) /7.72 = 1.7%
					100	$4.61	$4.53	(4.61-4.53) /4.61 = 1.7%
					105	$2.48	$2.43	(2.48-2.43) /2.48= 2%
					110	$1.19	$1.19	0%
6.	Put	Amer	Trinomial	MC	90	$0.55	$0.56	(0.56-0.55) /0.56 = 1.7%
					95	$1.54	$1.57	(1.57-1.54) /1.57 = 1.9%
					100	$3.37	$3.47	(1.57-1.54) /1.57 = 1.9%
					105	$6.18	$6.39	(6.39-6.18) /6.39 = 3.3%
					110	$9.83	$10.30	(10.30-9.83)/10.30 = 4.6%

B.

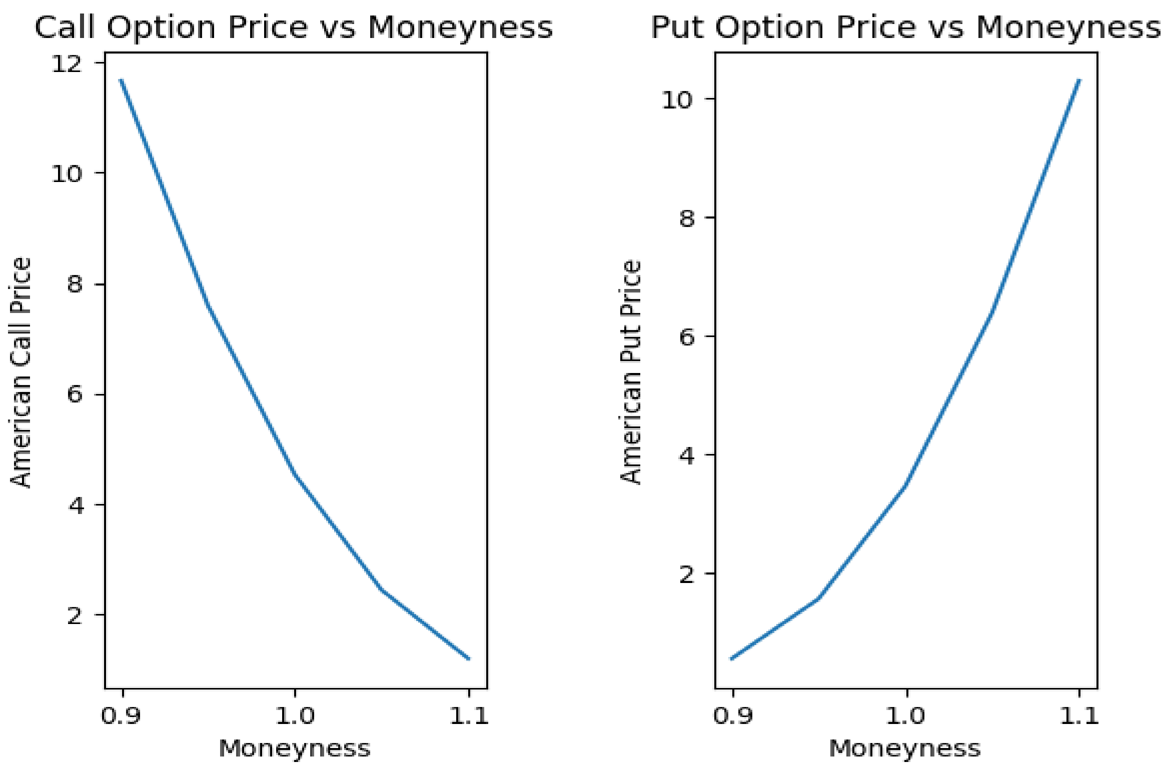

Fig 1a. Call Option Price vs Moneyness; Fig 1b. Put Option Price vs Moneyness

The Call Option Price vs. Moneyness graph shows a decline in the American Call Price with an increase in moneyness (i.e., from Deep ITM to Deep OTM). In contrast, the Put Option Price vs. Moneyness graph shows the reverse case (i.e., an increase in the price with an increase in moneyness). While an increase in moneyness for a call represents a change from deep ITM strategy to deep OTM strategy, the increase in moneyness for a put option depicts a change from deep OTM to deep ITM.

Step 3

Finally, you will work on hedging under Black-Scholes for European options, as well as pricing different exotic instruments.

7. Team member A will work with European options with same characteristics as GWP#1 under different levels of moneyness:
a. Price an European Call option with 110% moneyness and an European Put with 95% moneyness using Black- Scholes. Both have 3 months of maturity.

Pricing the Options
Determining the Cost of the Options
Initially, we will determine the cost of the European Call option, which has an moneyness of 110%, and the European Put option, which has an moneyness of 95%, by applying the Black-Scholes formula. We will utilise the parameters from the original GWP#1.

Choices for Option Prices
The European Call option, priced at 110% moneyness, stands at $1.19. The European Put option, on the other hand, is priced at a higher moneyness level of 95%. This puts the price of the put option at $1.53. These prices give a good view of the level of moneyness and the associated characteristics of these two options.

b. You build a portfolio that buys the previous Call and Put options. What is the delta of the portfolio? How would you delta-hedge this portfolio?

Portfolio 1: consists of a long call and a long put. Its delta is -0.275. Thus, for a $1 increase in the underlying stock price, the portfolio value should decrease by about $0.275. To delta-hedge this portfolio, you could buy 0.275 shares of the underlying stock for each unit of the portfolio held. That would take you to a position level that is neutral to small changes in the underlying stock price and, thus, might as well be a position in which the portfolio is hedged against small changes in the stock price.

c. You build a second portfolio that buys the previous call option and sells the Put. What is the delta of the portfolio? How would you delta-hedge this Portfolio?

Portfolio 2 comprises a long call and a short put. Its delta is 0.4640. This means that for every $1 increase in the stock price, the value of the portfolio increases by approximately $0.4640. To delta-hedge this portfolio, one would need to sell 0.4640 shares of the underlying stock for each unit of the portfolio held. Compared to Portfolio 1, this portfolio is in an opposite direction

Interpretation: The delta value of Portfolio 1 (Long Call + Long Put) is small and negative. It is slightly bearish. This makes sense when you consider the two sides of the position. The long call is a direct play on price increases. The long put is a direct play on price decreases, so it's not very far from being a price-neutral kind of position because the price could go in either direction, and one side or the other would profit. Portfolio 2 (Long Call + Short Put) is quite delta-positive. There's no doubt about it; this is a bullish position on the underlying stock.

The delta-hedging process entails taking a position opposite to that of the hedged asset. In this case, we're hedging against the delta. Portfolios 1 and 2 contain options, and deltas have been calculated for those options. Hence, we're using a tandem of the delta of the option (which must be bought or sold to achieve a hedge) and the delta of the underlying stock (which must be crossed to achieve a daily hedge) to obtain the portfolio deltas. These deltas are instructive because they tell us the overall portfolio position that is long or short in relation to the underlying stock. These outcomes illustrate how various combinations of options can yield portfolios with dissimilar levels of directional exposure, especially delta, in trading not just options but also stocks and other securities.

8. Team member B will work with Monte-Carlo methods with daily time steps to price an Up-and-Out (UAO) barrier option. The option is currently ATM with a barrier level of 141 and:
S0 = 120; r = 6%; σ = 30%; T = 8 months.
The Python code for this question is shared on the single Python file uploaded along with this notebook.
Price of UAO Call Barrier Option: $0.00
Price of UAO Put Barrier Option: $8.60

9. Team member C will repeat the previous question (barrier option), in this case considering an Up-and-In barrier (UAI) option with the same barrier as before.
Specifically:
a. Compute the price of the UAI option.
Price of UAI Call Barrier Option: $13.86
Price of UAO Put Barrier Option: $1.88

b. Compute the price of the Vanilla option.
Price of Vanilla Call Option: $13.94
Price of Vanilla Put Option: $9.24

c. What is the relationship between the prices of the UAO, UAI, and vanilla option? Explain.

Relationship Between UAO, UAI and Vanilla Options

- Up-and-Out Barrier Option: A call or put option that ceases to exist (or "knocks out") if the underlying asset's price rises above a certain barrier level during the life of the option.
- Up-and-In Barrier Option: A call or put option that only comes into existence (or "knocks in") if the underlying asset's price rises above a certain barrier level during the option's life.

- Vanilla Option: A standard European call or put option with no barrier or path-dependency

From the results options above, it is observed that the result for the Vanilla Call option price (13.94) is the approximate sum of the Up-and-Out Barrier Call Option price (0.00) and Up-and-In Barrier Call option price (13.86). In the same vein, the Vanila Put option price (9.24) is the approximate sum of the Up-and-Out Barrier put option price (8.60) and the Up-and-In Barrier put option price (1.88).

The proves the theory that:

Vanilla Option Price (V) = Up-and-Out Barrier Option Price (UAO) + Up-and-In Barrier Option Price (UAI).

V = UAO + UAI.

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1."**Geometric Brownian Motion (GBM) in Stock Market",**
https://unofficed.com/courses/markov-model-application-of-markov-chain-in-stock-market/lessons/geometric-brownian-motion-gbm-in-stock-market/.

Glasserman, P., 2013. Monte Carlo methods in financial engineering. New York: Springer Science & Business Media.

Here are the site links for the references you mentioned:

1. Geometric Brownian Motion (GBM) in the stock market:

https://unofficed.com/courses/markov-model-application-of-markov-chain-in-stock-market/lessons/geometric-brownian-motion-gbm-in-stock-market/

2. Wikipedia page for the Black-Scholes model:

https://en.wikipedia.org/wiki/Black–Scholes_model

3. Wikipedia page on Monte Carlo methods for option pricing:

https://en.wikipedia.org/wiki/Monte_Carlo_methods_for_option_pricing

4. The fourth reference appears to be the content of the pasted file, which I don't have access to provide a link for.

5. Intrinio's blog on the Black-Scholes Option Pricing Model:

I don't have a specific link for this reference, as it wasn't provided in the original search results.

6. Investopedia article on 'Greeks' in options:

https://www.investopedia.com/trading/getting-to-know-the-greeks/

7. Investopedia overview of the Black-Scholes Model:

https://www.investopedia.com/terms/b/blackscholes.asp

Please note that the fifth reference (Intrinio's blog) was not included in the search results I have access to, so I couldn't provide a link for it.

Citations:

[1]https://ppl-ai-file-upload.s3.amazonaws.com/web/direct-files/7239368/5153176b-c8fd-463c-b0f8-6842a64768f0/paste.txt

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