CSCI/MATH 3180, Lab Assignment #3

NOTE: <u>LABS MUST BE INDEPENDENT WORK.</u>

Name:______________________________

Part I. Use Maple to calculate the values of the analytical solutions for the following ODE's. You do NOT need to solve the differential equations; solutions are given here.

1) $x' = \frac{x}{1+t}$ with $x(0)= 1$ Find $x(2)$. (Analytical solution: $x = t + 1$) =**<u>3</u>**

2) $x' = t + x$ with $x(0) = 1$. Find $x(2)$. (Analytical solution: $x = -t + 2e^t - 1$) =**<u>11.77811220</u>**

3) $x' = t - x$ with $x(2) = 4$. Find $x(4)$. (Analytical solution: $x = t + 3e^{2-t} - 1$) =**<u>3.406005850</u>**

4) $x' = \frac{t-x}{t+x}$ with $x(2)= 1$ Find $x(4)$. (Analytical solution: $x = \sqrt{2t^2 + 1} - t$) =**<u>1.744562647</u>**

Part II. 1. Given $x' = \frac{t-x}{t+x}$ with $x(2) = 1$, find $x(4)$ using the Euler's method with h = 1.

Note: x is a function of t and x' is a function of t and x. In other words, $x'(t) = f(t, x(t))$.

Show all work. (No calculators!!!)

Compute the approximate value of $x(3)$.

*X(t+h)=x(t)+h*x'(t)*

$X(2+1)=x(2)+h*(\frac{t-x(t)}{t+x(t)})$

$= 1+ 1* (\frac{2-x(2)}{2+x(2)})$

$= 1+ 1* (\frac{2-1}{2+1})$

$= 1+ 1* (\frac{1}{3})$

$= \frac{4}{3}$

x(3) $\boxed{4/3}$ ≈ (in reduced fraction)

Compute the approximate value of $x(4)$.

$X(3+1)=x(4)=x(3)+h*(\frac{t-x(t)}{t+x(t)})$

$= \frac{4}{3} + 1 * (\frac{3-x(2)}{3+x(2)})$

$= \frac{4}{3} + 1 * (\frac{3-\frac{4}{3}}{3+\frac{4}{3}})$

$= \frac{4}{3} + 1 * (\frac{9-4}{9+4})$

$= \frac{4}{3} + \frac{5}{13}$

$=\frac{67}{39}$

x(4) ≈ $\boxed{\frac{67}{39}}$ (in reduced fraction)

x(4) ≈ $\boxed{1.717948718}$ (in decimal)

2. Given *x*' = *t* − *x* with *x*(2) = 4, find the approximate value of *x*(4) using the Heun's method with *h* = 1.
Note: *x* is a function of *t* and *x*' is a function of *t* and *x*. In other words, *x*'(*t*) = *f*(*t*, *x*(*t*)).

The Heun's method is based on `x(t+h) = x(t) + (h/2)*[f(t, x(t)) + f(t+h, x(t+h))]`, where *x*(*t*+*h*) on the right- hand side is computed **first** using the **Euler's method**.

The Heun's Method is a two-step process and can be written as

```
Step 1. K = x(t) + h*f(t, x(t))
Step 2. x(t+h) = x(t) + ½*h*[f(t, x(t)) + f(t+h, K)]
```

a) Compute the approximate value of *x*(3).
Show work for Step 1.

```
k= x(t) + h*f(t, x(t))
k=x(2) + h * (t-x(t))
k = 4 + 1*(2-4)
k  = 4 – 2 =2
```

Show work for Step 2.

```
x(t+h) = x(t) + ½*h*[f(t, x(t)) + f(t+h, K)]
```

X(2+1)=x(3)= x(2) *+1/2*1*[* t-x(t) + ((t+h)- K]

=4+ 0.5[(2-4)+(2+1)-2)*

=7/2

$x(3) \approx$ $\boxed{7/2}$ (in fraction)

b) Compute the approximate value of $x(4)$.

Show work for Step 1.

k=x(4)=x(3+1)= x(t)+h(t-x(t)

= 7/2 +1*(3-7/2)

=3

Show work for Step 2.

X(t+h)=x(t)+1/2(t-x(t) + (t+h)-k)

= 7/2 +0.5(3-7/2 + (3+1)-3)

=15/4

$x(4) \approx$ $\boxed{15/4}$ (in reduced fraction) $x(4) \approx$ $\boxed{3.75}$ (in decimal)

c) What is the analytical solution for $x(4)$? Use Maple. $\boxed{3.406005850}$

evalf((4 + 3*exp(2 - 4)) - 1);

3.406005850

What is the absolute error in the approximation in b)? $\boxed{0.41}$

Show work!

Absolute error=|analytical solution – Euler method x(4)|

=|3.41-3|=0.41

What is the relative error in the approximation in b)? $\boxed{0.12}$

Show work!

Relative error=absolute error/analytic solution

=0.41/3.41=0.12

d) Write a C++ function that implements the Heun's Method and test it to verify your answers.

1) Test your program using h = 1.

What does your program output as the approximation for $x(4)$? 3.75

What does your program output as the absolute error? 0.406006

What does your program output as the relative error? 0.119203

2) Test your program the second time using h = 0.1.

What does your program output as the approximation for $x(4)$? 2.1525

What does your program output as the absolute error? 1.26101

What does your program output as the relative error? 0.37023

Part III.

1. Create a C++ program and name it YourLastName3.cpp.
2. Write a driver that tests the three methods to find numerical solutions to the ODE's given in Part I.
3. Write a separate function for each of the three methods: Euler's method, Heun's Method, Runge-Kutta's Method
4. Write a separate function for each of the ODE's (**four** functions in total) given in Par I, 1), 2), 3), and 4).
5. All floating point arithmetic will be double precision.
6. Input to the main program: See the **sample run** on the next page for details.

   Choice of an ODE and its analytical solution

   Interval, $[a, b] = [t_0, t_N]$

   Initial value, $x(\mathrm{a}) = x(t_0)$

   Integer N, the number of sub intervals

Note: *The analytical solution can be hardcoded in the program*.

7. Program output:

1) A table containing **10** rows with the following values (**first nine and the last**).

General format

t_0	$x(t_0)$
t_1	$x(t_1)$
t_2	$x(t_2)$
t_3	$x(t_3)$
t_4	$x(t_4)$
t_5	$x(t_5)$
t_6	$x(t_6)$
t_7	$x(t_7)$
t_8	$x(t_8)$
t_9	$x(t_9)$
$\boldsymbol{t}$**N**	$\boldsymbol{x(t_N)}$

N =10

t_0	$x(t_0)$
t_1	$x(t_1)$
t_2	$x(t_2)$
t_3	$x(t_3)$
t_4	$x(t_4)$
t_5	$x(t_5)$
t_6	$x(t_6)$
t_7	$x(t_7)$
t_8	$x(t_8)$
t_9	$x(t_9)$
$\boldsymbol{T}$**10**	$\boldsymbol{x(t_{10})}$

N = 100

t_0	$x(t_0)$
t_1	$x(t_1)$
t_2	$x(t_2)$
t_3	$x(t_3)$
t_4	$x(t_4)$
t_5	$x(t_5)$
t_6	$x(t_6)$
t_7	$x(t_7)$
t_8	$x(t_8)$
t_9	$x(t_9)$
$\boldsymbol{T}$**100**	$\boldsymbol{x(t_{100})}$

2) **The relative error with $\boldsymbol{x(t_N)}$**

8. Test the three methods on the four ODE's with 1) N= 10 and 2) N= 100. Analyze your output and write a short report including the following

- For each of the ODE's
  - Program input
  - Program output
- Your conclusion/findings
- Save your report as **YourLastName3_report**.

<u>Submission:</u> Submit the following in a zipped folder to the Dropbox on D2L.

YourLastName3.mw – *Maple worksheet*
YourLastName3_Part2.pdf
YourLastName3.cpp
YourLastName3_report.pdf -- *report on the experiment*

***Confirm** your submission; multiple submissions are allowed, but the last submission will be graded.*

<u>Sample Run</u>

```
Which ODE would you like to use?
1. x' = x / (1 + t)
2. x' = t + x
```

```
3. x' = t - x
4. x' = (t - x) / (t + x)

Choice: 3
```

`Enter the analytical solution: 3.406005850` **(*This can be hardcoded*.)**

```
Enter the lower bound of the interval (a): 2
Enter the upper bound of the interval (b): 4
Enter the initial value: 4
Enter the number of sub intervals: 100


Euler's Method:
2.0200000000
3.9600000000
2.0400000000
3.9212000000
2.0600000000
3.8835760000
2.0800000000
3.8471044800
2.1000000000
3.8117623904
2.1200000000
3.7775271426
2.1400000000
3.7443765997
2.1600000000
3.7122890677
2.1800000000
3.6812432864
4.0000000000
3.3978586677   Relative
Error: 0.0023920048

Heun's Method:
2.0200000000
3.9606000000
2.0400000000
3.9223761200
2.0600000000
3.8853050728
2.0800000000
3.8493640324
2.1000000000
3.8145306245
2.1200000000
3.7807829182
```

```
2.1400000000
3.7480994164
2.1600000000
3.7164590480
2.1800000000
3.6858411588
4.0000000000
3.4060608060   Relative
Error: 0.0000161350

Runge-Kutta Method:
2.0200000000
3.9605960200
2.0400000000
3.9223683176
2.0600000000
3.8852936010
2.0800000000
3.8493490395
2.1000000000
3.8145122545
2.1200000000
3.7807613106
2.1400000000
3.7480747067
2.1600000000
3.7164313675
2.1800000000
3.6858106348
4.0000000000
3.4060058508   Relative
Error: 0.0000000002
 Press any key to continue
. . .
```

EVALUATION RUBRIC

Part I		Maple worksheet	__/1
Part II		Showed all work and answers are correct.	__/10
Part III	1	Solve the assigned problem using methods described in program description. Functions for the three methods are implemented and called properly. Four separate functions are implemented and called properly. Input to the main program meets the requirements. Program output meets the requirements.	__/8

	2	Compilation/Execution Compile without errors and execute without crashing. Work for all data and produce correct answers.	__/1
	3	Main Comment Block includes the following. file name due date author course # program description input output	__/0.5
	4	Documentation, indentation, and white space usage Meaning variable names are used and they are briefly described. Each section of statements in the program is well documented. Proper INDENTATION is used to make the program easier to read. WHITE SPACES are used in appropriate places for readability.	__/0.5
	5	Contents of report • For each of the data sets ○ Program input ○ Program output • Your conclusion/findings	__/4
		Contents of zipped folder contains the Maple worksheet. Contains work and answers for Part II. contains the C++ program. contains the report.	
		Lab Demo	
		TOTAL	__/25