GCLC 2022
(Geometry Constructions → LaTeX Converter)
Manual

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Chapter 1

Briefly About GCLC

What is GCLC? **GCLC** (from "Geometry Constructions → LaTeX converter") is a tool for visualizing and teaching geometry, and for producing mathematical illustrations. Its basic purpose is converting descriptions of mathematical objects (written in the GCL language) into digital figures. **GCLC** provides easy-to-use support for many geometrical constructions, isometric transformations, conics, and parametric curves. The basic idea behind **GCLC** is that constructions are formal procedures, rather than drawings. Thus, in **GCLC**, producing mathematical illustrations is based on "describing figures" rather than of "drawing figures". This approach stresses the fact that geometrical constructions are abstract, formal procedures and not figures. A figure can be generated on the basis of abstract description, in Cartesian model of a plane. These digital figures can be displayed and exported to LaTeX or some other format.

Although **GCLC** was initially built as a tool for converting formal descriptions of geometric constructions into LaTeX form, now it is much more than that. For instance, there is support for symbolic expressions, for drawing parametric curves, for program loops, user-defined procedures, etc; built-in theorem provers can automatically prove a range of complex theorems; the graphical interface makes **GCLC** a tool for teaching geometry, and other mathematical fields as well.

The main purposes of GCLC:

- producing digital mathematical illustrations of high quality;
- usage in mathematical education and as a research tool;
- storing mathematical contents;
- studing automated geometrical reasoning.

The main features of GCLC:

- freely available;
- support for a range of elementary and advanced constructions, and isometric transformations;
- support for symbolic expressions, second order curves, parametric curves, loops, user-defined procedures, etc.

- user-friendly interface, interactive work, animations, tracing points, watch window ("geometry calculator"), and other tools;
- easy drawing of trees;
- built-in theorem provers, capable of proving many complex theorems (in traditional geometry style or in algebraic style);
- very simple, very easy to use, very small in size;
- export of high quality figures into LaTeX, EPS, SVG, bitmap format;
- import from JavaView JVX format;
- available from http://www.matf.bg.ac.rs/~janicic/gclc and from EMIS (The European Mathematical Information Service) servers: http://www.emis.de/misc/software/gclc/.

Implementation and platforms: There are command-line versions and versions with graphical user interface (GUI) of **GCLC** for Windows and for Linux. The version with graphical user interface provides a range of additional functionalities, including interactive work, animations, traces, "watch window", etc. It gives **GCLC** a new, graphic user-friendly interface, and introduces some new features which are not available in the command-line version. It is a kind of an "Integrated Development Environment" or IDE for **GCLC**. The version of **GCLC** with GUI for Windows is called **WinGCLC** or **gclc-gui**, the version of **GCLC** with GUI for Linux is called **gclc-gui**.

GCLC can be also used via **GeoThms** (joint work with Pedro Quaresma, University of Coimbra), a web-based framework for constructive geometry (http://hilbert.mat.uc.pt/~geothms).

GCLC program is implemented in the C++ programming language.

Author: **GCLC** is being developed at the Faculty of Mathematics, University of Belgrade, by Predrag Janičić and, in some parts, by Predrag Janičić and his collaborators:

- Ivan Trajković (University of Belgrade, Serbia) — a co-author of the graphical interface for **WinGCLC** 2003;
- prof. Pedro Quaresma (University of Coimbra, Portugal) — a co-author of the theorem prover based on the area method built into **GCLC**.
- Goran Predović (University of Belgrade, Serbia) — the main author of the theorem prover based on the Wu's method and Gröbner based method built into **GCLC**.
- prof. Pedro Quaresma (University of Coimbra, Portugal), Jelena Tomašević (University of Belgrade, Serbia), and Milena Vujošević-Janičić (University of Belgrade, Serbia) — co-authors of the XML support for **GCLC**.
- Luka Tomašević (University of Belgrade, Serbia) — the main author of the support for graph drawing.
- prof. Konrad Polthier and Klaus Hildebrandt (Technical University, Berlin, Germany) — coauthors of JavaView → **GCLC** converter).

Version history: **GCLC** programs is under development since 1995. and had several releases since then. It has thousands of users and has been used for producing digital illustrations for a number of books and journal volumes and in a number of different high-school and university courses.

What others said about GCLC/WinGCLC: "... program **WinGCLC**... is a very useful, impressive professional academic geometry program." (from an anonymous review for "Teaching Mathematics and its Applications")

References: More on the background of **GCLC** can be found in [6, 9, 5, 10, 7, 8].

1.1 Comments and Bugs Report

Please send your comments and/or noticed bugs to the following e-mail address: `janicic@matf.bg.ac.rs`. Your feedback would be very much appreciated and would help in improving the future releases of **GCLC**.

1.2 Copyright Notice

If you use **GCLC**, please let me know by sending an e-mail to Predrag Janičić (`janicic@matf.bg.ac.rs`). You will be put on the **GCLC** mailing list and be informed about new releases of **GCLC**.

If you used **GCLC** for producing figures for your book, article, thesis, I would be happy to hear about that.

Your feedback would be very much appreciated and would help in improving the future releases of **GCLC**.

Chapter 2

Quick Start

In **GCLC** one describes mathematical objects in the GCL language. This description can be visualized within the version with GUI (or within VIEW previewer, see p 85) or can be converted into some other format, e.g., LaTeX format. In this chapter, we describe how to run **GCLC** and we give one very simple figure description and discuss how it can be processed and give an illustration in LaTeX format.

2.1 Installation

There is no installation required for **GCLC**— just unzip the distribution archive (to a folder of your choice) and you can run the program. For convenience, you can add the path to this folder to the system path, so you can run **GCLC** from any folder. You can associate **GCLC** (GUI version) with `.gcl` files, so you can always open them with **GCLC**.

When the archive is unpacked, in the root folder there will be executable programs – a command line version (`gclc`) and a version with GUI (`GCLC`), and LaTeX packages `gclc.sty` and `gclcproofs.sty` for processing figures and proofs generated by **GCLC** (in a simple LaTeX format). In addition, there will be the following folders:

- `manual` with the manual file and additional reference papers;
- `samples` with a range of `.gcl` samples, organized in the following subfolders:
  - `basic_samples` with basic samples for **GCLC**;
  - `samples_prover` with samples for the theorem prover;
  - `samples_gui` with samples specific for GUI version;
- `tools` with additional tools (`view` and `jv2gcl`) (not included in the version for Linux);
- `working_example` with a self-contained example ready to be processed by LaTeX;

- `LaTeX_packages` with LaTeX packages (developed by other authors) required for the prover output or for support for colors.
- `XML_support` XML suite for different processing of XML files generated by **GCLC**.

2.2 First Example

Using **GCLC** is very simple. Like many other programs, **GCLC** has its document type — `*.gcl` document type. `*.gcl` file is nothing more than a plain text file (it has no special formatting inside), containing a list of GCL commands.

Consider the following text:

```
point A 40 85
point B 35 20
point C 95 20

cmark_lt A
cmark_lb B
cmark_rb C

drawsegment A B
drawsegment B C
drawsegment C A
```

It describes a triangle `ABC` via GCL commands. The command `point A 40 85` introduces a point `A` with coordinates $(40, 85)$, given in millimeters (the point $(0, 0)$ is in the left-bottom corner of the picture and the picture has the default size $140mm \times 100mm$). The command `cmark_lt A` denotes the point `A` by a small circle and prints its name in left-top direction. The command `drawsegment A B` draws the segment `AB`. More details on GCL commands can be found in Chapter 3.

If you are using the command-line version of **GCLC**, type the above text (GCL code) in any text editor and save it under the name, say, `quick.gcl`. The figure in LaTeX format can be generated using the following command:

```
> gclc quick.gcl quick.pic
```

where `quick.pic` is the name of a resulting file (in the simple LaTeX format[1]).

Within the GUI version, you can type the above code directly to the built-in editor, save the file under the name, say, `quick.gcl`, and press the button *Build* in the toolbar (or choose the option *Picture/Build* from the menu). Then, you can export the picture to LaTeX format by selecting the option File/Export to.../LaTeX (and choosing the name, say, `quick.pic`). More details about the GUI version can be found in Chapter 4.

The picture (contained in `quick.pic`) can be included in your LaTeX document using the command:

```
\input{quick.pic}
```

[1] Within this chapter, we comment only on the simple LaTeX format, supported by `gclc.sty`. However, **GCLC** can export to other LaTeX formats, see Chapter 5.

in an appropriate position in your LaTeX document. In addition, you have to include (by the LaTeX command `\usepackage{gclc}`) the package `gclc` (provided within the `gclc` distribution) in the preamble of your document,[2] and then you can process your LaTeX document as usual. If everything is ok, within your LaTeX document you will get the illustration as shown in Figure 2.1. More details about export to LaTeX can be found in Chapter 5.

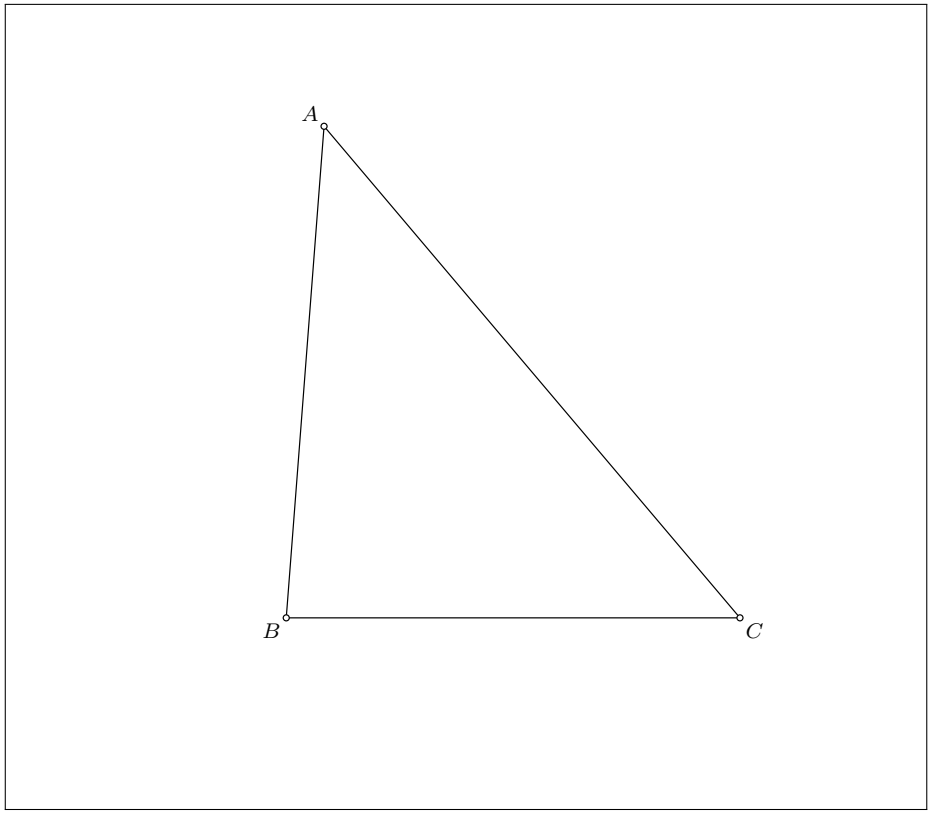

Figure 2.1: Illustration generated from the given **GCLC** code

2.3 Basic Syntax Rules

The syntax of the GCL language is very simple. Commands, identifiers, constants etc. must be separated by at least one tab or space symbol or a new line. Usually, each new command (with its argument) is in separate line and empty lines separate different parts of the construction.

2.4 Basic Objects

There are five types of objects in the GCL language: NUMBER, POINT, LINE, CIRCLE and CONIC. They are represented in the following manner:

[2] You also have to put the file `gclc.sty` (providing the `gclc` package) in the current folder (where your LaTeX document is) or in the folder with other LaTeX packages.

NUMBER n	(n)
POINT (x, y)	(x, y)
LINE $ax + by + c = 0$	(a, b, c)
CIRCLE $(x - x_0)(x - x_0) + (y - y_0)(y - y_0) = r^2$	(x_0, y_0, r)
CONIC $ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$	(a, b, c, d, e, f)

While processing an input file, **GCLC** generates the transcript file `gclc.log` (in the current directory) with the list of all warnings and the list of all defined objects (with their names and parameters). Instead of writing to the log file, the GUI version shows this list in its output window.

2.5 Geometrical Constructions

Geometrical constructions are the main area of **GCLC**. A geometrical construction is a sequence of specific, primitive construction steps. These primitive construction steps are also called *elementary constructions* and they are:

- construction (by *ruler*) of a line such that two given points belong to it;
- construction of a point such that it is the intersection of two lines (if such a point exist);
- construction (by *compass*) of a circle such that its center is one given point and such that the second given point belongs to it;
- construction of intersections between a given line and a given circle (if such points exist).

By using the set of primitive constructions, one can define more involved, compound constructions (e.g., construction of right angle, construction of the segment midpoint, construction of the segment bisector etc.). In describing geometrical constructions, it is usual to use higher level constructions as well as the primitive ones.

GCLC follows the idea of formal constructions. It provides easy-to-use support for all primitive constructions, but also for a range of higher-level constructions. (Although motivated by the formal geometrical constructions, **GCLC** provides a support for some non-constructible objects too — for instance, in **GCLC** it is possible to determine/use a point obtained by rotation for 1°, although it is not possible to construct that point by ruler and compass).

2.6 Basic Ideas

There is a need of distinguishing abstract (i.e., formal, axiomatic) nature of geometrical objects and their usual models. A geometrical construction is a mere procedure of abstract steps and not a picture. However, for each (Euclidean) construction, there is its counterpart in the standard Cartesian model. While a construction is an abstract procedure, in order to make its usual representation in Cartesian model of Euclidean plane, we still have to make some link between these two. For instance, given three vertices of a triangle we can construct a center of its inscribed circle (by using primitive constructions), but in order to represent this construction in Cartesian plane, we have to take three particular

Cartesian points as vertices of the triangle. A figure description in **GCLC** is usually made by a list of definitions of several fixed points (defined in terms of Cartesian plane, i.e., by pairs of coordinates) and then a list of construction steps based on these points. Normally, there should be very few such fixed points and all other points should depend on them. Afterwards, if one wants to vary a figure, he/she would usually change only coordinates of fixed points and all other objects will be recalculated automatically. For instance, if points `A` and `B` are given by their coordinates, never introduce their midpoint `M` also by coordinates, but always via the command `midpoint M A B`. This would give your **GCLC** descriptions flexibility and would better reflect the mathematical/geometrical meaning of the figure.

Chapter 3

GCLC Language

In the GC language, there are several entities: commands (source code statements), objects (scalar, point, line, conic), and constants. A source code line will generally have a command and identifiers (as handles for an object or variable) and possibly constants (including constant text enclosed by brackets). The syntax requires that entities be separated by at least one tab or space symbol. Commands should be separated by at least one tab or space symbol, or, for better readability, by new line.

The commands fall into ten categories, and the identifiers will be one of five types of objects in **GCLC**: NUMBER, POINT, LINE, CIRCLE, and CONIC. The types are not attached to variables explicitly, but implicitly (with respect to the given context).

Notation conventions. This document uses the following notation to clarify that an identifier is of a particular type. When writing the source code, always leave out the `<` and `>` marks.

- `<n_id>` a constant (a decimal number) or a simple numerical variable (of type NUMBER);
- `<p_id>` an identifier associated with a POINT;
- `<l_id>` an identifier associated with a LINE;
- `<circle_id>` an identifier associated with a CIRCLE;
- `<conic_id>` an identifier associated with a CONIC;
- `<text>` Constant text string beginning with the symbol `{` and ending with the symbol `}`.
- `<exp>` denotes an arbitrary expression.

Identifiers can be one to 99 characters. Underscores, single-quotes or braces are permissible, but not white spaces. Identifiers are case-sensitive. Names like `A_1` or `Q_{a}'` (in usual TEX form) can be used.

3.1 Basic Definition Commands

The first parameter for each of these commands is the identifier, the rest are constants or variables. The identifier `<id>` can have a previous definition which is ignored, it just gets a new type and new value.

- `number <n_id0> <n_id1>`: Definition of a number. The object `<n_id0>` is defined or re-defined as type NUMBER, and can be used in commands as a segment length, measure of an angle (in degrees) or as a command specific parameter (but cannot be used where a POINT, LINE, CIRCLE or CONIC is expected). The variable's value can be changed by another `number` command, or by `expression` command. `<n_id1>` can be a constant or another number identifier. Example: `number left_bottom_x 80`.

- `point <p_id> <n_id1> <n_id2>`: Definition of a point. `<p_id>` is defined or re-defined as type POINT, where its x-coordinate value becomes `<n_id1>` and its y-coordinate value becomes `<n_id2>`. `<n_id1>` and `<n_id2>` can be constants or number variables.

- `point <p_id> <n_id1> <n_id2> <n_id3> <n_id4>`: Extended definition of a point within support of animations (relevant only for GUI version, see 3.11), where (`<n_id1>`, `<n_id2>`) is the starting location for the point, and (`<n_id3>`,`<n_id4>`) is the final location. `<n_id1>`, `<n_id2>`, `<n_id3>`, `<n_id4>` can be constants or number variables.

- `line <l_id> <p_id1> <p_id2>`: Definition of a line. Identifier `<l_id>` gets type LINE, determined by (already defined) points `<p_id1>` and `<p_id2>`.

- `circle <c_id> <p_id1> <p_id2>`: Definition of a circle. Identifier `<c_id>` gets type CIRCLE, and represents a circle determined by two points: the first point (`<p_id1>`) is the center and the second (`<p_id2>`) is anywhere on the circle.

- `set_equal <id1> <id2>`: The object `<id1>` gets the type and the value of the object `<id2>`.

3.2 Basic Constructions Commands

- `intersec <p_id> <l_id1> <l_id2>` The object with the specified identifier `<p_id>` becomes POINT and gets coordinates of the intersection of two given lines.

  This command can also be used in this form:

  `intersec <p_id> <p_id1> <p_id2> <p_id3> <p_id4>`

  The object with the specified identifier `<p_id>` becomes POINT and gets coordinates of the intersection of two given lines given by `<p_id1> <p_id2>` and by `<p_id3> <p_id4>`.

  The full name, `intersection`, can also be used for this command.

- `intersec2 <p_id1> <p_id2> <id1> <id2>` The objects with the specified identifiers `<p_id1>` and `<p_id2>` become POINTs and get coordinates

of the intersection of two given circles, of a given circle and a line, or of a given line and a circle.

The full name, `intersection2`, can also be used for this command.

- `midpoint <p_id> <p_id1> <p_id2>` The object with the specified identifier `<p_id>` becomes POINT and gets coordinates of the midpoint of the segment determined by two given points.

- `med <l_id> <p_id1> <p_id2>` The object with the specified identifier `<l_id>` becomes LINE and gets the parameters of the line that bisects (and is perpendicular to) the segment determined by the two given points.

  The full name, `mediatrice`, can also be used for this command.

- `bis <l_id> <p_id1> <p_id2> <p_id3>` The object with the specified identifier `<l_id>` becomes LINE and gets the parameters of the line that bisects the angle determined by three given points. Example:

  ```
  bis s A B C
  ```

  makes object `s` to become LINE with parameters of the bisector of the angle ∠`ABC`.

  The full name, `bisector`, can also be used for this command.

- `perp <l_id> <p_id1> <l_id1>` The object with the specified identifier `<l_id>` becomes LINE and gets the parameters of the line that is perpendicular to the given line and contains the given point.

  The full name, `perpendicular`, can also be used for this command.

- `foot <p_id> <p_id1> <l_id1>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the foot of the perpendicular from the point `<p_id1>` to the line `<l_id1>`.

- `parallel <l_id> <p_id1> <l_id1>` The object with the specified identifier `<l_id>` becomes LINE and gets the parameters of the line that is parallel to the given line and contains the given point.

- `getcenter <p_id> <c_id>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the center of the given circle.

- `onsegment <p_id> <p_id1> <p_id2>` The object with the specified identifier `<p_id>` becomes POINT, placed randomly on the line segment determined by the two given points.

- `online <p_id> <p_id1> <p_id2>` The object with the specified identifier `<p_id>` becomes POINT, placed randomly on the line determined by the two given points (more precisely, a random point is chosen between points X and Y such that X is symmetrical to `<p_id1>` with respect to `<p_id2>` and Y is symmetrical to `<p_id2>` with respect to `<p_id1>`). (This command is suitable for describing constructions with properties to be proved by the theorem provers.)

- `oncircle <p_id> <p_id1> <p_id2>` The object with the specified identifier `<p_id>` becomes POINT, placed randomly on the circle with center `<p_id1>` and with the point `<p_id2>`. (This command is suitable for describing constructions with properties to be proved by the theorem provers.)

3.3 Transformation Commands

- `translate <p_id> <p_id1> <p_id2> <p_id3>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the point that is an image of the third point (`<p_id3>`) in a translation for the vector determined by the first and the second given point. Example:

  ```
  translate A2 X Y A1
  ```

  makes object `A2` to become POINT such that $\mathcal{T}$(`A1`)=`A2`, where $\mathcal{T}$ is translation for the vector `XY`.

- `rotate <p_id> <p_id1> <n> <p_id2>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the point that is an image of the second given point in a rotation around the first given point and for the given (positive or negative) angle (determined by the given constant or NUMBER). Example:

  ```
  rotate A2 O 90 A1
  ```

  makes object `A2` to become POINT such that $\mathcal{R}$(`A1`)=`A2`, where $\mathcal{R}$ is rotation around the point `O` for the angle of 90°.

- `rotateonellipse <p_id> <p_id1> <p_id2> <p_id3> <n> <p_id4>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the point `Y` such that the angle `X <p_id1> Y` is equal to `n`, where `X` is the intersection of the half-line `<p_id1> <p_id4>` and the ellipse determined by `<p_id1> <p_id2> <p_id3>`.

- `sim <p_id> <id1> <p_id>` The object with the specified identifier `<p_id>` becomes POINT and gets the parameters of the point that is an image of the given point in a half-turn, line-reflection or inversion (depending of the type (POINT, LINE, or CIRCLE) of the second argument). Example:

  If `l` is LINE,

  ```
  sim A2 l A1
  ```

  makes object `A2` to become POINT such that $\mathcal{S}$(`A1`)=`A2` where $\mathcal{S}$ is line reflection determined by the line `l`.

  The full name, `symmetrical`, can also be used for this command.

- `turtle <p_id> <p_id1> <p_id2> <n1> <n2>` The object with the specified identifier `<p_id>` becomes POINT. Its segment length from `<p_id2>` will be `<n2>`. The segment determined by `<p_id>` and `<p_id2>` will make an angle `<n1>` (in degrees) with the segment from `<p_id1>` to `<p_id2>`. Example:

  ```
  turtle A X Y 90 10.00
  ```

  makes object `A` to become POINT such that `YA=10.00` and ∠`XYA`=90°.

- `towards <p_id> <p_id1> <p_id2> <n1>` The object with the specified identifier `<p_id>` becomes POINT, placed on the line determined by the two given points. The distance from `<p_id1>` to the new point `<p_id>` is a fraction `<n1>` of the distance from `<p_id1>` to `<p_id2>`. Thus, if $0 <$ `<n1>` < 1, then the point `<p_id>` will be between points `<p_id1>` and `<p_id1>`; if `<n1>` > 1, the point `<p_id2>` will be between points `<p_id1>` and `<p_id>`; if `<n1>` < 0, the point `<p_id1>` will be between points `<p_id2>` and `<p_id>`. Example:

  ```
  towards O A B 0.9
  ```

  makes object `O` to become POINT such that `AO`$=0.9 \cdot$`AB` and `O` is between `A` and `B`.

3.4 Calculations, Expressions, Arrays, and Control Structures

- `getx <n_id> <p_id>`: The object with the specified identifier `<n_id>` becomes NUMBER and gets the value of the x-coordinate of the given point `<p_id>`.

- `gety <n_id> <p_id>` The object with the specified identifier `<n_id>` becomes NUMBER and gets the value of the y-coordinate of the given point `<p_id>`.

- `distance <n_id> <p_id1> <p_id2>`: The object with the specified identifier `<n_id>` becomes NUMBER and gets the value of the distance between two given points.

- `angle <n_id> <p_id1> <p_id2> <p_id3>`: The object with the specified identifier `<n_id>` becomes NUMBER and gets the measure of the angle determined by three given points. Example:

  ```
  angle alpha A B C
  ```

  makes object `alpha` to get type NUMBER and its value will be the measure of the (oriented) angle $\angle$`ABC` (in degrees).

- `angle_o <n_id> <p_id1> <p_id2> <p_id3>`: The same as the command `angle`, but takes orientation into account, so the angle can have positive and negative values. Therefore, this version is compatible with the command `rotate`.

- `random <n_id>` The object with the specified identifier `<n_id>` gets type NUMBER and gets a (pseudo)random value between 0 and 1.

- `expression <n_id> {exp}`: The object with the specified identifier `<n_id>` gets type NUMBER and gets the value of the expression `exp`. Example:

  ```
  expression e {sin(3)*(5+2)}
  ```

  After the above command, `e` will have the value 0.366352.

  Defined variables of the type NUMBER can be used in expressions. No other variables can be used in expressions. For instance,

`expression e {n+5}`

makes object `e` to become NUMBER equal to `n`+5, if `n` is of the type NUMBER.

The following standard functions, operators and relations are supported in expressions: `+` (addition), `-` (subtraction), `*` (multiplication), `/` (division), `==` (equality), `!=` (inequality), `<`, `<=`, `>`, `>=`, `&&` (and), `||` (or), `abs`, `ceil` (rounding up), `floor` (rounding down), `sin`, `cos`, `tan` (with arguments expressed in radians), `sinh`, `cosh`, `tanh`, `asin`, `acos`, `atan`, `sqrt`, `exp`, `pow` (exponentiation), `log`, `log10`, `min` (two arguments), `max` (two arguments).

For example,

`expression m { pow(n+1,2)}`

makes object `m` equal $(n+1)^2$ (note that the operator `^` is not used for exponentiation).

The `ite` operator (from *if-then-else*) is also supported. For example,

`expression E {ite(n>0,1,2)}`

makes object `E` to become NUMBER equal to 1 if `n` is greater than zero, and 2 otherwise.

Blank spaces in expressions are allowed and ignored.

- `array <c_id> { <n_id0> <n_id1> ... <n_idk> }`: Definition of an (multidimensional) array. The values `<n_id0> <n_id1> ... <n_idk>` are dimensions of the array. There can be up to 10 dimensions. All elements of the array initially have type NUMBER and value 0. Indexing is 1-based, i.e., the first element of the array has all indices equal 1. Indices are written in separate angle brackets. Examples:

  `array A { 4 3 }`

  defines $4 \times 3 = 12$ elements of the array A — $A[1][1]$, $A[1][2]$, ..., $A[4][3]$. All these elements initially have the type NUMBER and value 0, but both of these can be changed, as for any other variable. So, different elements of the same array can have different types.

  Indices of an array element can be arbitrary expressions, that can also involve other array elements (of type NUMBER). For instance, if all elements of an (one-dimensional) array A are numbers, one can use the following construction: $A[5 + A[5]]$ (in any position that requires a number).

  An array with the same name can be defined more than once. If the numbers of dimensions are same, and if all dimensions are same, then all old elements are reset to have type NUMBER and value 0. If some dimensions are different, new elements may be added (if some new dimensions are greater then the old ones), but old elements (those not covered by the new definition) are never destroyed (even if some new dimensions are less then the old ones). If the numbers of dimensions (in two definitions) are different, then these two arrays are considered different and there are no resetting of the old elements.

- `while {<exp>} { <while-block> }`: `<while-block>` is a sequence of commands. This sequence will be repeatedly executed as long as `<exp>` condition is true (nonzero). Example:

```
point A 0 0
number n 0
while { n<30 }
{
    point B n 30
    drawsegment A B
    expression n { n+1 }
}
```

Both syntax and run-time errors encountered within a while-block are reported only as `Invalid while block` error and no other (more detailed) information on the error is provided. Also, all warnings encountered within a while-block are suppressed and are not written to the log.

The sequence of commands in the while-block behaves as any **GCLC** sequence. It shares the defined variables and the environment (defined by commands `ang_picture` and `ang_origin` etc) with the outer **GCLC** context.

If the `<exp>` condition is never fulfilled, this leads to non-termination (i.e., infinite loop). In order to prevent this, the system enables only a limited number (10000) of executions of blocks within while-loops. If this number is exceeded, then the error `Too many while-block executions (more than 10000). Possible infinite loop` is reported and the processing is stopped.

Procedures cannot be defined within while-blocks.

- `if_then_else {<exp>} { <then-block> } { <else-block> }`:

  `<then-block>` is a sequence of commands. This sequence will be executed if `<exp>` condition is true (nonzero). `<else-block>` is a sequence of commands. This sequence will be executed if `<exp>` condition is false (zero). Example:

```
distance d1 C A
distance d2 C B
if_then_else { d1<d2 }
{
    drawsegment A C
}
{
    drawsegment B C
}
```

Both syntax and run-time errors encountered within `<then-block>` and `<else-block>` are reported only as `Invalid if-then-else block` error and no other (more detailed) information on the error is provided. Also, all warnings encountered within a if-then-else-block are suppressed and are not written to the log.

The sequence of commands in the blocks behaves as any **GCLC** sequence. It shares the defined variables and the environment (defined by commands `ang_picture` and `ang_origin` etc) with the outer **GCLC** context.

Procedures cannot be defined within if-then-else-blocks.

- `procedure <name> { <arguments> } { <block of commands> }`:

  `<name>` is the name of the procedure. `<arguments>` is a list of the procedure's arguments. Arguments are separated by blank spaces. In **GCLC**, arguments are passed by their names, which means that they may be changed by the procedure. `<block of commands>` is a sequence of commands. It inherits the environment form the outer context, but not the variables from the outer context. Within a block, only arguments and variables defined within it can be used. The definition of a procedure must precede calling it. Procedures cannot be defined within while-blocks or within definitions of other procedures.

  Example:

```
procedure drawtriangle { X Y Z }
{
    drawsegment X
    drawsegment Y
    drawsegment Z
}
```

- `call <name> { <arguments> }` `<name>` is the name of the procedure. `<arguments>` is a list of the arguments that will be passed to the procedure. Arguments are separated by blank spaces. In **GCLC**, variables are passed to procedures as arguments by names and they may be changed by a procedure. Argument of a procedure call can also be a constant (and, of course, it is passed by value). If a variable that is argument is not defined before (i.e., with an intention that it receives the resulting value of the function), by default it get the type NUMBER and the value 0. If a single variable is used for several arguments, it will get the value of the last of such arguments (when returning from the procedure).

  Procedures can be called from other procedures.

  Example:

```
call drawtriangle { A B C }
```

- `include <file_name>` Reads/consults the contents of another `.gcl` file. After this command, variables and procedures defined in that another file can be used (as they were defined within the current file). Both syntax and run-time errors encountered within the consulted file are reported only as `Invalid include file` error and no other (more detailed) information on the error is provided. Also, all warnings encountered within the consulted file are suppressed and are not written to the log.

3.5 Drawing Commands

All drawn figures are clipped against the defined picture area. The current area can be changed by the `area` command.

- `drawpoint <p_id>` Generates a (export–specific) command for drawing the specified point.
- `drawsegment <p_id1> <p_id2>` Generates a command for drawing the segment determined by endpoints named `<p_id1>` and `<p_id2>`.
- `drawdashsegment <p_id1> <p_id2>` Generates commands for drawing the dashed segment connecting points named `<p_id1>` and `<p_id2>`. The length of dashes can be changed by the `dash` command.
- `drawline <l_id>` Generates a command for drawing the given line.
- `drawline <p_id1> <p_id2>` Generates a command for drawing the line determined by the points `<p_id1>` and `<p_id2>`.
- `drawdashline <l_id>` Generates a command for drawing the given line dashed.
- `drawdashline <p_id1> <p_id2>` Generates a command for drawing the dashed line determined by the points `<p_id1>` and `<p_id2>`.
- `drawvector <p_id1> <p_id2>` Generates a command for drawing the vector determined by points `<p_id1>` and `<p_id2>`.
- `drawarrow <p_id1> <p_id2> <n>` Generates commands for drawing an arrow on the line segment `<p_id1> <p_id2>` (the line segment itself is not drawn). The ratio between the distance from `<p_id1>` to the end of the arrow and the distance from `<p_id1>` to `<p_id2>` is equal to `<n>`. The default shape of the arrow can be changed by the command `arrowstyle`.
- `drawcircle <c_id>` Generates commands for drawing the given circle.

  In figures exported to the simple LaTeX format, **GCLC** draws circles and arcs segment by segment. The number of segments can be changed by the `circleprecision` command. By the default, a circle of radius 10mm has 72 segments, while the number of segments depends (linearly) on the circle size.
- `drawcircle <p_id1> <p_id2>` Generates commands for drawing the circle determined by center `<p_id1>` and one point `<p_id2>`.

  In figures exported the simple LaTeX format, **GCLC** draws circles and arcs segment by segment. The number of segments can be changed by the `circleprecision` command.
- `drawdashcircle <p_id1> <p_id2>` Generates commands for drawing the circle determined by center `<p_id1>` and one point `<p_id2>`, and made of dashes. In this mode, **GCLC** draws circles arc by arc and every third arc will not be drawn, so the length of these arcs can be changed by the `circleprecision` command.

- `drawarc <p_id1> <p_id2> <n>` Generates commands for drawing the arc determined by center `<p_id1>`, one point `<p_id2>` and the measure of angle equal `<n>` (`<n>` is constant or NUMBER).

- `drawarc_p <p_id1> <p_id2> <n>` The same as `drawarc`, but always draws the arc in positive (counterclockwise) direction.

- `drawdasharc <p_id1> <p_id2> <n>` Generates commands for drawing the dashed arc determined by center `<p_id1>`, one point `<p_id2>` and the measure of angle equal `<n>` (`<n>` is constant or NUMBER). **GCLC** draws arcs segment by segment and by this command every third segment will not be drawn, so the length of dashes by command `circleprecision`.

- `drawdasharc_p <p_id1> <p_id2> <n>` The same as `drawdasharc`, but always draws the arc in positive (counterclockwise) direction.

- `drawellipse <p_id1> <p_id2> <p_id3>` Generates commands for drawing the ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis. **GCLC** draws ellipses segment by segment. The number of segments can be changed by the `circleprecision` command.

- `drawdashellipse <p_id1> <p_id2> <p_id3>` Generates commands for drawing the dashed ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis. Picture of ellipse is made of dash segments. **GCLC** draws circles and ellipses segment by segment and for this dash command every third segment will not be drawn, so the length of dashes can be changed by the `circleprecision` command.

- `drawellipsearc <p_id1> <p_id2> <p_id3> <n>` Generates commands for drawing the arc (with a starting point `<p_id2>`) of the ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis and `<n>` is the measure of the angle (`<n>` is constant or NUMBER). There is a similar command `drawellipsearc1`.

- `drawdashellipsearc <p_id1> <p_id2> <p_id3> <n>` Generates commands for drawing the arc (with a starting point `<p_id2>`) of the ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis and `<n>` is the measure of the angle (`<n>` is constant or NUMBER). **GCLC** draws arcs segment by segment and in this arc every third segment will not be drawn, so the length of dashes can be changed by the `circleprecision` command. There is a similar command `drawdashellipsearc1`.

- `drawellipsearc1 <p_id1> <p_id2> <p_id3> <p_id4> <n>` Generates commands for drawing the arc (with a starting point `<p_id4>`) of the ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis and `<n>` is the measure of the angle (`<n>` is constant or CONSTANT).

- `drawdashellipsearc1 <p_id1> <p_id2> <p_id3> <p_id4> <n>` Generates commands for drawing the arc (with a starting point `<p_id4>`) of the ellipse determined by center `<p_id1>` and two points `<p_id2>` and `<p_id3>`, such that a line determined by points `<p_id1>` and `<p_id2>` is one of the ellipse's axis and `<n>` is the measure of the angle (`<n>` is constant or NUMBER). **GCLC** draws arcs segment by segment and in this arc every third segment will not be drawn, so the length of dashes can be changed by the `circleprecision` command.

- `drawellipsearc2 <p_id1> <p_id2> <p_id3> <n1> <n2>` Generates commands for drawing the elliptical arc `X Y`, where `X` and `Y` are points on the ellipse determined by the points `<p_id1> <p_id2> <p_id3>`, the angle `<p_id2> <p_id1> X` is equal to `n1`, and the angle `Y <p_id1> X` is equal to `n2`.

- `drawdashellipsearc2 <p_id1> <p_id2> <p_id3> <p_id4> <n>` Generates commands for drawing the elliptical arc `X Y`, where `X` and `Y` are points on the ellipse determined by `<p_id1> <p_id2> <p_id3>`, the angle `<p_id2> <p_id1> X` is equal to `n1`, the angle `Y <p_id1> X` is equal to `n2`. **GCLC** draws arcs segment by segment and in this arc every third segment will not be drawn, so the length of dashes can be changed by the `circleprecision` command.

- `drawbezier3 <p_id1> <p_id2> <p_id3>` Generates commands for drawing the quadratic Bézier curve determined by points `<p_id1>`, `<p_id2>`, `<p_id3>` (it goes from `<p_id1>` to `<p_id3>`, while `<p_id2>` is a control point). **GCLC** draws Bézier curves segment by segment. The number of segments can be changed by the `bezierprecision` command.

- `drawdashbezier3 <p_id1> <p_id2> <p_id3>` Generates commands for drawing the quadratic Bézier curve determined by points `<p_id1>`, `<p_id2>`, `<p_id3>` (it goes from `<p_id1>` to `<p_id3>`, while `<p_id2>` is a control point). **GCLC** draws Bézier curves segment by segment and in this mode every third segment will not be drawn, so the length of dashes can be changed by the `bezierprecision` command.

- `drawbezier4 <p_id1> <p_id2> <p_id3> <p_id4>` Generates commands for drawing the cubic Bézier curve determined by points `<p_id1>`, `<p_id2>`, `<p_id3>`, `<p_id4>` (it goes from `<p_id1>` to `<p_id4>`, while `<p_id2>` and `<p_id3>` are control points). **GCLC** draws Bézier curves segment by segment. The number of segments can be changed by the `bezierprecision` command.

- `drawdashbezier4 <p_id1> <p_id2> <p_id3> <p_id4>` Generates commands for drawing the cubic Bézier curve determined by points `<p_id1>`, `<p_id2>`, `<p_id3>`, `<p_id4>` (it goes from `<p_id1>` to `<p_id4>`, while `<p_id2>` and `<p_id3>` are control points). **GCLC** draws Bézier curves segment by segment and in this mode every third segment will not be drawn, so the length of dashes can be changed by the `bezierprecision` command.

- `drawpolygon <p_id> <p_id> <n>` Generates commands for drawing the regular polygon determined by center `<p_id1>`, one vertex `<p_id2>`, and number of sides `<n>` (`<n>` is constant or NUMBER).

- `drawtree <p_id> <n1> <n2> <n3> <n4> <tree_description>` Generates commands for drawing the tree with the given point `<p_id>` as a root. The numerical parameters `<n1>` and `<n2>` give the width and height of the tree (in millimeters), `<n3>` determines the style for drawing tree, and `<n4>` gives a rotation angle (in degrees) with the root as a center, and a vertical, top-down direction as a reference direction (corresponding the angle 0°). There are four drawing styles (`1`, `2`, `3` and `4`).

  A tree description is of the form `{ node_name <subtree_1> ... <subtree_n>`. All node names are printed in appropriate positions. If a tree node should not be labelled, its name should start with the symbol `_`. Empty subtree, written `{ }` is not drawn or labelled, but it takes one position of subtrees in the same level.

  All tree nodes are defined as POINTs and get names built from the name of the reference point and their label. In the example given below, one can use points `Proot`, `Pleft`, etc.

  Example:

```
drawtree P 90 70 1 10
{
    root
    { left
        { }
        { left-right }
    }
    { right
        { _
            { a }
            { b }
            { c }
        }
        { right-right }
    }
}
```

- `drawgraph_a <p_id> <n1> <n2> <list_of_nodes> <list_of_edges>`

  Generates commands for drawing the graph using the arc-layered method.[1] The given reference point `<p_id>` will be the center of the graph image. The numerical parameter `<n1>` gives the width of the graph image, while `<n2>` gives a rotation angle (in degrees) with the point `<p_id>` as the center.

  The list of nodes `<list_of_nodes>` is of the form `{ node_name1 node_name2 ... node_name_n }`. All node names in the

[1] The main author of support for graph drawing is Luka Tomašević (University of Belgrade).

figure are printed in left-top positions. If a graph node should not be labelled, its name should start with the symbol `_`.

The list of edges `<list_of_edges>` is of the form
`{ node_name1_1 node_name1_2 ... node_name_n_1 node_name_n_2 }`, where all node names must already appear in the list of nodes.

If the graph is not connected, then none of its nodes or edges will not be drawn.

All graph nodes are defined as POINTs and get names built from the name of the reference point and their label. In the example given below, one can use points `P_a`, `Pb`, etc.

Example:

```
point P 30 50
drawgraph_a P 40 0
{ _a b c d e }
{
  _a b
  _a c
  _a d
  b  d
  b  e
}
```

- `drawgraph_b <id> <list_of_nodes> <list_of_edges>` Generates commands for drawing the graph using the barycenter method. The given reference name `<id>` is used just to identify graph nodes, it does not need to refer to any point or other object.

  The list of nodes `<list_of_nodes>` is of the form
  `{ node_name1 p_id1 node_name2 p_id2 ... node_name_n p_id_n}`.

  The node names represent the names of the graph nodes, while the point identifiers associate graph nodes to already defined points. If a graph node is not initially associated to an already defined point, then the point identifier should be `_`, or can begin with `_`. The set of used defined points (serving as fixed nodes) must form a convex polygon.

  All node names in the figure are printed in left-top positions. If a graph node should not be labelled, its name should start with the symbol `_`.

  The list of edges `<list_of_edges>` is of the form
  `{ node_name1_1 node_name1_2 ... node_name_n_1 node_name_n_2 }`, where all node names must already appear in the list of nodes.

  If the graph is not connected, then none of its nodes or edges will not be drawn.

  All graph nodes are defined as POINTs and get names built from the name of the reference point and their label. In the example given below, one can use points `P_a`, `Pb`, etc.

  Example:

```
point A 80 50
point B 100 50
point C 90 55
drawgraph_b G
{ _a A
  b  B
  c  C
  d  _
}
{
  _a b
  _a c
  _a d
  b  d
  b  c
}
```

- `filltriangle <p_id1> <p_id2> <p_id3>` Fills the triangle determined by the given points with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillrectangle <p_id1> <p_id2>` Fills the rectangle determined by the given points (`<p_id1>` is the left-bottom corner, `p_id2` is the right-top corner) with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillcircle <c_id>` Fills the circle `<c_id>` with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillcircle <p_id1> <p_id2>` Fills the circle determined by the given points (`<p_id1>` is the center, `p_id2` lies on the circle) with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillellipse <p_id1> <n_id1> <n_id2>` Fills the ellipse determined by the given points (`<p_id1>` is the center, `<n_id1>` is the half-width, and `<n_id2>` is the half-height of the ellipse) with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillarc <p_id1> <p_id2> <n>` Fills the circular arc determined by the given points (`<p_id1>` is the center, `p_id2` lies on the circle, `n` is the measure of the angle) with the current color. This command is ignored when exporting to the simple LaTeX format.
- `fillarc0 <p_id1> <p_id2> <n>` The same as `fillarc`, except that the area determined by the circle center and the arc endpoints is not filled.
- `fillellipsearc <p_id1> <n1> <n2> <n3> <n4>` Fills the elliptical arc determined by the given points (`<p_id1>` is the center of the ellipse with axes parallel to coordinate axes, `<n1>` is the half-width, `<n2>` is the half-height of the ellipse, `n3` is the start central angle, and `n4` is the measure

of the angle that corresponds to the arc). This command is ignored when exporting to the simple LaTeX format.

- `fillellipsearc0 <p_id1> <n1> <n2> <n3> <n4>` This command has the same effect as `fillellipsearc`, except that the area determined by the ellipse center and the arc endpoints is not filled.

3.6 Labelling and Printing Commands

- `cmark <p_id>` Denotes the given point by a small empty circle (with radius `0.4`mm) at its coordinates.

- `cmark_lt <p_id>`

  `cmark_l  <p_id>`

  `cmark_lb <p_id>`

  `cmark_t  <p_id>`

  `cmark_b  <p_id>`

  `cmark_rt <p_id>`

  `cmark_r  <p_id>`

  `cmark_rb <p_id>`

  Generates commands for denoting the given point by its name and by small empty circle (with radius 0.4mm) at its coordinates. The name of the point is written (in LaTeX mode — in size `\footnotesize`) in one of eight directions (left-top, left, left-bottom, top, bottom, right-top, right, right-bottom).

- `mark <p_id>` Generates commands for printing the name of the given point at its coordinates (without denoting it by a circle).

- `mark_lt <p_id>`

  `mark_l  <p_id>`

  `mark_lb <p_id>`

  `mark_t  <p_id>`

  `mark_b  <p_id>`

  `mark_rt <p_id>`

  `mark_r  <p_id>`

  `mark_rb <p_id>`

  Generates commands for printing the name of the given point at its coordinates (without denoting it by a circle) in one of eight directions. These commands could be used for denoting lines and circles, too (of course, first, some point with a line or circle name has to be defined).

- `printat  <p_id> <text>` Generates commands for printing given text at coordinates of the given point. Text must begin with symbol `{` and end with symbol `}`. These two symbols are not part of the text and will not be printed. Text can have at most 100 characters. Text can include

all characters including `{`, `}` and blank space. In LaTeX mode, the text is printed in math mode, so for ordinary text, LaTeX command `\mbox{...}` should be used.

`printat_lt <p_id> <text>`

`printat_l  <p_id> <text>`

`printat_lb <p_id> <text>`

`printat_t  <p_id> <text>`

`printat_b  <p_id> <text>`

`printat_rt <p_id> <text>`

`printat_r  <p_id> <text>`

`printat_rb <p_id> <text>`

Generates commands for printing given text in one of eight directions with respect to the given point. Example:

```
printat_l A {A=S_a(B) \mbox{($a$ is the bisector of $AB$)}}
```

will generate a command for printing the given text left from the point `A`.

- `printvalueat <p_id> <id>` Generates commands for printing value of a given object `<id>` at coordinates of the given point `<p_id>`. The object `<id>` can be of any type. The value is printed in the format given in Section 2.4. In LaTeX mode, the text is printed in math mode, so for ordinary text, LaTeX command `\mbox{...}` should be used.

  `printvalueat_lt <p_id> <id>`

  `printvalueat_l  <p_id> <id>`

  `printvalueat_lb <p_id> <id>`

  `printvalueat_t  <p_id> <id>`

  `printvalueat_b  <p_id> <id>`

  `printvalueat_rt <p_id> <id>`

  `printvalueat_r  <p_id> <id>`

  `printvalueat_rb <p_id> <id>`

  Generates commands for printing value of a given object `<id>` in one of eight directions with respect to the given point `<p_id>`. Example:

  ```
  printvalueat_b A A
  ```

  will generate a command for printing the coordinates of the point `A` on bottom from the point `A`.

3.7 Low Level Commands

- `%` A comment is marked by the symbol `%` (like in TeX). Characters in the line after the symbol `%` will not be read.

- `dim <n1> <n2>` Defines dimensions of the picture. This command can be at any position in the figure description file. If there is more than one occurrence of this command, only the first one is used. The default dimensions of a picture are 140mm×100mm. The rectangle defined by this command also defines the visible area.

- `area <n1> <n2> <n3> <n4>` Defines the visible area of the picture. The area is defined by the lower-left corner (the first two numbers) and its upper-right corner (the second pair of numbers). There is always at most one active area. All objects are clipped with respect to the active, current area. If there is no defined area, the default area is the whole of the picture.

- `color <n1> <n2> <n3>` Changes the current color. The parameters are RGB (red/green/blue) components of the color. Each of them should range between 0 and 255. For instance, `255 0 0` defines (pure) red color, `0 255 0` defines (pure) green color, `0 0 255` defines (pure) blue color, `255 255 0` defines yellow, `255 0 255` defines magenta, `0 255 255` defines cyan, `127 127 127` defines grey color, `0 0 0` defines black and `255 255 255` white color.

  Figures using this command exported to LaTeX require using the package `color` in your LaTeX document. Note that this support for colors might not work properly in conjunctions with some LaTeX distributions (i.e., with some DVI drivers).

- `background <n1> <n2> <n3>` Sets the background color. The parameters are RGB (red/green/blue) components of the color. This command is ignored when exporting to the simple LaTeX format.

- `fontsize <n>` Changes the current font size. Font size is given in pts. The default value is 8. In export to LaTeX, this command can change the current fontsize to one of the values: `\tiny` (1pt-5pt), `\scriptsize` (6pt-7pt), `\footnotesize` (8pt), `\small` (9pt), `\normalsize` (10pt), `\large` (11pt-12pt), `\Large` (13pt-14pt), `\LARGE` (15pt-17pt), `\huge` (18pt-20pt), `\Huge` (over 21pt).

- `arrowstyle <n1> <n2> <n3>` Defines a shape of arrows drawn by the command `drawarrow`. The angle between outer line segments in arrows is given, in degrees, by `<n1>` (the default value is 15°, maximal value is 180°). The length of outer line segments is given, in millimeters, by `<n2>` (the default value is 3). Inner line segments meet the central line at the point that is determined by `<n3>` — if X is the intersection of the central line with the line determined by the endpoints of the outer line segments, if Y the point where the inner line segments meet the central line, and if Z is the endpoint of the arrow, then $XY/XZ =$ `<n3>` (the default value is 0.667).

- `circleprecision <n>` When exporting to LaTeX, **GCLC** draws circles and ellipses segment by segment. The default value is 72 segments for a circle of a radius 10mm, more for larger circles (while there is linear dependency). The number of segments can be changed by this command. It will linearly depend on the circle radius, but it will be no less than the

given value <n>. This command is irrelevant for export into formats other than LaTeX.

- `bezierprecision <n>` **GCLC** draws Bézier curves segment by segment. The default value is 36 segments for a curve. The number of segments can be changed by this command.

- `linethickness <n>` The line thickness can be changed by this command. Thickness is expressed in millimeters. The default value is 0.16mm. If the given value is negative, then the line thickness is product of the absolute value of the argument and the default value.

- `double` This command makes all subsequent lines to be drawn with double thickness.

- `normal` This command makes all subsequent lines to be drawn with normal thickness.

- `dash <n>` The length of dash lines (in dash drawn lines and line segments) can be changed by this command. Length is expressed in millimeters. The default value is 1.5mm. The length of space between dash lines is <n>/2. This command cancels the effect of previous `dash` and `dashstyle` commands.

- `dashstyle <n1> <n2> <n3> <n4>` This command provides more expressive way of describing dashed lines and segments layout than given by the command `dash`. The pattern defined by this command is as follows: <n1> dash line — <n2> empty space — <n3> dash line — <n4> empty space. All lengths are expressed in millimeters. The default values are: 1.5 0.75 1.5 0.75. This command cancels the effect of previous `dash` and `dashstyle` commands.

- `dmc <n>` The distance between point and its name or associated text can be changed by this command. The default value is 1mm.

- `mcr <n>` The radius of circle marking point can be changed by this command. The default value is 0.4mm.

- `mcp <n>` When exporting to the simple LaTeX format, draws circles marking points segment by segment. The default value is 9 segments for a circle. The number of segments can be changed by this command. This command is irrelevant for export into formats other than LaTeX.

- `mcp <n>` When exporting to the simple LaTeX format, draws circles marking points segment by segment. The default value is 9 segments for a circle. The number of segments can be changed by this command. This command is irrelevant for export into formats other than LaTeX.

- `export_to_latex {<text>}` When exporting to the simple LaTeX format, PSTricks LaTeX format, or Ti*k*Z LaTeX format, the given text is directly exported to the output file (normally this should be a sequence of LaTeX commands).

- `export_to_latex {<text>}` When exporting to the simple LaTeX format, PSTricks LaTeX format, or Ti*k*Z LaTeX format, the given text is directly exported to the output file (normally this should be a sequence of LaTeX commands).
- `export_to_simple\_latex {<text>}` When exporting to the simple LaTeX format, the given text is directly exported to the output file.
- `export_to_pstricks {<text>}` When exporting to PSTricks LaTeX format, the given text is directly exported to the output file.
- `export_to_tikz {<text>}` When exporting to Ti*k*Z LaTeX format, the given text is directly exported to the output file.
- `export_to_eps {<text>}` When exporting to EPS format, the given text is directly exported to the output file.
- `export_to_svg {<text>}` When exporting to SVG format, the given text is directly exported to the output file.

3.8 Cartesian Commands

- `ang_picture <n1> <n2> <n3> <n4>` Defines a rectangular area for Cartesian picture (`ang` is from "ANalytical Geometry"). The first two parameters determine its lower left corner, and last two parameters determine its upper right corner. If there is no defined area, the default area is empty (i.e., it is defined by lower left and upper right corner equal to $(0, 0)$). This area is relevant only for Cartesian commands (`ang_...`).
- `ang_origin <n1> <n2>` Defines an origin of the coordinate system. The default value is $(0, 0)$.
- `ang_unit <n>` Defines the unit of the coordinate system in millimeters. The default value is 10mm.
- `ang_scale <n1> <n2>` Defines the scale between y and x coordinates.

  The parameter `<n1>` determines if the coordinate system is regular (value 1) or logarithmic (value 2). In logarithmic system, x axis is set on $y = 1$.

  The parameter `<n2>` determines the multiplication factor for y coordinates. It must be positive. If a negative value or zero is given, the value 1 is assumed.

  These values determines the coordinate system and all consequent drawings. The default values are 1 and 1.

  Note that, if logarithmic system is used, the commands doing with lines (`ang_tangent`, `ang_drawline`, `ang_drawline_p`) will not work properly. Other commands (including, for instance, commands `ang_drawdashconic` and `ang_draw_parametric_curve`) can be used.
- `ang_drawsystem_p <n1> <n2>  <n3> <n4> <n5>` Generates commands for drawing coordinate axes.

The parameter <n1> controls denoting the integer points on the axes: with parameter value 1, integer points are denoted by small circles, with value 2 by small dashes, and with value 3 they are not denoted at all.

The parameter <n2> controls the step for denoting integer points on x axis. For instance, if <n2> is equal to 1, then each integer point on x axis is denoted (in a way defined by the parameter <n1>); if <n2> is equal to 2, then every second integer point is denoted.

The parameter <n3> controls the step for denoting integer points on y axis.

The parameter <n4> controls denoting the axes: with parameter value 1, the axes are denoted by x and y, and with value 2 the axes are not denoted.

The parameter <n5> controls drawing arrows at the endpoints of the system: with parameter value 1, there are arrows at both positive and negative endpoints, with parameter value 2, there are arrows only at positive, and with parameter value 3, there are no arrows at all.

This command can replace all other variants of `ang_drawsystem...` commands (however, they are kept for simplicity and for the reasons of vertical compatibility).

- `ang_drawsystem` Generates commands for drawing the axes and denotes (by small circles) integer points on them.
- `ang_drawsystem0` Generates commands for drawing the axes and doesn't denote integer points on them.
- `ang_drawsystem1` Generates commands for drawing the axes and denotes by small dashes integer points on them.
- `ang_drawsystem_a` Same as `ang_drawsystem`, but denotes the axes by x and y.
- `ang_drawsystem0_a` Same as `ang_drawsystem0`, but denotes the axes by x and y.
- `ang_drawsystem1_a` Same as `ang_drawsystem1`, but denotes the axes by x and y.
- `ang_point <p_id> <n1> <n2>` Definition of an `ang` point. A difference from the standard command `point` is that the `ang_point` command introduces point by coordinates given with respect to a defined origin and unit of Cartesian system.
- `ang_getx <n_id> <p_id>` The object with the specified identifier <n_id> becomes NUMBER and gets the value of the x-coordinate of the given point <p_id> in the active Cartesian system (<p_id> was not necessarily defined by the command `ang_point`).
- `ang_gety <n_id> <p_id>` The object with the specified identifier <n_id> becomes NUMBER and gets the value of the y-coordinate of the given point <p_id> in the active Cartesian system (<p_id> was not necessarily defined by the command `ang_point`).

- `ang_line <l_id> <n1> <n2> <n3>` Introduces a line `<l_id>` by given parameters in the form $ax + by + c = 0$, with respect to a defined origin and unit.

- `ang_conic <conic_id> <n1> <n2> <n3> <n4> <n5> <n6>` Definition of a conic. A conic `<conic_id>` is determined by the given parameters a, b, c, d, e and f in the following form: $ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$, with respect to a defined origin and unit. An object defined in such a manner gets type CONIC, and can not be used as NUMBER, POINT, LINE or CIRCLE (unless its type is changed by another definition).

- `ang_intersec2 <p_id1> <p_id2> <l_id> <conic_id>`

  `ang_intersec2 <p_id1> <p_id2> <conic_id> <l_id>`

  Objects with specified identifiers `<id1>` and `<id2>` become POINTs and get coordinates of the intersection of a given line and a conic, or of a given conic and line. If there is just one intersection point, then both `<id1>` and `<id2>` have the same value. If there are no intersection points, then the program reports a run-time error.

  The full name, `ang_intersec2`, can also be used for this command.

- `ang_tangent <l_id> <p_id> <conic_id>` Introduces a line `<l_id>` which is tangent to a conic `<conic_id>` at a point `<p_id>`.

- `ang_drawline <l_id>` Generates commands for drawing a line `<l_id>` within the defined area.

- `ang_drawline_p <p_id1> <p_id2>` Generates commands for drawing the line determined by points `<p_id1>` and `<p_id2>`, within the defined area.

- `ang_drawconic <conic_id>`

  Generates commands for drawing the conic `<conic_id>`.

- `ang_drawdashconic <conic_id>`

  Generates commands for drawing the conic `<conic_id>` and made of dash segments. **GCLC** draws conics segment by segment and by this command every third segment will not be drawn, and you can change length of dash segments by the command `'conicprecision'`. The default number of segments is `144` on x-axis.

- `ang_draw_parametric_curve <id>`
  `{<exp1>;<exp2>;<exp3>}{<exp4>;<exp5>}`

  The object with the specified identifier `<id>` becomes NUMBER and serves as a (iterating) curve parameter. A curve is being drawn in iterations. `<exp1>` is the initial value for the parameter, `<exp2>` is the (*while*) condition that the (changing) parameter has to meet, and `<exp2>` is the expression for recalculating the parameter in each iteration. The pair `<exp4>`, `<exp5>` determines the pair (x, y) of coordinates of a point on the curve. In building expressions, one can use the same functions, operators and relations as in command `expression`, described in §3.4.

  Examples:

`ang_draw_parametric_curve x {-5;x<10;x+0.1}{x;x*sin(x)}`

`ang_draw_parametric_curve t {0;t<30;t+0.3}{sin(t)*t;-cos(t)*t}`

The curve drawing is reset if in some iteration an undefined expression is encountered. For instance,

`ang_draw_parametric_curve x {-5;x<10;x+0.1}{x;d/x}`

will produce two pieces of a line with the discontinuity point at $(0, 0)$. However,

`ang_draw_parametric_curve x {-5;x<10;x+0.3}{x;d/x}`

will produce a single line since the discontinuity point $(0, 0)$ is missed. The system does not determine parameter values for which a resulting point is undefined, but can only detect such situation if encountered in some iteration. However, in order to provide expected output, the system does not draw segment between subsequent point on the line if both of them are out of the picture area.

As explained above, the system skips points in which expressions giving (x, y) is not defined. So, it is not reported if one of these expressions is not defined in some iterations. Moreover, it is also not reported if expressions involve undefined functions or are ill-formed. Because of that, in case of a problem, it is a good practice to check the expressions separately, as arguments to the `expression` command.

- `ang_conicprecision` **GCLC** draws conics segment by segment. The default value is `144` segments on x-axis. This number can be changed by this command.

- `ang_plot_data <n> { <sequence of coordinates> }` Draws the graph of the function given by its points. The number `<n>` is equal to 0 if the points are not to be denoted, and is equal to 1 if the points are to be denoted by small circles. The points are given as a sequence of their coordinates.

  Example:

```
ang_plot_data 1
{
    1.0 1.0
    2.0 1.0
    3.0 2.0
}
```

3.9 3D Cartesian Commands

- `ang3d_picture <n1> <n2> <n3> <n4>` Defines a rectangular area for 3D Cartesian picture. The first two parameters determine its lower left corner, and last two parameters determine its upper right corner. If there is no defined area, the default area is empty (i.e., it is defined by lower left and upper right corner equal to $(0, 0)$).

- `ang3d_origin <n1> <n2> <n3> <n4>` Defines a 3D Cartesian coordinate system, shown using normal projection. `<n1>` and `<n2>` give a position of the origin of the system. The default value is $(0, 0)$. `<n3>` and `<n4>` give the angles (in radians) that determine the viewing angle to the coordinate system. In the default position of the system, x axis is screen-horizontal with respect to the screen, y axis is perpendicular to the screen, z axis is screen-vertical. This default position corresponds to the values 0 and 0 of `<n3>` and `<n4>`. For other values, the rotation around the z axis, for `<n3>` radians, is first applied (the new and the old x axes build the angle `<n3>`). After that, the rotation around the default x axis, for `<n4>` radians is applied (the new and the old z axes build the angle `<n4>`).

- `ang3d_unit <n>` Defines the unit of the coordinate system in millimeters. The default value is 10mm.

- `ang3d_scale <n1> <n2> <n3>` Defines the scale between y and x coordinates and between z and x coordinates.

  The parameter `<n1>` determines if the coordinate system is regular (value 1) or logarithmic (value 2). In logarithmic system, x and y axes are set on $z = 1$.

  The parameter `<n2>` determines the multiplication factor for y coordinates. It must be positive. If a negative value or zero is given, the value 1 is assumed.

  The parameter `<n3>` determines the multiplication factor for z coordinates. It must be positive. If a negative value or zero is given, the value 1 is assumed.

  These values determines the coordinate system and all consequent drawings. The default values are 1, 1, and 1.

  Note that, if logarithmic system is used, the commands doing with lines will not work properly.

- `ang3d_axes_drawing_range <n1> <n2> <n3> <n4> <n5> <n6>` Sets the range for drawing awes. On x axis the interval [¡n1¿ ¡n2¿] will be drawn, on y axis the segment [¡n3¿ ¡n4¿], and on z axis the segment [¡n5¿ ¡n6¿]. This command should be used before drawing the system (i.e., before the command `ang3d_drawsystem_p`.

- `ang3d_drawsystem_p <n1> <n2> <n3> <n4> <n5> <n6>` Generates commands for drawing coordinate axes.

  The parameter `<n1>` controls denoting the integer points on the axes: with parameter value 1, integer points are denoted by small circles, with value 2 by small dashes, and with value 3 they are not denoted at all.

  The parameter `<n2>` controls the step for denoting integer points on x axis. For instance, if `<n2>` is equal to 1, then each integer point on x axis is denoted (in a way defined by the parameter `<n1>`); if `<n2>` is equal to 2, then every second integer point is denoted.

  The parameter `<n3>` controls the step for denoting integer points on y axis.

The parameter <n4> controls the step for denoting integer points on z axis.

The parameter <n5> controls denoting the axes: with parameter value 1, the axes are denoted by x, y, and z, and with value 2 the axes are not denoted.

The parameter <n6> controls drawing arrows at the endpoints of the system: with parameter value 1, there are arrows at both positive and negative endpoints, with parameter value 2, there are arrows only at positive, and with parameter value 3, there are no arrows at all.

- ang3d_point <p_id> <n1> <n2> <n3> Definition of an ang3d point by coordinates in the current 3D Cartesian system.

- ang3d_getx <n_id> <p_id> The object with the specified identifier <n_id> becomes NUMBER and gets the value of the x-coordinate of the given point <p_id> in the 3D Cartesian system (and it gets the value 0 if <p_id> was not defined by the command ang3d_point).

- ang3d_gety <n_id> <p_id> The object with the specified identifier <n_id> becomes NUMBER and gets the value of the y-coordinate of the given point <p_id> in the 3D Cartesian system (and it gets the value 0 if <p_id> was not defined by the command ang3d_point).

- ang3d_getz <n_id> <p_id> The object with the specified identifier <n_id> becomes NUMBER and gets the value of the z-coordinate of the given point <p_id> in the 3D Cartesian system (and it gets the value 0 if <p_id> was not defined by the command ang3d_point).

- ang3d_drawline_p <p_id1> <p_id2> Generates commands for drawing the line determined by points <p_id1> and <p_id2>, within the defined area.

- ang3d_draw_parametric_surface <id1> <id2>
  {<exp1>;<exp2>;<exp3>}
  {<exp4>;<exp5>;<exp6>}
  {<exp7>;<exp8>;<exp9>}

  This commands draws a surface given by two parameters. The objects with the specified identifiers <id1> and <id2> become NUMBERs and serve as a (iterating) surface parameters. A surface is drawn in iterations. For the first parameter (id1), <exp1> is the initial value, <exp2> is the (*while*) condition that the (changing) parameter has to meet, and <exp2> is the expression for recalculating the parameter in each iteration. For the second parameter (id2), <exp4> is the initial value, <exp5> is the (*while*) condition that the (changing) parameter has to meet, and <exp6> is the expression for recalculating the parameter in each iteration. The triple <exp7>, <exp8>, <exp9> determines the triple (x, y, z) of coordinates of a point on the surface. In building expressions, one can use the same functions, operators and relations as in command expression, described in §3.4.

  Example (drawing a torus):

```
ang3d_draw_parametric_surface u v
    {0; u<=6.43; u+0.15}
    {0; v<=6.43; v+0.15}
    { (4+2*sin(v))*cos(u); (4+2*sin(v))*sin(u); b*cos(v) }
```

Drawing of the surface is reset if in some iteration an undefined expression is encountered (see explanation for `ang_draw_parametric_curve` command)

- `ang3d_draw_parametric_curve <id>`
  `{<exp1>;<exp2>;<exp3>}{<exp4>;<exp5>;<exp6>}`

  The object with the specified identifier `<id>` becomes NUMBER and serves as a (iterating) curve parameter. A curve is being drawn in iterations. `<exp1>` is the initial value for the parameter, `<exp2>` is the (*while*) condition that the (changing) parameter has to meet, and `<exp2>` is the expression for recalculating the parameter in each iteration. The triple `<exp4>`, `<exp5>`,`<exp6>` determines the triple (x, y, z) of coordinates of a point on the curve. In building expressions, one can use the same functions, operators and relations as in command `expression`, described in §3.4.

  Example:

```
ang3d_draw_parametric_curve u
    {0; u<=5; u+0.05}
    { u*sin(u*u); u*cos(u*u); u }
```

  Drawing of the curve reset if in some iteration an undefined expression is encountered (see explanation for `ang_draw_parametric_curve` command)

3.10 Layers

- `layer <n>` Declares the layer: all subsequent drawing, labelling commands, commands for changing color or line thickness, etc. will be considered to belong to this layer. Layers are irrelevant for construction commands. The parameter `<n>` can have values between 0 and 1000.

- `hide\_layer <n>` Hides contents of the layer `<n>` will not be shown. The parameter `<n>` can have values between 0 and 1000.

- `hide_layers_from <n>` Hides contents of all layers higher than `<n>`, including the layer `<n>`.

- `hide_layers_to <n>` Hides contents of all layers lower than `<n>`, including the layer `<n>`.

3.11 Support for Animations

Animations are supported in GUI version of **GCLC** version only.

Within animations, the entire `gcl` descriptions runs for every frame. The only difference between frames occurs with moving points. If the animation is defined to have, say, 100 frames, then there will be 100 runs of the script with 100 tuples of values for moving points. The first run (giving the first frame) will give the output for the initial values for all points. The 100th run will give the output for the ending values for all points. Each moving point moves uniformly and linearly from its initial to its ending position. Having all frames built, they are just shown one by one.

- `animation_frames <n1> <n2>` Defines the total number of frames in the animation and the number of frames per second. The default value for the total number of frames is 0. This command is ignored when producing e.g.,LaTeX or bitmap images. The command is used only in animation mode in the version with the graphical user interface. In other modes/releases this command has no effects.

  This command should not be used within while-blocks.

- `point <p_id> <n1> <n2> <n3> <n4>` This is a five-arguments version of the `point` command. The point will move from the position determined by the first pair of coordinates to the position determined by the second pair of coordinates. The command is used in this form only in animation mode in the GUI version. In other modes, the last two arguments are ignored.

  `trace <p_id> <n1> <n2> <n3>` Defines the point for which the *trace* (or *locus*) will be drawn during the animation. The three given numbers give RGB components of the color for the trace.

  The command is used only in animation mode in the GUI version. In other modes/releases this command has no effects.

  Tracing spans all animation frames so it can have only a global scope. Thus, it is ignored if invoked within an user-defined procedure or a conditional block of commands.

  Traces for a selected frame are also exported to output files.

3.12 Support for Theorem Provers

Theorem proving is supported in both the command-line and in the GUI version. There are several automated theorem provers built-in **GCLC**, the specific prover is selected either by a command line parameter (for **GCLC**) or by appropriate button (in the GUI version).

- `prove { <statement> }` After the processing of construction description, the theorem prover will be called on the given statement. The output of the prover will be exported to LaTeX format or to XML format, in the current directory.

- `prooflevel <n>` Controls the output level of the prover. A level can be an integer between 0 and 7. The default value is 1.
- `prooflimit <n>` Controls the maximal number of proof steps. The default value is 10000.
- `prover_timeout <n>` Sets the time (in seconds) available to the prover. The default value is 10 seconds.
- `theorem_name <n>` Sets the name of the theorem (it is used in prover's output documents).

More detailed description of the build-in theorem provers are given in Chapter 6.

Chapter 4

Graphical User Interface

The graphical user interface for **GCLC** does not provide only a a new, user-friendly interface, but also introduces some features which are not available in command-line version (e.g., moving points, building and playing animations, tracing points, watch window etc). It is a kind of an "Integrated Development Environment" or IDE for **GCLC**.

4.1 An Overview of the Graphical Interface

The graphical interface consists of these main parts (see Figure 4.1):

1 Editor window.

2 Picture window

3 Debug/Log window

4 Menus and toolbar

5 Status bar

6 Watch window

Editor window is provided here for writing source files (i.e., descriptions of a geometric construction) in it. It has standard features which are expected from an text editor, find/replace, and some additional features like coloring syntax and error/warning locating.

The Picture window is used for showing the processed figure. One can interactively work in this window by selecting and moving fixed points, zooming the picture, etc. (Note: "Fixed point" is a point defined by GCL command `point`.)

Debug/Log window is a notifying window. It displays messages about the status of the most recent operation such as compiling, exporting etc.

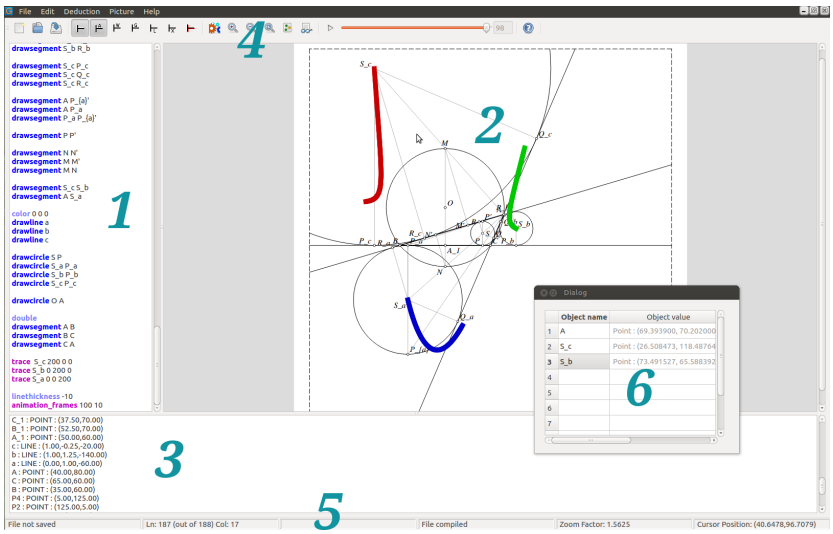

Figure 4.1: Screenshot of **gclc-gui**

Menus and toolbars are standard parts of almost every program with GUI program. There is a standard menu with a toolbar that provides shortcuts for menu items. Most items on the menu have an equivalent on some toolbar. The menu items can also be activated by appropriate keyboard shortcuts — for instance, Build (compiles a picture or builds animation) can be activated by the keys *Ctrl+B*.

- The toolbar has a number of items, including those for creating, opening and saving files, for zooming picture in and out, for theorem proving (for turning on and off the prover based on the area method, for turning on and off the prover based on Wu's method, for turning on and off the prover based on Gröbner bases method, for turning deduction control on and off, for generating proofs in LaTeX, for generating proofs in XML.
- The status bar displays some real-time information like current line and column in editor window, current x and y coordinates of the mouse pointer in the picture window, whether the file has been saved or processed, and the zoom factor.

4.2 Features for Interactive Work

Creating/Opening: In the beginning, one can created a new or open an existing `*.gcl` file. From the *File* menu, select the item *New* and the blank new document window should appear. Type a few GCL commands in the editor window. For instance, try this set of commands:

```
point A 20 20
point B 90 30
```

```
point C 40 90

drawsegment A B
drawsegment B C
drawsegment C A

cmark_b A
cmark_b B
cmark_t C
```

If you are not familiar with GCL, explore some of the samples and learn about commands of GCL.

Editing: You can type GCL commands in the editor window just like in any other programming language.

Building: After entering of the source, select the item *Build* from the *Picture* menu and start the compilation and building of the picture. If there were no errors, a figure corresponding to the description given by the source code will appear in the picture window (see Figure 4.2).

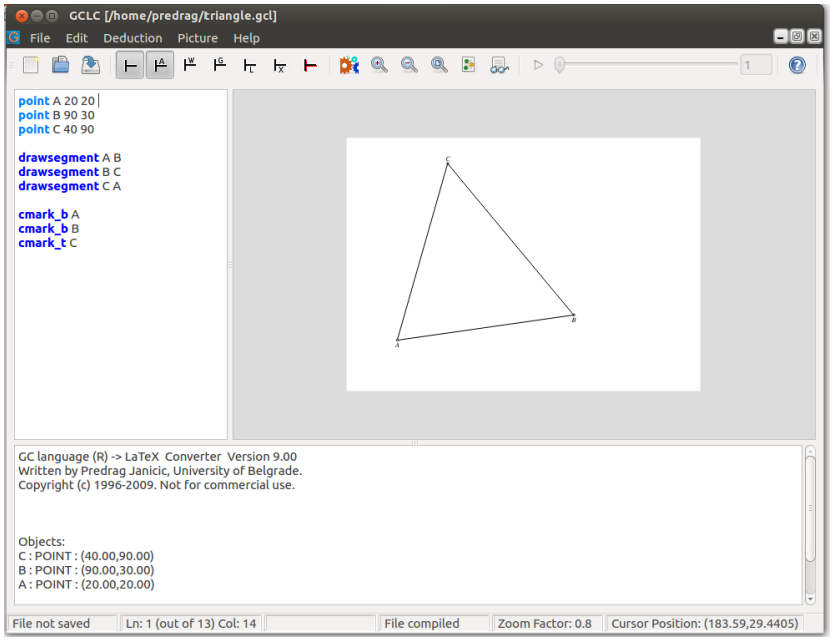

Figure 4.2: After building the image

Modifying the picture: Modifying the figure can be achieved by editing the source manually (which gives more control over the picture), or by moving "free" points directly on the picture. After successful compilation, you can select the item *Show Free Points* from the *Picture* menu. Now, every free point will have a green circular mark which can be selected (picked-up)

and dragged to another position (also changing the coordinates of the point in the editor). In a similar way, destination points for animations can be dragged (while they have orange circular marks) (see Figure 4.3).

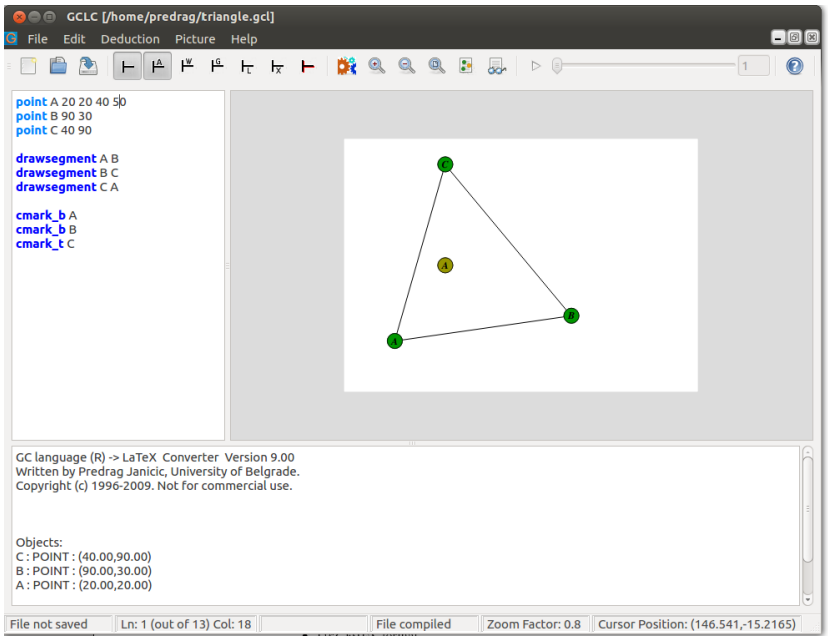

Figure 4.3: Moving a fixed point

Export: GUI GCLC can export a compiled picture (or a particular frame from an animation) to the following formats:

- simple LaTeX format (supported by `gclc.sty`
- PSTricks LaTeX format
- Ti*k*Z LaTeX format
- Bitmap
- EPS (Encapsulated PostScript)
- SVG (Scalable Vector Graphics)
- raster formats: bitmap, PNG, JPG
- XML (textual description of the figure given as XML file).

Also, an animation can be exported to a sequence of bitmaps (which can be further used for making animations by some other tool). These features are available in the *File/Export* menu. More about export options can be found in Chapter 5.

Import: GUI **GCLC** can import JavaView .jvx files. This feature is available in the *File/Import* from menu.

Theorem proving: Options for the built-in theorem proving can be accessed via the option group *Source* or via the *Source toolbar*. If the source file contains a conjecture, **GCLC** will automatically invoke the built-in theorem prover, if the option *Theorem proving* is checked (i.e., if this button is turned on). If the option *Deductive control* is checked, when a run-time error occurs (during processing the source), **GCLC** will run the prover to check whether this error occurs in all situations (i.e., whether it is a deductive consequence of the given construction) (see Section 6.7). Theorem prover can generate proofs in LaTeX and in XML form. If at least one of the options *Theorem proving* and *Deductive control* is checked, then the options *Generate Proofs in LaTeX* and *Generate Proofs in* XML can be selected. If neither of these two options were selected, then the prover works without generating files with the proofs. More about the theorem proving and the deduction control can be found in Chapter 6.

Miscellaneous: GUI **GCLC** provides some additional useful features, including

- Watch window: Gives information about type and values of specified objects in the generated picture or in any particular animation frame.

Chapter 5

Exporting Options

On the basis of a (valid) figure description in the file `in.gcl`, a figure can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.pic -option
```

where `out.pic` is the name of a resulting file (it can have any extension, not necessarily `.pic`), and `-option` controls the output format. If the option is omitted, the output format is the simple LaTeX format (supported by `gclc.sty`). For PSTricks LaTeX format, `-pst` is used, for Ti*k*Z LaTeX format, `-tikz` is used, for EPS, `-eps` is used, for SVG, `-svg` is used, and for XML, the option `-xml` is used. Command line version of **GCLC** does not support export to bitmap.

Within the GUI version of **GCLC**, a picture exported in some of supported formats can be obtained by selecting one of the option available in the group *File/Export to...* (see Chapter 4). When exporting a particular frame from an animation, traces are also saved to output files.

File exported to vector formats (LaTeX, SVG, XML) contain comments — explanations for portions of code for easier understanding and modifying.

By using the `export_to...` **GCLC** commands, arbitrary text can be exported directly to files containing specific vector formats (LaTeX, SVG, XML).

Note that mathematical formulae written within a **GCLC** file will be formatted properly only when exported to LaTeX formats. If a figure is exported to, say, EPS format, it will not have properly formatted formulae. In order to obtain an EPS file with properly formatted formulae, one must first export the file to LaTeX, then process a LaTeX file that contains only this figure, then export the obtain document to a PS document, and then finally, convert the PS to a EPS file.

When exported to LaTeX, the text is printed in math mode and in the size `footnotesize`.

5.1 Export to Simple LaTeX format

By "simple export LaTeX format" we mean the format based on the `gclc.sty` package. Good sides of this choice are: there is just one file required (`gclc.sty`), it can be stored in standard folder for LaTeX packages or in the current folder, it can be easily shared with others to make LaTeX documents portable, or it can

even be copied to LaTeX documents (since it is just few lines long). This package insignificantly slows down processing LaTeX documents that use it. Bad sides of choosing this LaTeX format are: in most systems producing PDF documents has to be performed via PostScript format. In addition, this format does not support some features of **GCLC**, like filling polygons.

5.1.1 Generating LaTeX Files and GCLC.STY

On the basis of a (valid) figure description in the file `in.gcl`, a picture in LaTeX format can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.pic
```

where `out.pic` is the name of a resulting file (it is not necessary for it to have the extension `.pic`). If the extension of the input file is not given, then the assumed extension is `.gcl`. If the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.pic`).

Within the GUI version of **GCLC**, a picture exported in LaTeX format can be obtained by selecting the option *File/Export to.../LaTeX*.

The picture can be included in a LaTeX document using the command:

```
\input{figures/out.pic}
```

in appropriate position in your LaTeX document.

The package `gclc` must be included (by `\usepackage{gclc}`) in the preamble of your document, or you can add the following lines in the preamble:

```
\def\gclcpicture#1{%
   \def\gclcline##1##2##3##4##5##6##7{%
      \put(##1,##2){\special{em:point #1##3}}%
      \put(##4,##5){\special{em:point #1##6}}%
      \special{em:line #1##3,#1##6,##7mm}}}
\gclcpicture{1}
```

If the command `color` is used in the GCL file, a LaTeX document including the corresponding picture (`.pic` file) should also use the package `color`.[1]

5.1.2 Changing LaTeX File Directly

A position of the picture in your document can be changed by changing (within any editor) appropriate parameters in the **GCLC** output file. For example,

```
\begin{picture}(140.00,100.00)
```

can be changed to

```
\begin{picture}(180.00,100.00)(10.00,10.00)
```

in order to change picture dimensions and its position. (All values in both input and output file are expressed in units of 1 millimeter.)

[1] For convenience, the package `color.sty` (developed by other authors), is distributed with **GCLC**.

5.1.3 Handling More Pictures on a Page

If you want to put two or more pictures generated by **GCLC** on one page, you might need to take care about point numbers, otherwise DVI... programs might report the warning `duplicate point numbers` and only your first picture on a page will be drawn correctly. This is so because (many) DVI programs can (in an appropriate mode) address a limited number of points each of which has its own index. These indices can go from 1 to 32767. If there are several pictures, each of their points should have an index which is unique; indices of points in different pictures thus could differ in the first digit. So, before each, there should be one `\gclcpicture` command with the number of the picture (you can omit `\gclcpicture{1}` before the first picture):

```
\input{sample1.pic}
\gclcpicture{1}
\input{sample2.pic}
```

This way, point indices are built by concatenating picture indices and original point indices. Add command `\gclcpicture{1}` after a sequence of pictures supposed to be on the same page. Note that if pictures have thousands of points this approach can fail (for instance, if there are three pictures on a page, each with 3000 points).

There is also another solution for this problem: **GCLC** can create pictures with indices of points starting with a given number. Therefore, for example, if **GCLC** reports `Ending point number: 99` after successfully processing an input file, starting point number for the second picture should be 100 or more:

```
> gclc in.gcl out.pic 100
```

If this parameter is omitted, the starting point number is 1.

5.1.4 Batch Processing

If **GCLC** (the command line version) is called the option `-b`, then the batch processing is used. It expects a LaTeX file with blocks of **GCLC** commands, given in `gclc` environment, and it outputs a single LaTeX file with **GCLC** commands replaced by corresponding LaTeX drawing commands (based on the `gclc.sty` style). This output file then should be processed (only) by LaTeX processor. If the option `-b` is used, then all other options are ignored (the theorem prover and deduction control are off, and only exporting to LaTeX is enabled).

The batch processing is used as in the following example:

```
> gclc in.tex out.tex -b
```

If the extension of the input file is not given, then the assumed extension is `.tex`. If the option `-b` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in-gclc.tex`).

For example, an input LaTeX file (`in.tex`) can be

```
\documentclass{article}
\usepackage{gclc}

\begin{document}

Here goes a triangle...

\begin{figure}[h]
    \begin{gclc}
        dim 100 35
        point A 10 10
        point B 80 10
        point C 30 30
        drawsegment A B
        drawsegment A C
        drawsegment B C
    \end{gclc}
\caption{Triangle}
\end{figure}

...and here goes a parametric curve...

\begin{gclc}
    dim 100 35
    ang_picture 0 0 100 35
    ang_origin 10 10
    ang_drawsystem
    ang_draw_parametric_curve x
        { 0; x<8; x+0.05}
        { x; sin(pow(x,2))*cos(x) }
\end{gclc}

\end{document}
```

5.2 Export to PSTricks LaTeX format

PSTricks is a collection of PostScript-based TeX macros that is compatible with LaTeX (but also most TeX macro packages, including plaint TeX). It enables creating high quality graphics in an inline manner. PSTricks package is not distributed with **GCLC**.[2] Good sides of PSTricks choice are: it is very popular, very expressive, it is understandable and can be relatively easily modified by hand. Bad sides of choosing this LaTeX format are: it takes several megabytes, it required installing dozens of LaTeX packages, it is impossible to share LaTeX documents with those who do not have PSTricks installed, it slows down a bit processing LaTeX documents that use it, in order to produce PDF document, an intermediate PostScript document has to be generated.

On the basis of a (valid) figure description in the file `in.gcl`, a picture in PSTricks format can be generated by the command-line version of **GCLC** using

[2] It can be found on `http://tug.org/PSTricks`.

the following command:

```
> gclc in.gcl out.pst -pst
```

where `out.pst` is the name of a resulting file. If the extension of the input file is not given, then the assumed extension is `.gcl`. If the option `-pst` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.pst`).

Within the GUI version of **GCLC**, a picture exported in PSTricks format can be obtained by selecting the option *File/Export to.../LaTeX-PSTricks*.

A picture in PSTricks format can be included in a LaTeX document using the command:

```
\input{out.pst}
```

in appropriate position in your LaTeX document or the whole picture in PSTricks format can be copied to the document (the package `pstricks` must be included.

Batch processing is supported for this format. If **GCLC** (the command line version) is called the option `-b`, then the batch processing is used. It expects a LaTeX file with blocks of **GCLC** commands, given in `gclc` environment, and it outputs a single LaTeX file with **GCLC** commands replaced by corresponding LaTeX PSTricks drawing commands. This output file then should be processed (only) by LaTeX processor. If the option `-b` is used, then all other options are ignored (the theorem prover and deduction control are off).

The batch processing is used as in the following example:

```
> gclc in.tex out.tex -b -pst
```

5.3 Export to Ti*k*Z LaTeX format

The PGF package (PGF is supposed to mean „portable graphics format“), is a package for creating graphics in an inline manner. It defines a number of TeX commands that draw graphics. Ti*k*Z is the natural frontend for PGF. It gives access to all features of PGF, but it is intended to be easy to use. The syntax is a mixture of METAFONT and PSTRICKS and some additional ideas. The PGF package, supporting Ti*k*Z format, is not distributed with **GCLC**.[3] Good sides of Ti*k*Z choice are: it is very expressive, it is understandable and can be relatively easily modified by hand, it enables directly producing PDF documents with figures directly from the LaTeX source (there is no need for PostScript intermediate step). Bad sides of choosing this LaTeX format are: it takes several megabytes, it required installing dozens of LaTeX packages, it is impossible to share LaTeX documents with those who do not have PGF installed, it slows down processing LaTeX documents that use it.

On the basis of a (valid) figure description in the file `in.gcl`, a picture in Ti*k*Z format can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.tkz -tikz
```

[3] It can be found on `http://sourceforge.net/projects/pgf`.

where `out.tkz` is the name of a resulting file. If the extension of the input file is not given, then the assumed extension is `.gcl`. If the option `-tikz` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.tkz`).

Within the GUI version of **GCLC**, a picture exported in Ti*k*Z format can be obtained by selecting the option *File/Export to.../LaTeX-TikZ*.

A picture in Ti*k*Z format can be included in a LaTeX document using the command:

```
\input{out.tkz}
```

in appropriate position in your LaTeX document or the whole picture in Ti*k*Z format can be copied to the document (the package `tikz` must be included).

Batch processing is supported for this format. If **GCLC** (the command line version) is called the option `-b`, then the batch processing is used. It expects a LaTeX file with blocks of **GCLC** commands, given in `gclc` environment, and it outputs a single LaTeX file with **GCLC** commands replaced by corresponding LaTeX Ti*k*Z drawing commands. This output file then should be processed (only) by LaTeX processor. If the option `-b` is used, then all other options are ignored (the theorem prover and deduction control are off).

The batch processing is used as in the following example:

```
> gclc in.tex out.tex -b -tikz
```

5.4 Export to Raster-based Formats and Export to Sequences of Images

Within **WinGCLC**/**gclc-gui**, there is available export to bitmap format (option File/Export to.../LaTeX). The picture can be generated for the following resolutions 75, 150, 300 and 600 DPI. The new GUI version supports also export to JPEG and PNG formats.

Within the GUI version, an animation can be exported to a sequence of bitmaps (option File/Export to...) which can be further used for making animations by some other tool.

5.5 Export to EPS Format

On the basis of a (valid) figure description in the file `in.gcl`, a picture in EPS (Encapsulated PostScript Format) format can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.eps -eps
```

where `out.eps` is the name of a resulting file. If the extension of the input file is not given, then the assumed extension is `.gcl`. If the option `-eps` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.eps`).

Within the GUI version of **GCLC**, a picture exported in EPS format can be obtained by selecting the option *File/Export to.../EPS*.

A picture in EPS format can be included in a LaTeX document using the command:

```
\includegraphics[width=0.5\textwidth]{figures/out.eps}
```

in appropriate position in your LaTeX document (the package `graphicx` must be included (by `\usepackage{graphicx}`) in the preamble of your document).

5.6 Export to SVG Format

On the basis of a (valid) figure description in the file `in.gcl`, a picture in SVG (Scalable Vector Graphics) format can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.svg -svg
```

where `out.svg` is the name of a resulting file. If the extension of the input file is not given, then the assumed extension is `.gcl`. If the option `-svg` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.svg`).

Within the GUI version of **GCLC**, a picture exported in SVG format can be obtained by selecting the option *File/Export to.../SVG*. A picture in SVG format can be directly open by modern web browsers. For more details about XML and SVG see Chapter 7.

5.7 Export to XML Format

On the basis of a (valid) figure description in the file `in.gcl`, a textual description as XML file can be generated by the command-line version of **GCLC** using the following command:

```
> gclc in.gcl out.xml -xml
```

where `out.xml` is the name of a resulting file. If the extension of the input file is not given, then the assumed extension is `.gcl`. If the option `-xml` is used and if the name of the output file is not given, then the assumed name is built from the input file name (e.g., `in.xml`).

Within the GUI version of **GCLC**, a figure description exported in XML format can be obtained by selecting the option *File/Export to.../XML*. A figure description in XML format can be directly open by modern web browsers. For more details about XML and SVG see Chapter 7.

5.8 Generating PostScript and PDF Documents

LaTeX files with figures generated by **GCLC** should be normally converted to DVI format (by the LaTeX processor), and then to PostScript (by some DVI to PostScript converter, e.g., `dvi2ps`).

LaTeX files with figures generated by **GCLC** might not properly get converted to PDF documents in some environments (neither by LaTeX to PDF processor, nor by DVI to PDF converter, e.g., `dvi2pdf`). If this is the case, then first produce the DVI file, then convert it to PostScript, and then, finally, to PDF (by some PostScript to PDF converter, e.g., `ps2pdf`).

Chapter 6

Theorem Prover

There are three theorem provers built into **GCLC**:

- a theorem prover based on the Chou's *area method*[1] [4]; the prover produces traditional (i.e., geometric, not algebraic, coordinate-based), readable proofs; the proofs are expressed in terms of higher-level geometry lemmas and expression simplifications.
- theorem provers based on the Wu's method [12, 3] and on the Gröbner bases method [1, 2];[2] these provers are algebraic theorem provers; they are based on manipulating polynomials and they do not produce traditional geometrical proofs.

The theorem prover to be used is selected in the following way:

- in the command line version, by an appropriate parameter: `-a` for the area method (default), `-w` for the Wu's method, `-g` for the Gröbner bases method.
- in the GUI version, the theorem prover is selected by checking appropriate button in the toolbar or by checking the option *Deduction Control*.

All provers can prove a range of non-trivial theorems, including theorems due to Ceva, Menelaus, Gauss, Pappus, Thales etc, but they are still a subject of further improvements.

Support for the provers involves only a few commands:

- `prove` (for providing a conjecture);
- `prooflevel` (for setting the level of proof details);
- `prooflimit` (for setting maximal size of a proof).
- `prover_timeout` (for setting the timeout for the prover).

[1] This theorem prover based on the area method was developed in collaboration with prof. Pedro Quaresma, Department of Mathematics, University of Coimbra, Portugal. This work was partially supported by CISUC/FCT, Centro International de Matemática (CIM), under the programme "Research in Pairs", while on visit to Coimbra University under the Coimbra Group Hospitality Scheme.

[2] The main author of these theorem provers is Goran Predović (University of Belgrade).

- `theorem_name` (for setting the theorem's name).

The provers work in both command line version and in GUI version (and they do not use any specific functionalities of the GUI). Proofs of theorems can be generated in LaTeX or in XML form and saved in a file. For the area method, each deduction step is accompanied by its semantics counterpart — corresponding numeric values in Cartesian plane.

The theorem provers are very efficient. Many conjectures are proved in only milliseconds. However, some conjecture may take several seconds, several minutes, or in some specific cases even several hours. The maximal number of proof steps can be set by the command `prooflimit`. The default value is 10000 proof steps.[3] If the prover perform more proof steps, the proving process is stopped. Similarly, the time available to the prover (in seconds) can be set by the command `prover_timeout`. The default value is 10 seconds.

6.1 Introductory Example

The theorem prover is tightly integrated into **GCLC**. This means that one can use the prover to reason about a **GCLC** construction (i.e., about objects introduced in it), without any required adaptations required for the deduction process. Of course, only the conjecture itself has to be added.

The example **GCLC** code given in Figure 6.1 describes a triangle and midpoints of two of triangle's sides. This **GCLC** code produces the Figure 6.2. It holds that the lines AB and A_1B_1 are parallel and this can be proved by the theorem prover. The conjecture „AB and A_1B_1 are parallel“ is given as argument to the `prove` command in the following way:

```
prove { parallel A B A_1 B_1 }
```

At the end of the processing of the **GCLC** file, the theorem prover is invoked; it can produce a proof in LaTeX and in XML form (in the current directory) and, within the **GCLC** report, a report about the proving process: whether the conjecture was proved or disproved, data about CPU time spent, etc.

6.2 Basic Sorts of Conjectures

Statements for the basic sorts of conjectures are given in the following table:

points A and B are identical:	`identical A B`
points A, B, C are collinear:	`collinear A B C`
AB is perpendicular to CD:	`perpendicular A B C D`
AB is parallel to CD:	`parallel A B C D`
O is the midpoint of AB:	`midpoint O A B`
AB has the same length as CD:	`same_length A B C D`
points A, B, C, D are harmonic:	`harmonic A B C D`

All these sorts of conjectures can also be expressed in terms of geometry quantities. Geometry quantities provide more general way for stating conjectures.

[3] On a modern PC computer, 10000 steps are performed in less then 1 minute.

```
point A 20 10
point B 70 10
point C 35 40

midpoint B_1 B C
midpoint A_1 A C

drawsegment A B
drawsegment A C
drawsegment B C
drawsegment A_1 B_1

cmark_b A
cmark_b B
cmark_t C
cmark_l A_1
cmark_r B_1

prove { parallel A B A_1 B_1 }
```

Figure 6.1: Description of a triangle and midpoints of two of triangle's sides and the conjecture of midpoint theorem

6.3 Geometry Quantities and Stating Conjectures

The theorem prover deals with the following geometry quantities:

ratio of directed segments: for four collinear points P, Q, A, and B such that $A \neq B$, it is the ratio $\frac{\overrightarrow{PQ}}{\overrightarrow{AB}}$;

signed area: it is the signed area S_{ABC} of a triangle ABC or the signed area S_{ABCD} of a quadrilateral $ABCD$;

Pythagoras difference: for three points, P_{ABC} is defined as follows:

$$P_{ABC} = AB^2 + CB^2 - AC^2 \ .$$

Pythagoras difference for four points, P_{ABCD} is defined as follows:

$$P_{ABCD} = P_{ABD} - P_{CBD} \ .$$

real number: it is a real number, constant.

In **GCLC**, geometry quantities are written as in the following examples:

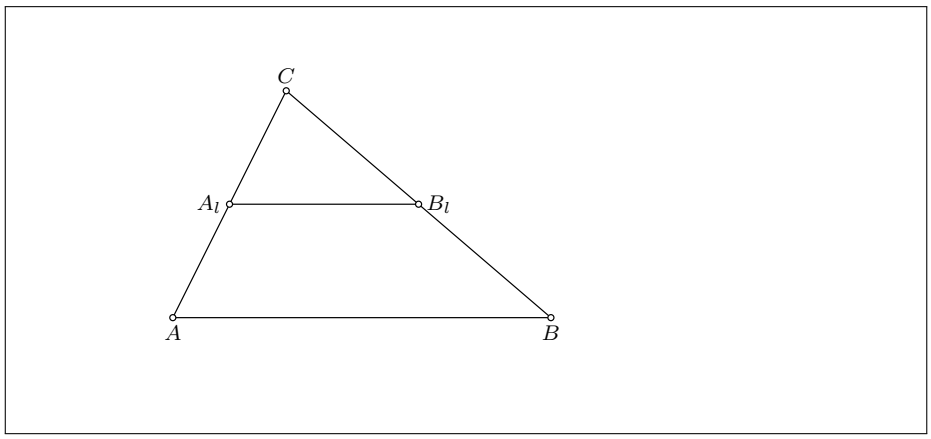

Figure 6.2: Illustration generated from the **GCLC** code from Figure 6.1

ratio of directed segments	$\frac{\overrightarrow{PQ}}{\overrightarrow{AB}}$	`sratio P Q A B`
signed area (arity 3)	S_{ABC}	`signed_area3 A B C`
signed area (arity 4)	S_{ABCD}	`signed_area4 A B C D`
Pythagoras difference (arity 3)	P_{ABC}	`pythagoras_difference3 A B C`
Pythagoras difference (arity 4)	P_{ABCD}	`pythagoras_difference4 A B C D`

A conjecture to be proved is given as argument to the `prove` command. It has to be some of the basic sorts of conjectures (see Section 6.2), or it has to be of the form $L = R$, where L and R are expressions over geometry quantities. The conjecture can involve geometry quantities (only) over points already introduced (by a subset of commands) within the current construction. Geometry quantities can be combined together into more complex terms by operators for addition, multiplication and division. Operators are written in textual form as in the following table:

=	equality
+	sum
·	mult
/	ratio

The conjecture and all its subterms are written in prefix form, with brackets if needed. For instance,

$$S_{A_1B_1A} = S_{A_1B_1B}$$

is given to be proved in the following way:

```
prove { equal { signed_area3 A_1 B_1 A }
              { signed_area3 A_1 B_1 B }
      }
```

and

$$\left(\left(\frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right) = 1$$

is given to be proved in the following way:

```
prove { equal { mult { mult { sratio A F F B }
                            { sratio B D D C } }
                     { sratio C E E A } }
              1 }
```

A range of geometry conjectures can be stated in terms of geometry quantities. The representations for the basic sorts of conjectures is are given in the following table:

points A and B are identical	iff	$P_{ABA} = 0$
points A, B, C are collinear	iff	$S_{ABC} = 0$
AB is perpendicular to CD	iff	$P_{ACD} = P_{BCD}$
AB is parallel to CD	iff	$S_{ACD} = S_{BCD}$
O is the midpoint of AB	iff	$\frac{\overrightarrow{AO}}{\overrightarrow{OB}} = 1$
AB has the same length as CD	iff	$P_{ABA} = P_{CDC}$
points A, B, C, D are harmonic	iff	$\frac{\overrightarrow{AC}}{\overrightarrow{CB}} = \frac{\overrightarrow{DA}}{\overrightarrow{DB}}$

Note that the command

```
prove { parallel A B A_1 B_1 }
```

(from Section 6.1) is equivalent to

```
prove { equal { signed_area3 A_1 B_1 A }
              { signed_area3 A_1 B_1 B }
      }
```

The prover transforms the basic sorts of conjectures into statements given in terms of geometric quantities: if a conjecture of a basic sort is given, the very first step in the proof is its formulation in terms of geometric quantities.

The conjecture can involve geometry quantities only over points and lines already introduced within the current construction, and by using (only) the following commands:

- `point`
- `line`
- `intersec`
- `midpoint`
- `med`
- `perp`
- `foot`
- `parallel`
- `translate`
- `towards`

- `online`

The prover cannot prove conjectures about object constructed by using some other commands. For instance, if a line a is constructed by the command `bis`, then the prove cannot prove conjectures involving a or involving points constructed by using a.

6.4 Area Method

One of the theorem prover built into **GCLC** is based on the algorithm described in [4]. The basic idea of the algorithm is to express a theorem in terms of geometry quantities, to eliminate (by appropriate lemmas) all occurrences of constructed point and to simplify the expression, yielding a trivial equality.

6.4.1 Underlying Constructions

In [4], a construction is expressed in terms of commands `INTER`, `PRATIO`, `TRATIO` and `FOOT`:

`INTER Y ln1 ln2` : Point Y is the intersection of line ln_1 and line ln_2.

`PRATIO Y W U V r` : Point `Y` is a point such that $\overline{WY} = r\overline{UV}$, where r is a real number.

`TRATIO Y U V r` : Point Y is a point on line l, such that $r = \frac{UY}{UV}$, where r is a real number and l is a line such that U lies on l and l is perpendicular to UV.

`FOOT Y P U V` : Point Y is a foot from P to line UV (i.e., YP is perpendicular to UV and Y lies on UV.

For each point X constructed by the above constructions and for each geometry quantity g involving X, there is a suitable lemma that enables replacing g by an expression with no occurrences of X. Thanks to these lemmas, all constructed points can be eliminated from the conjecture.

6.4.2 Integration of Algorithm and Auxiliary Points

In order to be tightly integrated into **GCLC**, the prover uses standard **GCLC** construction commands and, if needed, transforms them internally into form required by the algorithm and/or introduces some auxiliary points:

`midpoint` is expressed in terms of `PRATIO`, it does not introduce new points;

`foot` is expressed in terms of `FOOT`, it does not introduce new points;

`med` introduces two auxiliary points: for instance, `med m A B` introduce a point M_m as the midpoint of AB and a point T_m on the bisector of AB (such that `TRATIO` T_m M_m `A 1`); the line m is then determined by the points M_m and T_m;

`perp` introduces one auxiliary point: if A lies on the line q, then `perp p A q` introduces a point T_p on a line perpendicular to q (such that `TRATIO` $\mathtt{T_p}$ `A` $\mathtt{Q_1}$ `1`; where the line q is determined by points Q_1 and Q_2); in this case, the line p is determined by the points A and T_p; if A does not lie on the line q, then `perp p A q` introduce a point F_p which is a foot of the normal from A to the line q; in this case, the line p is determined by the points A and F_p;

`parallel` introduces one auxiliary point: for instance, `parallel p A q` introduces a point P_p on a line parallel to q (such that `PRATIO` $\mathtt{P_p}$ `A` $\mathtt{Q_1}$ $\mathtt{Q_2}$ `1`; the line p is then determined by the points A and P_p;

`translate` is expressed in terms of `PRATIO`, it does not introduce new points;

`towards` is expressed in terms of `PRATIO`, it does not introduce new points;

`online` is expressed in terms of `PRATIO`, it does not introduce new points, but introduces a (indeterminate) constant r: for instance, `online X A B` is interpreted as `PRATIO X A A B r`.

Definitions of auxiliary points are given at the beginning of the proof.

6.4.3 Non-degenerative Conditions and Lemmas

Some constructions are possible only if certain conditions are met. For instance, the construction `inter X a b` is possible only if the lines `a` and `b` are not parallel. For such constructions *non-degenerative conditions* are store for future possible use and listed at the end of the proof.

Some non-degenerative conditions can also be introduced during the proving process:

- some lemmas have two cases (for instance, „if A belongs to CD“ and „if A does not belong to CD“); if a condition for one case can be proved (as a lemma), then that case is applied, otherwise, a condition for one case (the one of the form $L \neq R$) is assumed and introduced as a non-degenerative condition.

- in the cancellation rule, if all summands on both sides of the equality have the same multiplication factor X, the rule tries to prove (as a lemma) that $X = 0$; if this fails, a condition $X \neq 0$ is assumed and introduced as a non-degenerative condition and the equality is cancelled by X.

Lemmas are being proved as separate conjectures, but, of course, sharing the construction and non-degenerative conditions with outer context.

6.4.4 Structure of Algorithm

The algorithm has one main *while* loop — it process the sequence of all (relevant) constructions in backward manner (from last to first construction step) and transforms the current goal as follows:

- the current goal is initially the given conjecture;

- *while* there are construction steps do:
  - apply geometric simplifications to the current goal;
  - apply algebraic simplifications to the current goal;
  - if the current construction step introduce a new point P, then eliminate (using the elimination rules) one of occurrences of P (from the current goal) and go to the top of the while loop; otherwise, go to next construction step.
- apply geometric simplifications to the current goal;
- apply algebraic simplifications to the current goal.
- if the current goal is a equality trivially true, then the conjecture has been proved, if the current goal is a equality trivially false, then the conjecture has been disproved, otherwise, the conjecture has been neither proved nor disproved.

The reasoning steps, as seen from the above overall algorithm, are divided into three groups:

algebraic simplifications: applies simplification rewrite rule (not directly related to geometry) such as:

$$\begin{array}{rcl} x+0 & \rightarrow & x \\ 0+x & \rightarrow & x \\ x\cdot 1 & \rightarrow & x \\ x\cdot 0 & \rightarrow & 0 \\ \frac{x}{y}+\frac{u}{v} & \rightarrow & \frac{x\cdot v+u\cdot y}{y\cdot v} \\ \cdots & & \end{array}$$

geometric simplifications: applies simplification rewrite rule, directly related to geometry quantities such as:

$$\begin{array}{rcl} S_{AAB} & \rightarrow & 0 \\ S_{ABC} & \rightarrow & S_{BCA} \\ P_{AAB} & \rightarrow & 0 \\ \cdots & & \end{array}$$

elimination simplifications: applies elimination lemmas for eliminating constructed points for the current goal; for instance, if the point Y is introduced by as the intersection of lines l_1 (determined by U and V) and l_2 (determined by P and Q), then Y can be eliminated from expression of the form $\frac{\overrightarrow{AY}}{\overrightarrow{CD}}$ using the following equality:

$$\frac{\overrightarrow{AY}}{\overrightarrow{CD}} = \begin{cases} \frac{S_{APQ}}{S_{CPDQ}}, & \text{if } A \in UV \\ \frac{S_{AUV}}{S_{CUDV}}, & \text{if } A \notin UV \end{cases}$$

Full details about all used lemmas and rewrite rules are given in the technical report "Framework for constructive geometry (based on the area method)" (written by Pedro Quaresma and Predrag Janičić) [11]. This technical report is a part of **GCLC** distribution.

6.4.5 Scope

The theorem prover can prove *any* geometry theorem expressed in terms of geometry quantities, and involving only points introduced by using the commands `point`, `line`, `intersec`, `midpoint`, `med`, `perp`, `foot`, `parallel`, `translate`, `towards`, `online`. This can be proved following the ideas from [4]. However, some of the proofs can be very long and can take lot of time.

6.5 Wu's Method and Gröbner Bases Method

One of the theorem prover built into **GCLC** is based on the Wu's method [12, 3] and one is based on the Gröbner bases method [1, 2]. These methods are algebraic methods and they prove geometrical statements by manipulating polynomials corresponding to the constructions and given conjectures. These methods can also detect non-degenerative conditions, described in Section 6.4.

6.6 Prover Output

Whenever there was a conjecture given in the input GCL file, the prover produces a short report. The prover can also generate proofs in LaTeX format, or in XML format. The level of details given in generated proof can be controlled by the user.

6.6.1 Prover's Short Report

The prover produces a short report (if there was a conjecture given in the GCL file). In the command line version, this short report is shown and written in the log file, while the GUI version shows this report in its output window. For the area method, this report consists of information on number of steps performed, on CPU time spent and whether or not the conjecture has been proved. For example:

```
Number of elimination proof steps:      3
Number of geometric proof steps:        6
Number of algebraic proof steps:       23
Total number of proof steps:           32

Time spent by the prover: 0.002 seconds
The conjecture successfully proved.
The prover output is written in the file ceva_proof.tex.
```

6.6.2 Controlling Level of Output

For the area method, the level of generated proof output is controlled by the command `prooflevel`. This command has one argument (an integer from 0 to 7) which provides the output level:

0 : no output (except the statement);

1 : elimination steps plus grouped geometric steps and algebraic steps;

2 : elimination steps plus geometric steps plus grouped algebraic steps;

3 : as level 2, plus statements of lemmas;

4 : as level 3, plus elimination steps plus grouped geometric steps and algebraic steps in lemmas;

5 : as level 4, plus geometric steps in lemmas;

6 : as level 5, plus algebraic steps at proof level 0;

7 : as level 6, plus algebraic steps in lemmas.

The default output level is 1.

For the algebraic theorem provers, the output is always given in a standard form and the command `prooflevel` is ignored.

6.6.3 Proofs in LaTeX format

The proof in LaTeX form is generated by the command line version of **GCLC**, if the option `-eps` (for export to EPS format) is used, or, if there no export option is used (when the figure is exported to LaTeX format).

The proof in LaTeX form is generated by the GUI version of **GCLC**, if the option *Deduction/Proof Export to LaTeX* is checked.

In these cases, the proof is exported to the file `name_proof.tex` (in the current directory, `name` is the name of the input file). If there is no `prove` command within the construction, then the file with a proof will not be created.

For proofs generated by the area method, at the beginning of the proof, the auxiliary points are defined, for instance:

Let M_a^0 be the midpoint of the segment BC. Let T_a^1 be the point on bisector of the segment BC (such that `TRATIO` T_a^1 M_a^0 B 1).

The proof consists of *proof steps*. In each proof step, the current goal is changed. For each proof step, there is an explanation and (optionally) its semantics counterpart. This semantic information is calculated for concrete points used in the construction (note that these coordinates are never used in the proof itself); it can serve as a semantic test, especially for conjectures for which is not

known whether or not they are theorems. Proof step are enumerated. For example:

$\left(\left(\frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right)$	$= 1$	by the statement (value 1=1)
$\left(\left(\left(-1 \cdot \frac{\overrightarrow{AF}}{\overrightarrow{BF}}\right) \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right)$	$= 1$	by geometric simplifications (value 1=1)

Lemmas are proved within the main proof (making nested proof levels), and the beginning and the end of a proof for a lemma is marked by a horizontal solid line.

At the end of a proof, it is reported if the conjecture is proved ("Q.E.D." — lat. Quod Erat Demonstrandum — which was required to prove), if the conjecture is disproved (if it was shown that it is invalid), or neither of these two.

At the end of the main proof all non-degenerative conditions are listed. For instance:

$S_{M_a^0 M_b^2 T_b^3} \neq S_{T_a^1 M_b^2 T_b^3}$ i.e., lines $M_a^0 T_a^1$ and $M_b^2 T_b^3$ are not parallel (construction based assumption)

For proofs generated by the Wu's and Gröbner bases method, the output consist of a summary of processing steps over the relevant set of polynomials.

See one complete example in the appendix (Section E.5).

Modifying LaTeX Output

The prover exports proofs in LaTeX form. These LaTeX files require LaTeX package `gclcproof.sty`.

Once generated, the proof in LaTeX form can still be parameterized. Namely, data in the file can be formatted in several ways, giving different layouts. Also, semantics part can be omitted. Parameters for proof formatting are given as options for the package `gclcproof.sty`. There are three parameters:

style: defines the layout; available choices are:

- `portrait` — uses the package `longtable` to generate a multi-page table.[4]
- `portraitbreqn` — uses the package `breqn` to try to break the equations. Problems with extra large fractions.
- `landscape` — uses the packages `lscape`, `amsmath` (with option `leqno`), and `breqn`, to generate the list of equations in landscape mode, with numbers on the left, and with automatic equation breaking.

The default value for style is `portrait`.

size: defines the font size in the proof. It uses the LaTeX font size names of, from `tiny` up to `large`. The default value is `small`.

[4]For convenience, the required packages `breqn.sty`, `flexisym.sty`, `mathstyle.sty`, `cmbase.sim` (developed by other authors), are distributed with **GCLC**. The packages `longtable.sty`, `lscape,sty`, `amsmath.sty` (developed by other authors) are not distributed with **GCLC**, but are widely available, most often as part of a TeX distribution.

semantic values: controls if the semantics values will be shown. If the option `semantics` is used, the semantic values of both sides of equations will be shown. The default value is NULL, i.e., without any value. The semantics values are only generated by the area method.

According to the above,
`\usepackage{gclcproof}`
is equivalent to
`\usepackage[portrait,small]{gclcproof}`
(which gives a proof in portrait style, small size, without semantics style).
For example,

- `\usepackage[tiny]{gclcproof}`

  retains the portrait mode (default value), but decrease the size of the fonts used in the proof to tiny. No semantic values displayed.

- `\usepackage[portraitbreqn,sematics]{gclcproof}`

  uses the `breqn` package of the AMSTeX to try to break automatically the equations. Portrait form, small size fonts, semantic values displayed. Note: the `breqn` package may fail (not being able to process the document) if the proof contains large fractions, `breqn` it is unable to break them into lines.

- `\usepackage[landscape]{gclcproof}`

  uses the packages `lscape`, `amsmath` (with option `leqno`), and `breqn` to produce a proof in landscape mode. It is the mode that provides the larger environment for proofs, so in some proofs it will be the only one to be able to display the proofs without overlapping of the different elements. Note: the rotation of the proof to landscape mode it is done with Postscript specials, so it may not display properly in some viewers. The size of the fonts it is small, and the sematic values are not displayed.

In the generated proofs, there is always `\usepackage{gclcproof}` used (in the preamble) and the layout can be changed by using options for this command, as discussed above.

6.6.4 Proofs in XML format

The proof in XML form is generated by the command line version of **GCLC**, if the option `-xml` (for export to XML format) or if the option `-svg` (for export to SVG format) is used.

The proof in XML form is generated by the GUI version of **GCLC**, if the option *Deduction/Proofs Export to XML* is checked.

In these cases, the proof is exported to the file `name_proof.xml` (in the current directory, `name` is the name of the input file). If there is no `prove` command within the construction, then the file with a proof will not be created.

Proofs stored in XML are formatted analogously as in LaTeX format.

The proofs in XML format fulfil restrictions posed by `GeoCons_proof.dtd` (as stated in the line `<!DOCTYPE figure SYSTEM "GeoCons_proof.dtd">` in each of these files). For any XML file, it can be checked if it meets these restrictions.

It can be done using a XML processor, such as `AltovaXML` (copyrighted by Altova GmbH). For instance:

```
> AltovaXML /v GeoCons_proof.dtd thm_proof.xml
```

verifies if the file `thm_proof.xml` is valid.

A proof in XML format (valid with respect to `GeoCons_proof.dtd`) can be converted to a HTML form, for example:

```
>AltovaXML.exe /xslt1 GeoCons_proof.xsl /in thm_proof.xml /out thm_proof.html
```

A file with a proof in XML format can also be open directly by web browsers. See one example in the appendix (Section E.5).

For more details about proofs represented in XML format, see Chapter 7.

6.7 Automatic Verification of Regular Constructions

A geometrical construction is associated with some fixed points (with concrete Cartesian coordinates). In such environments, some constructions (e.g., if they attempt to use intersection of parallel lines), but the question if such construction is always illegal or it is illegal only for given particular fixed points is not trivial (if a construction is never illegal, i.e., if it is always possible, we will call it *regular*). For answering such question, one has to use deductive reasoning, and not only semantic check for the special case. Consider one simple example: given (by Cartesian coordinates) three fixed distinct points A, B, C, we can construct a point D as an image of the point C in translation $\mathcal{T}_{AB}$ (in terms of **GCLC** commands: `translate D A B C`); later on, if we try to construct an intersection of lines AC and BD, we will discover that there is no such intersection (since these two lines are parallel). This holds not for some specific points A, B, C, with D determined as above, but for all triples of points A, B, C. So, this construction is illegal, and moreover, it is illegal not only for a given special case, but always.

The system for automated testing whether a construction is regular or illegal is built into **GCLC** and is based on the built-in theorem provers. All built-in theorem provers can be used for this purpose. This verification mechanism can be switched on or off:

- in the command line version, the verification mechanism is turned on by using the option `-d`.
- in the GUI version, the verification mechanism is turned on by checking the button *Deduction Control* in the toolbar or the option *Deduction/Deduction Control*.

If the verification mechanism is turned on, while processing the input file (with a description of a geometrical construction), **GCLC** provides to the theorem prover all construction steps performed. When **GCLC** encounters a construction step that cannot be performed (e.g., two identical points do not determine a line), it reports that the step is illegal with respect to a given set of fixed points, and then it invokes the theorem prover. The prover is run on

the critical conjecture (e.g., it tries to prove that two points are identical) and, if successful, it reports that the construction step is always illegal/impossible. The prover generates the proof for the critical construction:

- in the command line version: in the format selected for conjectures (see Chapter 5);
- in the GUI version: in LaTeX and XML format if the options, respectively, *Proof Export to LaTeX* and *Proof Export to* XML are checked.

Realm. The verification deductive-check system currently covers the following critical commands and constructions:

`line` — construction of a line given two points (error if the two points are identical);

`med` — construction of a segment-bisector given two endpoints (error if the two points are identical);

`bis` — constructing an angle-bisector of the angle determined by three points A, B, C (error if A and B, or C and B are identical);

`intersec` — constructing an intersection of lines a and b (error if the two lines are parallel);

`angle` — calculating an angle determined by three points A, B, C (error if A and B, or C and B are identical);

Geometry objects that are subject to deductive verification have to be made within using the following commands:

- `point`
- `line`
- `intersec`
- `midpoint`
- `med`
- `perp`
- `foot`
- `parallel`
- `translate`
- `towards`
- `online`

which are internally transformed into primitive constructions of the area method. For more details see [11].

It is worth pointing out that although **GCLC** has support for a large number of constructions, only few of them can be illegal. The above list of critical constructions almost exhaust them. The only possible illegal constructions that are not covered by the current version of our system are constructions of intersection points of circle and line, and of two circles. Corresponding geometry conjectures cannot be generally handled by the area method and the built-in theorem prover.

Chapter 7

XML Support

GCLC has XML support, both for processing figures and proofs.[1] XML is format suitable for storing descriptions of geometrical constructions and proofs and as interchange format:

- instead of raw, plain text representation, geometrical constructions are stored in strictly structured files; these files are easy to parse, process, and convert into different forms and formats;
- input/output tasks are supported by generic, external tools and different geometry tools will communicate easily;
- growing corpora of geometrical constructions will be unified and accessible to users of different geometry tools;
- easier communication and exchange of material with the rest of mathematical and computer science community;
- there is a wide and growing support for XML;
- different sorts of presentation (text form, LaTeX form, HTML) easily enabled;
- strict content validation of documents with respect to given restrictions.

7.1 XML

Extensible Markup Language (XML) is a simple, very flexible text format derived from SGML (ISO 8879) and with data structured using tags. Originally designed to meet the challenges of large-scale electronic publishing, XML is also playing an increasingly important role in the exchange of a wide variety of data on the Web and elsewhere[2]. It is called extensible because it is not a fixed format like HTML (a single, predefined markup language), instead the tags indicate the semantic structure of the data, rather than only its layout in a browser.

[1] XML support was developed in collaboration with prof. Pedro Quaresma, Department of Mathematics, University of Coimbra, Portugal, Jelena Tomašević (University of Belgrade, Serbia), and Milena Vujošević-Janičić (University of Belgrade, Serbia).

[2] http://www.w3.org/XML/

However, XML is not just for Web pages: it can be used to store any kind of structured information, and to enclose or encapsulate information in order to pass it between different computing systems. An XML document can carry both presentation (i.e., plausible visualisation) and content information. XML is a project of the World Wide Web Consortium (W3C) and is a public format — it is not a proprietary development of any company. Almost all browsers that are currently in use support XML natively.

Data type definitions (DTDs) provide a formal specification of the constraints on the structure of data presented in XML form. A DTD is given as a formal description in XML declaration syntax. It sets out what names are to be used for the different types of element, where they may occur, and how they all fit together. This formal description enables automatic verification ("validation") of whether a given document meets the given syntactical restrictions. This way, groups sharing data with similar meanings can agree on different sets of tags and DTDs for representing different kinds of data.

Extensible stylesheet language transformation (XSLT) is a document processing language that is used to transform the input XML documents i.e., the input files to the desired output documents. An XSLT style-sheet declares a set of rules (templates) for an XSLT processor to use when interpreting the contents of an input XML document. These rules tell the XSLT processor how that data should be presented — as an XML document, as an HTML document, as plain text, or in some other form.

Scalable Vector Graphics (SVG) is a language for describing two-dimensional graphics and graphical applications in XML[3]. As XML, SVG is a W3C recommendation.

7.2 XML Suite

GCLCXML suite for geometrical constructions and geometrical proofs consists of the following components and tools:

- XML-based format for representing geometrical constructions with corresponding DTD, `GeoCons.dtd`;
- the option for conversion **GCLC** files to XML-based form;
- the option for generating figures from **GCLC** in SVG form;
- the converter (implemented as XSLT file, `GeoConsGCLC.xlst`) from XML-based form to **GCLC** format;
- the converter (implemented as a XSLT file, `GeoConsHTML.xlst`) from XML-based form to a simple, readable HTML form (with syntax colouring features, provided for better readability);
- the converter (implemented as a XSLT file, `GeoConsNL.xlst`) from XML-based form to a natural language form (currently, only for English language);

[3] http://www.w3.org/Graphics/SVG/

- the XML-based format for representing proofs of properties of geometrical constructions with a corresponding DTD, `GeoCons_proof.dtd`; the format is adapted for the area-method;
- the option for generating proofs in XML-based form;
- the converter (implemented as a XSLT file, `GeoCons_proof.xslt`) for proofs from XML-based form to a simple, readable HTML form (with syntax colouring features, and other features for better readability).

7.3 Using XML Tools

A regular `.gcl` file (file without errors) can be converted to XML form in the following way:

- in the command-line version — using the option `-xml`:

```
> gclc example.gcl example.xml -xml
```

- in the GUI version of **GCLC**, by selecting the option *File/Export to.../XML*.

Note that the XML file obtained in this way is not an image file, but also textual representation of the construction described in the input file.

XML files obtained this way fulfil restrictions posed by `GeoCons.dtd` (as stated in the line `<!DOCTYPE figure SYSTEM "GeoCons.dtd">` in each of these files). For any XML, it can be checked if it meets these restrictions. It can be done using a XML processor, such as `AltovaXML` (copyrighted by Altova GmbH). For instance:

```
> AltovaXML /v GeoCons.dtd example.xml
```

verifies if the file `example.xml` is valid.

A XML file with geometrical contents (valid with respect to `GeoCons.dtd`) can be converted to

- **GCLC** format, for example:

```
>AltovaXML.exe /xslt1 GeoConsGCLC.xsl /in example.xml /out example1.gcl
```

- to a simple, readable HTML form, for example:

```
>AltovaXML.exe /xslt1 GeoConsHTML.xsl /in example.xml /out example.html
```

- to a natural language (English) form (formatted in HTML):

```
>AltovaXML.exe /xslt1 GeoConsNL.xsl /in example.xml /out example.html
```

The geometrical XML files can be open directly by web browsers: For instance, if the second line in the XML file reads:

`<?xml-stylesheet href="GeoConsNL.xsl" type="text/xsl"?>`

then the web-browser will render the contents of the file according to the file `GeoConsNL.xsl` (see the example given in Figure 7.1).

GCLC files can be used for generating figures in SVG format:

Figure 7.1: Natural language presentation of a figure description stored in XML form

- in the command-line version — using the option `-svg`:

  ```
  > gclc example.gcl example.svg -svg
  ```

- in the GUI version of **GCLC**, by selecting the option *File/Export to.../SVG*.

A picture in SVG format can be directly open by modern web browsers.
For `.gcl` files with conjectures, **GCLC** can generate proofs in XML form:

- in the command-line version — if the option `-xmls` or the option `-svg` is used;
- in the GUI version of **GCLC**, by checking the option *Source/Generate proof in XML* or the corresponding button in the source toolbar.

The XML files obtained in the above described way fulfil restrictions posed by `GeoCons_proof.dtd` (as stated in the line `<!DOCTYPE figure SYSTEM "GeoCons_proof.dtd">` in each of these files). For any XML, it can be verified if it meets these restrictions (by analogy with verifying files with constructions).

A XML file with a proof contents (valid with respect to `GeoCons_proof.dtd`) can be converted to a HTML form, using `GeoCons_proof.xsl`, for example:

```
>AltovaXML.exe /xslt1 GeoCons_proof.xsl /in proof.xml /out proof.html
```

XML files with proofs can be also open directly by web browsers see one example in the appendix (Section E.5).

Appendix A

List of Errors and Warnings

```
Syntax error: Number expected.
Syntax error: Two decimal points in number.
Syntax error: Identifier expected.
Syntax error: Identifier or number expected.
Syntax error: Undefined variable.
Syntax error: Wrong variable type.
Syntax error: Symbol '{' expected.
Syntax error: Symbol '}' expected.
Syntax error: Unrecognized definition or command.
Invalid or non-defined expression.
Symbol ';' expected.
Invalid while block.
Invalid if-then-else block.
Invalid procedure definition.
Too many while-block executions (more than 10000). Possible infinite loop.
The conjecture given to prove is ill-formed or includes a point
that is not constructed by commands supported by the prover.
Syntax error: Unknown procedure.
Syntax error: Parameters lists not matched.
Syntax error: Procedures cannot be defined within while-blocks or procedures
Syntax error: Invalid tree description.
Syntax error: Invalid graph description.
Invalid include file.

Input error: Invalid input.

Run-time error: Bad definition. Can not determine line.
Run-time error: Bad definition. Circle radius too small.
Run-time error: Bad definition. Can not determine intersection.
Run-time error: Bad definition. Can not determine angle.
Run-time error: Bad definition. Cannot determine ellipse.
Run-time error: Cannot export data.

Memory error: Cannot allocate enough memory.
```

```
Warning: Changing variable value
Warning: Changing variable type
```

Appendix B

Version History

1995 First version of **GCLC** (© 1995-2022 Predrag Janičić);

1996 First public release of **GCLC** (**GCLC** v1.0) ;

1998 **GCLC** v2.0 : Cartesian commands (including conics) added. Full vertical compatibility.

2000 **GCLC** v2.1 : Several bugs fixed.

2003 **GCLC** v3.0 : Commands

`dim`,

`area`,

`color`,

`drawline`,

`getx`,

`gety`,

`fontsize`,

`animation_frames`,

`trace`

added. Full vertical compatibility (for input format). New export format and new LaTeX style is used (`gclc.sty`). Since this version, commands `cmark` do not have to precede other drawing commands.

WinGCLC— first release (graphical interface: © 2003-2009 Ivan Trajković, Predrag Janičić);

2004 **GCLC** v3.1: Several small bugs fixed.

WinGCLC 2004: export now supports exporting an animation to a sequence of bitmaps, which can then be used for generating animated gif (by some other tool) or the animation in some other format. Import from JavaView `.jvx` format is integrated in **WinGCLC**.

2005 GCLC v4.0: Commands

`expression`,

`while`,

`random`,

`ang_draw_parametric_curve`,

`ang_drawsystem1`,

`ang_drawsystem_a`,

`ang_drawsystem0_a`,

`ang_drawsystem1_a`

added. Full vertical compatibility. A bug in drawing negative arcs fixed.

WinGCLC 2005: no changes in the Windows interface.

2006 GCLC v5.0: Theorem prover tightly built-in. Full vertical compatibility.

The commands

`foot`,

`online`,

`angle_o`,

`drawdashline`

`ang_drawsystem_p`,

`ang_scale`,

`prove`,

`prooflevel`,

`prooflimit`,

added. If not defined, `area` (a visible part of the picture) is, by default, the whole of the picture (which is relevant for `drawline` and `drawdashline` as they are applied only if there is defined `area` — now always). There is support for colors for LaTeX files, i.e., the command `color` is relevant for exporting to LaTeX too (not only for exporting to bitmap). The new manual and the help file.

WinGCLC 2006: The behavior of changing line thickness (by the commands `linethickness`, `normal`, `double`) is fixed and improved for **WinGCLC** and for export to bitmap. New application icon and new file type icon.

2006 GCLC v6.0: Full vertical compatibility. Added options for export to EPS (Encapsulated PostScript) and SVG. Full support for XML (both for construction descriptions and proofs). Added options for deductive verification of constructions.

The following commands were added:

`procedure`

`call`

(providing support for user-defined procedures),

`ang3d_picture`,

`ang3d_origin`,

`ang3d_unit`,

`ang3d_scale`,

`ang3d_point`,

`ang3d_axes_drawing_range`,

`ang3d_drawline_p`,

`ang3d_drawsystem_p`,

`ang3d_draw_parametric_surface`,

`ang3d_draw_parametric_curve`,

(providing support for 3D Cartesian system) and

`drawarc_p`

`dradashwarc_p`

`set_equal`

`printvalueat_rb <p_id> <id>`

`background`

`ang_plot_data`.

For the commands `ang_intersec2`, `bis`, `intersec`, `intersec2`, `med`, `perp`, `sim`, full names (`ang_intersection2`, `bisector`, `intersection`, `intersection2`, `mediatrice`, `perpendicular`, `symmetrical`) can also be used.

Using the theorem prover is simpler. Instead of stating conjectures in terms of geometric quantities, one can also use the following basic sorts of conjectures: `identical A B`, `collinear A B C`, `perpendicular A B C D`, `parallel A B C D`, `midpoint O A B`, `same_length A B C D`, `harmonic A B C D`.

The sequence of commands in while-blocks now shares both the defined variables and the environment with the outer context (In previous versions, the environment, defined by commands `ang_picture` and `ang_origin` etc) was reset in while-blocks).

In **WinGCLC**, there are new icons for easier use of the theorem prover.

In **WinGCLC** viewer and bitmap images (as well as in newly supported formats, EPS and SVG) circles and arcs are not represented by small segments, but by proper circles and arcs.

WinGCLC 2006.1: New icons for easier using of the theorem prover.

When exporting a particular frame from an animation, traces are also saved to output files.

2007 **GCLC** 7.0: Several small fixes were made and the following commands were added:

`drawtree`

`if_then_else`

WinGCLC 2007: no changes in the Windows interface.

Support for hyperbolic geometry (through Poincaré disc model) is given via a sample file, as illustration of the mechanism of user-defined procedures (`sample21_hyp.gcl`).

2007.1 GCLC 7.1: Several small fixes were made.

Support for arrays (and the command `array`) added.

Support for export to another LaTeX formats — PSTricks and Ti*k*Z.

Support for including other files (and the command `include`) added.

Arguments for procedure calls can now also be constants.

If a tree node should not be labelled, its name should start with the symbol `_` (in the previous version, its name had to be just the symbol `_`). All tree nodes introduced by `drawtree` can be used as points.

If there is no defined area for `ang_picture` (or `ang3d_picture`), the default area is empty.

File exported to vector formats (LaTeX, SVG, XML) contain comments — explanations for portions of code for easier understanding and modifying.

In the command line version, assumed file names are supported.

The batch processing option added.

The following commands were added:

`filltriangle`

`fillrectangle`

`fillcircle`

`fillellipse`

`fillarc`

`fillellipsearc`

`dashstyle`

`drawarrow`

`arrowstyle`

`drawbezier3`

`drawdashbezier3`

`drawbezier4`

`drawdashbezier4`

`bezierprecision`

`drawellipsearc1`

`drawdashellipsearc1`

`drawellipsearc2`

`drawdashellipsearc2`

`rotateonellipse`

`onsegment`

`oncircle`

`mcp`

WinGCLC 2007.1: no changes in the Windows interface.

2008 GCLC 8.0:

New theorem provers added: one based on the Wu's method and one based on the Gröbner bases method.

New form of `intersection` command (intersection of lines determined by the given four points).

The following commands were added:

`prover_timeout`

`theorem_name`

`layer`

`hide_layer`

`hide_layers_from`

`hide_layers_to`

`ang_getx`

`ang_gety`

`ang3d_getx`

`ang3d_gety`

`ang3d_getz`

`fillellipsearc0`

`fillarc0`

Several bugs fixed.

WinGCLC 2008: New buttons added for additional theorem provers.

2008 GCLC 8.1:

The following commands were added:

`drawgraph_a`

`drawgraph_b`

2009 GCLC 9.0:

Several small fixes were made.

The following commands were added:

`getcenter`

`export_to_latex`

`export_to_simple_latex`

`export_to_pstricks`

`export_to_tikz`

`export_to_eps`

`export_to_svg`

New version of support for hyperbolic geometry (through Poincaré disc model) is given via a new version of the sample file `sample21_hyp.gcl` (co-authored with Zoran Lučić).

WinGCLC 2009: no changes in the Windows interface.

2015 Completely new versions of graphical user interfaces for Windows and Linux. Only minor bug-fixes in the engine, vertical compatibility kept.

2020 Code revision, speed-ups, minor bugs fixed, minor changes in output in the PSTricks and SVG formats, and in the output of algebraic based automated theore provers. The code turns open-source.

2022 Minor changes and fixes.

Appendix C

Additional Modules

There are several additional modules that going with the basic **GCLC** program:

- **WinGCLC/gclc-gui**: **GCLC** with graphical user interface.
- Simple previewer VIEW for **GCLC** output files (there is no need to "latex" them) (© 1996-2003 Predrag Janičić). Supports both output formats (old, based on `emlines.sty` and new, based on `gclc.sty`). A picture `out.pic` generated by **GCLC** can be seen by:

  ```
  > view out.pic
  ```

  Program VIEW has the following commands (they are similar to commands in TEX viewers):

  - → right ;
  - ← left ;
  - ↑ up ;
  - ↓ down ;
  - + zoom-in ;
  - − zoom-out ;
  - `C` increase step ;
  - `F` decrease step ;
  - `Q` quit.

- JavaView to **GCLC** converter JV2GCL (© 2002 Predrag Janičić): converts files from JavaView to **GCLC** format.[1]

[1] This program was developed in collaboration with prof. Konrad Polthier and Klaus Hildebrandt from Mathematical Institute, Technical University, Berlin. This work was partially supported by DAAD grant form my visit to Technical University, Berlin (2003).

Appendix D

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I am grateful to:

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- Prof. Pedro Quaresma (University of Coimbra), Jelena Tomašević, and Milena Vujošević-Janičić, coauthors of the XML support for **GCLC** (2006);
- Prof. Bruno Buchberger (RISC, University of Linz, Austria), for kindly inviting me to visit RISC and to present **GCLC** there (2006).
- EMIS (The European Mathematical Information Service) for mirroring **GCLC** web page at http://www.emis.de/misc/software/gclc/ and other EMIS locations.
- James Fry (New Albany, Indiana, USA) for careful revision of the **GCLC**/-**WinGCLC** manual and help file and for many useful insights and comments (2005);

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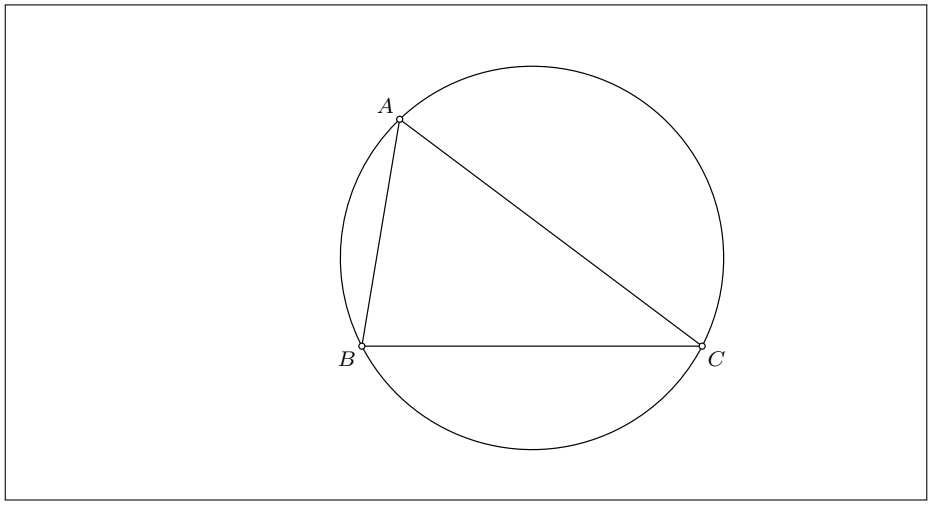
- All **GCLC** users for their support, feedback and suggestions.

Appendix E

Examples

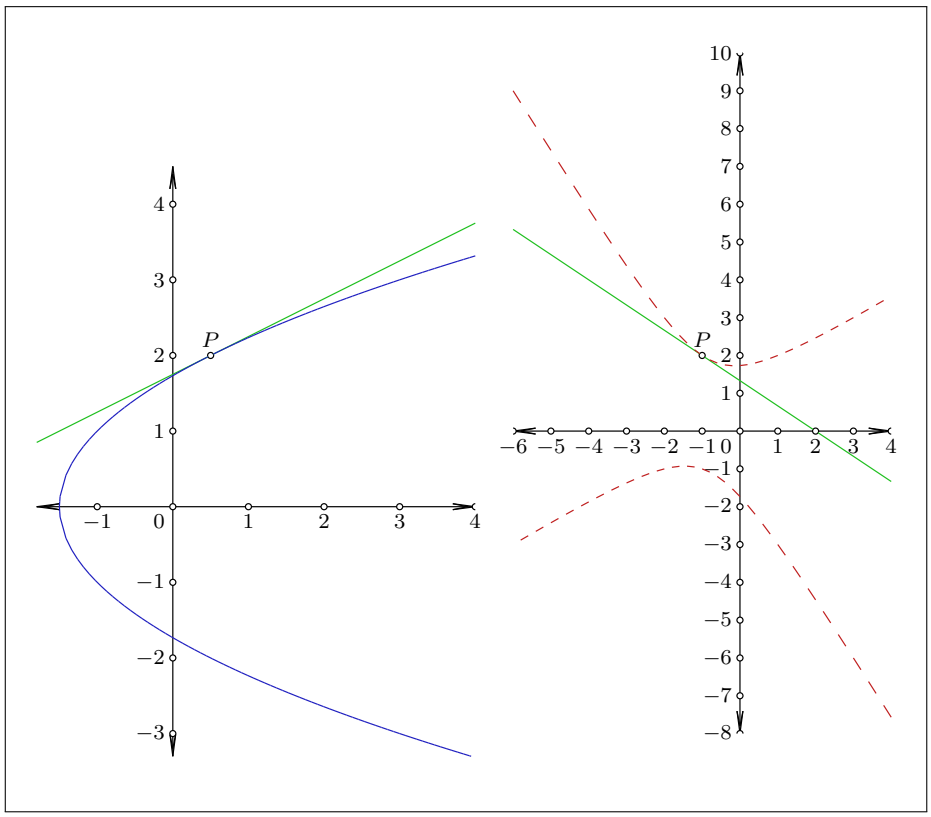
E.1 Example (Simple Triangle)

```
point A 50 45
point B 45 15
point C 90 15

cmark_lt A
cmark_lb B
cmark_rb C

drawsegment A B
drawsegment B C
drawsegment C A

med a C B
med b A C
intersec O a b
drawcircle O A
```



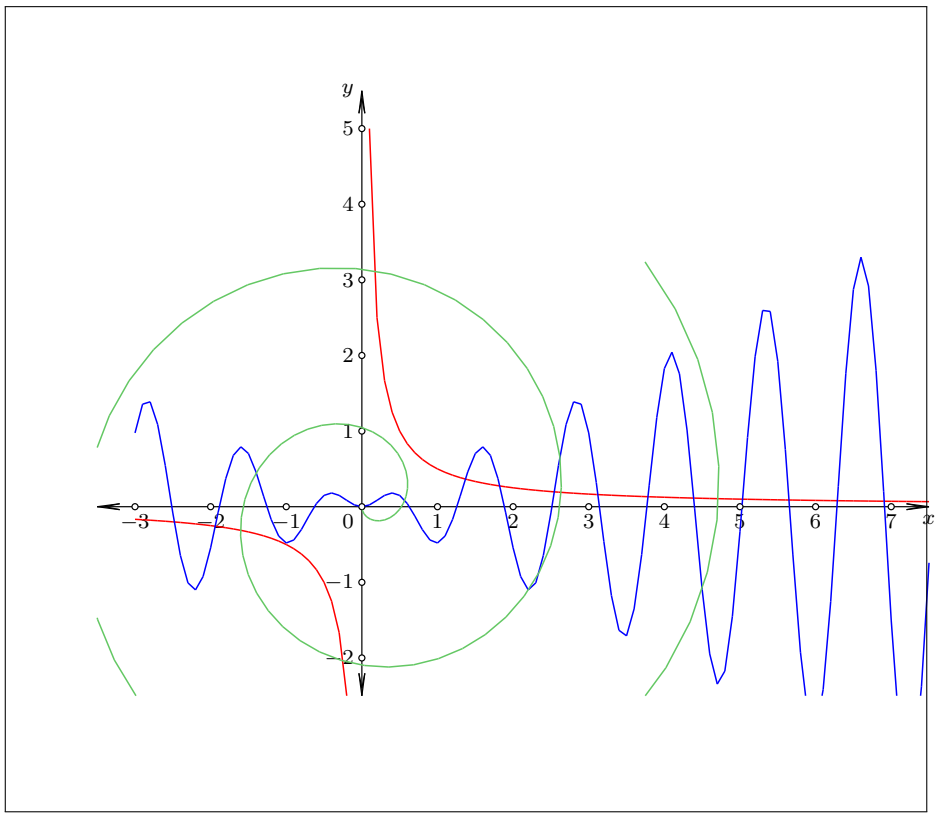
E.2 Example (Conics)

```
ang_picture 0 0 80 80
ang_origin 20.0 35.0
ang_drawsystem

ang_conic h 0 0 1 -1 0 -3

ang_conicprecision 75

ang_point A1 2 2
ang_point A2 3 2
line l A1 A2
ang_intersec2 P P2 h l

cmark_t P
ang_tangent p P h
color 32 192 32
ang_drawline p
color 32 32 192
ang_drawconic h
color 0 0 0

% ------------------------------------------------

ang_unit 5
ang_picture 85 5 140 100
ang_origin 115.0 45.0
ang_drawsystem

ang_conic h -1 0.5 1 -1 0 -3

ang_conicprecision 50

ang_point A1 2 2
ang_point A2 3 2
line l A1 A2
ang_intersec2 P P2 h l

cmark_t P
ang_tangent p P h
color 32 192 32
ang_drawline p
color 192 32 32
ang_drawdashconic h
```


P
P

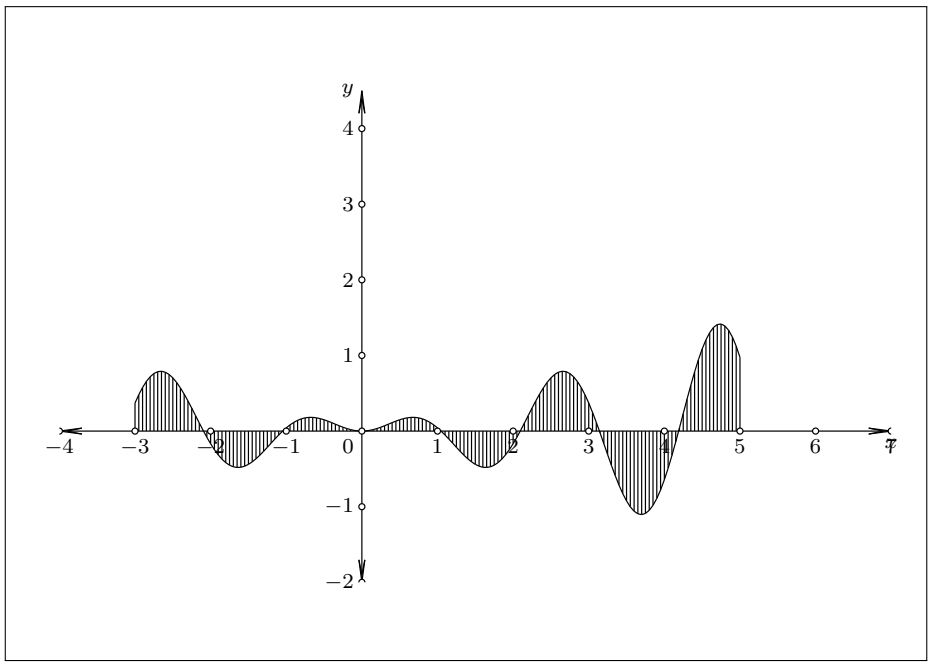
E.3 Example (Parametric Curves)

```
ang_picture 5 5 112 92
ang_origin 45 35
ang_drawsystem_a

point X -0.5 0 0.5 0
point O 0 0
distance d X O

color 255 0 0
ang_draw_parametric_curve x {-3;x<8;x+0.1}{x;d/x}

color 0 0 255
ang_draw_parametric_curve x {-3;x<8;x+0.1}{x;d*x*sin(10*d*x)}

color 100 200 100
ang_draw_parametric_curve t {0;t<30;t+0.3}{sin(d*t)*t/6;-cos(d*t)*t/6}

animation_frames 50 5
```


y
x
5
4
3
2
1
0
−1
−2
−3
−2
−1
1
2
3
4
5
6
7

E.4 Example (While-loop)

```
dim 120 80

ang_picture 5 5 115 70
ang_origin 45 25
ang_drawsystem_a

ang_draw_parametric_curve x {-3;x<5;x+0.05}{x;0.3*x*sin(3*x)}

% while loop in conjunction with parametric curves
number x -3
while { x<5 }
{
    ang_picture 10 10 100 70
    ang_origin 45 25

    expression y { 0.3*x*sin(3*x) }
    ang_point A x 0
    ang_point B x y
    drawsegment A B

    expression x { x+0.05}
}
```


y
4
3
2
1
−1
−2
−4
−3
−2
−1
0
1
2
3
4
5
6
7

E.5 Example (Ceva's theorem)

```
point A 30 10
point B 80 10
point C 60 90
point P 55 55

line a B C
line b A C
line c A B

line pa P A
line pb P B
line pc P C

intersec D a pa
intersec E b pb
intersec F c pc

drawsegment A B
drawsegment A C
drawsegment B C

drawsegment A D
drawsegment B E
drawsegment C F

cmark_b A
cmark_b B
cmark_t C
cmark_rt D
cmark_b F
cmark_lt E
cmark_r P

prove { equal { mult { mult { sratio A F F B }
                           { sratio B D D C } }
                     { sratio C E E A } }
              1 }
```

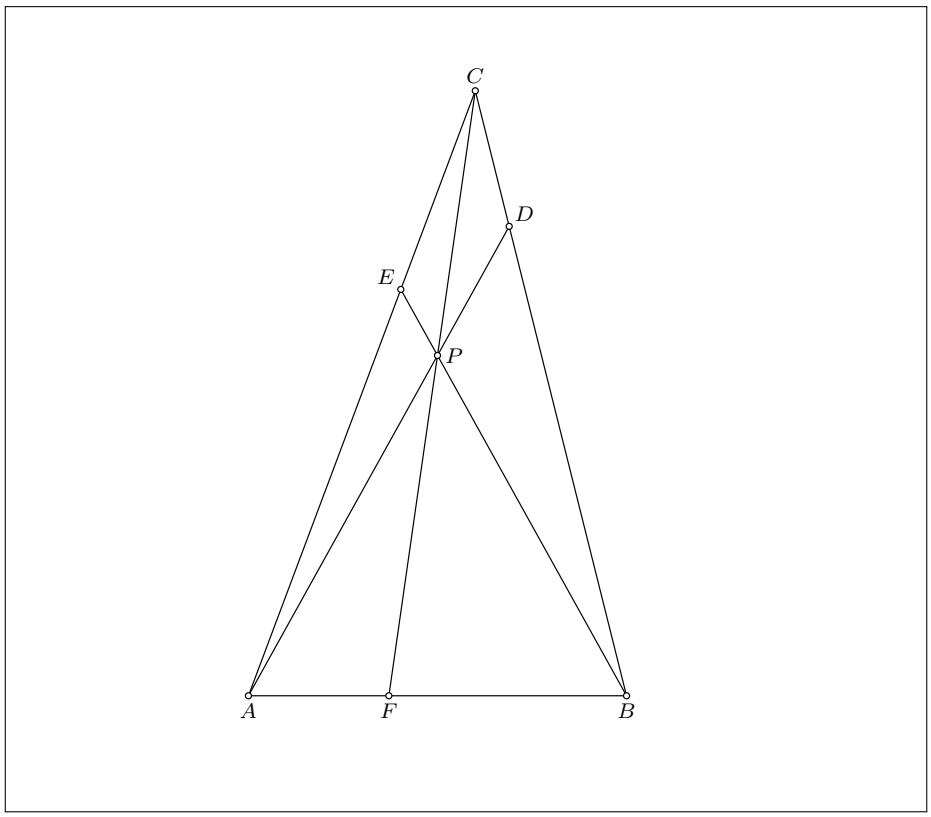


Figure E.1: Output for Example E

LaTeX output of the prover based on the area method:

$$\left(\left(\frac{\overrightarrow{AF}}{\overrightarrow{FB}}\cdot\frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right)\cdot\frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right) = 1 \quad \text{by the statement (value 1=1)} \quad (0)$$

$$\left(\left(\left(-1\cdot\frac{\overrightarrow{AF}}{\overrightarrow{BF}}\right)\cdot\frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right)\cdot\frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right) = 1 \quad \text{by geometric simplifications (value 1=1)} \quad (1)$$

$$\left(-1\cdot\left(\frac{\overrightarrow{AF}}{\overrightarrow{BF}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{DC}}\cdot\frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right)\right)\right) = 1 \quad \text{by algebraic simplifications (value 1=1)} \quad (2)$$

$$\left(-1\cdot\left(\frac{S_{APC}}{S_{BPC}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{DC}}\cdot\frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right)\right)\right) = 1 \quad \text{by Lemma 8 (point } F \text{ eliminated) (value 1=1)} \quad (3)$$

$$\left(-1\cdot\left(\frac{S_{APC}}{S_{BPC}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{DC}}\cdot\left(-1\cdot\frac{\overrightarrow{CE}}{\overrightarrow{AE}}\right)\right)\right)\right) = 1 \quad \text{by geometric simplifications (value 1=1)} \quad (4)$$

$\frac{\left(S_{APC} \cdot \left(\frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AE}}\right)\right)}{S_{BPC}}$	=	1	by algebraic simplifications (value 1=1)	(5)
$\frac{\left(S_{APC} \cdot \left(\frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{S_{CPB}}{S_{APB}}\right)\right)}{S_{BPC}}$	=	1	by Lemma 8 (point E eliminated) (value 1=1)	(6)
$\frac{\left(S_{APC} \cdot \left(\left(-1 \cdot \frac{\overrightarrow{BD}}{\overrightarrow{CD}}\right) \cdot \frac{S_{CPB}}{S_{APB}}\right)\right)}{(-1 \cdot S_{CPB})}$	=	1	by geometric simplifications (value 1=1)	(7)
$\frac{\left(S_{APC} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{CD}}\right)}{S_{APB}}$	=	1	by algebraic simplifications (value 1=1)	(8)
$\frac{\left(S_{APC} \cdot \frac{S_{BPA}}{S_{CPA}}\right)}{S_{APB}}$	=	1	by Lemma 8 (point D eliminated) (value 1=1)	(9)
$\frac{\left(S_{APC} \cdot \frac{S_{BPA}}{(-1 \cdot S_{APC})}\right)}{(-1 \cdot S_{BPA})}$	=	1	by geometric simplifications (value 1=1)	(10)
1	=	1	by algebraic simplifications (value 1=1)	(11)

Q.E.D.

NDG conditions are:

$S_{BPA} \neq S_{CPA}$ i.e., lines BC and PA are not parallel (construction based assumption)

$S_{APB} \neq S_{CPB}$ i.e., lines AC and PB are not parallel (construction based assumption)

$S_{APC} \neq S_{BPC}$ i.e., lines AB and PC are not parallel (construction based assumption)

$P_{FBF} \neq 0$ i.e., points F and B are not identical (conjecture based assumption)

$P_{DCD} \neq 0$ i.e., points D and C are not identical (conjecture based assumption)

$P_{EAE} \neq 0$ i.e., points E and A are not identical (conjecture based assumption)

Number of elimination proof steps: 3

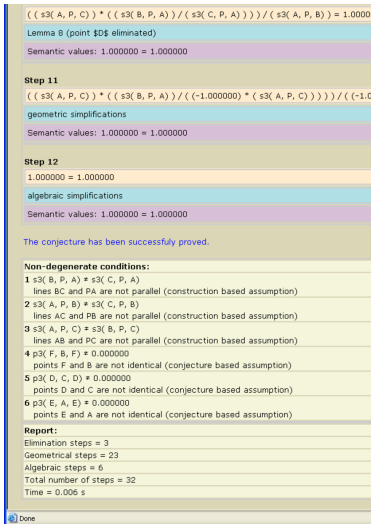
Number of geometric proof steps: 6

Number of algebraic proof steps: 23

Total number of proof steps: 32

Time spent by the prover: 0.002 seconds

A fragment of the XML output of the prover based on the area method:



A fragment of the LaTeX output of the prover based on the Wu's method:

Creating polynomials from hypotheses

- Point A
  no condition
- Point B
  no condition
- Point C
  no condition
- Point P
  no condition
- Line a: B C
  - point B is on the line (B, C)
    no condition
  - point C is on the line (B, C)
    no condition
- Line b: A C
  - point A is on the line (A, C)
    no condition

  - point C is on the line (A, C)
    no condition

- Line c: $A\ B$
  - point A is on the line (A, B)
    no condition
  - point B is on the line (A, B)
    no condition

- Line pa: $P\ A$
  - point P is on the line (P, A)
    no condition
  - point A is on the line (P, A)
    no condition

- Line pb: $P\ B$
  - point P is on the line (P, B)
    no condition
  - point B is on the line (P, B)
    no condition

- Line pc: $P\ C$
  - point P is on the line (P, C)
    no condition
  - point C is on the line (P, C)
    no condition

- Intersection of lines, D: $a\ pa$
  - point D is on the line (B, C)
    $$p_{33} = -u_3 x_2 + (u_2 - u_1)x_1 + u_3 u_1$$
  - point D is on the line (P, A)
    $$p_{34} = u_5 x_2 - u_4 x_1$$

- Intersection of lines, E: $b\ pb$
  - point E is on the line (A, C)
    $$p_{35} = -u_3 x_4 + u_2 x_3$$
  - point E is on the line (P, B)
    $$p_{36} = u_5 x_4 + (-u_4 + u_1)x_3 - u_5 u_1$$

- Intersection of lines, F: $c\ pc$
  - point F is on the line (A, B) — true by the construction
    no condition
  - point F is on the line (P, C)
    $$p_{37} = (u_5 - u_3)x_6 + (-u_5 u_2 + u_4 u_3)$$

Creating polynomial from the conjecture

- Processing given conjecture(s).

Conjecture 1:

$$p_{38} = -2x_6x_3x_1 + u_3x_6x_3 + u_3x_6x_1 + u_1x_3x_1 - u_3u_1x_3$$

Invoking the theorem prover

The used proving method is Wu's method.
The input system is:

$$\begin{aligned} p_0 &= -u_3x_2 + (u_2 - u_1)x_1 + u_3u_1 \\ p_1 &= u_5x_2 - u_4x_1 \\ p_2 &= -u_3x_4 + u_2x_3 \\ p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\ p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3) \end{aligned}$$

Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable is 2.

Minimal degrees: 3 polynomials with degree 1 and 2 polynomials with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_0 &= -u_3x_2 + (u_2 - u_1)x_1 + u_3u_1 \\ p_1 &= u_5x_2 - u_4x_1 \\ p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\ p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\ p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3) \end{aligned}$$

Triangulation, step 3

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

Triangulation, step 4

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable is 2.

Minimal degrees: 1 polynomials with degree 1 and 0 polynomials with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1.

Finished a triangulation step, the current system is:

$$\begin{aligned}
p_0 &= (u_5u_2 - u_5u_1 - u_4u_3)x_1 + u_5u_3u_1 \\
p_1 &= u_5x_2 - u_4x_1 \\
p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\
p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\
p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3)
\end{aligned}$$

Triangulation, step 5

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_0 &= (u_5u_2 - u_5u_1 - u_4u_3)x_1 + u_5u_3u_1 \\
p_1 &= u_5x_2 - u_4x_1 \\
p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\
p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\
p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3)
\end{aligned}$$

Final remainder

Final remainder for conjecture 1

Calculating final remainder of the conclusion:

$$g = -2x_6x_3x_1 + u_3x_6x_3 + u_3x_6x_1 + u_1x_3x_1 - u_3u_1x_3$$

with respect to the triangular system.

1. Pseudo remainder with p_4 over variable x_6:

$$g = (-2u_5u_2 + u_5u_1 + 2u_4u_3 - u_3u_1)x_3x_1 +$$
$$(u_5u_3u_2 - u_5u_3u_1 - u_4u_3^2 + u_3^2u_1)x_3 + (u_5u_3u_2 - u_4u_3^2)x_1$$

2. Pseudo remainder with p_3 over variable x_4:

$$g = (-2u_5u_2 + u_5u_1 + 2u_4u_3 - u_3u_1)x_3x_1 +$$
$$(u_5u_3u_2 - u_5u_3u_1 - u_4u_3^2 + u_3^2u_1)x_3 + (u_5u_3u_2 - u_4u_3^2)x_1$$

3. Pseudo remainder with p_2 over variable x_3:

$$g = (u_5^2u_3u_2^2 - 2u_5^2u_3u_2u_1 + u_5^2u_3u_1^2 - 2u_5u_4u_3^2u_2 +$$
$$2u_5u_4u_3^2u_1 + u_5u_3^2u_2u_1 - u_5u_3^2u_1^2 + u_4^2u_3^3 - u_4u_3^3u_1)x_1 +$$
$$(u_5^2u_3^2u_2u_1 - u_5^2u_3^2u_1^2 - u_5u_4u_3^3u_1 + u_5u_3^3u_1^2)$$

4. Pseudo remainder with p_1 over variable x_2:

$$g = (u_5^2u_3u_2^2 - 2u_5^2u_3u_2u_1 + u_5^2u_3u_1^2 - 2u_5u_4u_3^2u_2 +$$
$$2u_5u_4u_3^2u_1 + u_5u_3^2u_2u_1 - u_5u_3^2u_1^2 + u_4^2u_3^3 - u_4u_3^3u_1)x_1 +$$
$$(u_5^2u_3^2u_2u_1 - u_5^2u_3^2u_1^2 - u_5u_4u_3^3u_1 + u_5u_3^3u_1^2)$$

5. Pseudo remainder with p_0 over variable x_1:

$$g = 0$$

Prover report

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.053 seconds.

NDG conditions are:

- $P_{FBF} \neq 0$ i.e., points F and B are not identical (conjecture based assumption).
- $P_{DCD} \neq 0$ i.e., points D and C are not identical (conjecture based assumption).
- $P_{EAE} \neq 0$ i.e., points E and A are not identical (conjecture based assumption).

Bibliography

[1] B. Buchberger. *An Algorithm for finding a basis for the residue class ring of a zero-dimensional polynomial ideal.* PhD thesis, Math. Inst. University of Innsbruck, Austria, 1965.

[2] B. Buchberger and F. Winkler, editors. *Gröbner Bases and Applications.* Cambridge University Press, 1998.

[3] S.-C. Chou. *Mechanical Geometry Theorem Proving.* D.Reidel Publishing Company, Dordrecht, 1988.

[4] S.-C. Chou, X.-S. Gao, and J.-Z. Zhang. Automated production of traditional proofs for constructive geometry theorems. In M. Vardi, editor, *Proceedings of the Eighth Annual IEEE Symposium on Logic in Computer Science LICS*, pages 48–56. IEEE Computer Society Press, June 1993.

[5] M. Djorić and P. Janičić. Constructions, instructions, interactions . *Teaching Mathematics and its Applications*, 23(2):69–88, 2004.

[6] P. Janičić. GCLC – A Tool for Constructive Euclidean Geometry and More than That. In N. Takayama, A. Iglesias, and J. Gutierrez, editors, *Proceedings of International Congress of Mathematical Software (ICMS 2006)*, volume 4151 of *Lecture Notes in Computer Science*, pages 58–73. Springer-Verlag, 2006.

[7] P. Janičić and P. Quaresma. System description: Gclcprover + GeoThms. In U. Furbach and N. Shankar, editors, *International Joint Conference on Automated Reasoning (IJCAR-2006)*, volume 4130 of *Lecture Notes in Artificial Intelligence*, pages 145–150. Springer-Verlag, 2006.

[8] P. Janičić. Geometry Constructions Language. *Journal of Automated Reasoning*, 44(1-2):3–24, 2010.

[9] P. Janičić and I. Trajković. WinGCLC — a Workbench for Formally Describing Figures. In *Proceedings of the 19 mth Spring Conference on Computer Graphics (SCCG 2003)*, pages 251–256, Budmerice, Slovakia, April, 24-26 2003. ACM Press, New York, USA.

[10] P. Quaresma and P. Janičić. Integrating dynamic geometry software, deduction systems, and theorem repositories. In J. Borwein and W. Farmer, editors, *Mathematical Knowledge Management (MKM-2006)*, volume 4108 of *Lecture Notes in Artificial Intelligence*, pages 280–294. Springer-Verlag, 2006.

[11] P. Quaresma and P. Janičić. Framework for the Constructive Geometry. Technical Report TR2006/001, Center for Informatics and Systems of the University of Coimbra, 2006.

[12] W.-T. Wu. On the decision problem and the mechanization of theorem proving in elementary geometry. *Scientia Sinica*, 21:157–179, 1978.

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