

The application of multi-objective PSO algorithm in energy-efficient optimization of metro systems

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Abstract—In order to reduce the energy consumption of urban rail transit, a hierarchical energy-saving optimization method is proposed. The upper layer properly allocates travel time of every interstation in whole line, and the lower layer optimizes the train target speed curve in different interstation. In the lower optimization scheme, the multi-objective target speed curve optimization mathematical model is established. Then by using multi-objective particle swarm optimization algorithm (MOPSO), both a set of optimal energy-saving speed curves and their corresponding energy consumptions were obtained. Least squares method was used to curve fitting data and mathematical model of the relationship between energy consumption and travel time for interstation is established. In the upper level optimization scheme, interstation travel time distribution optimization model was established, whose target is minimizing the whole line energy consumption. the travel time allocation of every interstation was optimized by gradient descent method. Finally, based on the real data of Yizhuang Line, Beijing Subway, the proposed optimization model was simulated and verified. As the simulation results show, in lower layer optimization the operation energy efficiency is greatly improved. In upper layer, interstation travel time distribution optimization, 10.7% energy consumption is reduced.

Keywords—metro systems; Energy-Efficient; MOPSO; Gradient Descent; driving strategy; travel time allocation

I. INTRODUCTION

In recent years, Metro systems has flourished and energy consumption has increased year after year. For example, Beijing has 19 subway lines with a total length of 574km. One of them, Subway Yizhuang Line, contains only 14 stations, but it consumes 42.53 million kWh of electricity per year, which is equivalent to the electricity demand of 22,000 households in Beijing[1]. Therefore, the research for energy-efficient method is valuable and meaningful. The energy consumption of railway system is composed of many parts and traction energy consumption occupies large proportion.

The traction energy consumption of subway train has negative correlation with section running time. For this reason, the energy performance of subway can be improved through the optimization for schedule timetable. In this aspect, many methods were proposed, such as improving brake regeneration energy utilization[2], making full use of redundancy running time of each section[3] and headway regularity optimization[4]. those methods make the subway energy consumption decreased in effect.

In other ways, optimization for target speed curve can also improve the traction energy consumption. In 1990, Howlett [5] used the maximum principle to prove that the optimal driving strategies on the straight road is the combination of the four modes contains maximum traction, coasting, speed-hold and maximum braking. optimal switching points for each mode can be obtained by the Hamiltonian function. In 2000, considering speed limit and road gradient, Eugene Khmelnitsky[6] established a energy-saving operation model for speed continuous variable train. The maximum principle analysis shows that the optimal solution is also consisted of five model maximum traction, cruise, idle and maximum braking under any and slope. Approaching. In 2010, Miyatake et al. [7] discussed the energy-saving strategies of trains considering regenerative braking, using dynamic programming, gradient method and quadratic programming to obtained the optimal energy-saving strategies, and comparing the advantages and faults of each method. Further more, Tang Haichuan [8], Zhong Weifeng [9] et al proposed to use the quadratic programming algorithm to optimize the train operation sequence.

In real circumstances, road gradient and speed limit conditions of subway are complex. Then mathematical analysis become cumbersome, so scholars began to resort to intelligent optimization algorithms which is flourish in recent years. In 1997, C.S.Chang[10] applied genetic algorithm to train operation optimization to obtained the optimal coasting operation segment, which can be applied to solve problems under different complex conditions. In 2016, Tang Tao [11], Gao Xurui[12], Cao Jiafeng [13], Huang Youneng[14] et al and others proposed different types of group intelligent optimization algorithms, which were applied in urban rail and high-speed rail scenarios.

Train operation optimization is a multi-objective optimization problem, which not only considers energy consumption, but also other factors such as running time error, passenger comfortable and parking accuracy. The traditional method is to optimize the weighted summation of multiple targets. If the weight coefficients are selected improperly, the mutual interference may happen between targets. In this paper, the multi-objective particle swarm optimization (MOPSO) is used to optimize the target speed curve of the train. The algorithm uses the Pareto dominance concept to judge the superiority of the solution, which can avoid the multi-objective confliction and ensure to obtain a more satisfactory solution. At the same time, the calculation result of MOPSO algorithm is

approximate Pareto optimal solution set, and the energy-time value of each solution is distributed in the vicinity of the energy-time relationship curve of each interstation, which is convenience to fitting the energy-time relationship curve of the section. Compared with the PSO algorithm, it is not necessary to set different planned running times to solve multiple times, which greatly improves efficiency.

In terms of interstation running time optimization, Huang Youneng and Ning Jingjie[14, 15], according to the time-energy curve of the inter-stations, proposed a method which use the fixed step time allocation algorithm to reallocated the running time of every interstation. In this paper, the method is improved, and the convex optimization mathematical model of interval running time allocation is established. The steepest descent method is used to solve the optimal solution of the model, which makes the solution result more accurate.

The content arrangement of this paper is as follows. Firstly, the motion equation of the train is analyzed, the multi-objective speed curve optimization mathematical model is established, and the evaluation function is established to quantify the optimization effect. Then the multi-objective particle swarm optimization algorithm based on Pareto optimal concept is applied to solve the model. Then, an optimization model of interval time allocation is established, and the steepest descent algorithm is used to optimize the allocation of each interstation running time to minimize the energy consumption of the whole line. Finally, in order to prove algorithm effective, there is a simulation using the data of the Beijing Metro Yizhuang line.

II. THE OPTIMIZATION MODEL OF TARGET SPEED CURVE

A. The kinetics analysis of train operation

The train is a complex dynamic system but can be simplified into a single particle for research [16]. The train is mainly subjected to traction, braking force, and additional resistance and gradient resistance during operation. The motion equation is shown in formula (1).

$$M \frac{dv}{dt} = F_a - F_b - r(v) \quad (1)$$

$$r(v) = (w_0 + w_i) \times g \times M / 1000 \quad (2)$$

$$w_0 = a + bv + cv^2 \quad (3)$$

Among them F_a is the traction force (KN), F_b the braking force (KN), and the $r(v)$ running resistance, which M represents the mass of the train (tons). The magnitude of trains traction and braking force is related to the speed v and is constrained by the traction braking characteristic curve which is shown in Figure 1. The resistance to the train can be denoted by equation (2), where w_0 is the basic resistance coefficient, the road gradient resistance coefficient, and the gravitational acceleration. Equation (3) is the Davis formulation [17] used to calculate the basic resistance coefficient, where a , b , and c are constants associated with the train's appearance and mechanical structure.

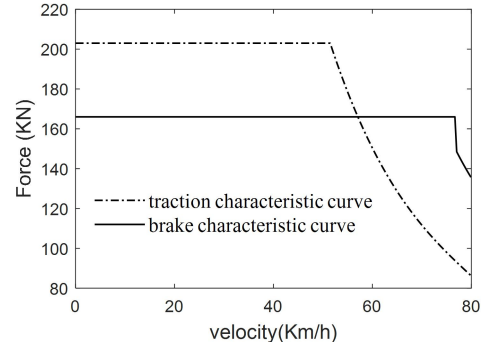


Fig. 1. Train traction characteristic curve and braking characteristic curve

The length of the interval is denoted as s , and the optimization goal during train operation is to shorten the running time T , reduce the total traction energy E , and reduce the impact rate $Jerk$. These indicators can be calculated from the formula (4, 5, 6).

$$E = \int_0^x F_a(x) + \rho F_b(x) dx \quad (4)$$

$$T = \int_0^s \frac{1}{v(x)} dx \quad (5)$$

$$Jerk = \frac{1}{T} \int_0^T \left| \frac{da}{dt} \right| dt \quad (6)$$

Where x represents the position during train operation and T represents the total time the train is running, ρ representing the efficiency of the regenerative braking feedback energy.

B. The evolution of optimization effect and optimization model

When optimizing the operating energy consumption, running time error and passenger comfortable, the membership function original from fuzzy logic can be used to quantify the optimization effect. This paper assumes it's tolerant that the deviations between train's actual running time and planned time is less than 5 s, and according to the comfort standard defined in ISO-2631, the comfort level is the highest when $Jerk < 3.15$. Therefore, the comfort function of the formula (7, 8, 9) is defined, where E_{max} and T_{min} is obtained from the minimum running time velocity curve of the interstation.

$$\mu(Jerk) = \begin{cases} 1 & Jerk < 3.15 \\ \exp(1 - \frac{Jerk}{3.15}) & Jerk > 3.15 \end{cases} \quad (7)$$

$$\mu(T) = \begin{cases} 1 - \frac{T - T_{min}}{T_{max} - T_{min}} & T_{min} < T < T_{max} \\ 0 & else \end{cases} \quad (8)$$

$$\mu(E) = \begin{cases} -1 + \exp\left(\frac{-E + E_{max}}{0.5E_{max}}\right) & E < E_{max} \\ 0 & E > E_{max} \end{cases} \quad (9)$$

Based on the kinematics equation of train, line constraints and objectives of operation optimization, a multi-objective optimization mathematical model of train operation is established, which is given by formula (10).

$$\begin{cases} \max f(u) = (\mu(E), \mu(T), \mu(Jerk)) \\ s.t. \quad u \in [-1, 1] \\ M \frac{dv}{dt} = F_a(u, v) - F_b(u, v) - r(v, s) \\ F_a \leq g_1(v), F_b \leq g_2(v) \\ v_0 = 0, v_d = 0, s_0 = 0, s_d = s \\ v_i \leq 0.95v_L, \forall 1 < i < n \end{cases} \quad (10)$$

Where u is the traction or braking rate of the train, so that the train can be in the four conditions of maximum traction, cruising, speed-hold and maximum braking. $g_1(v)$ and $g_2(v)$ represents the traction characteristic curve and represents the braking characteristic curve of the B type car respectively.

III. MOPSO ALGORITHM FOR TARGET SPEED CURVE OPTIMIZATION

The MOPSO algorithm is based on the PSO algorithm and contains many improvements. Firstly, the Pareto optimal concept was introduced into MOPSO algorithm and applied to the comparison of the priority between particles. The second is to add the external repository, to which Pareto optimal particles in the population will be preserved in each iteration. This is an elite strategy. Finally, the selection method for global best position is improved to make the final solution set more evenly distributed on the Pareto front. The final result of the MOPSO algorithm is a set of equilibrium solutions, called the Pareto optimal solution set [18]. In the Pareto optimal solution set, each solution has a unique advantage and is not subject to Pareto domination by other solutions. The classic MOPSO algorithm is the CMOPSO proposed by Coello et al[19]. The algorithm proves the advantages of MOPSO algorithm in solving multi-objective optimization through the test of classical problem sets.

A. Selection of decision variables

Select the position of the mode switch point as the decision variable. When the line speed limit is more complicated, operation mode which includes maximum traction, speed-hold and coast may be used multiple times. Therefore, the struct of target speed curve is composed of M stages, each stage includes three operation mode of maximum traction, speed-hold and coast, and finally ends with the maximum braking condition. So each solution contains $3M$ mode switch points. This paper names this method of selecting decision variable

SH (speed hold) control optimization. The form of the control variable is shown in equation (11).

$$X = [x_1, x_2, \dots, x_n] \quad (11)$$

Fig.2 shows a target speed curve containing six operating mode switch points. necessarily three conditions at a certain stage may not contain all three operation modes mentioned above, which can be achieved by reducing the mode continuous distance to zero as if this mode doesn't exist. As shown in Fig.2, the coast mode between x_2 and x_3 , the optimization algorithm will bring x_2 and x_3 close to each other, so that the mode of the segment is almost non-existent. This shows that this decision variable selection method has strong adaptability and can simulate various situations.

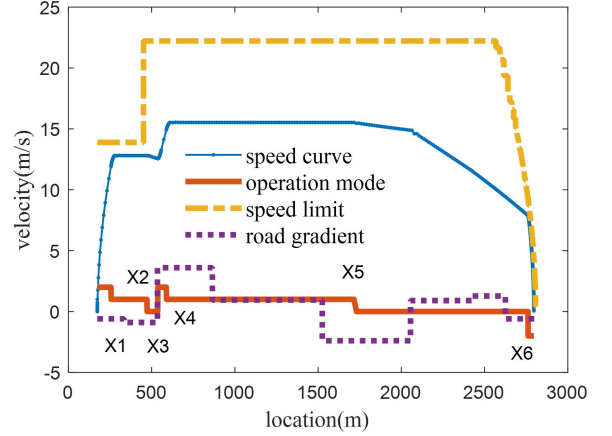


Fig. 2. schematic diagram of state switch point

B. Constraint conditions and design of fitness function

In order to make the optimal solution meet the requirements such as line speed limit and precise parking, a penalty function should be defined. Stopping Error (SE) and Over Speed Error (OE) are added to the target vector as a constraint indicator. When judging whether there is a Pareto dominance between two particles in a population, it can be compared in two levels. The constraint indicators (OE, SE) are prioritized for comparison to ensure that the final solution is feasible. If solution meet the constraint requirements, then the other optimization indicators are compared. The simulation calculation in this paper has numerical calculation accuracy error value, so this paper allows the generated target curve to have a certain parking error and does not affect the actual use. The requirements for precise parking can be optimized in the actual operation using the corresponding control algorithm [20]. Therefore, the membership function of parking error and over-speed limit is designed.

$$\mu(e_o) = \begin{cases} 1 & e_o \leq 0 \\ \exp(-e_o) & e_o > 0 \end{cases} \quad (12)$$

$$\mu(e_s) = \begin{cases} 1 & e_s \leq 0.5 \\ 1.25 / (1 + e_s^2) & e_s > 0.5 \end{cases} \quad (13)$$

The fitness function can be designed using the membership function, and the fitness function as shown in equation (14) is designed.

$$fit(E, T, Jerk, SE, OE) = \begin{bmatrix} \mu(E) \\ \mu(T) \\ \mu(Jerk) \\ \mu(e_s) \\ \mu(e_o) \end{bmatrix} \quad (14)$$

C. The basic process of the MOPSO algorithm

At the beginning of the algorithm, the particle swarm Pop and the historical optimal solution $Gbest$ are initialized. Then, the target vector corresponding to each particle is calculated, and the non-inferior solution which is not Pareto dominated by other solutions are added to the external repository Rep . After that, the target vector space is divided into multiple hypercubes, and the distribution density of the solution in the Rep in the hypercube is counted. Calculates the fitness of the hypercube, which is inversely related to the distribution density of the particles in the hypercube. Next, the iterative process begins, and each iteration is divided into 7 steps:

- a) Update the velocity of particles in a particle swarm using Equation (15):

$$VEL[i] = W \times VEL[i] + R_1 \times C_1 \times (Pbest[i] - Pop[i]) + R_2 \times C_2 \times (Rep[h] - Pop[i]) \quad (15)$$

Where W is the inertia factor, C_1 and C_2 are non-negative acceleration factors, and R_1 and R_2 are random numbers in the range $[0,1]$. A hypercube is selected using the roulette method according to the fitness of each hypercube, and then a particle h is randomly selected from the external solution contained in the cube to be taken as a global optimal solution into the formula (15) for calculation.

- b) Update each position of the particle according to formula (16)
 $Pop[i] = Pop[i] + VEL[i]$ (16)
- c) Check each swarm in the population, Judge whether the swarm keeps in search space, if not regenerate the swarm.
- d) Calculate the fitness vector of each particle in the population.
- e) The current position of the particle is compared with the best position $Pbest$ of the particle itself. If the current position is Pareto dominated by $Pbest$, the position in memory is maintained; otherwise, the current position replaces $Pbest$; if they do not dominate each other, one of them is randomly selected.
- f) Update the Rep . select the non-Pareto dominated solutions in the Population. Add them to the Rep and select the non-Pareto dominated solutions in Rep as a new Rep solution set.

- g) After the update is completed, the hypercube of the target vector space is re-divided and the fitness of each hypercube is calculated.
- h) Return to step a and increase the loop variable. If the loop reaches the maximum number of loops, terminate the algorithm.

D. Set the maximum iterations number of the algorithm

In the MOPSO algorithm, the fitness is a vector, so in the optimization process, the change of fitness appears as the movement of the Pareto frontier. If the dimension of the fitness vector is higher than 3, it is difficult to observe the Pareto frontier. So this paper uses the number of particles entering the Rep (the number of particles exchanged to Rep , EN) during each iteration as an optimization index. If after certain generations the EN is close to 0, the optimization result is mature, and the number of iterations can be set as the maximum number of iterations. As shown in Fig.3, in the optimization process of the MOPSO algorithm, as the number of iterations of the algorithm increases, the EN shows a downward trend. When the 150th generation is reached, the EN in the population is already close to 0, and stays nearby, indicating that the optimization results are mature and the algorithm can be terminated. So during the optimization process, the number of maximum iterations of the MOPSO algorithm can be set to 150.

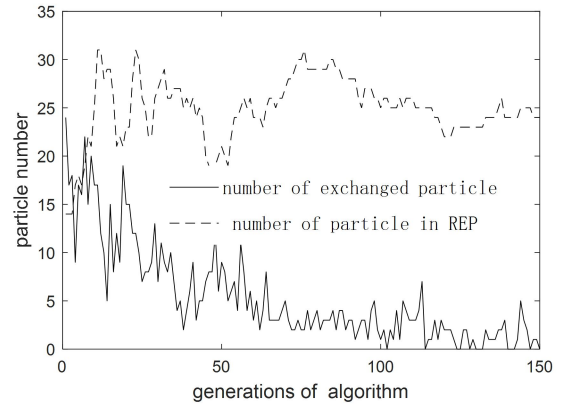


Fig. 3. The trend of exchanged particle number variation with the iteration generation

IV. OPTIMIZATION OF INTERSTATION PLANNING TIME ALLOCATION

A. Obtain relationship of minimum energy and consumption-planning time in the interstation

In the optimization of train traction energy consumption, the interstation running time and the minimum traction energy consumption are negatively correlated. When the interval running time is shortened, the minimum traction energy consumption will inevitably rise. There must be a lower limit of traction energy consumption at a certain operating time. Therefore, the obtained Pareto optimal solution set corresponding to each discrete time-energy value is distributed

in the boundary of the running time-traction energy consumption, ie, the minimum E - t relationship, which can be used to obtain minimum E - t relationship through the curve fitting.

In the III part, the optimized discrete time-energy value pairs are obtained in the optimization of the train running target speed curve. According to previous experience, it can be assumed that the mathematical model of the minimum energy-time curve in the interstation is as shown in equation (17).

$$\varepsilon(t) = \frac{A}{(t-B)^c} \quad (17)$$

In this model, the total energy consumption of the interval traction is represented, t is the interstation running time, and A , B , C ($A, B, C > 0$) are model parameters, which can be obtained by fitting. It is obviously that function of model is the a convex function. The correctness of the model will be verified in part V.

B. Modeling and optimization of planning time allocation for line interstation

The interstation runtime allocation problem of full line can be denoted as a constrained optimization model represented by equation (18). In this model, E , T and respectively represent the total energy consumption of the entire line and the total running time between stations.

$$\begin{cases} E(t_1, t_2, \dots, t_n) = \sum_{i=1}^n \varepsilon_i(t_i) \\ s.t. \sum_{i=1}^n t_i = T \end{cases} \quad (18)$$

For the additive function of the convex function can be also a convex function[21], and the established time distribution optimization model is a convex optimization problem. The constraint of fixed total time can be regarded as a penalty function and the unconstrained optimization model represented by formula (19) is obtained.

$$\begin{aligned} Q(t_1, t_2, \dots, t_n) &= E(t_1, t_2, \dots, t_n) \\ &+ (\sum_{i=1}^n t_i - T)^2 \end{aligned} \quad (19)$$

The optimal solution of model can be obtained by the steepest descent algorithm, and the search starting point is set to $T_0 = (t_1^{(0)}, t_2^{(0)}, \dots, t_n^{(0)})$, where is the minimum running time of the i -th station. The process of the steepest descent algorithm is denoted by formulation (20). In which $T^{(k)}$ represents result of the K th generation iteration, $d^{(k)}$ representing the steepest descent direction, λ_k indicating the search step length. Set the solution precision condition $\varepsilon = 1$, and the algorithm terminates after the condition is met.

$$\begin{aligned} T^{(k+1)} &= T^{(k)} + \lambda_k d^{(k)} \\ d^{(k)} &= -\nabla Q(T^{(k)}) \\ \lambda_k &: \text{ for } Q(T^{(k)} + \lambda_k d^{(k)}) \\ &= \min_{\lambda \geq 0} f(T^{(k)} + \lambda_k d^{(k)}) \end{aligned} \quad (20)$$

In the III part, the Pareto optimal solution set which contains many target speed curves corresponding to different running time is obtained. From them, one which's running time match the optimal planning time can selected and used for actual operation, and the overall optimization from the whole to the detail is realized. As result, the energy consumption of the entire line is optimized.

V. SIMULATION AND ANALYSIS

In order to verify the effect of the proposed algorithm, The data of the Beijing Subway Yizhuang Line was selected to give a simulation. The Yizhuang Line has a total length of 23.3km and passes through 14 stations along the way. The downward direction of the line starts at Songjiazhuang Station, ends in Yizhuang Station. The speed limit of the Yizhuang line is 80km/h, the minimum is 50km/h, and the maximum road gradient is 2.4%. The type-B train running in the Yizhuang line is the research object[22], and its parameters are shown in Tab.1.

TABLE I. THE PARAMETER OF THE TYPE-B TRAIN IN SIMULATION

Parameter category	Value
Train type	Type-B
Train Mass (ton)	194.295(AW0)
Base resistance parameter	a=2.031,b=0.0622,c=0.001807
Length of train (m)	120
Maximum acceleration (m/s ²)	1
Maximum deceleration (m/s ²)	1

A. Comparison between MOPSO and PSO algorithm

Based on the MATLAB platform, The PSO and the MOPSO algorithm are applicated to search for the optimal operation mode switch point. In order to improve the diversity of the population and enhance the search ability of the algorithm, the size of populations should not be too small, so the population of the two algorithms is set to 50. The scale of external repository Rep of the MOPSO algorithm is 50, which is consistent with the population's scale. When the number of iterations of the algorithm is 150 generations, the optimization effects of the MOPSO and the PSO algorithm are no longer changed, so the number of iterations of the algorithm is set to 150. The parameters of each algorithm are shown in the Tab.2.

TABLE II. THE PARAMETER OF THE PSO AND MOPSO ALGORITHM

Parameter name	PSO	MOPSO
Population scale	50	50
C_1	2.0	2.0
C_2	2.0	2.0
W	0.4-1.4	0.4-1.4
Total Iterator number N	150	150
Rep scale	-	50

Run PSO and MOPSO algorithm to optimize the train running process many times. Fig.5 shows the indicators of the optimal solution obtained for each optimization. It can be seen that the minimum energy consumption of the optimal solution obtained by the PSO algorithm has greater volatility. In contrast, the MOPSO algorithm has less volatility and stability. This paper thinks that this is because the PSO algorithm sums the fitness of multiple targets, resulting in mutual interference between the targets. It can also be seen from Fig. 5 that the optimal solution obtained by the MOPSO algorithm meet the requirements of the line speed limit requirement and the parking error. And the optimized running energy consumption is better than that obtained by the PSO algorithm, indicating that the optimization effect of MOPSO algorithm is better than that of PSO algorithm.

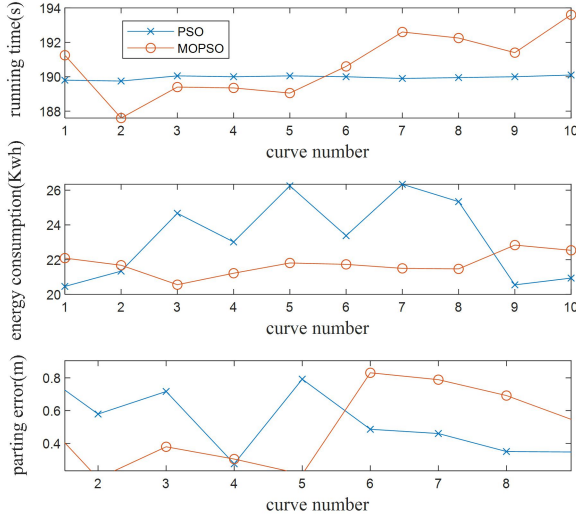


Fig. 4. The variation of optimal unit energy consumptions obtained by PSO and MOPSO algorithm every times

B. Interstation target speed curve optimization simulation and results

Taking the Songjiazhuang-Xiaocun section of Yiyuan Line as an example, the MOPSO algorithm is applied to obtain the optimized target speed curve. The algorithm parameters are consistent with above. The length of the section is 2806m, and the rate limit of the interval line is relatively complicated, which is suitable for testing the adaptability of the algorithm. The minimum running time and the planned running time for this interstation is 157s and 190s respectively. The target speed curves for the three cases of unoptimized, coasting optimization and Speed-Hold optimization proposed in this paper are shown in Fig.6.

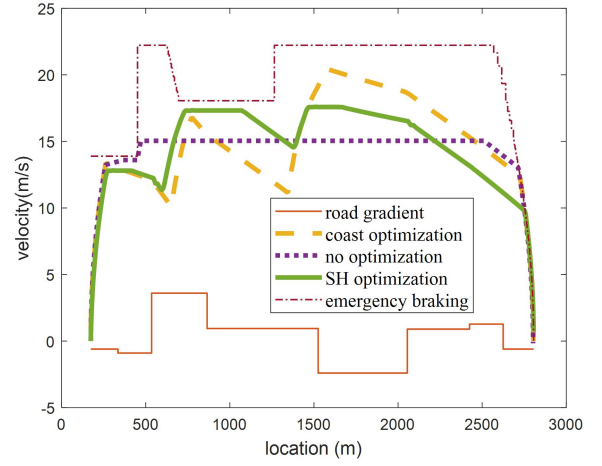


Fig. 5. The target speed curve obtained by three Methods which are no optimization, coast optimization and speed hold optimization

The values of parameters such as energy consumption and comfortable of three cases are shown in Table 2. Compared with the unoptimized, the proposed method reduce 23% operation energy consumption. The result obtained by Speed-Hold optimization is better in aspect such as energy efficient, comfortable and line shape, compared to inertial control optimization.

TABLE III. PERFORMANCE COMPARISON OF THREE OPTIMAL METHOD

Optimization indicator	Unoptimized	Coasting	Speed-Hold
Energy cost (Kw h)	25.46	20.81	19.59
Time(s)	192.3	191.5	190.7
Jerk(m/s ³)	0.0108	0.0478	0.0328
Energy efficient rate (%)	0	18.3	23.0

C. Interstation running time allocation optimization

The total planned running time of Yizhuang Line is 2100s, in which the parting time is 415s and the interstation running time is 1645s. Using the speed limit and gradient data of Yizhuang line, after simulation optimization, the energy-time corresponding to 13 interstation of Yizhuang line are obtained, ie A, B, C in formulation (19) Parameters. For example, the minimum energy consumption and time Pareto frontier point set of Wanyuan-Rongjingdong obtained by MOPSO algorithm is shown in Fig. 6. First, the formulation (17) is transformed into the form of the formula (21), and then the parameters are obtained by the least squares fitting method. The minimum energy consumption and time relationship can be denoted by the formula (22).

$$\ln(\varepsilon(t)) = \ln A - C \ln(t - B) \quad (21)$$

$$E = \frac{1706.5}{(T - 70)^{0.7323}} \quad (22)$$

After calculation, the residual of the fitting is $\sigma^2=0.0278$, set $\alpha=0.05$ as the parameter of Student's t test represented by formula (23) in the significance test. It can be concluded from $10610.3 > 2.3060$ that the fitting is valid.

$$|t| = \frac{|\hat{C}|}{\hat{\sigma}} \sqrt{S_{xx}} \geq 2.3060 \quad (23)$$

It can be seen from Fig.8 that formula (24) well fits the energy consumption-time curve.

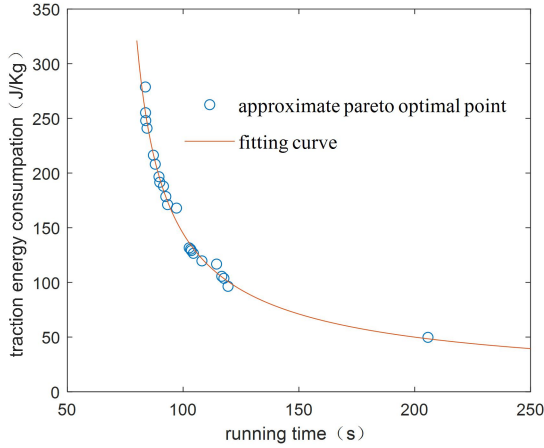


Fig. 6. Distribution of the Pareto frontier near the fitted curve

Combining with each interstation's relationship between energy consumption and time, this paper appulates the time-lapse method of the steepest descent method described in part IV to do optimization. The optimization results are shown in Fig.7 and Fig.8. According to the original planned interstation time allocation, the total energy consumption is 237.07Kwh. after optimization that is 213.23Kwh, which can reduce the energy consumption by 10.7%.

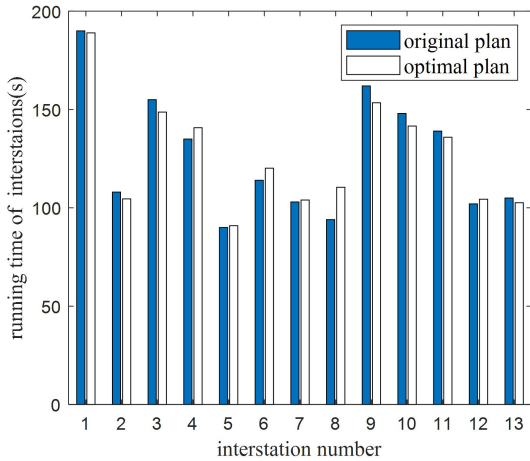


Fig. 7. Difference about interstation time allocation between primitive and optimized plan

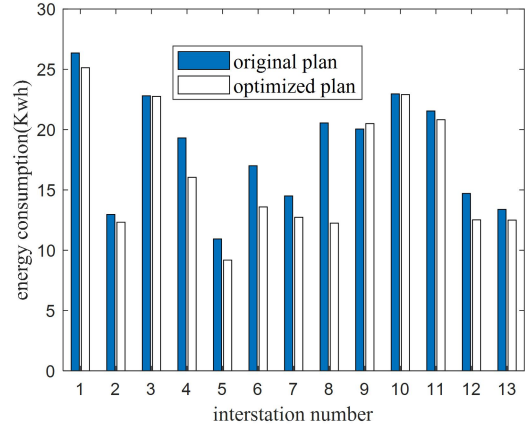


Fig. 8. Difference about energy consumption between primitive and optimized plan

VI. CONCLUSION

The research results of this thesis mainly include the following:

(1) Aiming at reducing energy consumption and time error, enhance passenger comfortable in urban rail transit, the corresponding evaluation functions of optimization effect are established. Based on this, a multi-objective optimization model for target speed curve is proposed to meet the actual requirements of train operation.

(2) This paper improves the Coast control, adds the Speed-Hold operation mode. The method can reduce energy consumption better. What's more, it can handle complex line speed limit, and improves passenger's comfort. The MOPSO algorithm is used to search for the optimal energy-efficient driving strategy, and Pareto is used to compare the priority of the solution, which avoids the problem of mutual interference between optimization targets. Compared with the PSO algorithm, the MOPSO algorithm achieves more stable results, better optimal results, and more efficient access to the energy-time relationship of the interstation.

(3) The energy-time relationship of the interval is obtained by the method of least squares fitting, and the validity of the fitting is proved by student's t test. The optimization model of interstation running time allocation of the whole line is established. The steepest descent algorithm is used to solve the optimal time allocation of each section of the line to achieve energy-efficient in whole line.

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