

$$\phi_{tt} - \phi_{xx} + 2\phi(\phi^2 - 1) = 0$$

$$\phi(x, t) = \phi(x - vt) = \phi(z)$$

$$(v^2 - 1)\phi'' + 2\phi(\phi^2 - 1) = 0 \mid \cdot \phi'$$

$$(v^2 - 1)\phi' \phi'' + 2\phi \phi' (\phi^2 - 1) = 0$$

$$(v^2 - 1)\phi'^2 \frac{1}{2} + \frac{1}{2}(\phi^2 - 1)^2 + C = 0$$

Граничные условия дают, что

$$\phi(\pm\infty) = \pm 1, \quad \phi'(\infty) = 0 \Rightarrow C = 0$$

$$(v^2 - 1)\phi'^2 + (\phi^2 - 1)^2 = 0$$

$$(1 - v^2)\phi'^2 = (\phi^2 - 1)^2$$

$$\phi' = \pm \frac{(\phi^2 - 1)}{\sqrt{(1 - v^2)}}$$

$$\phi = th \left(\mp \frac{(x - x_0) - v(t - t_0)}{\sqrt{(1 - v^2)}} \right)$$

Вычислим энергию

$$E = \int_{-\infty}^{\infty} \frac{1}{2} \left(\left(\frac{\partial \phi}{\partial t} \right)^2 + \left(\frac{\partial \phi}{\partial x} \right)^2 + (1 - \phi^2)^2 \right) dx = \frac{4}{3\sqrt{1 - v^2}}$$

Вычислим импульс

$$P = \int_{-\infty}^{\infty} \partial_t \phi \partial_x \phi dx = \frac{4v}{3\sqrt{1 - v^2}}$$