Proof of complexity of Label Setting Algorithm(LS)

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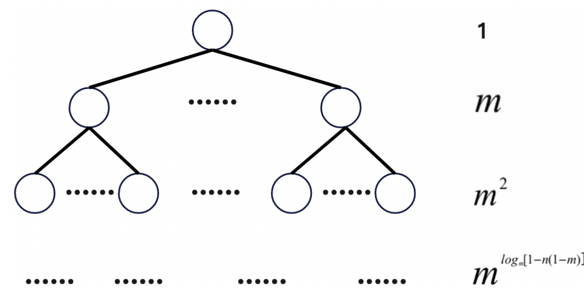
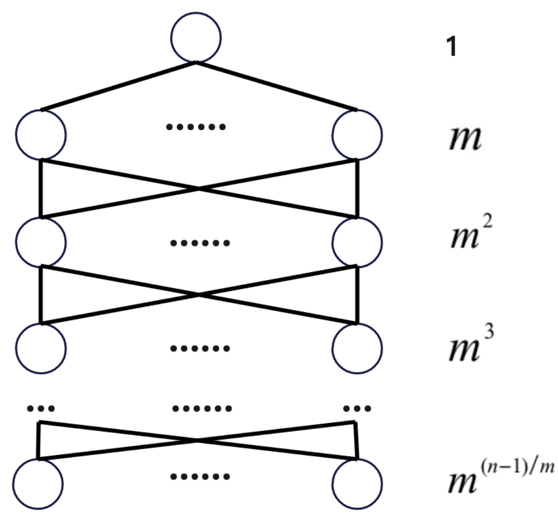
Theorem In graph G with n nodes, taking point s as the source point, the minimum complexity of LS to find the backup path of s to other nodes is O(n), the maximum is $O(m^{\frac{n}{m}})$, where m is the mean value of the out-degree of the node in G. Due to page limit.

Proof To calculate the worst case of the label setting algorithm, we increase the distance limit W to the maximum. So the label setting algorithm will not stop until every reachable destination is found. Then we assume that all nodes are reachable. This will produce a maximum number of labels. The number of labels in node i is the sum of labels of all nodes pointing to it. The node s is connected to m nodes. Each node of these m nodes has one label. There will be many situations that can be depicted as a tree diagram with the root node s. As the level of the tree increases, the total number of labels will increase. Then we consider two extreme situations from the best and worst perspectives. In Fig.1a, the best situation is that every node is connected to m new nodes, there are a total of m^2 new nodes, and the structure is an m-ary tree. Although the number of nodes in each layer has increased by m times, each node still has one label. So the total number of labels $T = 1 + m^2 + \ldots + m^{log_m[1-n(1-m)]-1} = O(n)$. In Fig.1b, the worst case is that these m nodes are connected to m new nodes. Each of the new nodes is connected by m nodes in the upper layer, and the number of labels for each new node is m, then the total number of labels in this layer is $m \times m$. So the next layer is still m new nodes and the number of each label is m^2. The total number of labels is m^3. So the total number of labels of this most complex structure $T = 1\times1+1\times m+m\times m+m^2\times m+m^3\times m^+\ldots+m^{\frac{n-1}{m}} = (1-m^{\frac{n-1}{m}+1})/(1-m) = O(m^{\frac{n}{m}})$. So we prove that in the best case, the complexity of LS method is O(n), and in the worst case, the complexity is $O(m^{\frac{n}{m}})$. In this process, we can know that the structure of the topology has a large impact on the complexity of the LS method.



(a) Minimum number of layers



(b) Maximum number of layers

Fig. 1: Two extreme cases