

Bifurkace v obyčejných diferenciálních rovnicích

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Anotace

moje super anotace

Anotation

my awesome anotation

Klíčová slova

sjdls, sdjvlsd, lkstdlv, lkstda

Keywords

sův, kslr, jada.jda

Poděkování

Moc děkuji Váam

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Seznam obrázků

Přehled použitých značek

q_A – tepelný tok

k – součinitel kondukce

∇ – gradient, definován jako $\nabla = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial x} \right)$

T – aboslutní teplota

v_∞ – rychlost náběhu

c – rychlost

u – měrná vnitřní energie

h – měrná entalpie

p – tlak

g – gravitační zrychlení, $g \approx 9.81$

P – výkon

W – práce

Q – teplo

q – měrné teplo

$$[q_A] = W \cdot m^{-2}$$

$$[k] = W \cdot m^{-1} \cdot K^{-1}$$

$$[\nabla] = m^{-1}$$

$$[T] = K$$

$$[v_\infty] = m \cdot s^{-1}$$

$$[c] = m \cdot s^{-1}$$

$$[u] = J \cdot kg^{-1} \cdot K^{-1}$$

$$[h] = J \cdot kg^{-1} \cdot K^{-1}$$

$$[p] = N \cdot m^{-2}$$

$$[g] = m \cdot s^{-2}$$

$$[P] = W$$

$$[W] = J$$

$$[Q] = J$$

$$[q] = J \cdot kg^{-1}$$

Kapitola 1

Úvod

1.1 Introduction to Bifurcation in Ordinary Differential Equations

The study of bifurcation in ordinary differential equations (ODEs) is a cornerstone of dynamical systems theory, offering profound insights into the qualitative changes in system behavior as parameters vary. A bifurcation occurs when a small, smooth change in a system's parameters leads to a sudden and qualitative transformation in its long-term behavior. This phenomenon is pivotal for understanding complex dynamics in fields ranging from physics and biology to engineering and economics.

1.2 Motivation and Importance

Bifurcation theory provides a framework for analyzing how systems transition between different states, such as stability to instability or periodic to chaotic behavior. For example, it helps explain phenomena like the onset of turbulence in fluid dynamics, population dynamics in ecology, or even market crashes in economics. By identifying critical parameter values--bifurcation points--researchers can predict and control these transitions, making bifurcation analysis an essential tool for both theoretical exploration and practical applications.

1.3 Scope of Study

This thesis focuses on bifurcations in the context of ordinary differential equations, particularly nonlinear systems. The primary aim is to explore the mathematical structures underlying bifurcations, classify their types, and analyze their implications for system

stability and dynamics. Key topics include:

- **Types of Bifurcations:** Common bifurcations such as saddle-node, transcritical, pitchfork (supercritical and subcritical), and Hopf bifurcations will be examined. Each type represents a distinct mechanism by which system behavior changes qualitatively.
- **Analytical Techniques:** Methods for detecting and characterizing bifurcations, including stability analysis through eigenvalues of the Jacobian matrix and graphical approaches like bifurcation diagrams.
- **Applications:** Real-world examples where bifurcation theory provides critical insights into phenomena such as biological pattern formation, mechanical vibrations, or electrical circuit behavior.

1.4 Historical Context

The concept of bifurcation was first introduced by Henri Poincaré in 1885 during his pioneering work on dynamical systems. Since then, it has evolved into a comprehensive mathematical theory with significant contributions from researchers like Andronov, Hopf, and Smale. Modern advancements have extended its applicability to high-dimensional systems and complex networks.

1.5 Methodology

The thesis employs a combination of theoretical analysis and computational simulations to investigate bifurcations. Analytical methods will be used to derive conditions for the occurrence of bifurcations, while numerical tools will aid in visualizing bifurcation diagrams and exploring parameter spaces.

1.6 Structure of the Thesis

1. **Fundamentals of Bifurcation Theory:** An introduction to key concepts, including definitions, classifications, and mathematical formulations.
2. **Classic Examples:** Detailed exploration of well-known bifurcations with illustrative examples from one-dimensional ODEs.
3. **Advanced Topics:** Discussion on higher-dimensional systems, center manifold theory, and global bifurcations.

4. Applications: Case studies demonstrating the practical relevance of bifurcation analysis across various disciplines.
5. Conclusion: Summary of findings and potential directions for future research.

1.7 Significance

Understanding bifurcations in ODEs not only deepens our comprehension of nonlinear dynamics but also equips us with tools to predict and manage critical transitions in diverse systems. This thesis aims to contribute to this understanding by providing a detailed exploration of the mathematical underpinnings and practical implications of bifurcations.

Through this study, we hope to shed light on the intricate interplay between parameters and system behavior, offering insights that transcend disciplinary boundaries.

$$\iint_{\Omega} u(x, y, t) d\Omega = C$$

slkj*si* [1]

Příloha A

Code

Literatura

- [1] Eric Benoît. *Dynamic bifurcations: proceedings of a conference held in Luminy, France, March 5-10, 1990.* Springer, 2006.