

Task 1.

Implement the NOR Boolean logic gate using perceptron Neural Network. Inputs = x1, x2 and bias, weights should be fed into the perceptron with single Output = y. Display final weights and bias of each perceptron.

Task 2

Take the dataset of Diabetes 2

- Initialize a neural network with random weights.
- Calculate output of Neural Network:
- 1. Calculate squared error loss
- 2. Update network parameter using batch Mini Batch gradient descent optimizer function Implementation.
- 3. Display updated weight and bias values
- 4. Plot loss w.r.t. bias values

```
In [74]: import numpy as np
import matplotlib.pyplot as plt

# Task 1: Implement NOR Boolean Logic Gate using Perceptron

print("=*60)
print("TASK 1: NOR GATE USING PERCEPTRON")
print("=*60)

# NOR Truth Table:
# x1  x2  |  y
# 0   0   |  1
# 0   1   |  0
# 1   0   |  0
# 1   1   |  0

X = np.array([
    [0, 0],
    [0, 1],
    [1, 0],
    [1, 1]
])
y = np.array([1, 0, 0, 0])

print("\nNOR Truth Table:")
print("x1  x2  |  y")
print("-"*15)
for i in range(len(X)):
    print(f"{X[i][0]} {X[i][1]} | {y[i]}")

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TASK 1: NOR GATE USING PERCEPTRON
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```

```
NOR Truth Table:
x1  x2  |  y
-----
0   0   |  1
0   1   |  0
1   0   |  0
1   1   |  0
```

```
In # Step activation function
[75]: def step(z):
         return 1 if z >= 0 else 0

# Initialize weights and bias randomly
np.random.seed(42)
w = np.random.rand(2)
b = np.random.rand(1)

print(f"\nInitial weights: {w}")
print(f"Initial bias: {b}")

# Training parameters
epochs = 100
learning_rate = 0.1

print("\nTraining NOR Perceptron...")

# Training loop
for epoch in range(epochs):
    total_error = 0
    for i in range(X.shape[0]):
        # Forward pass
        z = np.dot(w, X[i]) + b
        y_pred = step(z)

        # Calculate error
        error = y[i] - y_pred
        total_error += abs(error)

        # Update weights and bias
        w += learning_rate * error * X[i]
        b += learning_rate * error

    # Stop if converged
    if total_error == 0:
        print(f"Converged at epoch {epoch + 1}")
        break
```

```
Initial weights: [0.37454012 0.95071431]
Initial bias: [0.73199394]
```

```
Training NOR Perceptron...
Converged at epoch 10
```

```
In # Display final weights and bias
[76]: print("\n" + "*60")
print("FINAL WEIGHTS AND BIAS FOR NOR PERCEPTRON")
print("*60)
print(f"\nTrained weights: {w}")
print(f"Trained bias: {b}")

# Test the trained perceptron
print("\nTesting NOR Perceptron:")
print("x1 x2 | Predicted | Actual")
print("-"*35)
for i in range(len(X)):
    z = np.dot(w, X[i]) + b
    prediction = step(z)
    print(f"{X[i][0]} {X[i][1]} | {prediction} | {y[i]}")

# Verify all predictions are correct
correct = sum([step(np.dot(w, X[i]) + b) == y[i] for i in range(len(X))])
print(f"\nAccuracy: {correct}/{len(X)} = {(correct/len(X))*100:.0f}%")
```

```
=====
FINAL WEIGHTS AND BIAS FOR NOR PERCEPTRON
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```

```
Trained weights: [-0.22545988 -0.04928569]
```

```
Trained bias: [0.03199394]
```

```
Testing NOR Perceptron:
```

x1	x2	Predicted	Actual
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

```
In [77]: print("\n" + "*60)
print("TASK 2: DIABETES DATASET WITH MINI-BATCH GRADIENT DESCENT")
print("*60)

import pandas as pd
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import load_diabetes

# Load Diabetes dataset
diabetes = load_diabetes()
X_diabetes = diabetes.data
y_diabetes = diabetes.target.reshape(-1, 1)

# Standardize features (already somewhat standardized, but ensuring consistency)
scaler_X = StandardScaler()
X_diabetes = scaler_X.fit_transform(X_diabetes)

# Standardize target for better training
scaler_y = StandardScaler()
y_diabetes = scaler_y.fit_transform(y_diabetes)

print(f"\nDataset shape: X={X_diabetes.shape}, y={y_diabetes.shape}")
print(f"Number of features: {X_diabetes.shape[1]}")
print(f"Number of samples: {X_diabetes.shape[0]}")
print(f"Feature names: {diabetes.feature_names}")
```

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TASK 2: DIABETES DATASET WITH MINI-BATCH GRADIENT DESCENT
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Dataset shape: X=(442, 10), y=(442, 1)
Number of features: 10
Number of samples: 442
Feature names: ['age', 'sex', 'bmi', 'bp', 's1', 's2', 's3', 's4', 's5', 's6']
```

```
In [78]: # Initialize Neural Network with random weights
input_size = X_diabetes.shape[1] # 10 features
hidden_size = 8
output_size = 1

np.random.seed(42)

# Layer 1: Input -> Hidden
W1 = np.random.randn(input_size, hidden_size) * 0.01
b1 = np.zeros((1, hidden_size))

# Layer 2: Hidden -> Output
W2 = np.random.randn(hidden_size, output_size) * 0.01
b2 = np.zeros((1, output_size))

print("\nInitial Network Architecture:")
print(f"Input Layer: {input_size} neurons")
print(f"Hidden Layer: {hidden_size} neurons")
print(f"Output Layer: {output_size} neuron")
print(f"\nW1 shape: {W1.shape}")
print(f"b1 shape: {b1.shape}")
print(f"W2 shape: {W2.shape}")
print(f"b2 shape: {b2.shape}")
```

```
Initial Network Architecture:
Input Layer: 10 neurons
Hidden Layer: 8 neurons
Output Layer: 1 neuron

W1 shape: (10, 8)
b1 shape: (1, 8)
W2 shape: (8, 1)
b2 shape: (1, 1)
```

```
In [79]: # Define activation functions
def relu(z):
    return np.maximum(0, z)

def relu_derivative(z):
    return (z > 0).astype(float)

# Forward propagation
def forward_pass(X, W1, b1, W2, b2):
    Z1 = np.dot(X, W1) + b1
    A1 = relu(Z1)
    Z2 = np.dot(A1, W2) + b2
    A2 = Z2 # Linear activation for regression

    cache = {'Z1': Z1, 'A1': A1, 'Z2': Z2, 'A2': A2}
    return A2, cache

# Calculate squared error loss (MSE)
def compute_loss(y_true, y_pred):
    m = y_true.shape[0]
    loss = (1 / (2 * m)) * np.sum((y_pred - y_true) ** 2)
    return loss

# Backward propagation
def backward_pass(X, y, cache, W1, W2):
    m = X.shape[0]

    A1 = cache['A1']
    A2 = cache['A2']
    Z1 = cache['Z1']

    # Output layer gradients
    dZ2 = (A2 - y)
    dW2 = (1 / m) * np.dot(A1.T, dZ2)
    db2 = (1 / m) * np.sum(dZ2, axis=0, keepdims=True)

    # Hidden layer gradients
    dA1 = np.dot(dZ2, W2.T)
    dZ1 = dA1 * relu_derivative(Z1)
    dW1 = (1 / m) * np.dot(X.T, dZ1)
    db1 = (1 / m) * np.sum(dZ1, axis=0, keepdims=True)

    gradients = {'dW1': dW1, 'db1': db1, 'dW2': dW2, 'db2': db2}
    return gradients

print("\nForward and Backward propagation functions defined.")
```

Forward and Backward propagation functions defined.

```
In [80]: # Mini-Batch Gradient Descent Optimizer with Momentum
def mini_batch_gradient_descent(X, y, W1, b1, W2, b2, batch_size=32,
                                 learning_rate=0.01, epochs=100, momentum=0.9):
    """
        Mini-Batch Gradient Descent Optimizer with Momentum
        Updates weights using gradients computed on mini-batches of data
    """
    m = X.shape[0]
    loss_history = []

    # Initialize velocity for momentum
    vW1 = np.zeros_like(W1)
    vb1 = np.zeros_like(b1)
    vW2 = np.zeros_like(W2)
    vb2 = np.zeros_like(b2)

    print("\nTraining with Mini-Batch Gradient Descent with Momentum...")
    print(f"Batch size: {batch_size}")
    print(f"Learning rate: {learning_rate}")
    print(f"Momentum: {momentum}")
    print(f"Epochs: {epochs}")
    print("-" * 60)

    for epoch in range(epochs):
        # Shuffle the data at the start of each epoch
        indices = np.random.permutation(m)
        X_shuffled = X[indices]
        y_shuffled = y[indices]

        epoch_loss = 0
        num_batches = 0

        # Process mini-batches
        for i in range(0, m, batch_size):
            # Get mini-batch
            X_batch = X_shuffled[i:i+batch_size]
            y_batch = y_shuffled[i:i+batch_size]

            # Forward propagation
            y_pred, cache = forward_pass(X_batch, W1, b1, W2, b2)

            # Compute loss
            batch_loss = compute_loss(y_batch, y_pred)
            epoch_loss += batch_loss
            num_batches += 1

            # Backward propagation
            gradients = backward_pass(X_batch, y_batch, cache, W1, W2)

            # Update velocity with momentum
            vW1 = momentum * vW1 - (1-momentum) * learning_rate * gradients['dW1']
            vb1 = momentum * vb1 - (1-momentum) * learning_rate * gradients['db1']
            vW2 = momentum * vW2 - (1-momentum) * learning_rate * gradients['dW2']
            vb2 = momentum * vb2 - (1-momentum) * learning_rate * gradients['db2']

            # Update parameters
            W1 += vW1
            b1 += vb1
            W2 += vW2
            b2 += vb2

        # Calculate average loss for the epoch
        avg_loss = epoch_loss / num_batches
        loss_history.append(avg_loss)

        # Print progress every 10 epochs
        if (epoch + 1) % 10 == 0:
            print(f"Epoch {epoch+1}/{epochs}, Loss: {avg_loss:.6f}")

parameters = {'W1': W1, 'b1': b1, 'W2': W2, 'b2': b2}
```

```
        return parameters, loss_history

    # Train the network with momentum
    trained_params, loss_history = mini_batch_gradient_descent(
        X_diabetes, y_diabetes, W1, b1, W2, b2,
        batch_size=32,
        learning_rate=0.01,
        epochs=100,
        momentum=0.9
    )
```

```
Training with Mini-Batch Gradient Descent with Momentum...
Batch size: 32
Learning rate: 0.01
Momentum: 0.9
Epochs: 100
-----
Epoch 10/100, Loss: 0.502542
Epoch 20/100, Loss: 0.494495
Epoch 30/100, Loss: 0.482979
Epoch 40/100, Loss: 0.410845
Epoch 50/100, Loss: 0.300766
Epoch 60/100, Loss: 0.259209
Epoch 70/100, Loss: 0.246893
Epoch 80/100, Loss: 0.241139
Epoch 90/100, Loss: 0.236405
Epoch 100/100, Loss: 0.235396
```

```
In [81]: # Display updated weights and bias values
print("\n" + "*60)
print("UPDATED WEIGHTS AND BIASES AFTER TRAINING")
print("*60)

print("\nLayer 1 (Input -> Hidden):")
print(f"W1 shape: {trained_params['W1'].shape}")
print(f"W1:\n{trained_params['W1']}")
print(f"\nb1 shape: {trained_params['b1'].shape}")
print(f"\nb1:\n{trained_params['b1']}")

print("\nLayer 2 (Hidden -> Output):")
print(f"W2 shape: {trained_params['W2'].shape}")
print(f"W2:\n{trained_params['W2']}")
print(f"\nb2 shape: {trained_params['b2'].shape}")
print(f"\nb2:\n{trained_params['b2']}")

print("\n" + "*60)
print(f"Initial Loss: {loss_history[0]:.6f}")
print(f"Final Loss: {loss_history[-1]:.6f}")
print(f"Loss Reduction: {loss_history[0] - loss_history[-1]:.6f}")
print("*60)

=====
UPDATED WEIGHTS AND BIASES AFTER TRAINING
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Layer 1 (Input -> Hidden):
W1 shape: (10, 8)
W1:
[[ 2.47342365e-02  1.28146223e-01  7.28478797e-03  2.91099200e-02
   2.59524287e-03  5.71769578e-02  2.14592242e-02  8.90493460e-03]
 [ 5.21467753e-02 -5.09483827e-02 -4.34969289e-03 -9.84868394e-03
   2.90145134e-02  1.61151871e-01 -1.55273897e-02 -4.53354771e-03]
 [-8.27471376e-02  4.85567088e-01 -3.15270329e-03  2.55053325e-02
  -1.76396623e-02 -2.14073554e-01  1.45929960e-02 -6.41571912e-03]
 [-3.85875790e-02  3.08167459e-01 -5.38085601e-03  2.79910522e-02
  -3.02718144e-02 -1.01180669e-01  3.54565249e-03  2.42816327e-02]
 [ 7.84345964e-04 -3.53513473e-02  1.06785430e-02 -1.86928742e-02
   3.41757099e-04 -2.37532289e-02 -1.38561420e-02  2.21928722e-03]
 [ 2.07599606e-02 -8.89996479e-02  8.52199391e-04 -1.62507773e-02
  -8.93091303e-03  3.39767186e-02 -5.91731211e-03  9.93522915e-03]
 [ 6.94875424e-02 -9.85802493e-02 -2.87789859e-03 -1.02952966e-02
   1.51782026e-02  1.80000957e-01  6.03910820e-04  5.57776995e-03]
 [-5.15245774e-02  6.84998519e-02  9.03726018e-03  1.30102660e-02
  -1.92691765e-02 -1.13672352e-01 -3.50361051e-03 -9.65482081e-03]
 [-1.00237102e-01  3.16674820e-01  7.68250356e-03  3.70585331e-02
  -4.28593217e-02 -3.24007915e-01  1.83540105e-02  2.23959808e-02]
 [-2.62553115e-03  1.64989562e-01 -2.13832095e-02  2.07345087e-02
  -2.98493949e-03 -5.88338550e-03  7.42584208e-03 -1.80666919e-02]]

b1 shape: (1, 8)
b1:
[[ 0.08075549  0.18981778 -0.00384552  0.01771259  0.02497907  0.23765902
   0.00519341  0.00028168]]

Layer 2 (Hidden -> Output):
W2 shape: (8, 1)
W2:
[[ -0.18848261]
 [ 0.72997806]
 [ 0.00240609]
 [ 0.06689957]
 [ -0.06830435]
 [ -0.53888969]
 [ 0.02534389]
 [ 0.01349131]]
```

```
b2 shape: (1, 1)
b2:
[[-0.14396657]]  
=====
Initial Loss: 0.498773
Final Loss: 0.235396
Loss Reduction: 0.263377
=====
```

```
In # Plot loss w.r.t. bias values
[82]: # We'll vary one bias value and see how loss changes

print("\nGenerating loss surface with respect to bias values...")

# Select one bias from b2 to vary (output layer bias)
bias_range = np.linspace(-2, 2, 100)
loss_vs_bias = []

# Keep all parameters constant except one bias value
W1_temp = trained_params['W1'].copy()
b1_temp = trained_params['b1'].copy()
W2_temp = trained_params['W2'].copy()
b2_temp = trained_params['b2'].copy()

# Store original bias value
original_bias = b2_temp[0, 0]

for bias_val in bias_range:
    # Set the bias to test value
    b2_temp[0, 0] = bias_val

    # Forward pass with this bias
    y_pred, _ = forward_pass(X_diabetes, W1_temp, b1_temp, W2_temp, b2_temp)

    # Compute loss
    loss = compute_loss(y_diabetes, y_pred)
    loss_vs_bias.append(loss)

# Restore original bias
b2_temp[0, 0] = original_bias

# Create the plot
plt.figure(figsize=(14, 5))

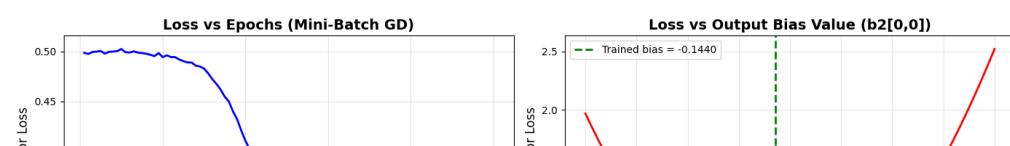
# Plot 1: Loss vs Epochs
plt.subplot(1, 2, 1)
plt.plot(range(1, len(loss_history) + 1), loss_history, linewidth=2, color='blue')
plt.title('Loss vs Epochs (Mini-Batch GD)', fontsize=14, fontweight='bold')
plt.xlabel('Epoch', fontsize=12)
plt.ylabel('Mean Squared Error Loss', fontsize=12)
plt.grid(True, alpha=0.3)

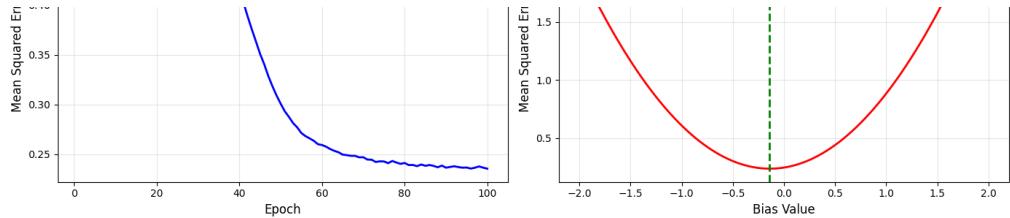
# Plot 2: Loss vs Bias Value
plt.subplot(1, 2, 2)
plt.plot(bias_range, loss_vs_bias, linewidth=2, color='red')
plt.axvline(x=original_bias, color='green', linestyle='--', linewidth=2,
            label=f'Trained bias = {original_bias:.4f}')
plt.title('Loss vs Output Bias Value (b2[0,0])', fontsize=14, fontweight='bold')
plt.xlabel('Bias Value', fontsize=12)
plt.ylabel('Mean Squared Error Loss', fontsize=12)
plt.legend()
plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()

print(f"\nOptimal bias value (after training): {original_bias:.6f}")
print(f"Loss at optimal bias: {compute_loss(y_diabetes, forward_pass(X_diabetes,
trained_params['W1'], trained_params['b1'], trained_params['W2'],
trained_params['b2'])[0]):.6f}")
```

Generating loss surface with respect to bias values...





Optimal bias value (after training): -0.143967
Loss at optimal bias: 0.235860

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