The Partition Problem: A Numerical Study Of The Wave Function And It's Wigner Function

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In the following document I will lay out some of the results

1 Definition

Let Z denote the n-tuple $Z = (z_1, ..., z_n)$ where $z_i \in \mathbb{Z}\ \forall i = 1, .., n$ and let ψ be the following normalizable wave function in a complex continuous Hilbert space, and it's projection to the position space be:

$$\langle x|\psi\rangle = \psi(x) = [\prod_{i=1}^{n} cos(\pi z_i x)]rect(2, x)$$

Where $rect(k, x) = \begin{cases} 1 & x \in [\frac{-k}{2}, \frac{k}{2}] \\ 0 & else \end{cases}$

1.1 Partition problem

From Wikipedia (https://en.wikipedia.org/wiki/Partition_problem):

"..the partition problem, or number partitioning, is the task of deciding whether a given multiset S of positive integers can be partitioned into two subsets S1 and S2 such that the sum of the numbers in S1 equals the sum of the numbers in S2.."

The problem of partitioning the set $S = \{a \,|\, \exists a \in Z : a = z_i \, for \, some \, i = 1, ..., n\}$ into N different combinations of subsets $S_1, S_2 \subset S$ can be solved by:

$$\int_{-1}^{1} \psi(x)dx = \frac{N}{2^n}$$

2 The Wave function

In this section we will see the wave function

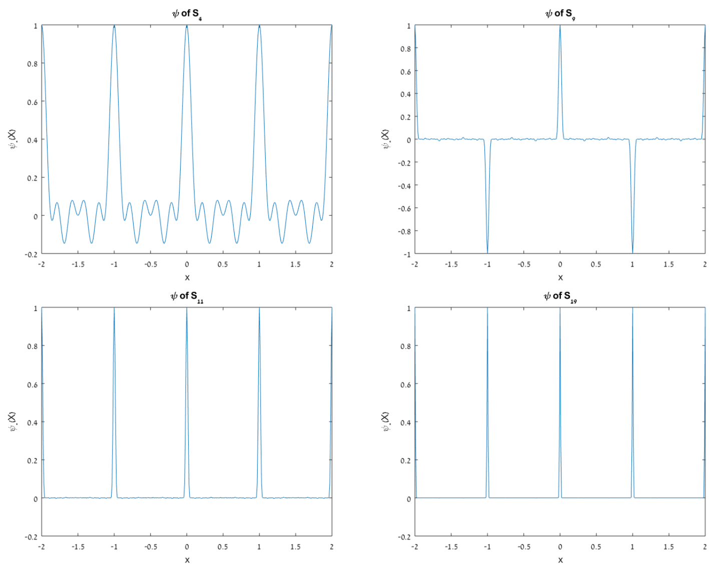

Figure 1: $\psi_*(x)$ for different sized S_k sets

$$\psi_*(x) = [\prod_{i=1}^{n} cos(\pi z_i x)]$$

for different sets $S_k = \{n | \, n \in \mathbb{N} \wedge n \leq k\}$ e.g. $S_3 = \{1, 2, 3\}$

2.1 Momentum Space

By projecting the wave function onto momentum space:

$$\langle p|\psi\rangle = \psi(p) = \int \psi(x) e^{-\frac{i2\pi px}{h}} dx$$

we are able to see that:

$$\psi(p = 0) = \frac{N}{2^n}$$

and so for $S_4 = \{1, 2, 3, 4\}$ we get only one partition so that

$$\psi(p = 0) = \frac{1}{2^4} = \frac{1}{16} = 0.0625$$

and so we see the following numerical result coincides :

3 Wigner Function

Let us present the Numeric result of the computation of the Wigner distribution of the S_{10} set:

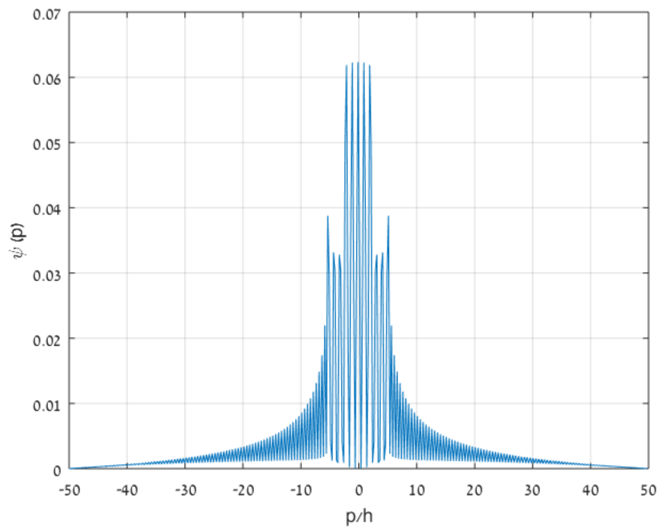

Figure 2: $\psi(p)$ of S_4

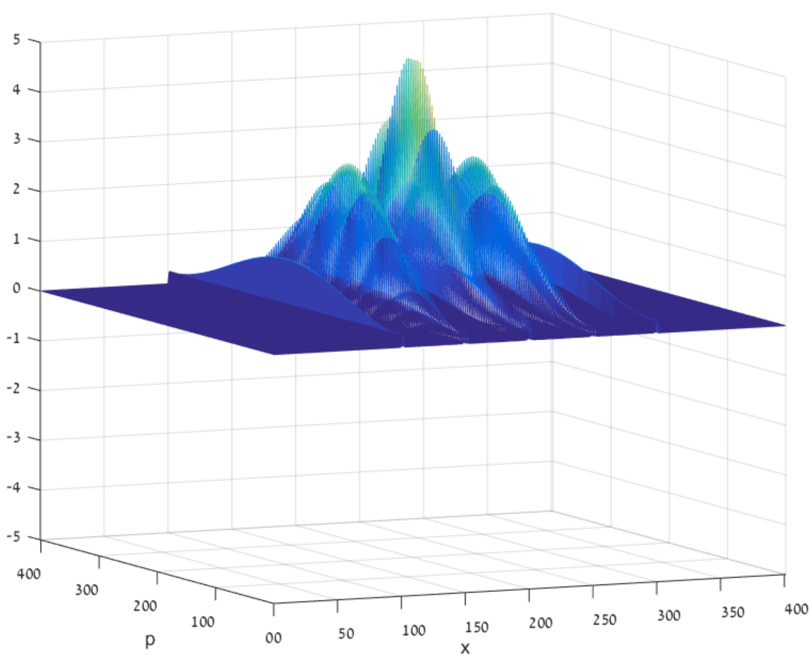

Figure 3: Wigner Function of S_{10}