Chapter 10 – Sound in Ducts

Slides to accompany lectures in

Vibro-Acoustic Design in Mechanical Systems

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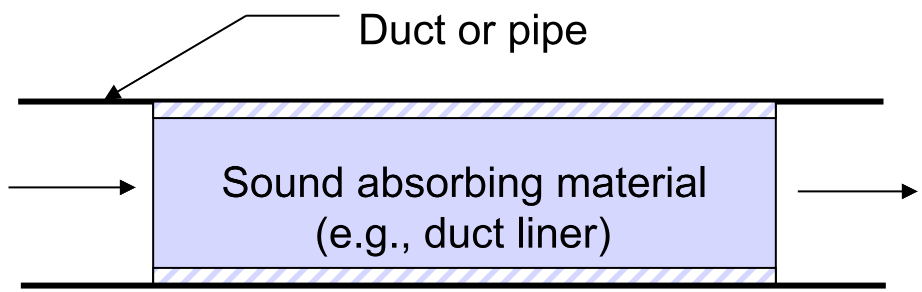
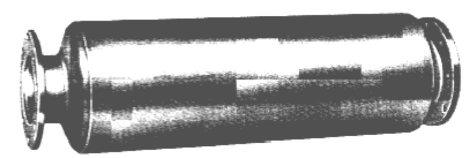
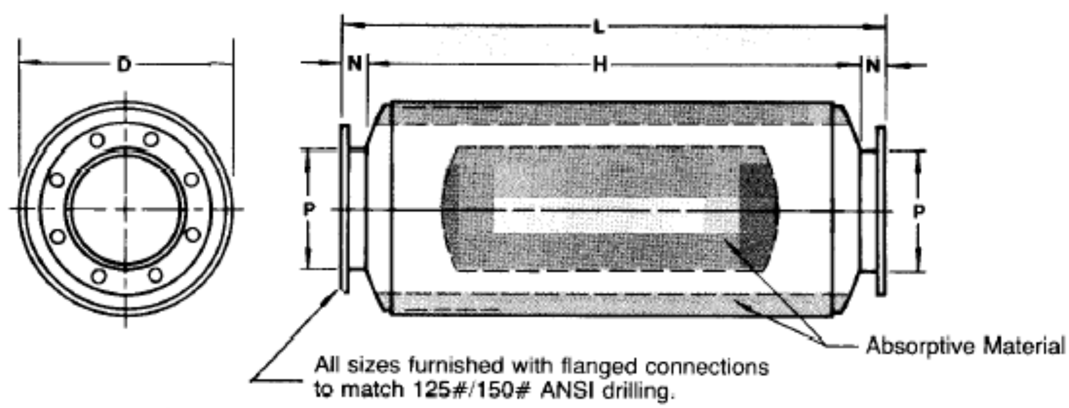
Types of Mufflers

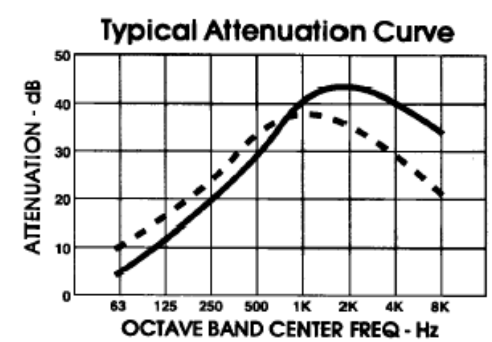
1. Dissipative (absorptive) silencer:





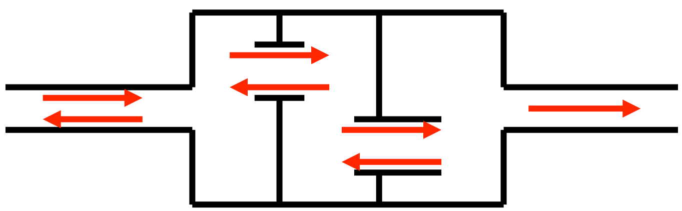
Sound is attenuated due to absorption (conversion to heat)



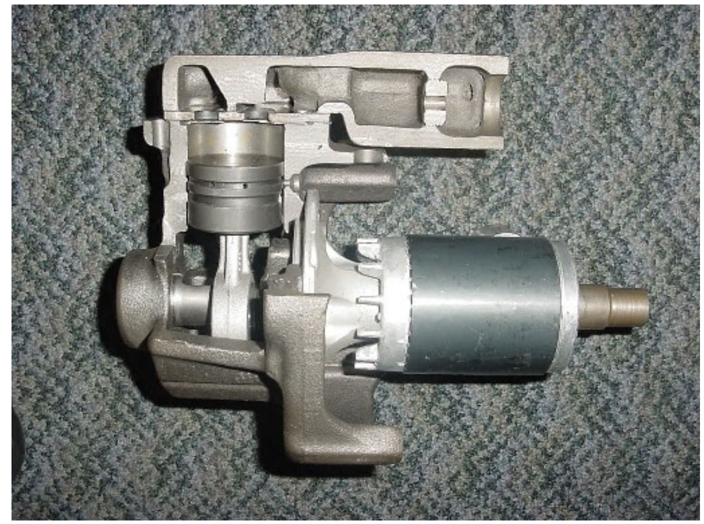


Types of Mufflers

2. Reactive muffler:

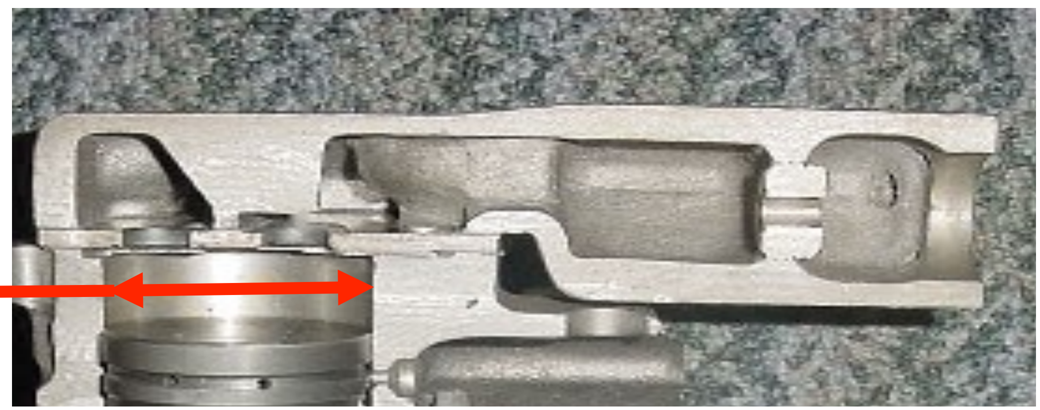


Sound is attenuated by reflection and "cancellation" of sound waves



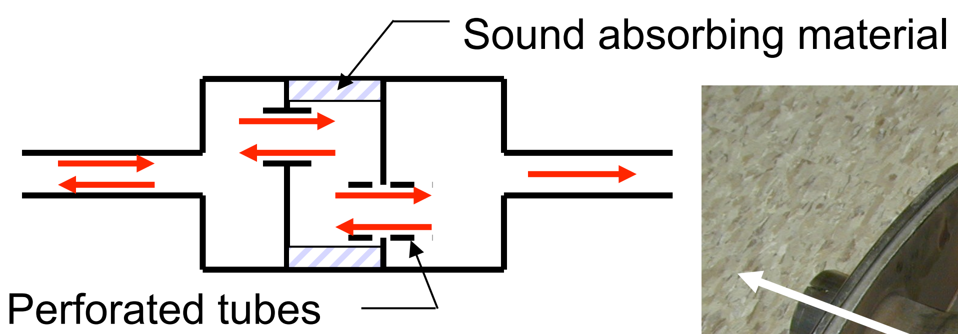
Compressor discharge details

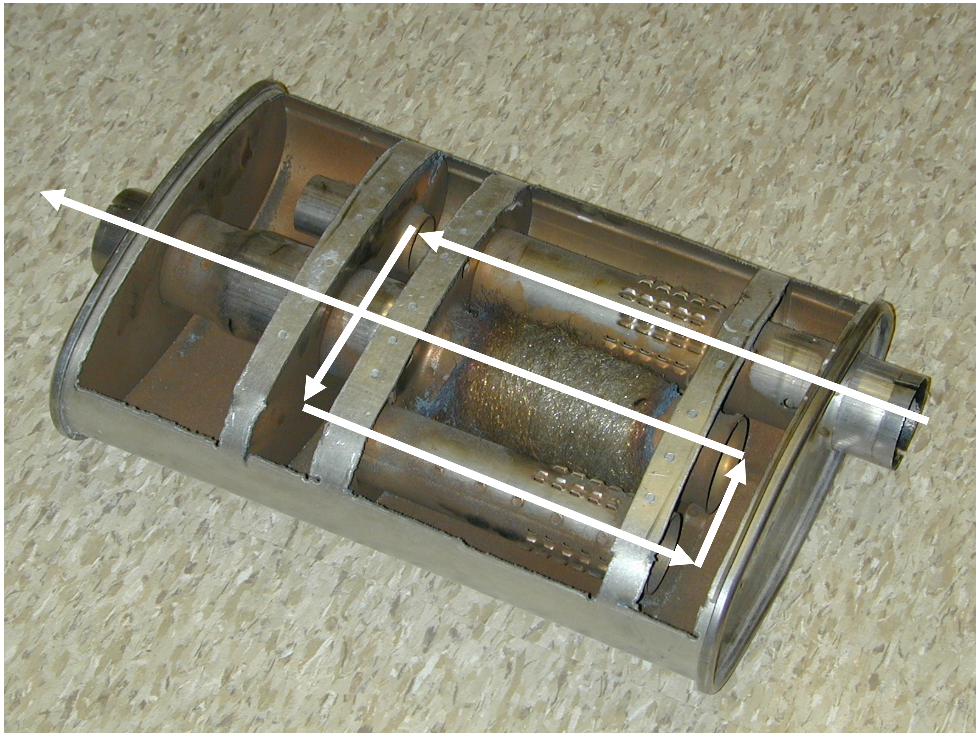
40 mm



Types of Mufflers

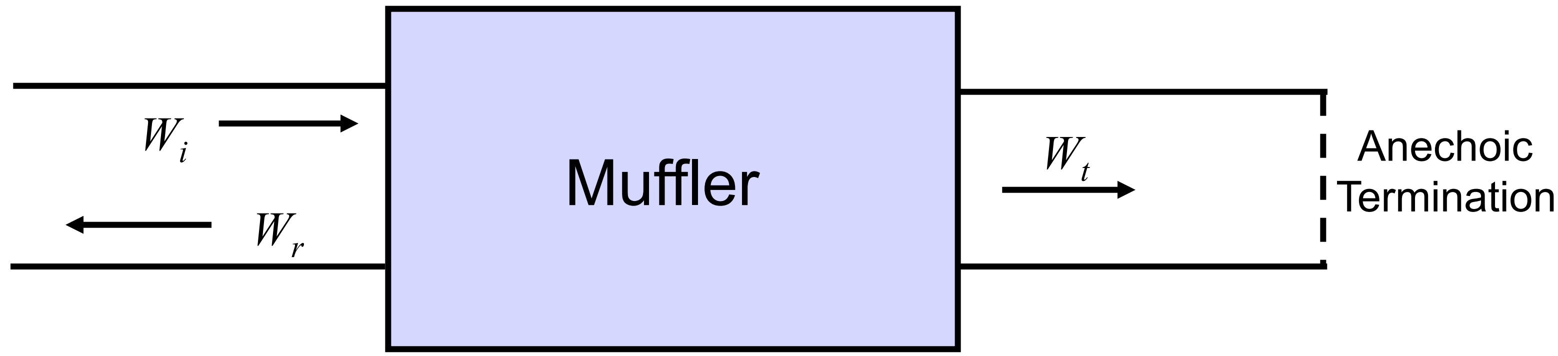
3. Combination reactive and dissipative muffler:





Sound is attenuated by reflection and "cancellation" of sound waves + absorption of sound

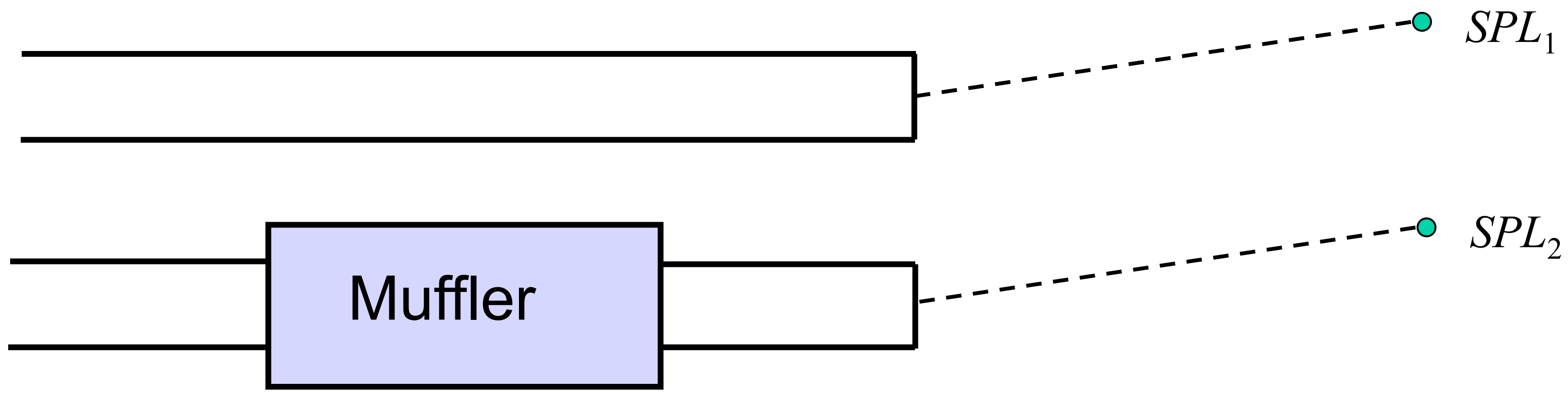
Performance Measures Transmission Loss



Transmission loss (TL) of the muffler:

$$TL(\mathrm{dB}) = 10\log_{10}\frac{W_i}{W_t}$$

Performance Measures Insertion Loss



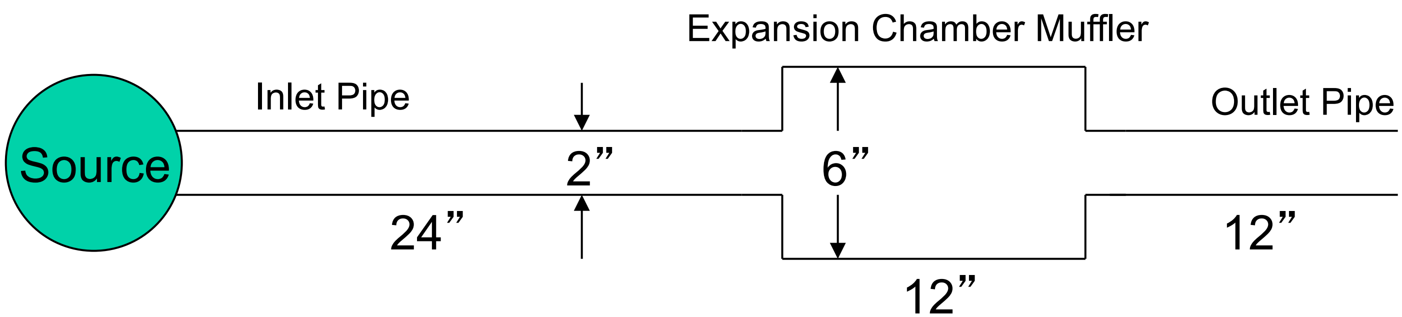
$$IL\ (\mathrm{dB}) = SPL_1 - SPL_2$$

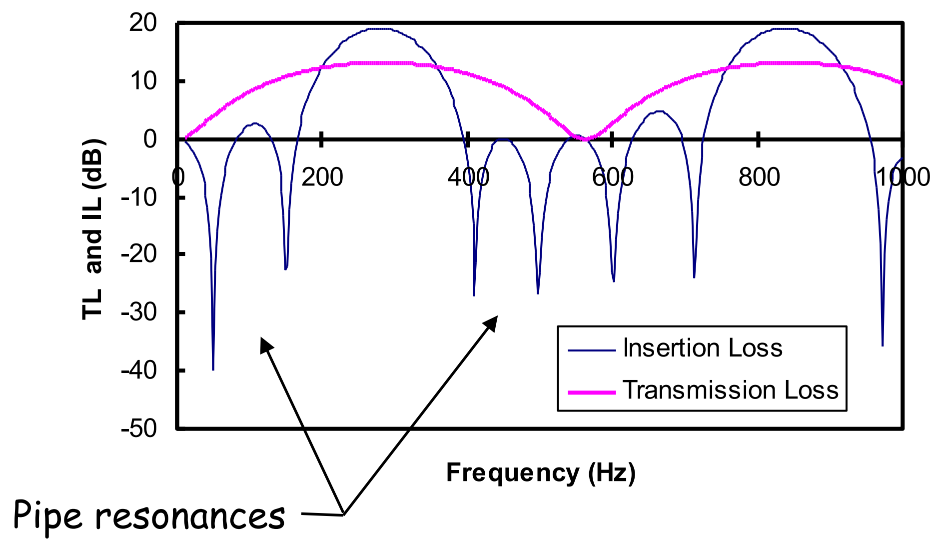
Insertion loss depends on :

- TL of muffler
- Lengths of pipes
- Termination (baffled vs. unbaffled)
- Source impedance

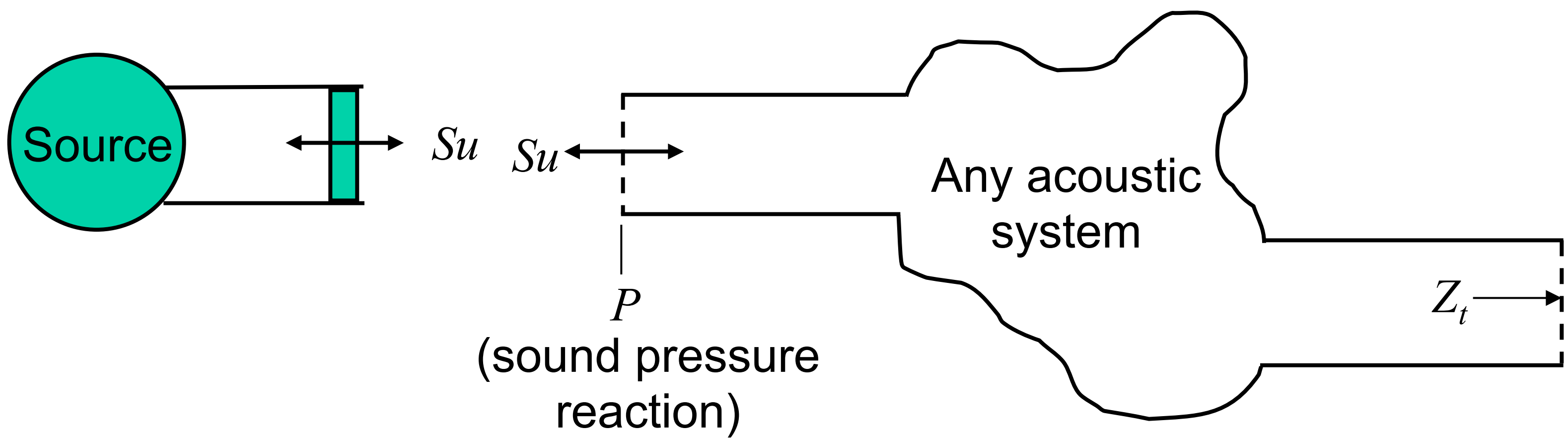
Note: TL is a property of the muffler; IL is a "system" performance measure.

Example TL and IL





Acoustic System Components



Input or load impedance $z = \dfrac{P}{Su} = r + jx$

Termination impedance $z_t = \dfrac{P_t}{Su_t} = r_t + jx_t$

Summary 1

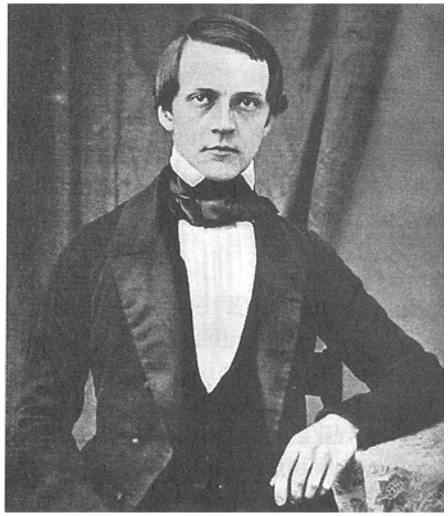
- Dissipative mufflers attenuate sound by converting sound energy to heat via viscosity and flow resistance – this process is called sound absorption.
- Common sound absorbing mechanisms used in dissipative mufflers are porous or fibrous materials or perforated tubes.
- Reactive mufflers attenuate sound by reflecting a portion of the incident sound waves back toward the source. This process is frequency selective and may result in unwanted resonances.
- Impedance concepts may be used to interpret reactive muffler behavior.

The Helmholtz Resonator

Named for:

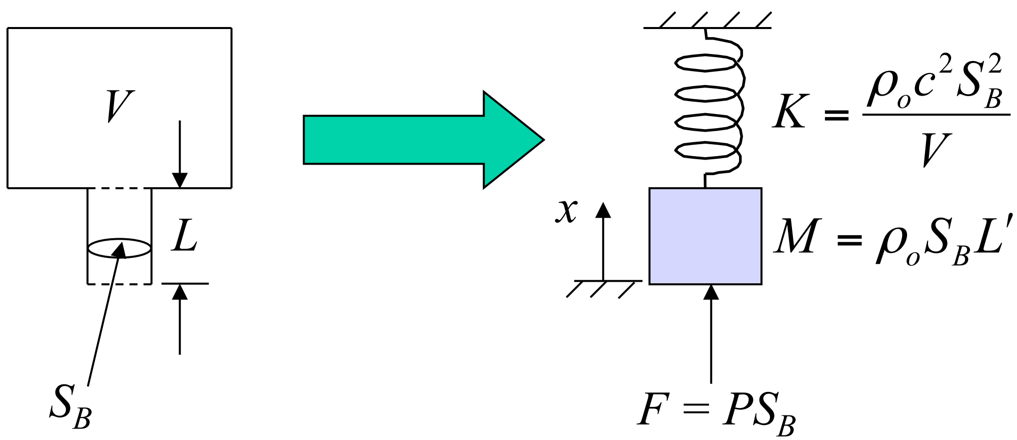
Hermann von Helmholtz, 1821-1894, German physicist, physician, anatomist, and physiologist.

Major work: Book, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, 1862.



von Helmholtz, 1848

Helmholtz Resonator Model



L' is the equivalent length of the neck (some air on either end also moves).

$$M\ddot{x} + Kx = PS_B \quad \ddot{x} = j\omega u_B \quad x = \frac{u_B}{j\omega}$$

Damping due to viscosity in the neck are neglected

$$j\left(\omega M - \frac{K}{\omega}\right)u_B = PS_B$$

$$z_B = \frac{P}{S_B u_B} = j\left(\frac{1}{S_B^2}\right)\left(\omega M - \frac{K}{\omega}\right)$$

$$z_B \to 0 \quad \text{when} \quad \omega = \sqrt{\frac{K}{M}} = c\sqrt{\frac{S_B}{L'V}}$$

(resonance frequency of the Helmholtz resonator)

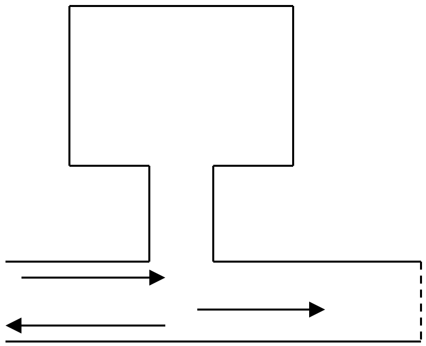
Helmholtz Resonator Example

A 12-oz (355 ml) bottle has a 2 cm diameter neck that is 8 cm long. What is the resonance frequency?

$$f_n = \frac{c}{2\pi}\sqrt{\frac{S_B}{L'V}} = \frac{343}{2\pi}\sqrt{\frac{\pi(0.02)^2/4}{(0.08)(355\times10^{-6})}}$$

$$f_n = 182 \text{ Hz}$$

Helmholtz Resonator as a Side Branch

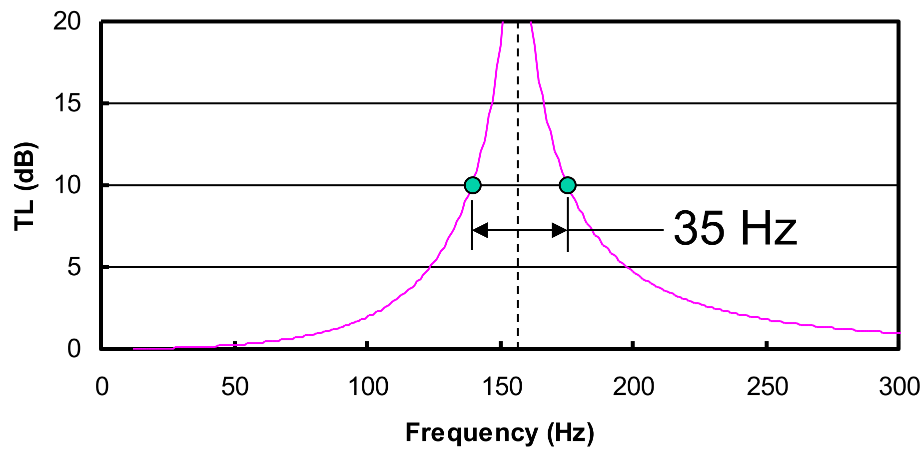


$$TL(\mathrm{dB}) = 10\log_{10}\left[1+\left(\frac{c/2S}{\omega L'/S_B - c^2/\omega V}\right)^2\right]$$

Anechoic termination

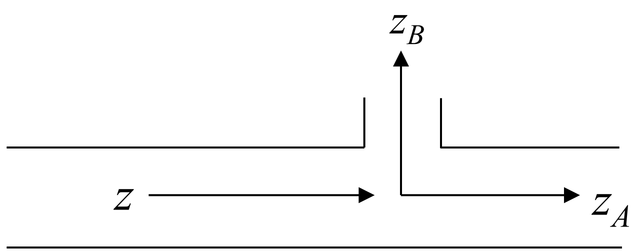
$V = 0.001\ \mathrm{m}^3$
$L = 25\ \mathrm{mm}$
$S_B = 2 \times 10^{-4}\ \mathrm{m}^2$
$S = 8 \times 10^{-4}\ \mathrm{m}^2$

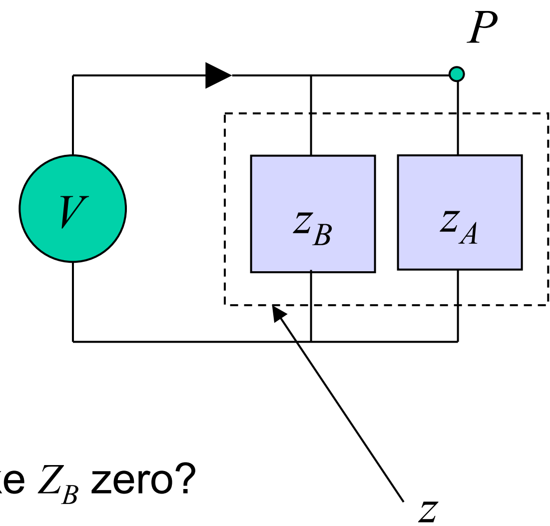
$f_n = 154\ \mathrm{Hz}$



Network Interpretation

(any system)





$$z = \frac{z_B z_A}{z_B + z_A}$$

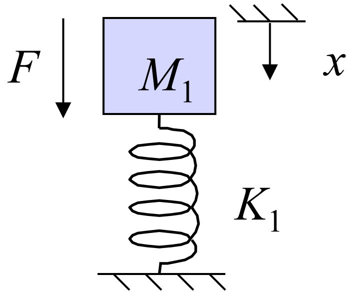
Can we make Z_B zero?

$$z_B = \frac{P}{S_B u_B} = j\left(\frac{1}{S_B^2}\right)\left(\omega M - \frac{K}{\omega}\right)$$

$$z_B \rightarrow 0 \quad \text{when} \quad \omega = \sqrt{\frac{K}{M}} = c\sqrt{\frac{S_B}{L'V}}$$

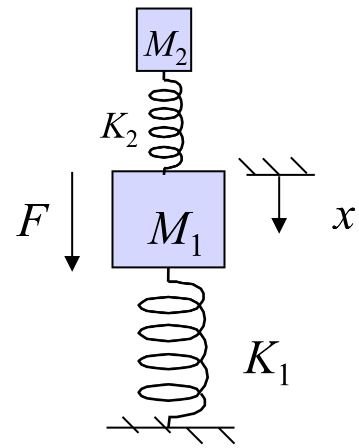
(Produces a short circuit and P is theoretically zero.)

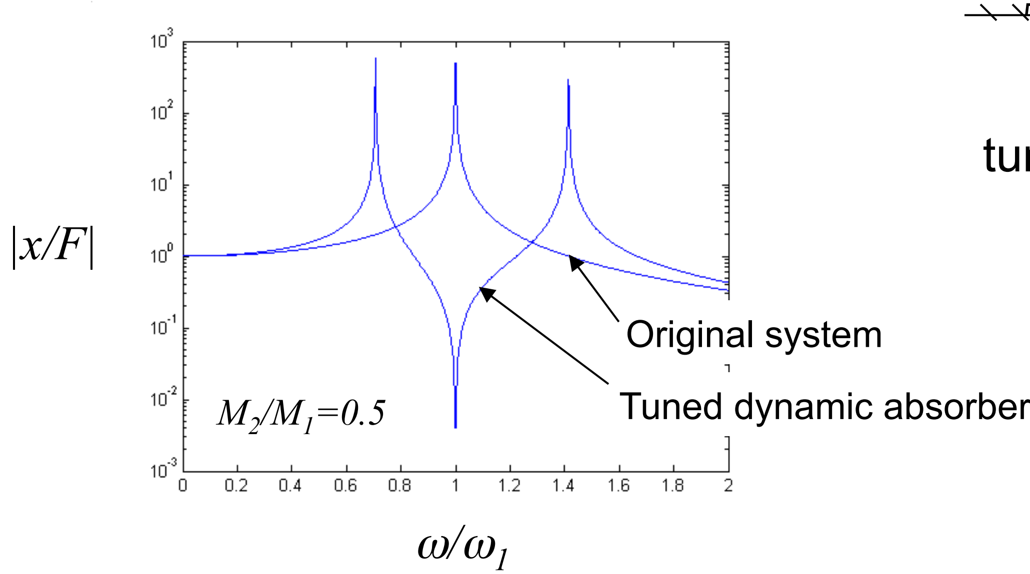
A Tuned Dynamic Absorber



Original System

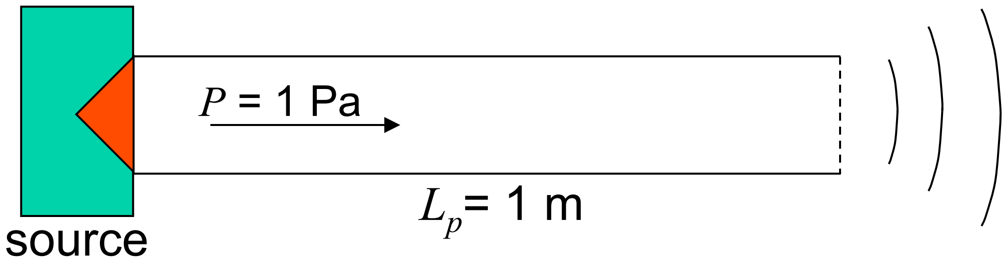
Tuned Dynamic Absorber

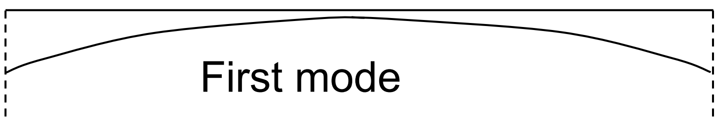




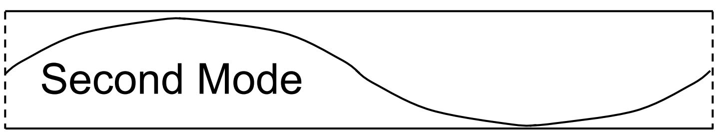
tune $\frac{K_2}{M_2} = \frac{K_1}{M_1}$

Resonances in an Open Pipe





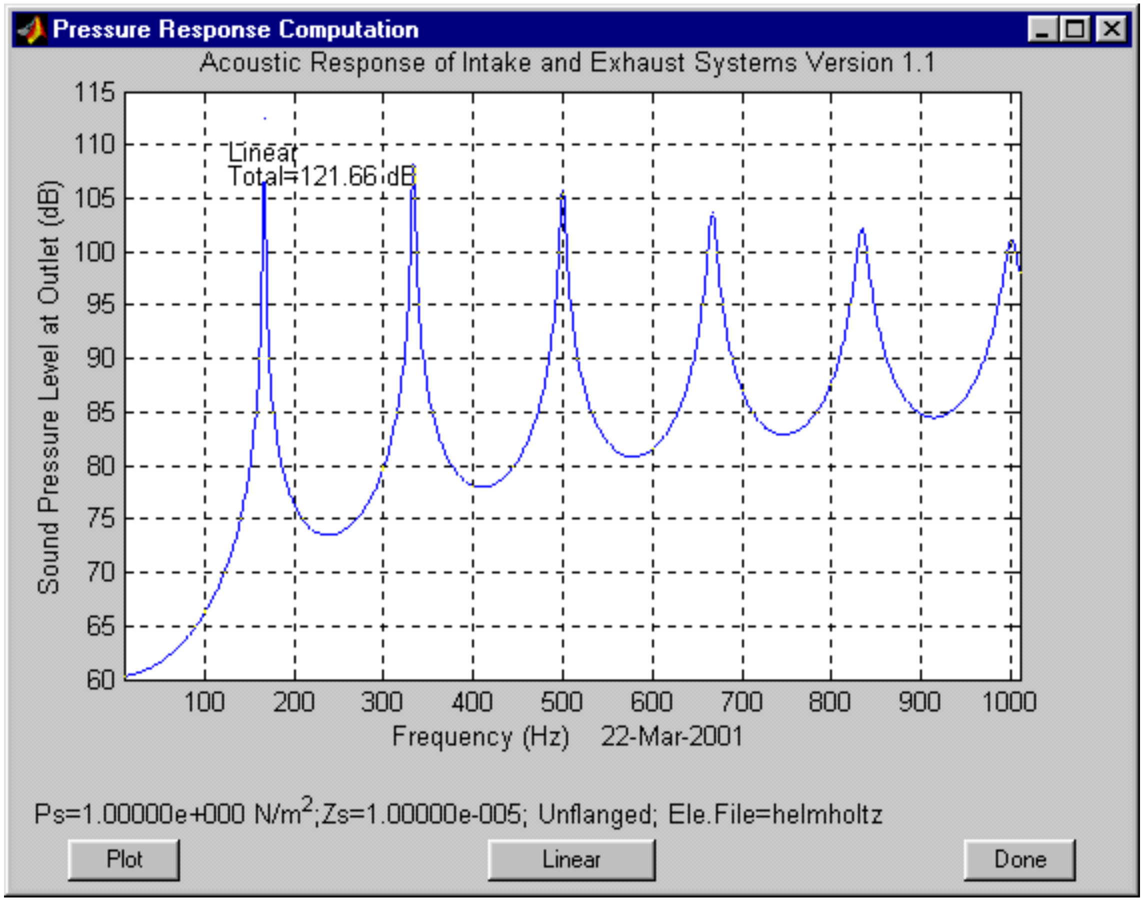
$$\lambda_1 = 2L_p = \frac{c}{f_1} \rightarrow f_1 = \frac{343}{2(1)} = 171.5 \text{ Hz}$$



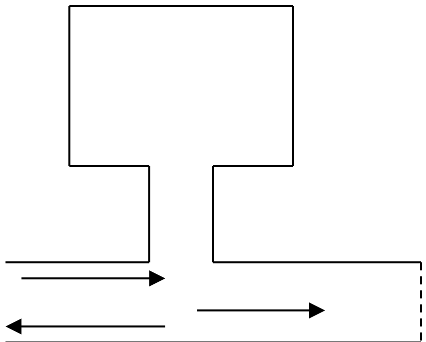
$$\lambda_2 = L_p = \frac{c}{f_2} \rightarrow f_2 = \frac{343}{1(1)} = 343 \text{ Hz}$$

etc.

SPL at Pipe Opening – No Resonator



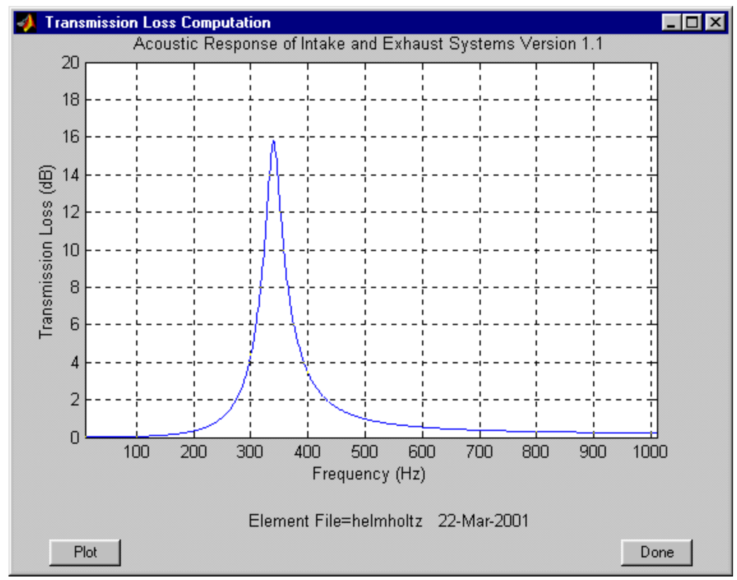
Example – HR Used as a Side Branch*



$$TL(\text{dB}) = 10\log_{10}\left[1 + \left(\frac{c/2S}{\omega L'/S_B - c^2/\omega V}\right)^2\right]$$

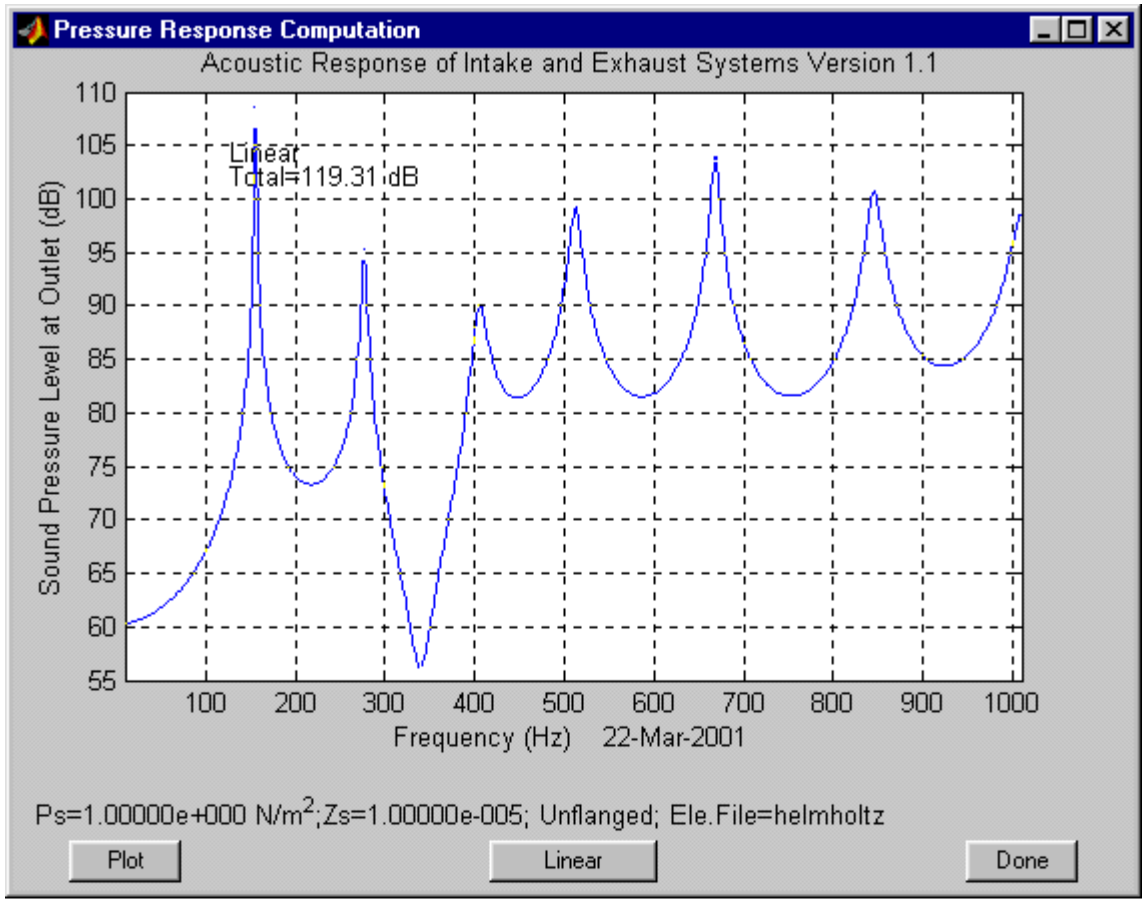
V = 750 cm^3
L = 2.5 cm (L'= 6.75 cm)
D_B = 5 cm (S_B= 19.6 cm^2)
D = 10 cm (S = 78.5 cm^2)

f_n = 340 Hz



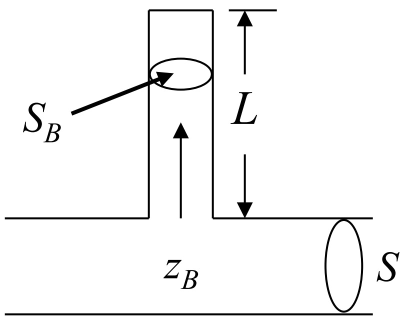
* e.g., engine intake systems

SPL at Pipe Opening – with Resonator



The Quarter Wave Resonator

The Quarter-Wave Resonator has an effect similar to the Helmholtz Resonator:



$$TL = 10\log_{10}\left(\frac{\tan^2(kl) + 4(S/S_B)^2}{4(S/S_B)^2}\right)$$

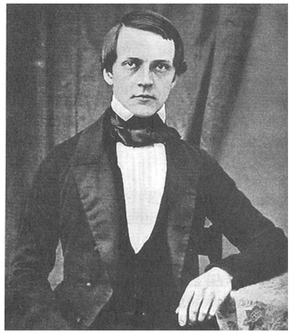
$$z_B = -\frac{j\rho_o c}{S_B}\cot(\omega L/c) = 0 \quad \text{when} \quad \omega L/c = n\pi/2 \qquad n = 1,3,5\ldots$$

$$\omega_n = \frac{n\pi c}{2L}$$

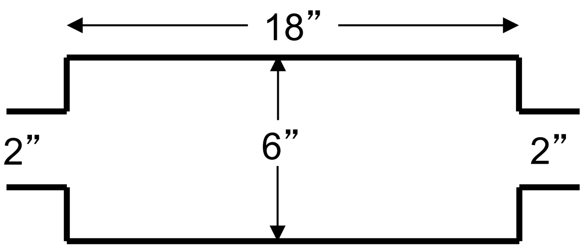
$$f_n = \frac{nc}{4L} \quad \text{or} \quad L = \frac{nc}{4f} = n\left(\frac{\lambda}{4}\right)$$

Summary 2

- The side-branch resonator is analogous to the tuned dynamic absorber.
- Resonators used as side branches attenuate sound in the main duct or pipe.
- The transmission loss is confined over a relatively narrow band of frequencies centered at the natural frequency of the resonator.

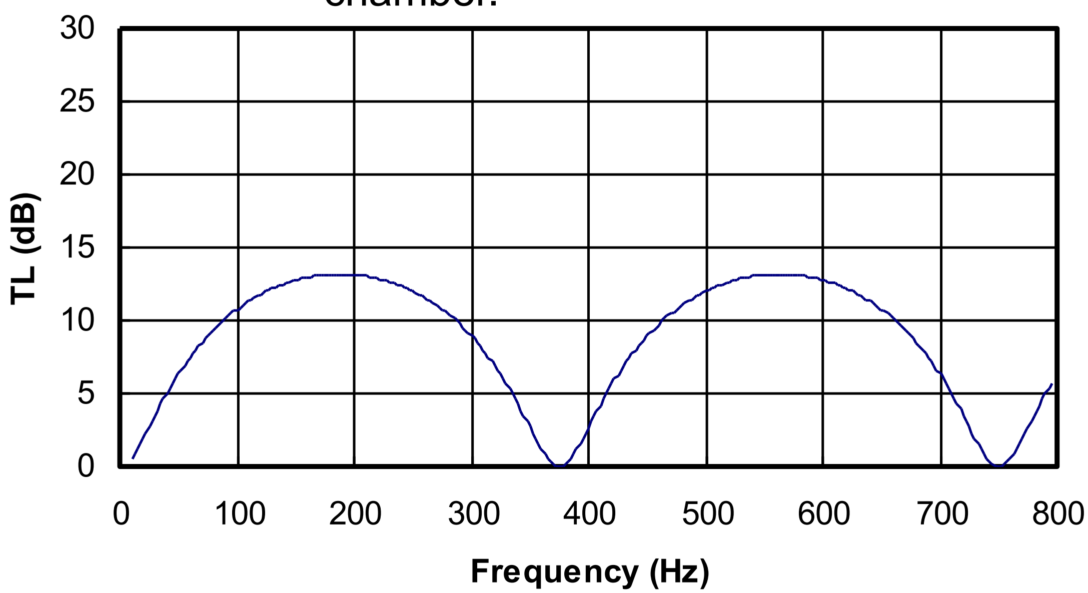


The Simple Expansion Chamber

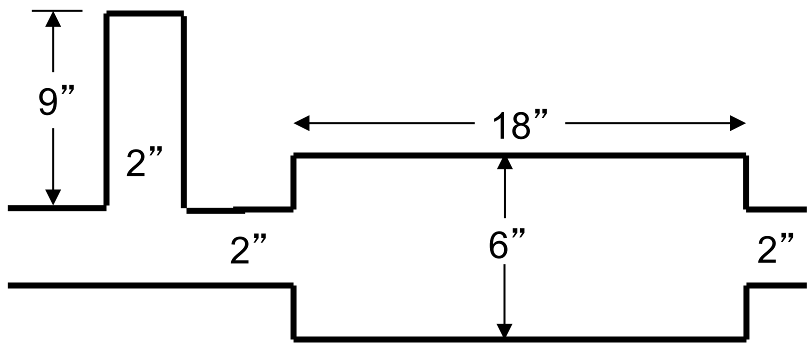


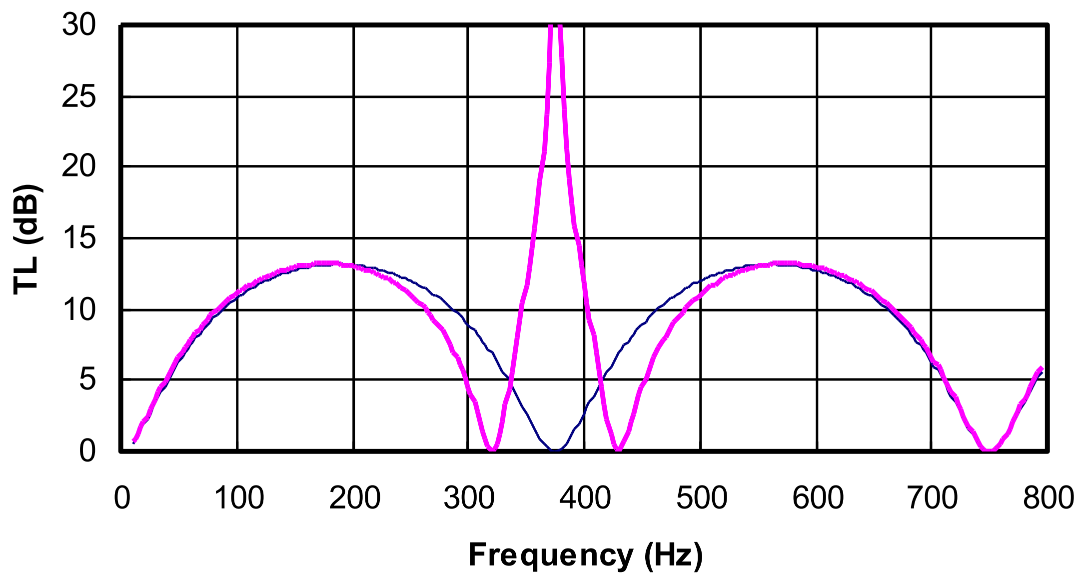
$$TL = 10\log_{10}\left[\frac{1}{4}\left(4\cos^2(kl) + \left(m + \frac{1}{m}\right)^2 \sin^2(kl)\right)\right]$$

where m is the expansion ratio (chamber area/pipe area) = 9 in this example and L is the length of the chamber.

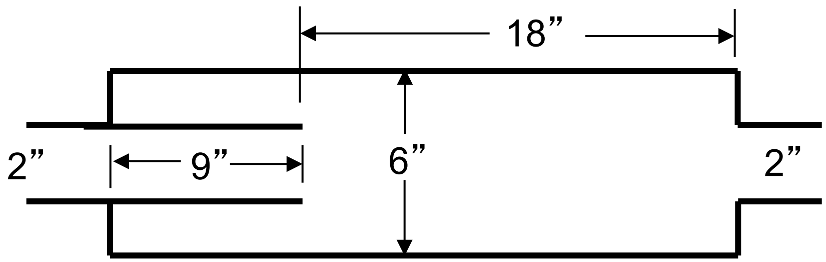


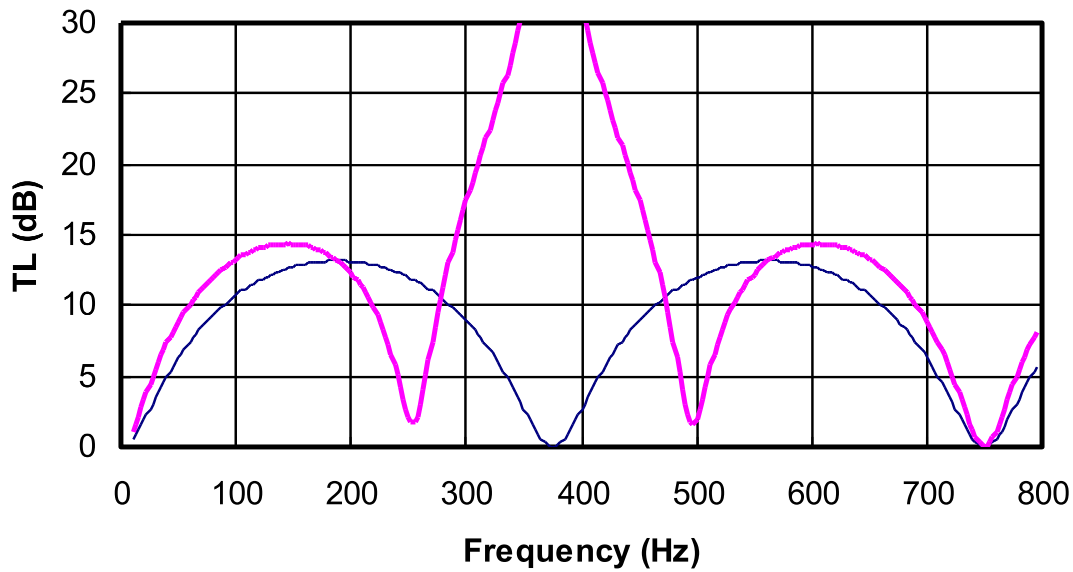
Quarter Wave Tube + Helmholtz Resonator





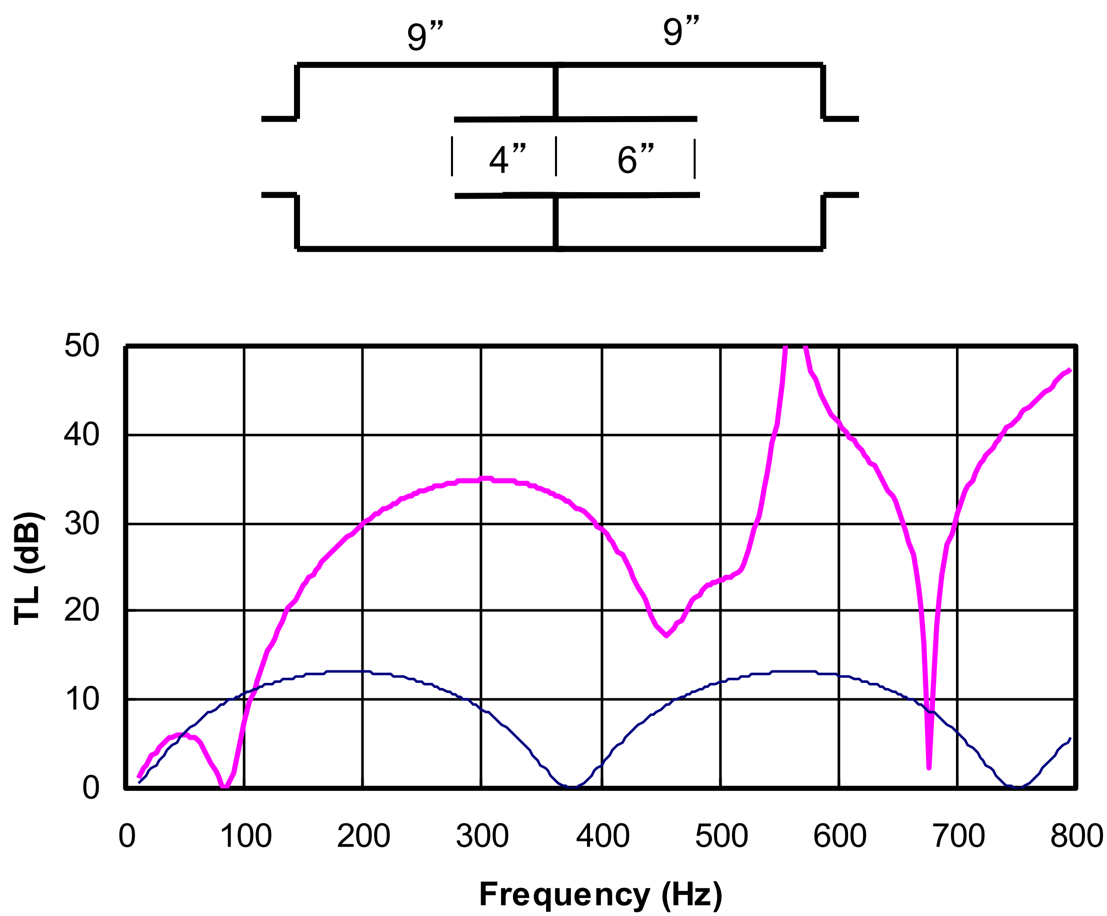
Extended Inlet Muffler





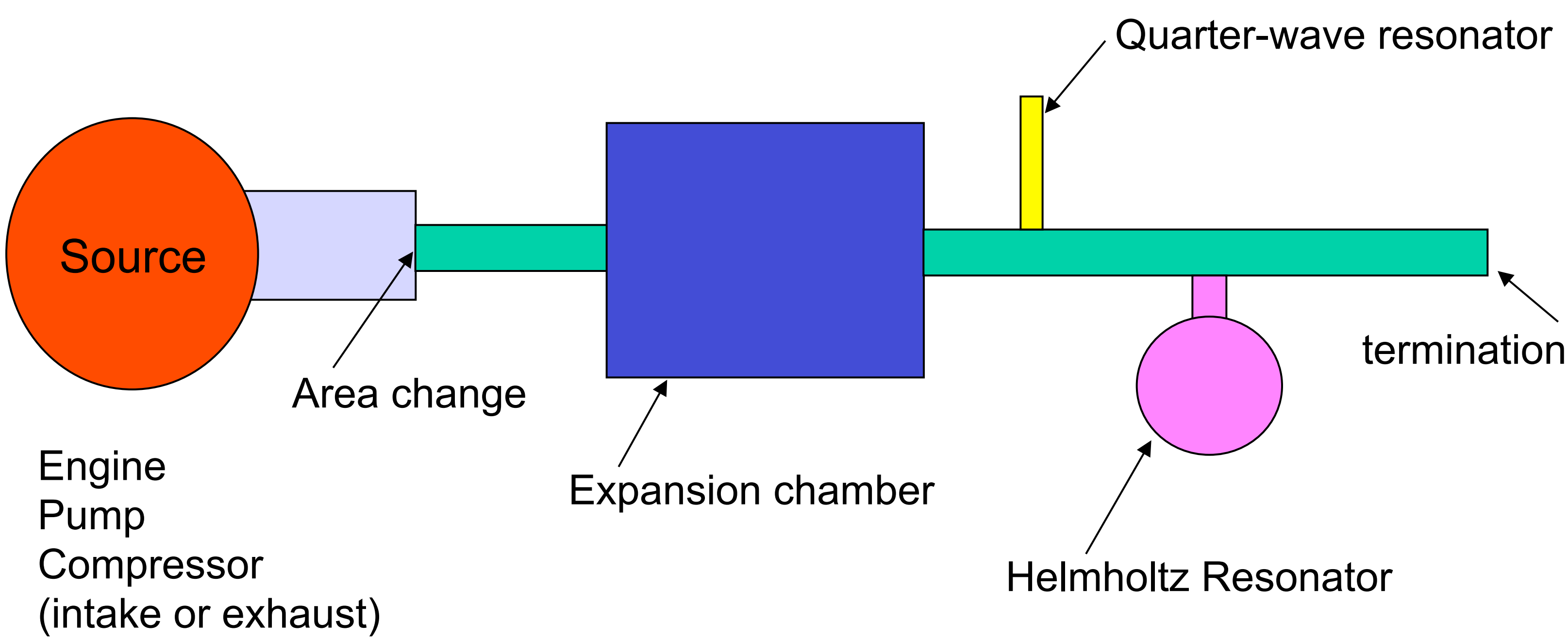
(same for extended outlet)

Two-Chamber Muffler



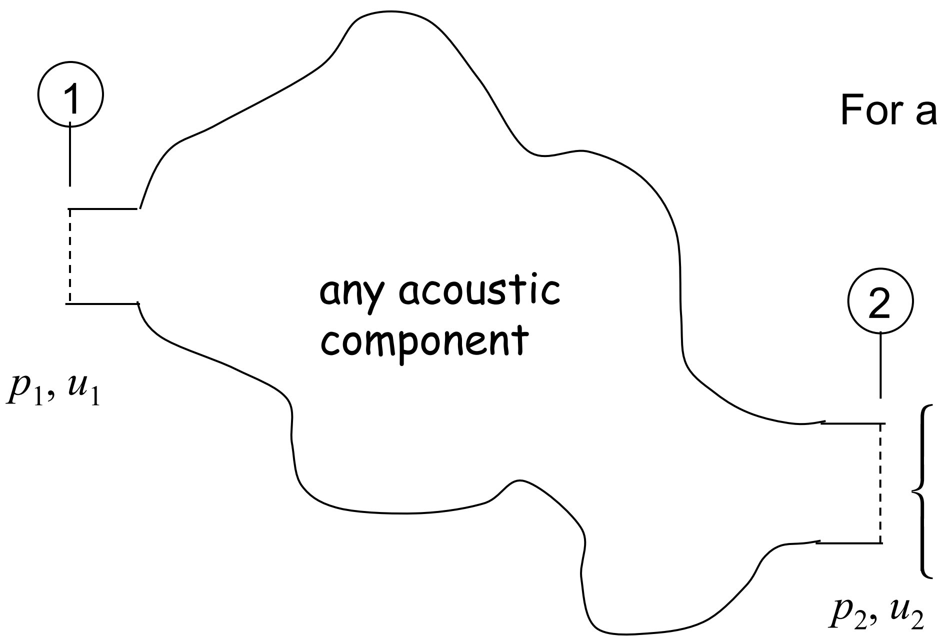
Complex System Modeling

We would like to predict the sound pressure level at the termination.



The Basic Idea

The sound pressure p and the particle velocity v are the acoustic state variables



For any passive, linear component:

$$p_1 = Ap_2 + BS_2u_2$$

$$S_1u_1 = Cp_2 + DS_2u_2$$

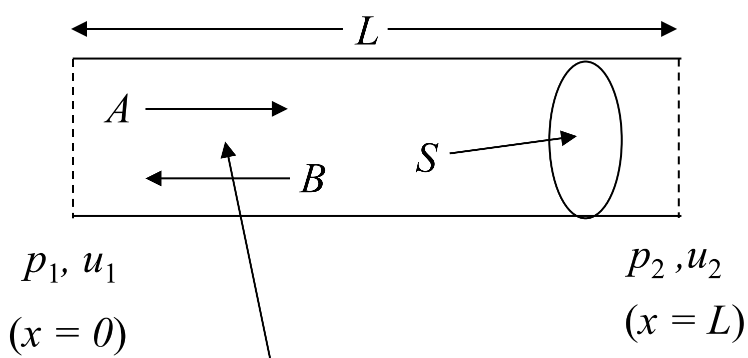
or

$$\begin{Bmatrix} p_1 \\ S_1u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2u_2 \end{Bmatrix}$$

Transfer, transmission, or four-pole matrix

(A, B, C, and D depend on the component)

The Straight Tube



must have plane waves

Solve for A, B in terms of p_1, u_1 then put into equations for p_2, u_2.

$$p(x) = Ae^{-jkx} + Be^{+jkx} \qquad u(x) = \frac{-1}{jk\rho_o c}\frac{dp}{dx}$$

$$p(0) = p_1 = A + B$$

$$u(0) = u_1 = \frac{A-B}{\rho_o c}$$

$$p(L) = p_2 = Ae^{-jkL} + Be^{+jkL}$$

$$u(L) = u_2 = \frac{Ae^{-jkL} - Be^{+jkL}}{\rho_o c}$$

$$p_1 = p_2\cos(kL) + u_2(j\rho_o c)\sin(kL)$$

$$u_1 = p_2(j/\rho_o c)\sin(kL) + u_2\cos(kL)$$

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} \cos(kL) & \dfrac{j\rho_o c}{S_2}\sin(kL) \\ \dfrac{jS_1}{\rho_o c}\sin(kL) & \dfrac{S_1}{S_2}\cos(kL) \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

(note that the determinant $A_1D_1 - B_1C_1 = 1$)

Combining Component Transfer Matrices

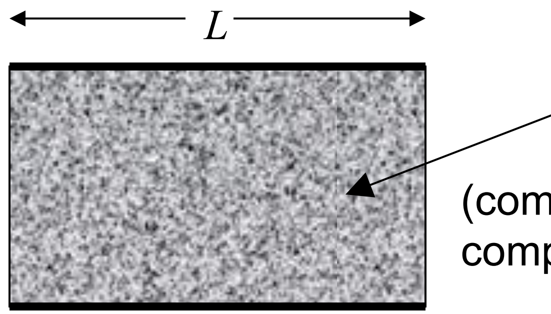
$$[T_i] = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}_{2\times 2}$$

Transfer matrix of i^{th} component

$$\begin{Bmatrix} p_n \\ v_n \end{Bmatrix} = [T_n]\cdots[T_i]\cdots[T_3][T_2][T_1]\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = [T_{\text{system}}]\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix}$$

$$[T_{\text{system}}] = \begin{bmatrix} A_{\text{system}} & B_{\text{system}} \\ C_{\text{system}} & D_{\text{system}} \end{bmatrix}_{2\times 2}$$

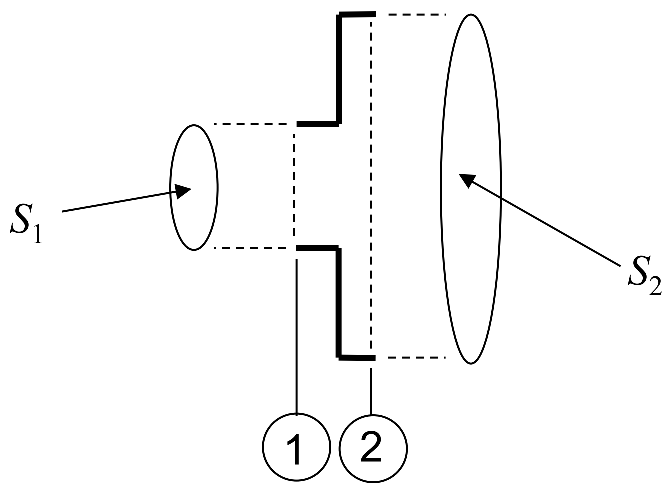
Straight Tube with Absorptive Material



(complex wave number and complex characteristic impedance)

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} \cos(k'L) & \dfrac{j z_c}{S_2}\sin(k'L) \\ \dfrac{j S_1}{z_c}\sin(k'L) & \dfrac{S_1}{S_2}\cos(k'L) \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

Area Change

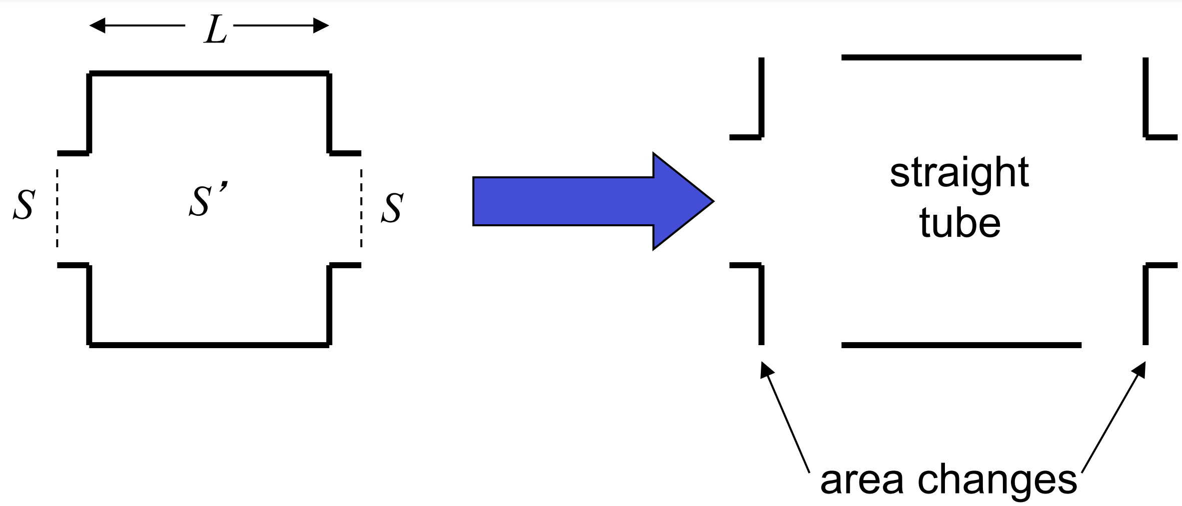


$$p_1 = p_2$$

$$S_1 u_1 = S_2 u_2$$

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

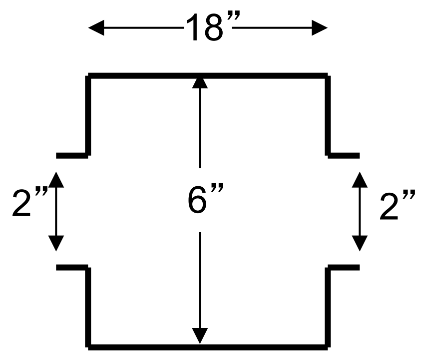
Expansion Chamber Muffler



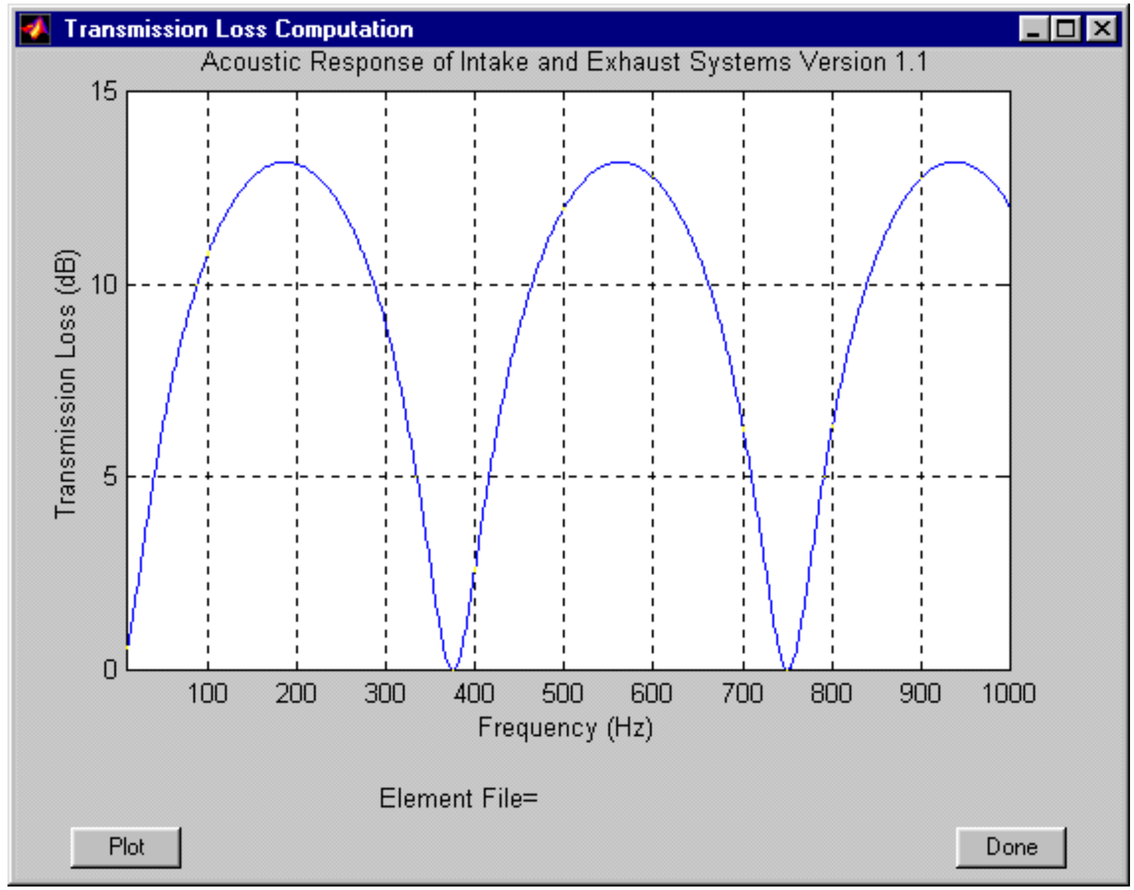
$$[T]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\cos(kL) & \dfrac{j\rho_o c}{S'}\sin(kL)\\ \dfrac{jS'}{\rho_o c}\sin(kL) & \cos(kL)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$[T]=\begin{bmatrix}\cos(kL) & \dfrac{j\rho_o c}{S'}\sin(kL)\\ \dfrac{jS'}{\rho_o c}\sin(kL) & \cos(kL)\end{bmatrix}$$

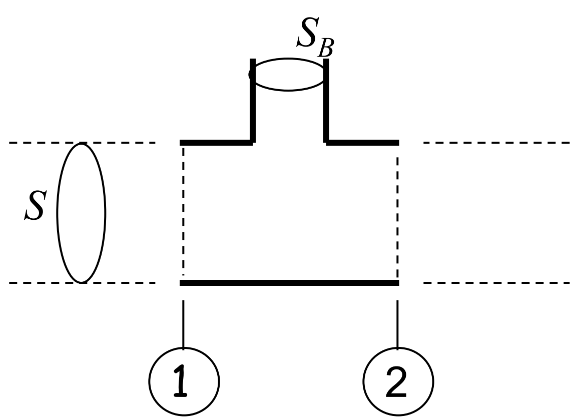
Expansion Chamber Muffler



$$\frac{S'}{S} = 9$$



Transfer Matrix of a Side Branch



$$p_1 = p_2 = p_B$$

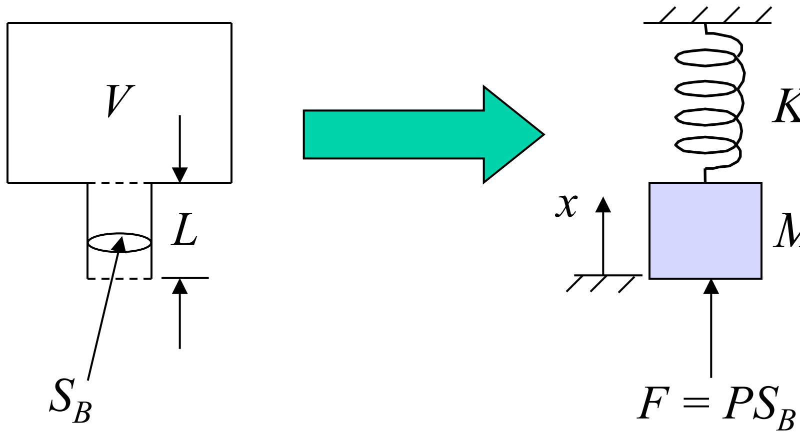
$$Su_1 = S_B u_B + Su_2$$

$$z_B = p_B / S_B u_B = p_2 / S_B u_B$$

$$Su_1 = \left(p_2 / z_B \right) + Su_2$$

$$\begin{Bmatrix} p_1 \\ Su_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/z_B & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ Su_2 \end{Bmatrix}$$

Helmholtz Resonator Model



$$K = \frac{\rho_o c^2 S_B^2}{V}$$

$$M = \rho_o S_B L'$$

L' is the equivalent length of the neck (some air on either end also moves).

$$M\ddot{x} + Kx = PS_B \qquad \ddot{x} = j\omega u_B \qquad x = \frac{u_B}{j\omega}$$

Damping due to viscosity in the neck are neglected

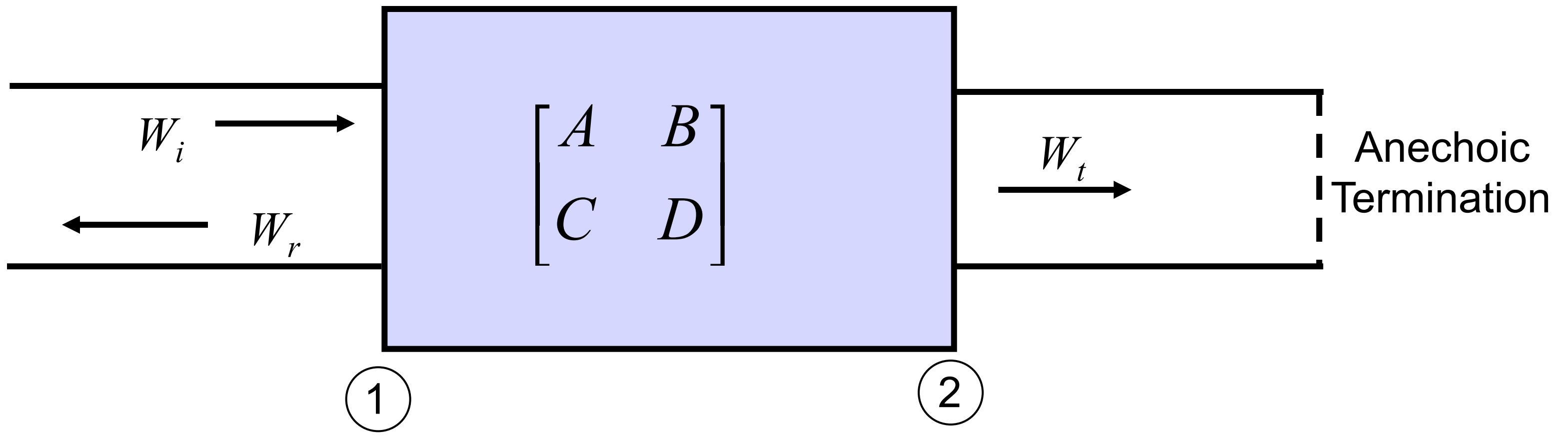
$$j\left(\omega M - \frac{K}{\omega}\right)u_B = PS_B$$

$$z_B = \frac{P}{S_B u_B} = j\left(\frac{1}{S_B^2}\right)\left(\omega M - \frac{K}{\omega}\right)$$

$$z_B \to 0 \quad \text{when} \quad \omega = \sqrt{\frac{K}{M}} = c\sqrt{\frac{S_B}{L'V}}$$

(resonance frequency of the Helmholtz resonator)

Performance Measures Transmission Loss

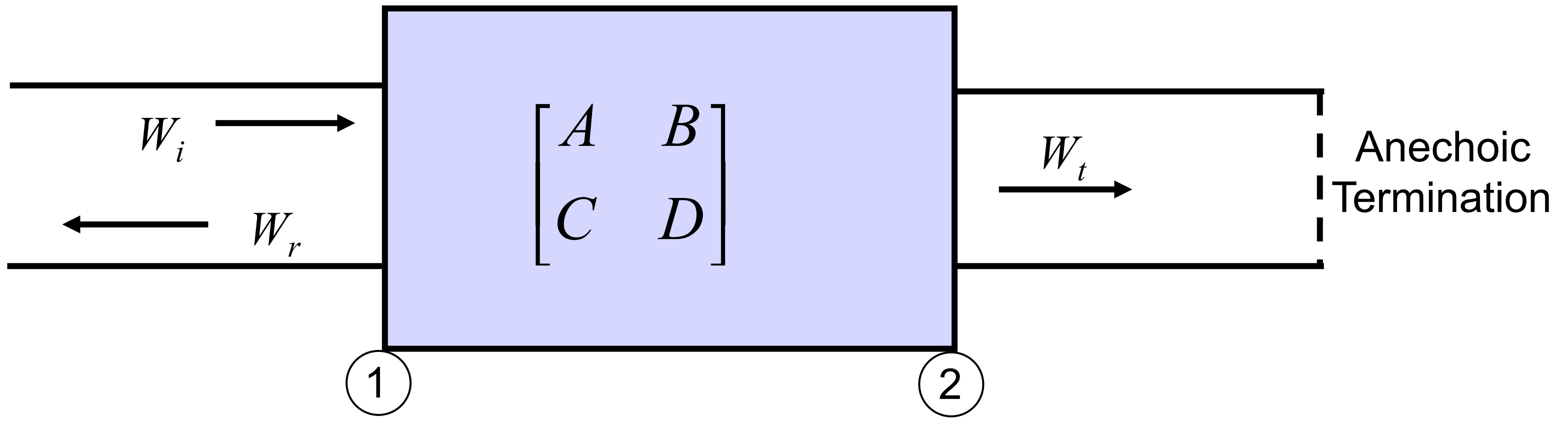


Transmission loss (TL) of the muffler:

$$TL(\mathrm{dB}) = 10\log_{10}\frac{W_i}{W_t}$$

$$TL = 10\log_{10}\left\{\frac{S_{in}}{4S_{out}}\left|A + \frac{S_{out}B}{\rho c} + \frac{\rho c C}{S_{in}} + \frac{S_{out}}{S_{in}}D\right|^2\right\}$$

Derivation Transmission Loss



$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

Express p_1, p_2, u_1 and u_2 in terms of incident reflected waves

$$p_1 = p_{+a} + p_{-a}$$

$$u_1 = \frac{p_{+a} - p_{-a}}{\rho c}$$

$$p_2 = p_{+b}$$

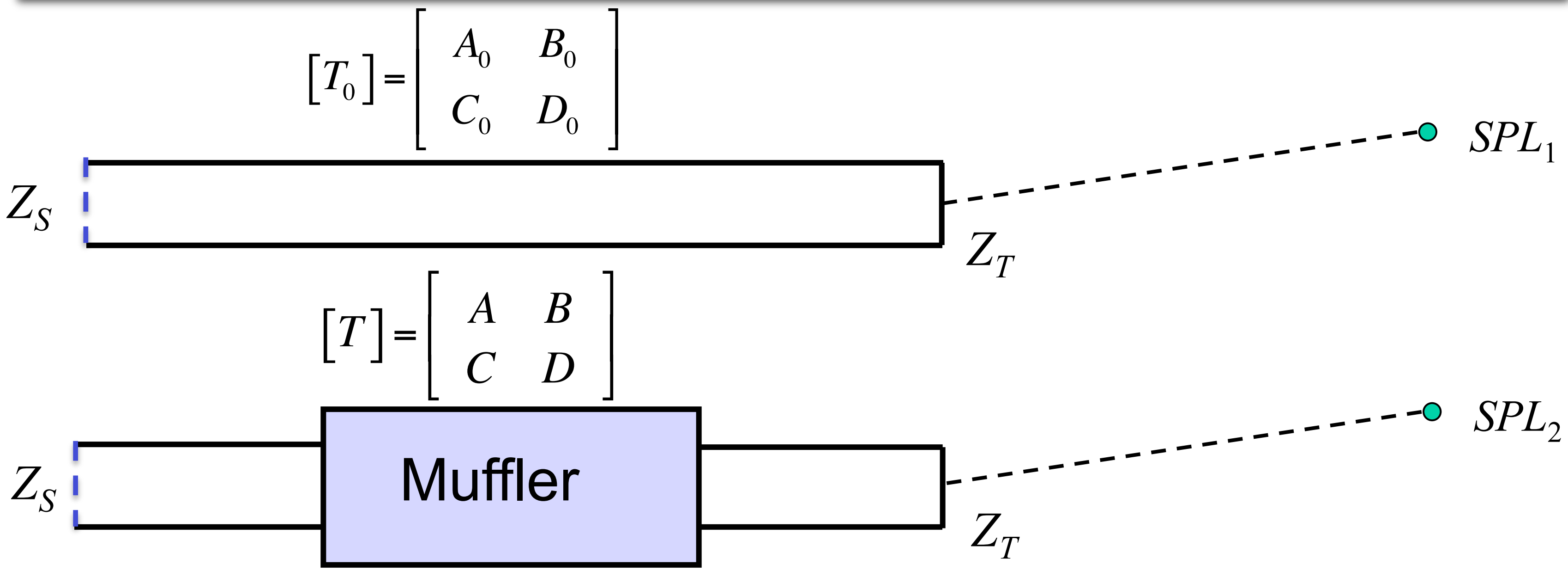
$$u_2 = \frac{p_{+b}}{\rho c}$$

$$W_i = \frac{p_{+a}^2}{\rho c} S_1$$

$$W_t = \frac{p_{+b}^2}{\rho c} S_2$$

$$TL = 10 \log \frac{W_i}{W_t}$$

Performance Measures Insertion Loss

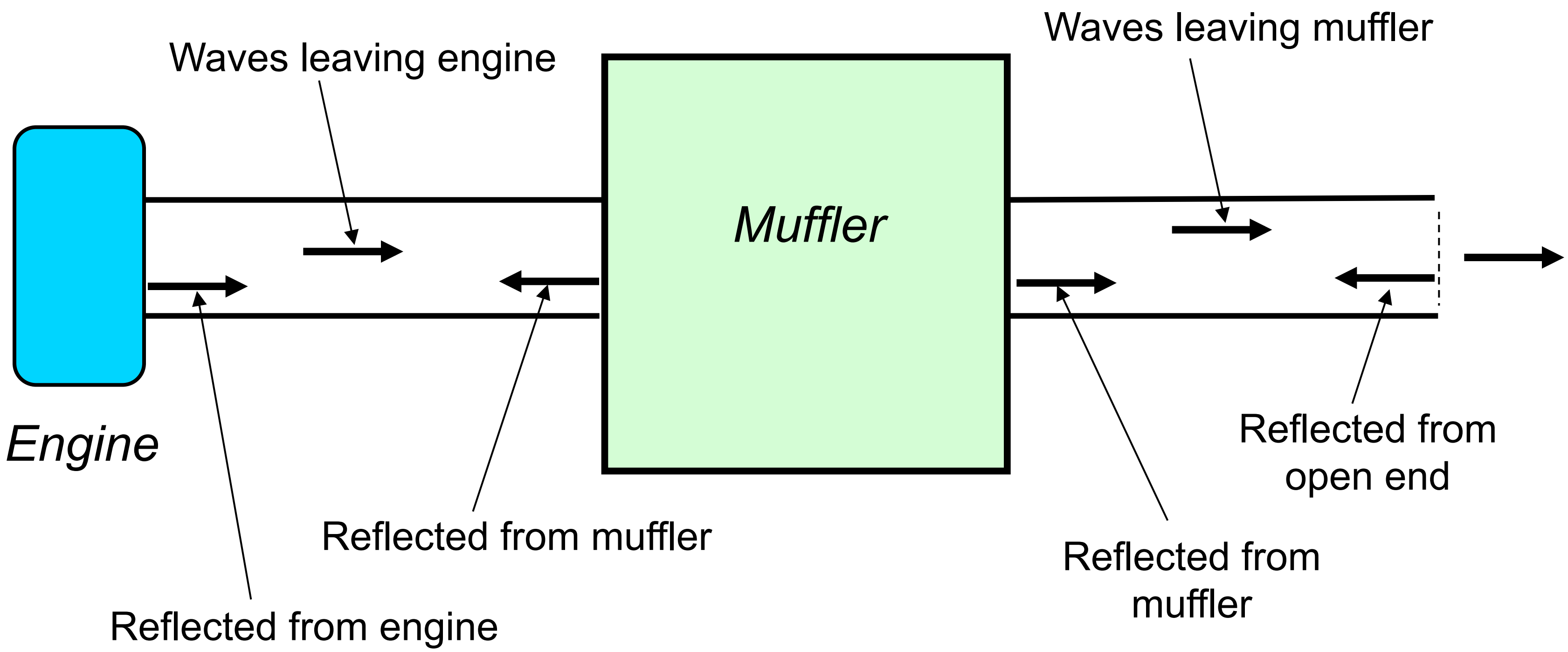


$$[T_0] = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix}$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

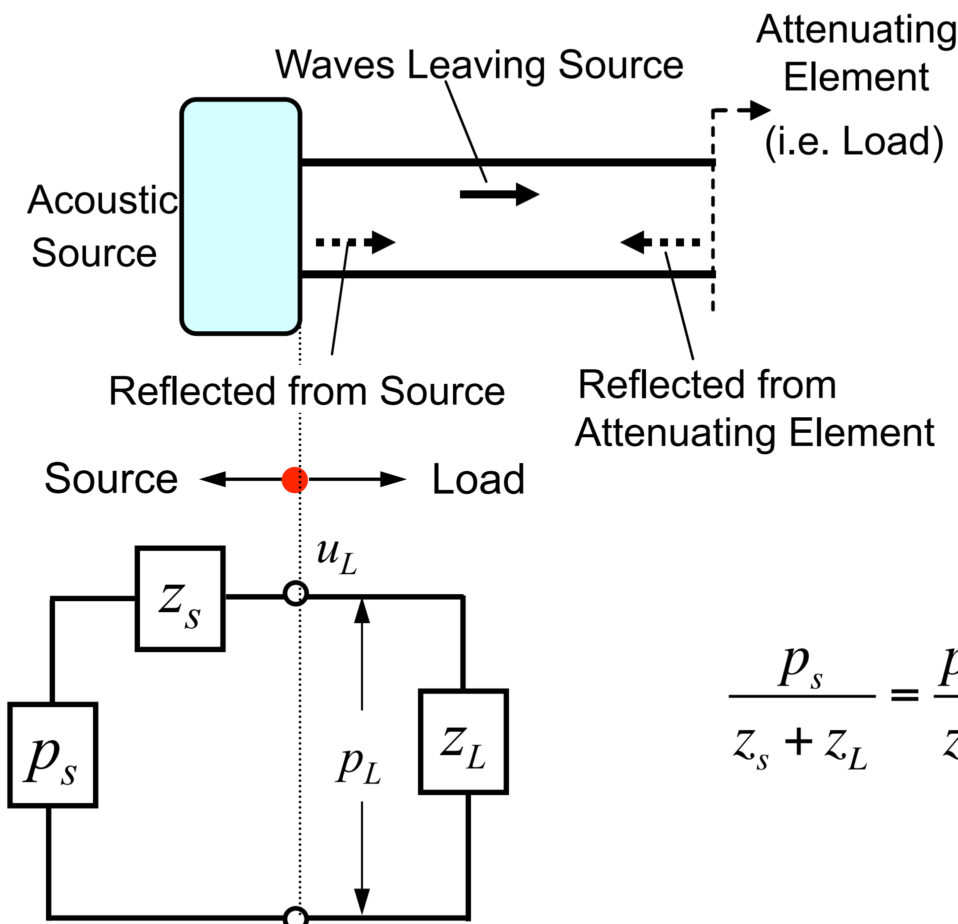
$$IL = 20\log_{10}\left\{\left|\frac{A/Z_S + B/Z_T Z_S + C + D/Z_T}{A_0/Z_S + B_0/Z_T Z_S + C_0 + D_0/Z_T}\right|\right\}$$

Sound Wave Reflections in Engines



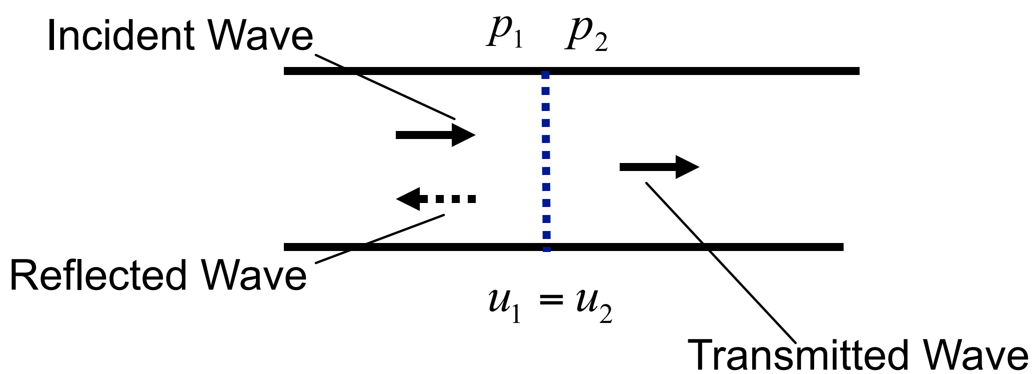
Resonances can form in the exhaust and tail pipes as well as within the muffler.

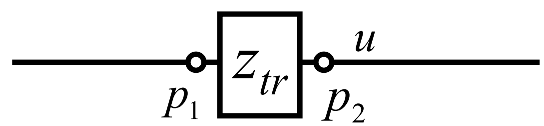
Source Impedance



$$\frac{p_s}{z_s + z_L} = \frac{p_L}{z_L}$$

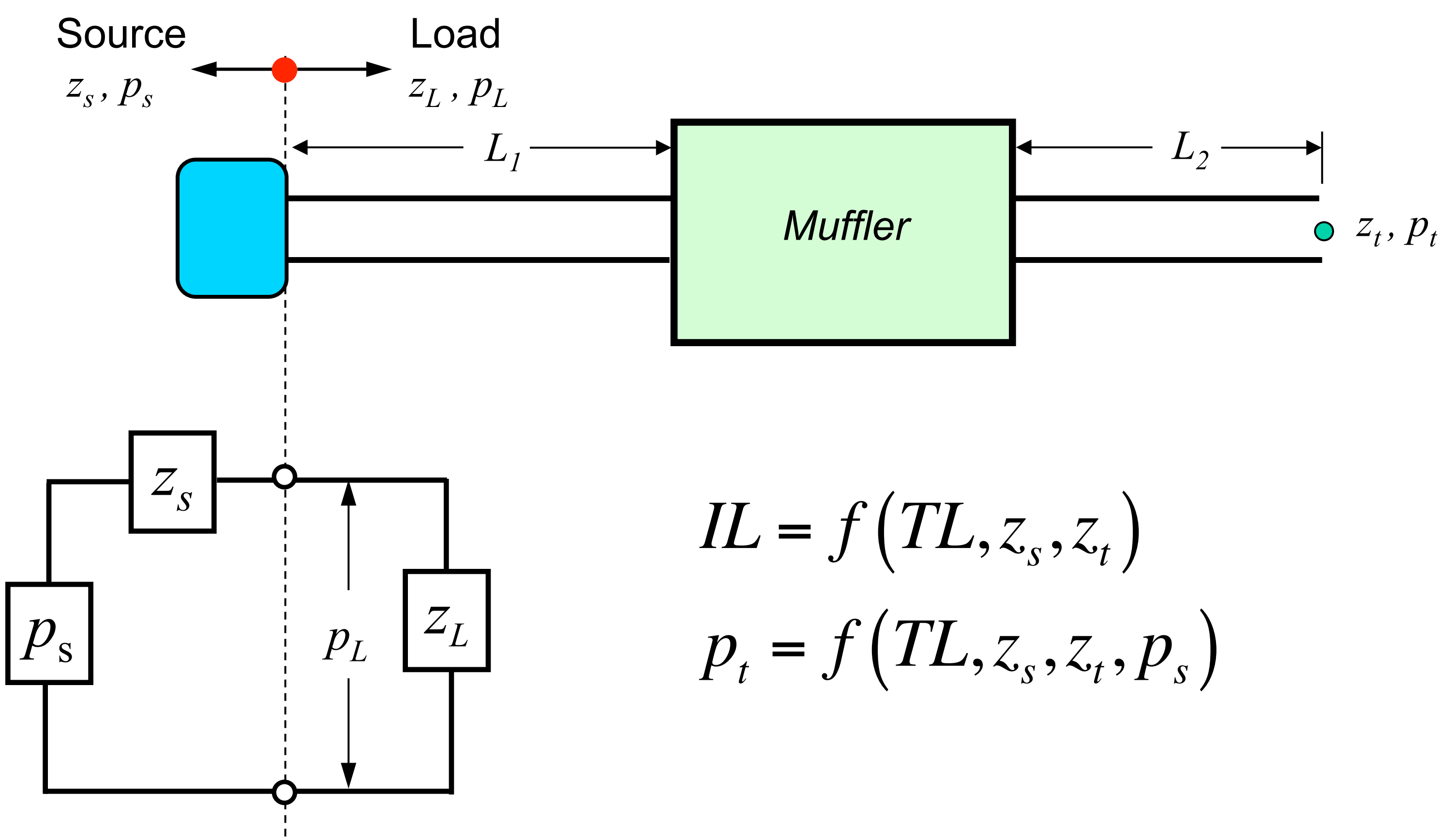
Transfer Impedance





$$z_{tr} = \frac{p_1 - p_2}{Su}$$

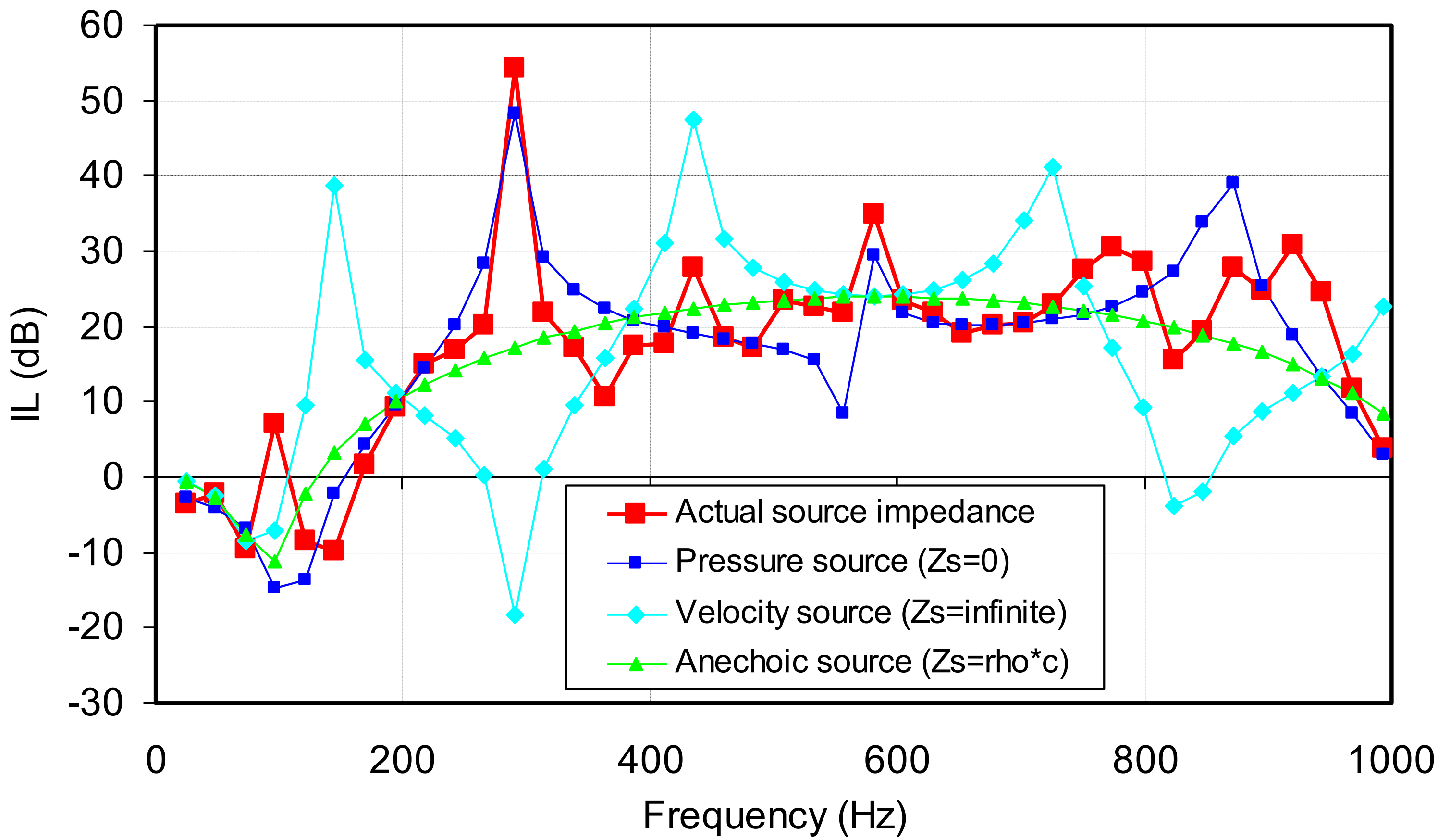
Source/Load Concept



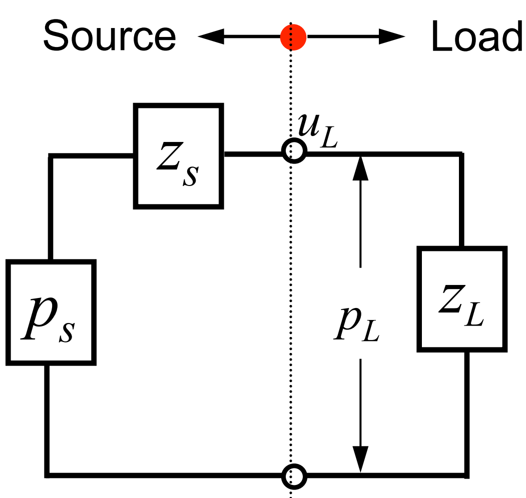
$$IL = f\left(TL, z_s, z_t\right)$$

$$p_t = f\left(TL, z_s, z_t, p_s\right)$$

Insertion Loss Prediction



Source Impedance Series Impedance



$$\frac{p_s}{z_s + z_L} = \frac{p_L}{z_L} = Su_L$$

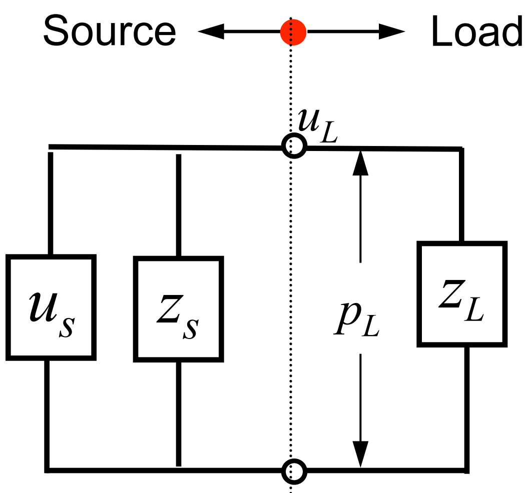
$$\Downarrow$$

$$p_s = Su_L z_s + p_L$$

$$\Downarrow$$

$$z_s = \frac{p_s - p_L}{Su_L}$$

Source Impedance Parallel Impedance

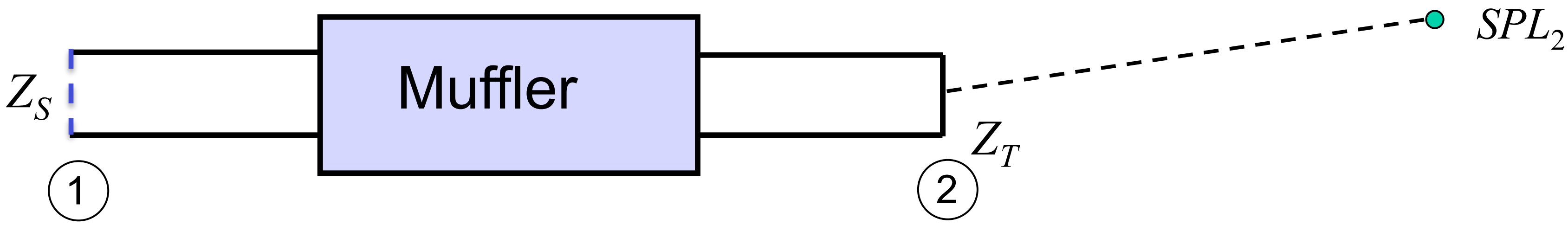


$$S u_L z_L = S u_s \frac{z_s z_L}{z_s + z_L} = p_L$$

$$\Downarrow$$

$$z_s = \frac{p_L}{S\left(u_s - u_L\right)}$$

Derivation Insertion Loss



$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

$$z_S = \frac{p_s - p_1}{S_1 u_1} \Rightarrow p_1 = p_s - S_1 u_1 z_s$$

$$z_T = \frac{p_2}{S_2 u_2} \Rightarrow S_2 u_2 = \frac{p_2}{z_T}$$

$$p_1 = A p_2 + \frac{B}{z_T} p_2$$

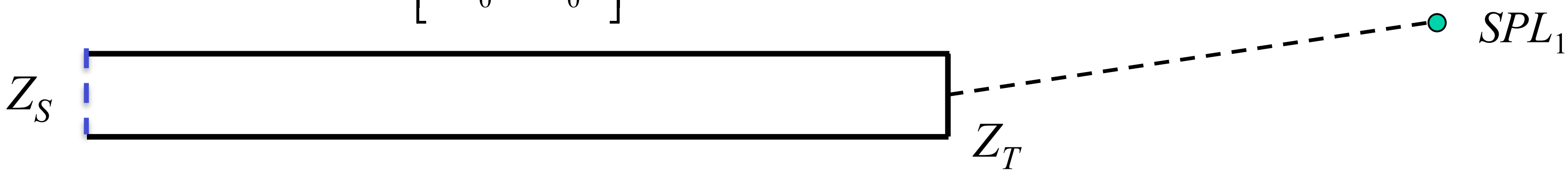
$$\begin{aligned} p_1 &= p_s - S_1 u_1 z_S \\ &= p_s - z_S \left(C p_2 + D \frac{p_2}{z_T} \right) \end{aligned}$$

$$A p_2 + \frac{B}{z_T} p_2 = p_s - z_S \left(C p_2 + D \frac{p_2}{z_T} \right)$$

$$\frac{p_2}{p_s} = \frac{1}{A + \frac{1}{z_T} B + z_S C + \frac{z_S}{z_T} D}$$

Derivation Insertion Loss

$$[T_0] = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix}$$



Determined in same manner as prior slide

$$\frac{p_2}{p_s} = \frac{1}{A_0 + \frac{1}{z_T} B_0 + z_S C_0 + \frac{z_S}{z_T} D_0}$$

Derivation Insertion Loss

$$IL = 20\log\left|\frac{p_{2,\text{no muffler}}}{p_{2,\text{muffler}}}\right|$$

$$\frac{p_{2,\text{no muffler}}}{p_s} = \frac{1}{A_0 + \frac{1}{z_T}B_0 + z_S C_0 + \frac{z_S}{z_T}D_0}$$

$$\frac{p_{2,\text{muffler}}}{p_s} = \frac{1}{A + \frac{1}{z_T}B + z_S C + \frac{z_S}{z_T}D}$$

$$IL = 20\log_{10}\left\{\left|\frac{A/Z_S + B/Z_T Z_S + C + D/Z_T}{A_0/Z_S + B_0/Z_T Z_S + C_0 + D_0/Z_T}\right|\right\}$$

Summary 3

- The transfer matrix method is based on plane wave (1-D) acoustic behavior (at component junctions).
- The transfer matrix method can be used to determine the system behavior from component "transfer matrices."
- Applicability is limited to cascaded (series) components and simple branch components (not applicable to successive branching and parallel systems).