Double inverted pendulum stabilization

around equilibrium point $\theta_1 = \theta_2 \approx 0$

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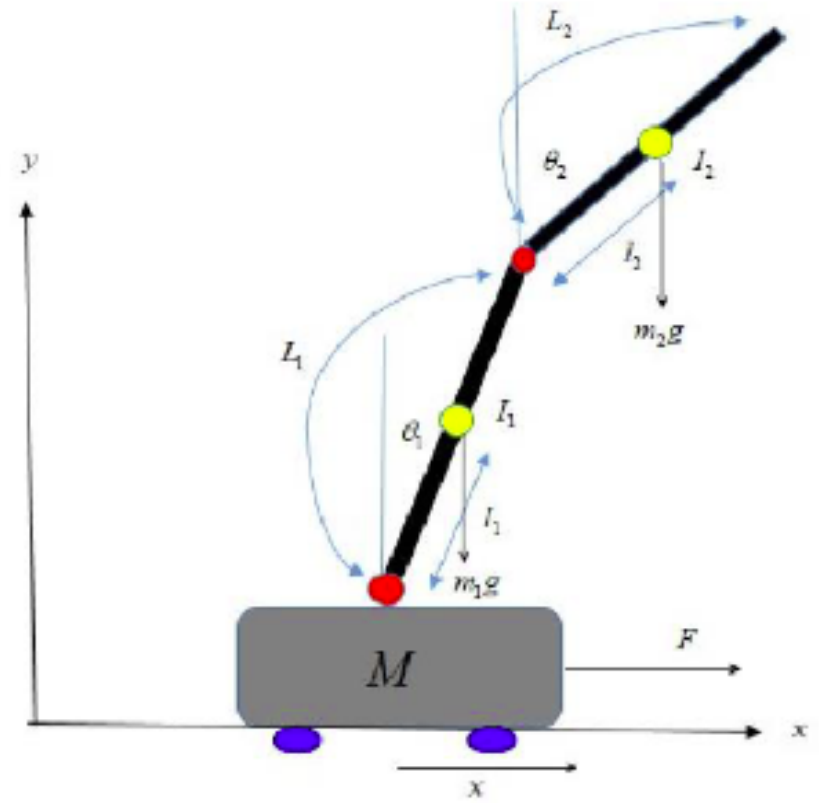
System description



System description

Parameters:

- L_1, L_2–lenghts of bottom and top pendulum $l_i = \frac{L_i}{2}$
- m_1, m_2– masses of the lower and upper pendulum, respectively
- $I_i = \frac{1}{3} m L_i^2$ – inertia moment
- θ_1, θ_2– angles of deviation from vertical of the lower and upper pendulum, respectively (state variables)
- M – mass of cartpole
- x – position of cartpole (state vatiable)
- F – applied force (control variable)
- $g = 9.81$ – gravitational constant

Equations of moution

Lagrange Function L:

$$L = T - V$$

T – kinetic energy
V– potential energy

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

Equations of moution

Linearized system $\theta_1 = \theta_2 \approx 0$:

$$\begin{pmatrix} (M + m_1 + m_2) & (m_1 l_1 + m_2 L_1) & m_2 l_2 \\ (m_1 l_1 + m_2 L_1) & (m_1 l_1^2 + m_2 L_1^2 + I_1) & m_2 l_2 L_1 \\ m_2 l_2 & m_2 L_1 l_2 & (m_2 l_2^2 + I_2) \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & -g\,(m_1 l_1 + m_2 L_1) & 0 \\ 0 & 0 & -g m_2 l_2 \end{pmatrix} \begin{pmatrix} x \\ \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} F \\ 0 \\ 0 \end{pmatrix}$$

Control Design

In the state space our equations transforms to:

$$\dot{X} = AX + BU$$

$$A = M^{-1}N, \quad B = M^{-1}F$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (M + m_1 + m_2) & (m_1 l_1 + m_2 L_1) & m_2 l_2 \\ 0 & 0 & 0 & (m_1 l_1 + m_2 L_1) & \left(m_1 l_1^2 + m_2 L_1^2 + I_1\right) & m_2 l_2 L_1 \\ 0 & 0 & 0 & m_2 l_2 & m_2 L_1 l_2 & \left(m_2 l_2^2 + I_2\right) \end{pmatrix}$$

Control Design

$$\dot{X} = AX + BU$$

$$A = M^{-1}N, \quad B = M^{-1}F$$

$$N = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g\left(m_1 l_1 + m_2 L_1\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & -g m_2 l_2 & 0 & 0 & 0 \end{pmatrix}$$

$$F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

Control Design

Then we use LQR control

$$J = \int xQx + uRu$$

$$Q = E,\ E - 6 \times 6\ identity\ matrix$$

$$R = 1$$

$$u = -KX$$

Results

```
# output is omitted for this code cell
scenario.run()
```

```
[11:36:26] INFO     runn. objective: 0.00, state est.: [1.  0.  0.  0.  0.1 0.1], observation: [1.
                    0.  0.  0.  0.1 0.1], action: [-4.05], value: 0.0000, time: 0.0000 (0.0%),
                    episode: 1/1, iteration: 1/1
```

```
           INFO     runn. objective: 0.00, state est.: [1.   0.02 0.01 0.03 0.09 0.04],
                    observation: [1.   0.02 0.01 0.03 0.09 0.04], action: [2.84], value: 0.0000,
                    time: 0.1300 (13.0%), episode: 1/1, iteration: 1/1
```

```
           INFO     runn. objective: 0.00, state est.: [ 1.01  0.02  0.01  0.13 -0.09 -0.01],
                    observation: [ 1.01  0.02  0.01  0.13 -0.09 -0.01], action: [1.64], value:
                    0.0000, time: 0.2500 (25.0%), episode: 1/1, iteration: 1/1
```

```
           INFO     runn. objective: 0.00, state est.: [ 1.03  0.    0.    0.17 -0.11 -0.06],
                    observation: [ 1.03  0.    0.    0.17 -0.11 -0.06], action: [0.09], value:
                    0.0000, time: 0.4100 (41.0%), episode: 1/1, iteration: 1/1
```

```
           INFO     runn. objective: 0.00, state est.: [ 1.06 -0.01 -0.01  0.15 -0.07 -0.06],
                    observation: [ 1.06 -0.01 -0.01  0.15 -0.07 -0.06], action: [-0.72], value:
                    0.0000, time: 0.5800 (58.0%), episode: 1/1, iteration: 1/1
```

```
           INFO     runn. objective: 0.00, state est.: [ 1.09 -0.02 -0.02  0.11 -0.03 -0.04],
                    observation: [ 1.09 -0.02 -0.02  0.11 -0.03 -0.04], action: [-1.02], value:
                    0.0000, time: 0.7500 (75.0%), episode: 1/1, iteration: 1/1
```

```
[11:36:27] INFO     runn. objective: 0.00, state est.: [ 1.1  -0.02 -0.02  0.06 -0.01 -0.02],
                    observation: [ 1.1  -0.02 -0.02  0.06 -0.01 -0.02], action: [-1.1], value:
                    0.0000, time: 0.9200 (92.0%), episode: 1/1, iteration: 1/1
```