Nonparametric Analysis of US Dairy Production and Consumption

Conformal prediction

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Contents

1 Load libraries and data

```r
library(mgcv)
library(conformalInference)
library(rgl)
library(dbscan)
library(pbapply)
library(beadplexr)
```

```r
data_path = file.path('data_updated_2021')
output_path = file.path('output')
data_infl <-
    read.table(
        file.path(data_path, 'production_facts_inflated.csv'),
        header = T,
        sep = ';'
    )
y = data_infl$avg_price_milk
n_b = n = length(y)
```

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2 Conformal prediction

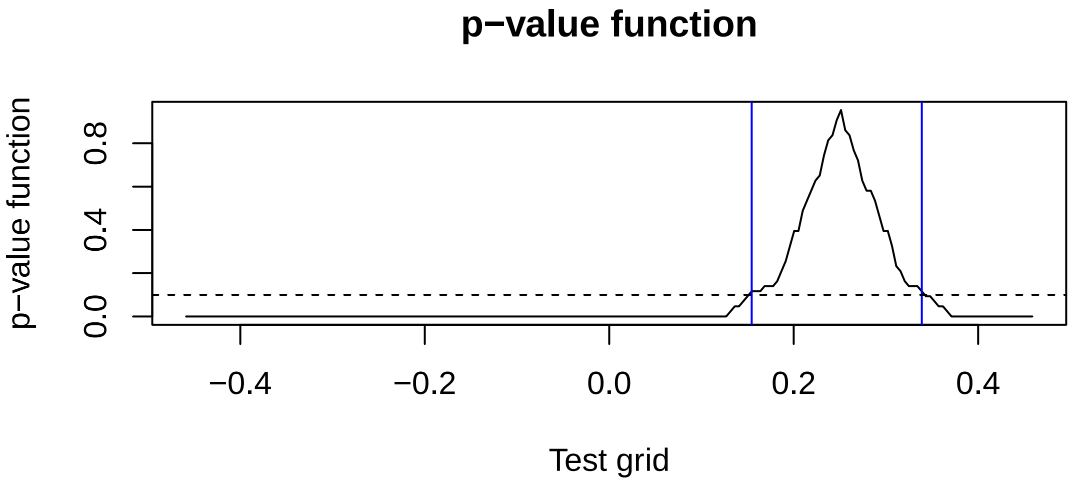
```
grid_factor = 1.25
n_grid = 200
alpha = 0.10
```

2.1 Using T Prediction Intervals

```
wrapper_full = function(grid_point) {
    aug_y = c(grid_point, y)
    mu = mean(aug_y)
    ncm = abs(mu - aug_y)
    sum((ncm[-1] >= ncm[1])) / (n + 1)
}

test_grid = seq(-grid_factor * max(abs(y)), +grid_factor * max(abs(y)),
                length.out = n_grid)

pval_fun = sapply(test_grid, wrapper_full)
index_in = pval_fun > alpha
pred_t_interval = range(test_grid[index_in])
```

Plot *p*-value function

```
plot_pval = function(test_grid, pval_fun, pred, alpha) {
    plot(
        test_grid,
        pval_fun,
        type = 'l',
        main = "p-value function",
        xlab = "Test grid",
        ylab = "p-value function"
    )
    abline(v = pred, col = 'blue')
    abline(h = alpha, lty = 2)
}

plot_pval(test_grid, pval_fun, pred_t_interval, alpha)
```

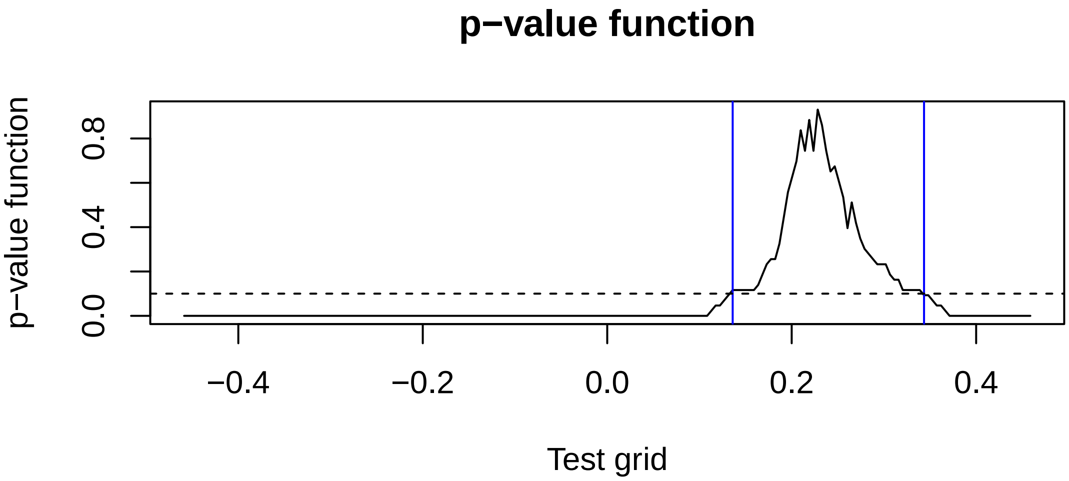


2.2 Using KNN distance

```
pval_fun = numeric(n_grid)
k_s = 0.46
wrapper_knn = function(grid_point) {
    aug_y = c(grid_point, y)
    ncm = kNNdist(matrix(aug_y), k_s * n)
    sum((ncm[-1] >= ncm[1])) / (n_b + 1)
}

pval_fun = sapply(test_grid, wrapper_knn)
index_in = pval_fun > alpha
pred_knn = test_grid[as.logical(c(0, abs(diff(index_in))))]
```

Plot p-value function

```
plot_pval(test_grid, pval_fun, pred_knn, alpha)
```

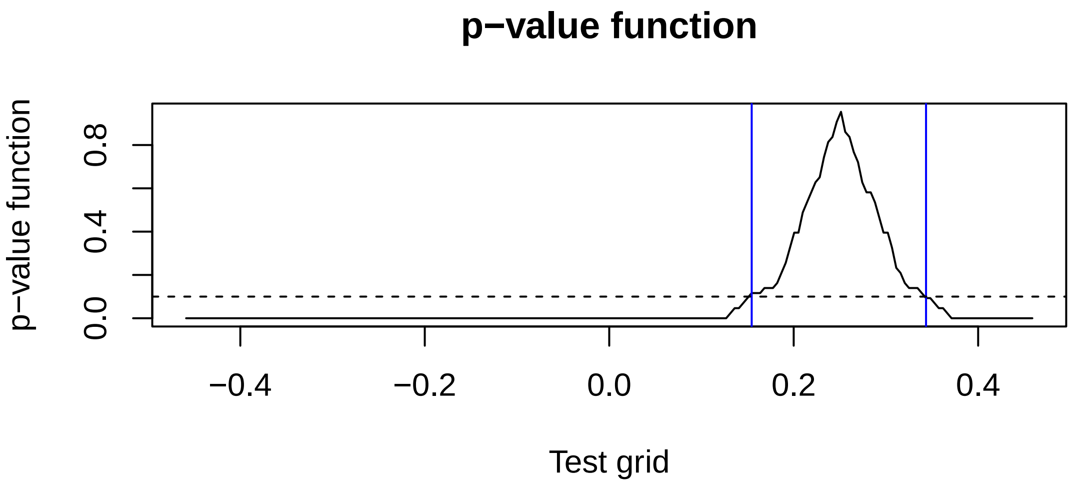


2.3 Using Mahalanobis distance

```
pval_fun = numeric(n_grid)
wrapper_mal = function(grid_point) {
    aug_y = c(grid_point, y)
    ncm = mahalanobis(matrix(aug_y), colMeans(matrix(aug_y)), cov(matrix(aug_y)))
    sum((ncm[-1] >= ncm[1])) / (n_b + 1)
}

pval_fun = sapply(test_grid, wrapper_mal)
index_in = pval_fun > alpha
pred_mahalanobis = test_grid[as.logical(c(0, abs(diff(index_in))))]
```

Plot p-value function

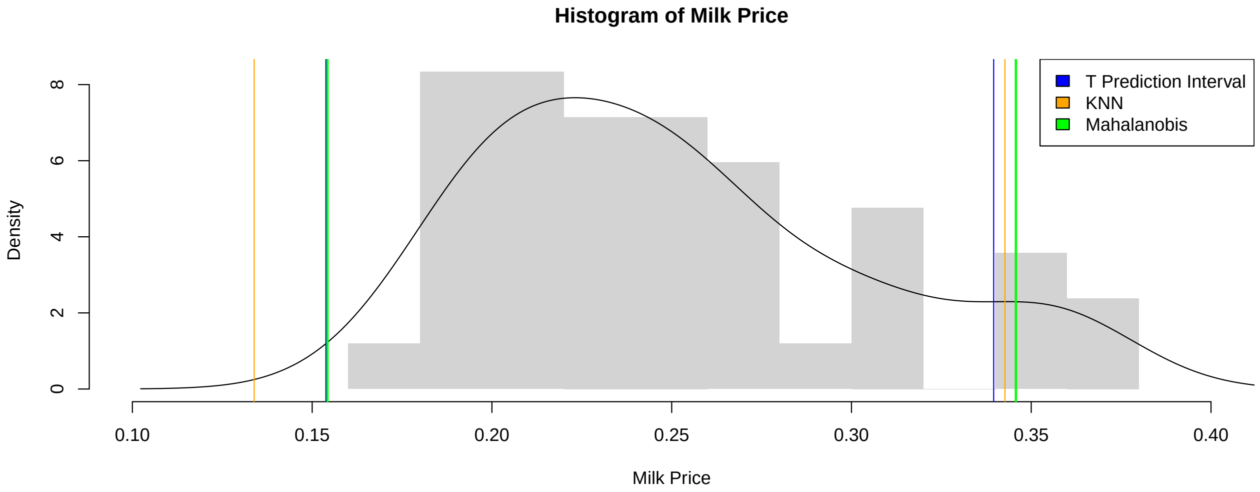
```
plot_pval(test_grid, pval_fun, pred_mahalanobis, alpha)
```



3 Show result

Plot histogram of target variable

```
hist(
    y,
    breaks = 10,
    freq = FALSE,
    main = 'Histogram of Milk Price',
    xlab = 'Milk Price',
    xlim = c(0.1, 0.4),
    border = NA
)
lines(density(y))

abline(v = jitter(pred_t_interval, amount=0.003), col = 'blue', lwd = 1)
abline(v = jitter(pred_knn, amount=0.003), col = 'orange', lwd = 1)
abline(v = jitter(pred_mahalanobis, amount=0.003), col = 'green', lwd = 2)

legend("topright",
       legend = c("T Prediction Interval", "KNN", "Mahalanobis"),
       fill = c("blue", "orange", "green"))
```



```r
result = data.frame(
    rbind(
        "T Prediction Interval"=pred_t_interval,
        "KNN"=pred_knn,
        "Mahalanobis"=pred_mahalanobis
    )
)
names(result) = c("LOWER", "UPPER")

#knitr::kable(result, format = "latex")
knitr::kable(result)
```

	LOWER	UPPER
T Prediction Interval	0.1544568	0.3388828
KNN	0.1360142	0.3434935
Mahalanobis	0.1544568	0.3434935