Characterizing a Spring Pendulum with Monte Carlo Methods

Dawn Sargent (Fairmont State University)

MAA Allegheny Mountain Section Conference, Penn State Behrend

6 April 2018

Special Thanks

FAIRMONT STATE UNIVERSITY
SCIENCE AND TECHNOLOGY

NASA

NVIDIA
GEFORCE GTX

The Problem

What makes a spring pendulum result in chaotic motion?

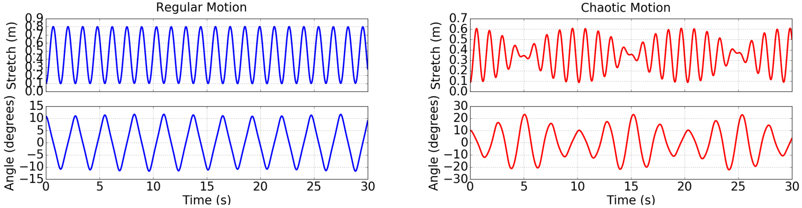
$$x'' + \frac{k}{m}x - (l+x)\theta'^2 - g\cos\theta = 0$$

$$\theta'' + \frac{g\sin\theta + 2x'\theta'}{l+x} = 0$$

Variable	Description	Distribution	Units
$x(0)$	initial stretch	$N(0.1, 0.01)$	m
$\theta(0)$	initial angle from vertical	$N(10, 0.6)$	deg
m	pendulum mass	$N(1, 0.1)$	kg
k	spring constant	$N(30, 0.25)$	N/m
g	acceleration due to gravity	$N(9.8, 0.1)$	m/s^2
l	unstretched length	$N(1, 0.1)$	m

Performance Metric

In order to identify what parameters effect the success of a trial, we must first determine what will define a success versus a failure.

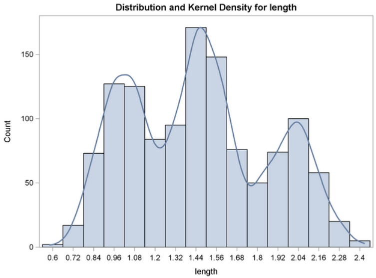


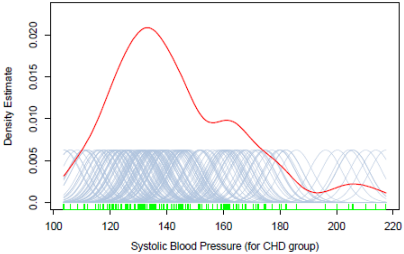
Motion	Gravity	Initial Angle	Initial Stretch	Length	Mass	Spring
Regular	9.57	10.85	0.099	0.8	1.4	29.895
Chaotic	9.73	10.16	0.088	1.065	1.058	29.79

Performance metric: If the max angle exceeds 21°, it is chaotic.

Kernel Density Estimate

What is a kernel density estimate?

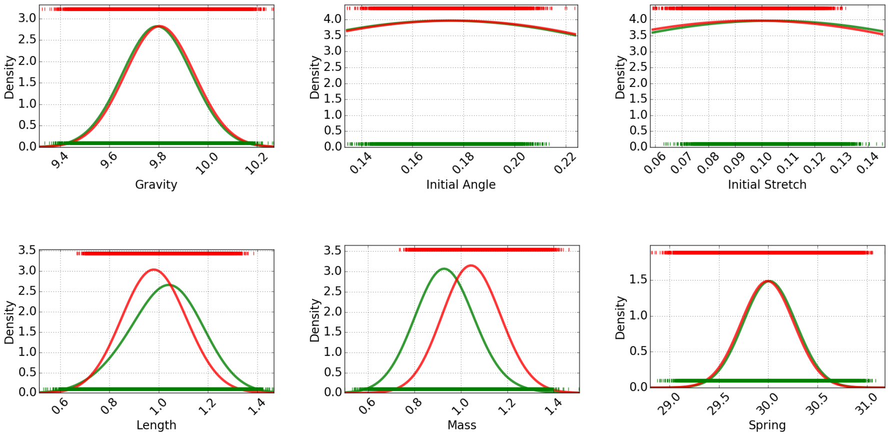




https://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm

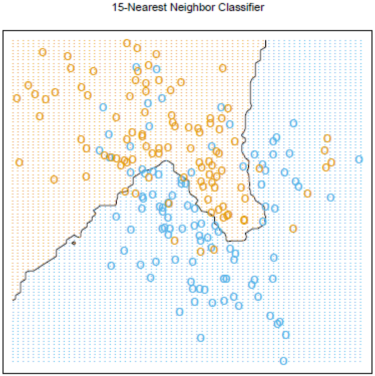
https://web.stanford.edu/ hastie/Papers/ESLII.pdf

KDE



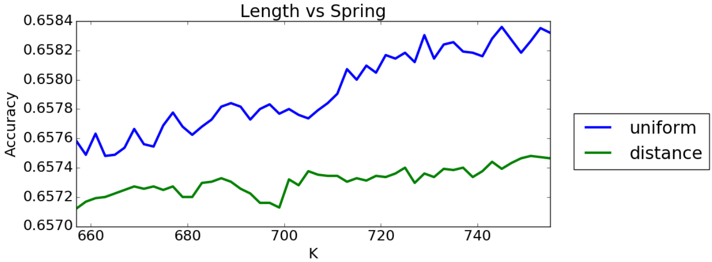
Density
Gravity
Initial Angle
Initial Stretch
Length
Mass
Spring

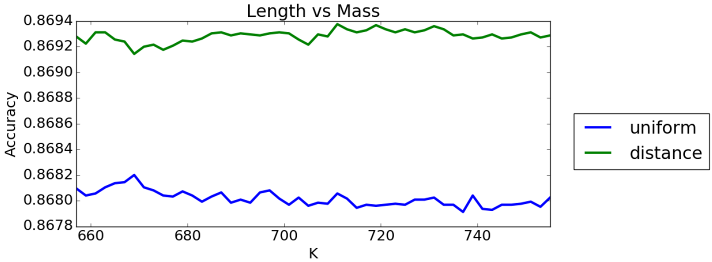
What is the k-nearest neighbor method?



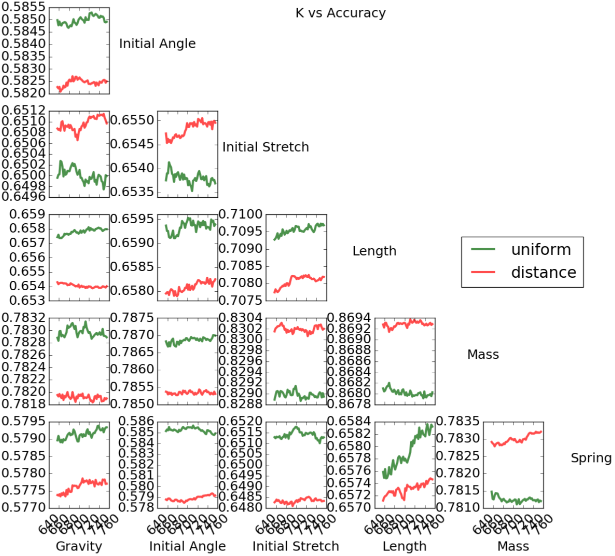
https://web.stanford.edu/ hastie/Papers/ESLII.pdf

KNN

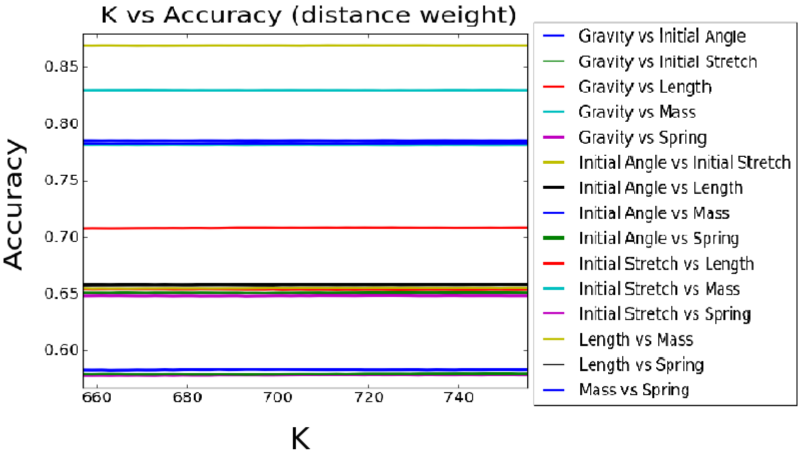

Length vs Spring
0.6584
0.6582
0.6580
0.6578
0.6576
0.6574
0.6572
0.6570
Accuracy
660
680
700
720
740
K
uniform
distance


Length vs Mass
0.8694
0.8692
0.8690
0.8688
0.8686
0.8684
0.8682
0.8680
0.8678
Accuracy
660
680
700
720
740
K
uniform
distance

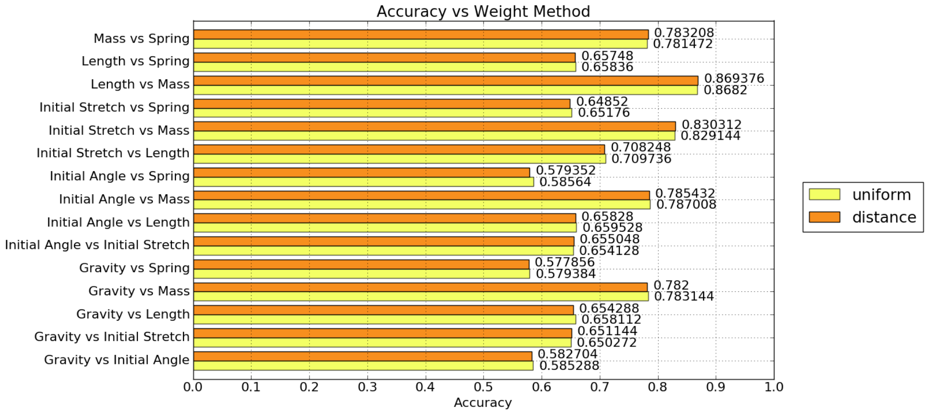
KNN

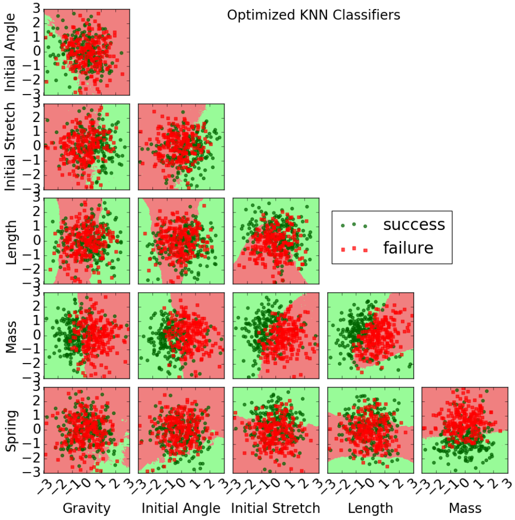

K vs Accuracy
Initial Angle
Initial Stretch
Length
Mass
Spring
uniform
distance
Gravity
Initial Angle
Initial Stretch
Length
Mass

KNN


K vs Accuracy (distance weight)
0.85
0.80
0.75
0.70
0.65
0.60
660
680
700
720
740
K
Accuracy
Gravity vs Initial Angle
Gravity vs Initial Stretch
Gravity vs Length
Gravity vs Mass
Gravity vs Spring
Initial Angle vs Initial Stretch
Initial Angle vs Length
Initial Angle vs Mass
Initial Angle vs Spring
Initial Stretch vs Length
Initial Stretch vs Mass
Initial Stretch vs Spring
Length vs Mass
Length vs Spring
Mass vs Spring

KNN


Accuracy vs Weight Method
Mass vs Spring
Length vs Spring
Length vs Mass
Initial Stretch vs Spring
Initial Stretch vs Mass
Initial Stretch vs Length
Initial Angle vs Spring
Initial Angle vs Mass
Initial Angle vs Length
Initial Angle vs Initial Stretch
Gravity vs Spring
Gravity vs Mass
Gravity vs Length
Gravity vs Initial Stretch
Gravity vs Initial Angle
0.783208
0.781472
0.65748
0.65836
0.869376
0.8682
0.64852
0.65176
0.830312
0.829144
0.708248
0.709736
0.579352
0.58564
0.785432
0.787008
0.65828
0.659528
0.655048
0.654128
0.577856
0.579384
0.782
0.783144
0.654288
0.658112
0.651144
0.650272
0.582704
0.585288
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Accuracy
uniform
distance


Optimized KNN Classifiers
success
failure
Initial Angle
Initial Stretch
Length
Mass
Spring
Gravity
Initial Angle
Initial Stretch
Length
Mass

Accuracy

Parameters	Accuracy
Mass vs Length	86.94%
Mass vs Initial Stretch	83.03%
Mass vs Initial Angle	78.70%
Mass vs Spring	78.32%
Mass vs Gravity	78.31%
Length vs Initial Stretch	70.97%
Initial Angle vs Length	65.95%
Length vs Spring	65.84%
Length vs Gravity	65.81%
Initial Angle vs Initial Stretch	65.50%
Initial Stretch vs Spring	65.18%
Initial Stretch vs Gravity	65.11%
Initial Angle vs Spring	58.56%
Initial Angle vs Gravity	58.53%
Spring vs Gravity	57.94%

Further Investigation

- Step sizes, computation time, and error
- Parallel processing