2019 International Conference on Electrical, Computer and Communication Engineering (ECCE), 7-9 February, 2019

PID-SMC controller for a 2-DOF planar robot

Umme Zakia[1], Mehrdad Moallem[2] and Carlo Menon[1], *Member, IEEE*

***Abstract*— Sliding mode controller (SMC) is considered as a robust control method. In this paper, a combination of SMC and traditional proportional-integral-differential (PID) control scheme for a 2-degree of freedom (2-DOF) planar manipulator is considered. Since the PID control does not require precise system modelling, a sliding mode surface for the SMC control is designed using the PID component parameters of the manipulator. The PID control adjusts the errors of the closed loop system and the SMC control ensures fast convergence towards the sliding mode surface. Hence, this hybrid controller possesses features of both control schemes and obtains better system stability.**

I. Introduction

Controlling systems in industrial environment is a continuous process and requires monitoring for quality of production management [1]. Closed loop systems are widely used in the industries where feedback mechanisms are implemented to adjust errors produced in the systems [1], [2]. PID controller is well known as a closed loop mechanism where proportional, integral and differential of the errors make the system adjustable and delivers the desired output [3].

Sliding mode controller is used in nonlinear systems that modifies dynamics of the system and forces the system to slide along the system's standard behavior. It is robust and efficient in maintaining stability for nonlinear dynamic systems. The characteristics and parameters of the SMC control design is governed by the sliding mode surface. To obtain the sliding mode, the system state has to satisfy the dynamic equation, and the sliding motion is ensured by infinite switching [4].

Control mechanism in industrial environments are largely designed with the popular PID controllers. Recent researches focus on sliding mode control designs which are compared with the traditional PID control. In recent years, researches are being carried out to study the performance improvements of trajectory control of robotic systems by combining these two controllers. In 2005, a chattering-free controller eliminates torque rippling problem for a 2- DOF direct-drive system and improves performance during position and trajectory tracking [5]. A sliding mode control scheme for a deep-sea manipulator is designed with self-adaptive velocity control and trajectory tracking and obtains good performance [6]. For a 2-DOF robot, a fuzzy tuning process is developed to adjust gain of PID and SMC controller where the dynamic equation is defined by a sliding surface [7]. An adaptive neuro-fuzzy inference system is proposed to solve inverse kinematics for controlling tracking of 3- DOF end-effector for rehabilitation [8]. A sliding mode controller based on sliding surface with PID for a 3- DOF spatial manipulator for tracking performance obtains higher accuracy and improved robustness [9]. A combination of adaptive PD and SMC controller shows stable system design with appropriate control law [10]. A non-linear sliding control algorithm for 6-DOF arm-like manipulator shows efficient trajectory tracking over the conventional PID control mechanism [11]. An SMC control for controlling major axes joint angles of the 3-DOF Puma manipulator obtains good tracking characteristics [12]. A non-linear PD with SMC control for a 3-DOF planar robot is designed for better tracking performances [13]. For pick-and-place tasks with accurate position control, a fast and robust sliding model controller is observed [14].

In this paper, we design a hybrid PID-SMC controller for a 2-DOF planar robot which has two revolute joints (RR). First, the PID controller is developed, then it is used as the sliding surface of the SMC controller with a sign function. The combined scheme provides fast adaptation of the uncertainty related to robot dynamics and better system stability. Our perspective is to study the effectiveness of the proposed scheme and to compare the performance of the PID controller with the proposed PID-SMC control scheme for the robot system. Simulations are performed and results show improvements of the PID-SMC controller over the conventional PID controller.

II. Background Study

A. 2-DOF Planar Manipulator

The dynamic equations of motion of a robot is described as,

$$B\,(q)\,q'' + C\,(q', q) + g\,(q) = F \tag{1}$$

where $B(q)$ is the mass inertia, $C(q)$ is the Coriolis force on the manipulator, $g(q)$ is the gravitational force acting on the manipulator and q'', q', q are acceleration, velocity and displacement of the joints of the manipulator respectively, and F is the joint driving torque [1], [2].

Our focus is to observe the performance of a hybrid PID-SMC controller and we have chosen a simple 2 -DOF planar robot, a classic multi-input multi-output (MIMO) system and a benchmark for assessment performance of nonlinear dynamic system. For simplicity, we assume that there are no external disturbances present in the system. The Lagrange equation to obtain the dynamics of a planar robot with two revolute joints is as follows

$$\mathcal{L} = KE - PE \tag{2}$$

where KE is the kinetic energy and PE is the potential energy of the robot. Fig.1 shows a 2-DOF planar manipulator that has two revolute joints with, $M_1 = M_2 =$ masses of link 1 and link 2, and $L_1 = L_2 =$ link length 1 and link length 2, respectively. Thus, we can write

$$x_1 = L_1 \sin\theta_1 \qquad (3),$$

$$y_1 = L_1 \cos\theta_1 \qquad (4),$$

Research supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), the Canadian Institutes of Health Research (CIHR) and the Canada Research Chair (CRC) program.

[1]Authors are with Menrva Research Group, Schools of Mechatronic systems Engineering and Engineering Science, Simon Fraser University, Metro Vancouver, BC, Canada (email: {uzakia, cmenon}@sfu.ca).

[2]Author is in Schools of Mechatronic systems Engineering, Simon Fraser University, Surrey, BC, Canada (email: mmoallem@sfu.ca).

*Corresponding author: C. Menon (email: cmenon@sfu.ca, phone: +1-778-782-9338; fax: +1-778-782-7514).

978-1-5386-9111-3/19/$31.00 ©2019 IEEE

$$x_2 = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \quad (5),$$

$$y_2 = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \quad (6).$$

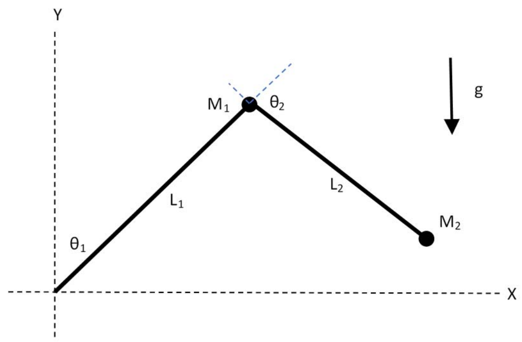

Fig. 1. 2-DOF (RR) Planar robot with two revolute joints.

B. *Dynamics of the 2-DOF Planar Manipulator*

The kinetic energy (*KE*) of the 2-DOF planar manipulator could be formed as follows

$$KE = \tfrac{1}{2} M_1 x_1'^2 + \tfrac{1}{2} M_1 y_1'^2 + \tfrac{1}{2} M_2 x_2'^2 + \tfrac{1}{2} M_2 y_2'^2 \quad (7)$$

Further simplification of (7) results in

$$KE = \tfrac{1}{2}(M_1+M_2)L_1^2\theta_1'^2 + \tfrac{1}{2} M_2 L_2^2\theta_1'^2 + M_2L_2^2\theta_1'\theta_2' + M_2L_2^2\theta_2'^2 + M_2L_1L_2\cos\theta_2(\theta_1'\theta_2' + \theta_1'^2) \quad (8)$$

The Potential Energy is given by

$$PE = M_1 g L_1\cos\theta_1 + M_2 g(L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)) \quad (9)$$

Next, we can obtain the Lagrange Dynamics based on (8) and (9) as follows

$$\mathcal{L} = \tfrac{1}{2}(M_1+M_2)L_1^2\theta_1'^2 + \tfrac{1}{2} M_2L_2^2\theta_1'^2 + M_2L_2^2\theta_1'\theta_2' + \tfrac{1}{2} M_2L_2^2\theta_2'^2 + M_2L_1L_2\cos\theta_2(\theta_1'\theta_2' + \theta_1'^2) - M_1gL_1\cos\theta_1 - M_2g(L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)) \quad (10)$$

Now, let us form the dynamics using the following

$$f_{\theta 1,2} = d/dt\,[\partial\mathcal{L} / \partial\theta'_{1,2}] - \partial\mathcal{L} / \partial\theta_{1,2} \quad (11)$$

The dynamic equations can be simplified as follows

$$((M_1+M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2)\,\theta_1'' + (M_2L_2^2 + M_2L_1L_2\cos\theta_2)\,\theta_2'' - M_2L_1L_2\sin\theta_2(2\theta_1'\theta_2' + \theta_2'^2) - (M_1+M_2)\,gL_1\sin\theta_1 - M_2gL_2\sin(\theta_1+\theta_2) = f_{\theta 1} \quad (12)$$

$$(M_2L_2^2 + M_2L_1L_2\cos\theta_2)\,\theta_1'' + M_2L_2^2\theta_2'' - M_2L_1L_2\sin\theta_2\theta_1'\theta'_2 - M_2gL_2\sin(\theta_1+\theta_2) = f_{\theta 2} \quad (13)$$

Finally, we define the motion of the robot by

$$B(q)\,q'' + C(q',q) + g(q) = F\,;$$

where

$$q = [\theta_1 \;\; \theta_2]^{\mathrm{T}} \quad (14),$$

$$B(q) = \begin{bmatrix} (M_1+M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2 & (M_2L_2^2 + M_2L_1L_2\cos\theta_2) \\ M_2L_2^2 + M_2L_1L_2\cos\theta_2 & M_2L_2^2 \end{bmatrix} \quad (15),$$

$$C(q',q) = \begin{bmatrix} -M_2L_1L_2\sin\theta_2\,(2\theta'_1\theta'_2 + \theta'^2_2) \\ -M_2L_1L_2\sin\theta_2\theta'_1\theta'_2 \end{bmatrix} \quad (16),$$

$$g(q) = \begin{bmatrix} -(M_1+M_2)\,gL_1\sin\theta_1 - M_2gL_2\sin(\theta_1+\theta_2) \\ -M_2gL_2\sin(\theta_1+\theta_2) \end{bmatrix} \quad (17),$$

and

$$F = [f_{\theta 1} \;\; f_{\theta 2}]^{\mathrm{T}} \quad (18).$$

The dynamics of the system can be expressed with the state-space equations. To this end, let

$$x_1 = \int e\,\theta_1\,dt,$$

which results in

$$x_1' = \theta_{1f} - \theta_1 \quad (19),$$

And, let

$$x_2 = \int e\,\theta_2\,dt$$

$$\Rightarrow \quad x_2' = \theta_{2f} - \theta_2 \quad (20).$$

Thus, the complete system equations are given by

$$\begin{cases} x_1' = \theta_{1f} - \theta_1 \\ x_2' = \theta_{2f} - \theta_2 \\ [\theta''_1 \;\; \theta''_2]\mathrm{T} = B(q)^{-1}[-C(q',q) - g(q)] + \\ \qquad [K_{p1}(\theta_{1f} - \theta_1) - K_{D1}\theta_1' + K_{I1}x_1 \\ \qquad K_{p2}(\theta_{2f} - \theta_2) - K_{D2}\theta_2' + K_{I2}x_2\,] \end{cases} \quad (21)$$

III. PROPOSED PID-SMC CONTROLLER

The system control scheme is based on the dynamics of the 2-DOF planar robot and the motion of the robot is based on the system equation (1). We can derive the accelerations of the joints as

$$B(q)\,q'' + C(q',q) + g(q) = F$$

$$\Rightarrow q'' = B(q)^{-1}[-C(q',q) - g(q) + F\,] \quad (22)$$

Let

$$F\sim = B(q)^{-1}F$$

$$\Rightarrow F = B(q)\,F\sim \quad (23)$$

Now, we redefine the system to the new input

$$F\sim = [f_1 \;\; f_2]^{\mathrm{T}} \quad (24)$$

Nevertheless, the torque inputs to the system are given by

$$[f_{\theta 1} \;\; f_{\theta 2}]^{\mathrm{T}} = B(q)\,[f_1 \;\; f_2]^{\mathrm{T}} \quad (25)$$

The error signals related to joint angles are given by

$$e(\theta_1) = \theta_{1f} - \theta_1 \quad (26),$$

$$e(\theta_2) = \theta_{2f} - \theta_2 \quad (27).$$

A. *Developing the PID Controller*

A proportional-Integral-differential (PID) controller for any input would have the general structure as below [1],[2]

$$f = K_P\,e + K_D\,e' + K_I\int e\,dt \quad (28)$$

where K_P, K_I, and K_D are the proportional, integral and derivative control gains of the PID controller.

For the 2-DOF RR planar robot, we have

$$f_1 = K_{P1}(\theta_{1f} - \theta_1) - K_{D1}\theta'_1 + K_{I1}\int e\,\theta_1\,dt \quad (29)$$

$$f_2 = K_{P2}(\theta_{2f} - \theta_2) - K_{D2}\theta'_2 + K_{I2}\int e\,\theta_2\,dt \quad (30)$$

Hence, the complete system equations with the PID control would be given by

$$q'' = B(q)^{-1}[-C(q',q) - g(q)] + F\sim \quad (31)$$

where,

$$F\sim = [f_1 \;\; f_2]^T$$
$$= [\,K_{P1}(\theta_{1f} - \theta_1) - K_{D1}\theta_1 + K_{I1}\textstyle\int e\,\theta_1\,dt$$
$$K_{P2}(\theta_{2f} - \theta_2) - K_{D2}\theta_2 + K_{I2}\textstyle\int e\,\theta_2\,dt] \quad (32)$$

B. Developing the SMC controller

The SMC controller is designed by establishing the sliding mode surface and a switching function responsible for reducing chattering effect. The sliding mode surface is built using the PID controller parameters such as

$$s = K_P e + K_D e' + K_I \textstyle\int e\,dt \quad (33)$$

Here, s is the sliding mode surface and K_P, K_D and K_I are the control gains of the PID control as defined earlier.

The control law of the hybrid PID-SMC is selected as follows

$$u = s + K_{sal}\, Sw\,(s, \sigma) \quad (34)$$

where K_{sal} is the control gain and Sw is the switching function. This function can be defined as follows

$$Sw\,(e, \sigma) = |e|^{\sigma}\, sign\,(e) \quad (35)$$

where σ is chosen between 0 to 1. The switching function combines features of the sign and the exponential function. When the system state is far from the sliding surface, it forces the system to reach the sliding surface quickly. In contrary, when the system state is near the sliding surface, it restricts the approach speed towards the surface. Thus, it can minimize the chattering effect of the system and achieves fast convergence towards stability.

IV. Simulations & Results

For the PID control gain scheme, the state variables x_1', x_2', θ_1'' and θ_2'' are used to represent the dynamics of the 2-DOF robot with MATLAB ode45 command. The simulations are carried out for the PID and the PID-SMC controllers with two sets of parameter values as stated in 'Parameter set 1' and 'Parameter set 2'. For 'Parameter set 1', the final positions are $[\theta_{1f} \;\; \theta_{2f}]^T = [\pi/2 \;\; -\pi/2]^T$ while the system has initial positions of $\theta_0 = [-\pi/2 \;\; \pi/2\,]^T$. For 'Parameter set 2', the final positions are $[\theta_{1f} \;\; \theta_{2f}]^T = [\pi/2 \;\; \pi]^T$ while the system has initial positions of $\theta_0 = [\pi \;\; \pi/2\,]^T$.

A. Simulating with Parameter set 1

The first set of values for the parameters for the PID control gains are selected as follows

$K_{P1}=15$, $K_{D1}=7$, $K_{I1}=10$,

$K_{P2}=15$, $K_{D2}=10$, $K_{I2}=10$.

In the 2-DOF robot design, units for mass and link length are kg and meter, respectively. For simplification, we have considered mass and link lengths as follows

$M_1=M_2=L_1=L_2=1$.

For the SMC control design, K_{sal} is set to 5 for the PID-SMC controller. The value of σ is selected by trial-and-error. It is observed that $\sigma = 0.3$ provides the best results for both PID and PID-SMC controllers.

1) States results of Parameter set 1 with PID control

The error forms of θ_1 & θ_2 for the PID controller is as follows

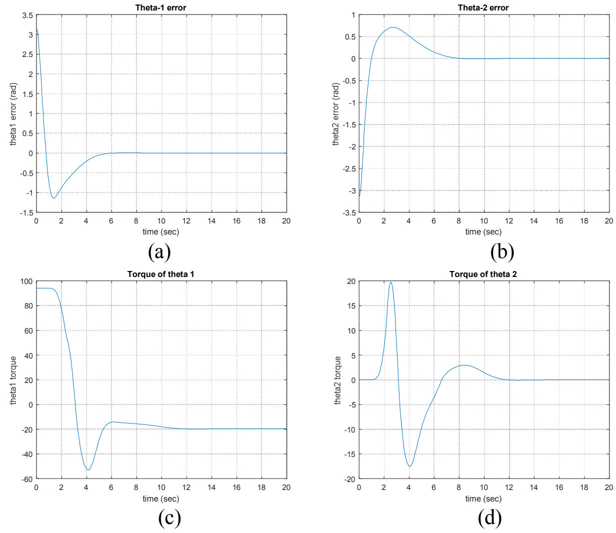

Fig. 2. Error forms of θ_1 (a) & θ_2 (b), Torque of θ_1 (c) & θ_2 (d) for the PID controller (for parameter set 1).

2) States results of Parameter *set 1 with PID-SMC control*

The error forms of θ_1 & θ_2 for the PID-SMC controller is as follows

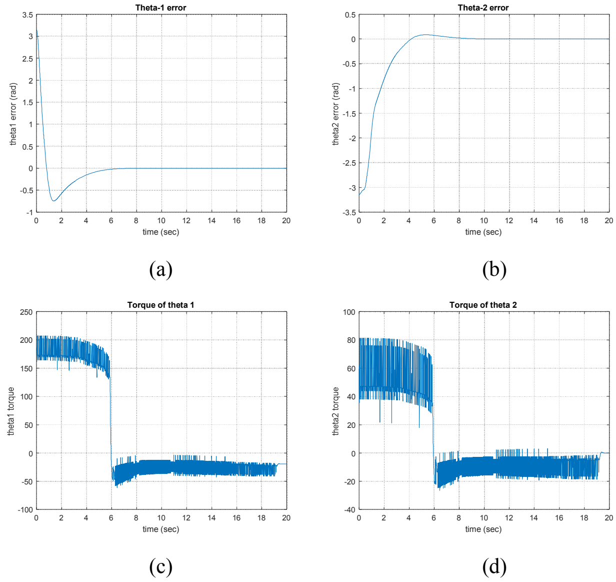

Fig. 3. Error forms of θ_1 (a) & θ_2 (b), Torque of θ_1 (c) & θ_2 (d) for the PID-SMC controller (for parameter set 1).

B. Simulating with Parameter set 2

The second set of values for the parameters are for the PID controller after the trial-and-error are set to as follows

$K_{P1}=30$, $K_{D1}=12$, $K_{I1}=20$,

$K_{P2}=32$, $K_{D2}=22$, $K_{I2}=30$.

Now the mass and link lengths of the 2-DOF robot are considered as

$M_1=5$, $M_2=5$ & $L_1=L_2=0.34$.

where units are in kg and meter.

Furthermore, we use the same values of *Ksal* and σ as mentioned in *Parameter set 1* for the PID-SMC control design such as *Ksal* = 5 and σ = 0.3.

3) States results of Parameter set 2 with PID control

The error forms of θ_1 & θ_2 for the PID controller is as follows

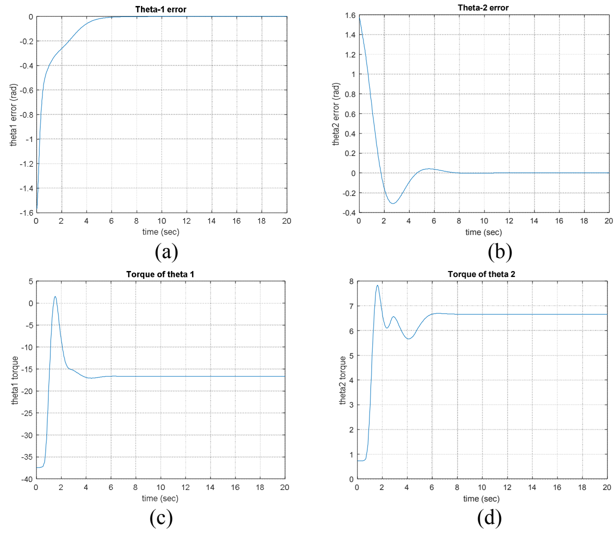

Fig. 4. Error forms of θ_1 (a) & θ_2 (b), Torque of θ_1 (c) & θ_2 (d) for the PID controller (for parameter set 2).

4) States results of Parameter set 2 with PID-SMC control

The error forms of θ_1 & θ_2 for the PID-SMC controller is as follows

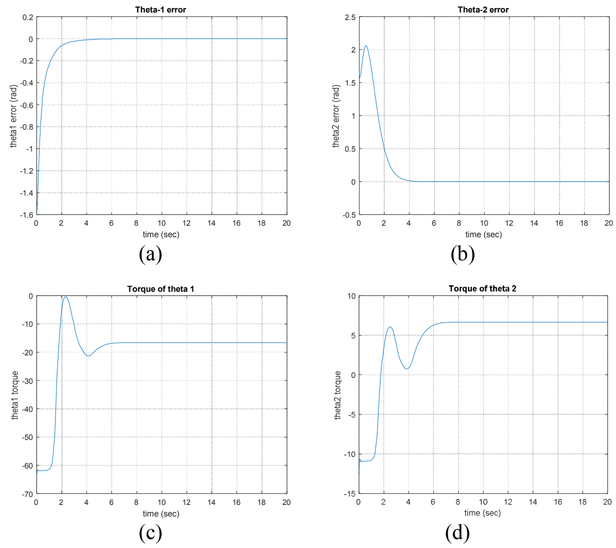

Fig. 5. Error forms of θ_1 (a) & θ_2(b), Torque of θ_1 (c) & θ_2(d) for the PID-SMC controller (for parameter set 2).

C. Discussion

PID control gains such as K_P, K_D and K_I are related to the direct error & speed development, interaction speed during state change and overall error cancelation, respectively. Though the PID controller produces stability through error cancellation in the closed loop control mechanism, slight deviations in the controller parameters may result in instability such as overshoot, oscillations etc. Similarly, the controller performance is dependent on the initial and final positions of the robot. Simulation results in Fig. 2 and Fig. 4 show the tracking performance and sensitiveness of the PID controller in favor of the discussion. Hence, designing PID control parameters needs careful adjustments for system operations such as undefined final positions, trajectory tracking, etc.

TABLE I. SIMULATION PARAMETERS

Simulation Trials		
Param.	***Parameter Set 1***	***Parameter Set 2***
Mass & Length	M_1=M_2=L_1=L_2=1.	M_1=5, M_2=5 & L_1=L_2=0.34.
PD component	K_{P1}=15, K_{D1}=7, K_{I1}=10, K_{P2}=15, K_{D2}=10, K_{I2}=10.	K_{P1}=30, K_{D1}=12, K_{I1}=20, K_{P2}=32, K_{D2}=22, K_{I2}=30.
SMC component	K_{sal} = 5 , σ = 0.3	K_{sal} = 5 , σ = 0.3
Position	Initial = [$\pi/2$ -$\pi/2$]T Final = [-$\pi/2$ $\pi/2$]T	Initial = [π $\pi/2$]T Final = [$\pi/2$ π]T

The PID-SMC controller, on the other hand, shows improvements by quickly converging the tracking errors to zero. Fig. 3 and Fig. 5 show that the manipulator is capable to rapidly trail the preferred path with the PID-SMC controller in comparison to the PID controller, as shown in Fig. 2 and Fig. 4. Chattering occurs when the θ_1 moves from $\pi/2$ to $-\pi/2$ and θ_2 moves from $-\pi/2$ to $\pi/2$ in 'parameter set 1', as shown in Fig. 3; while this effect is not present when θ_1 moves from π to $\pi/2$ and θ_2 moves from $\pi/2$ to π in 'parameter set 2', as shown in Fig. 5. This is because of the smooth trajectory between the joints while we simulate the 2-DOF robot with 'parameter set 2'. The PID-SMC controller allows faster settling time than the PID controller and shows more stability.

V. CONCLUSION

In this paper, a PID-SMC controller is implemented for a 2-DOF planar robot system with two revolute joints. The system dynamics is obtained with Lagrange method. Initially a PID controller is designed which becomes the sliding surface for the SMC controller. Simulation results illustrate that PID-SMC controller is well in tracking performance of the robotic arm compared with the traditional PID methods. The proposed PID-SMC control scheme will be applied on other spatial manipulator platforms in our future works.

REFERENCES

[1] Stanislaw H. ˙Zak, Systems and Control, Oxford University Press, Newyork, 2003.

[2] John J. Craig, Introduction to Robotics: Mechanics and Control, 3rd Ed., Pearson Education International, 2005.

[3] P. Tomei, "Adaptive PD Controller for Robot Manipulators," IEEE Trans. Robotics and automation, vol. 7, pp. 565–570, Aug. 1991.

[4] A. A. Pervozvanski , L. B. Freidovich, "Robust stabilization of robotic manipulators by PID controllers," Dynamics and Control, vol. 9, pp. 203–222, 1999.

[5] S. Bogosyan, P.R. Suravaram and M. Gokasan, "Sliding mode based rejection of load and torque ripple in a direct-drive 2 DOF robot arm ," 31st Annual Conference of IEEE Industrial Electronics Society, (IECON), Raleigh, NC, USA, Nov. 2005.

[6] G. H. Xu, Z. H. Xiao, Y. Guo, and X. B. Xiang, "Trajectory tracking for underwater manipulator using sliding mode control," IEEE International Conference on Robotics and Biomimetics (ROBIO), pp. 2127–2132, Sanya, China, Dec. 2007.

[7] D.I Vyas, J. Ohri and A. Patel, "Comparison of conventional & fuzzy based sliding mode PID controller for robot manipulator," International Conference on Individual and Collective Behaviors in Robotics (ICBR), Sousse, Tunisia, Dec. 2013.

[8] F. Sado, S. N. Sidek, and H. M. Yusuf, “Intelligent trajectory conversion and inverse dynamic control of a 3-DOF neuro-rehabilitation platform,” 10th Asian Control Conference (ASCC), Malaysia, Jun. 2015.

[9] Kaige Wan, Aiguo Wu, Haiting Liu, and Na Dong, “Trajectory Tracking of 3-DOF Spatial Robot Manipulator,” 12th World Congress on Intelligent Control and Automation (WCICA), Guilin, China, 2016.

[10] P. R. Ouyang, J. Tang, W.H. Yue, and S. Jayasinghe, “Adaptive PD plus Sliding Mode Control for Robotic Manipulator,” Proc. of IEEE International Conference on Advanced Intelligent Mechatronics (AIM), Banff, AB, Canada, July 2016.

[11] S. A. Ajwad, R. U. Islam, M. R. Azam, M. I. Ullah and J. Iqbal, “Sliding mode control of rigid-link anthropomorphic robotic arm,” 2nd International Conference on Robotics and Artificial Intelligence (ICRAI), Rawalpindi, Pakistan, Nov. 2016.

[12] S. Kasera, A. Kumar, and L. B. Prasad, “Trajectory Tracking of 3-DOF Industrial Robot Manipulator by Sliding Mode Control,” 4th IEEE Uttar Pradesh Section International Conference on Electrical, Computer and Electronics (UPCON) GLA University, Mathura, India, 2017.

[13] J. Tang, P.R. Ouyang, W.H. Yue, and H.M. Kang, “Nonlinear PD Sliding Mode Control for Robotic Manipulator,” IEEE International Conference on Advanced Intelligent Mechatronics (AIM), Munich, Germany, July, 2017.

[14] A. A. Mohammed and A. Eltayeb, “Dynamics and Control of a Two-link Manipulator using PID and Sliding Mode Control,” International Conference on Computer, Control, Electrical, and Electronics Engineering (ICCCEEE), Khartoum, Sudan, Aug. 2018.