Problem 42

The nth term of the sequence of triangle numbers is given by, tn = ½n(n+1); so the first ten triangle numbers are:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is 19 + 11 + 25 = 55 = t10. If the word value is a triangle number then we shall call the word a triangle word.

Using words.txt (right click and 'Save Link/Target As...'), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

Solution

A number k is triangular if $n^2 + n - 2\,k == 0$ has integer solutions - that is, if $\frac{-1+\sqrt{1+8\,k}}{2}$ is integer. Certainly $8\,k + 1$ is odd, so its square root is, and hence if the square root is integer then the entire solution is integer.

In[36]:= **triangleQ[k_] := IntegerQ[Sqrt[1 + 8 k]]**

In[37]:= **wordValue[str_] := Total[ToCharacterCode[ToUpperCase@str] - 64]**

I evaluated:
```
strings =
  "{" <> Import["https://projecteuler.net/project/words.txt"] <> "}" //
    ToExpression;
```
but for security/futureproofing, I use a Compress-ed version here, at the bottom of the page.

In[45]:= **Count[strings, _String ? (triangleQ@wordValue[#] &)]**

Out[45]= 162

```
strings = Uncompress[
  "1:eJxdXMmW4zhwHLu7a+2ef/CnUCIkoQUSHCxSaY4++D2ffLC/3ifnEgFqfClFYiPWRG6of/v3/
    yr/8b8vf/zx39//+OOP9J///T//1XwRM9VX/HmKK7VG/G06h/jCQeyO6BRSsYT2G+
    uE4p95CetQ3JY/THJZ4rC9GHMPWBqwVRY5xDmur7O7kZZvWx6DKHNezf0aoLuUcX2KQz3/
    ueNE2rK/SHgr91WMJ+
    GDJ8sFvBtmHFvM6oLT2RqjDRnqfEtMFyrCsiXn2xHmO1sjHEyEVXp0sOsY/DS9xjbWVyUr/
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```

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