

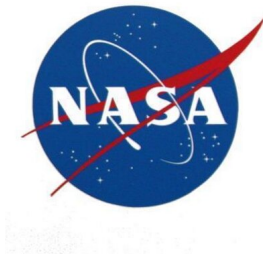
Characterizing a Spring Pendulum with Monte Carlo Methods

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Special Thanks



The Problem

What makes a spring pendulum result in chaotic motion?

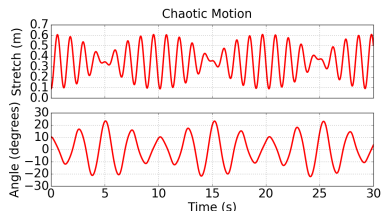
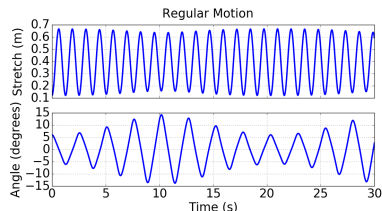
$$x'' + \frac{k}{m}x - (l+x)\theta'^2 - g \cos \theta = 0$$

$$\theta'' + \frac{g \sin \theta + 2x'\theta'}{l+x} = 0$$

Variable	Description	Distribution	Units
$x(0)$	initial stretch	$N(0.1, 0.01)$	m
$\theta(0)$	initial angle from vertical	$N(10, 0.6)$	deg
m	pendulum mass	$N(1, 0.1)$	kg
k	spring constant	$N(30, 0.25)$	N/m
g	acceleration due to gravity	$N(9.8, 0.1)$	m/s^2
l	unstretched length	$N(1, 0.1)$	m

Performance Metric

In order to identify what parameters effect the success of a trial, we must first determine what will define a success versus a failure.

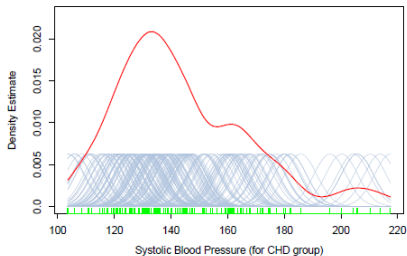
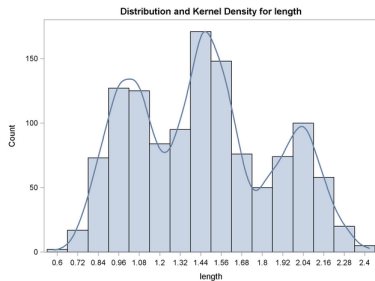


Motion	Gravity	Initial Angle	Initial Stretch	Length	Mass	Spring
Regular	9.82	6.31	0.121	0.8044	1.207	30.131
Chaotic	9.73	10.16	0.088	1.065	1.058	29.79

Performance metric: If the max angle exceeds 21° , it is chaotic.

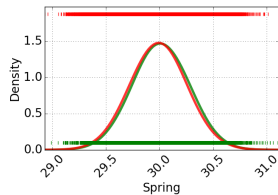
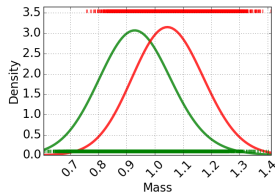
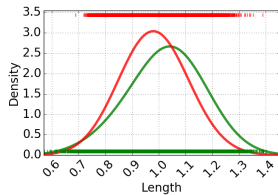
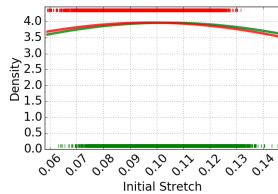
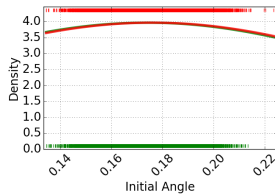
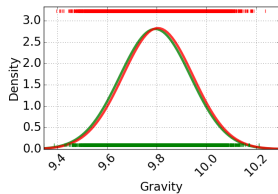
Kernel Density Estimate

What is a kernel density estimate?



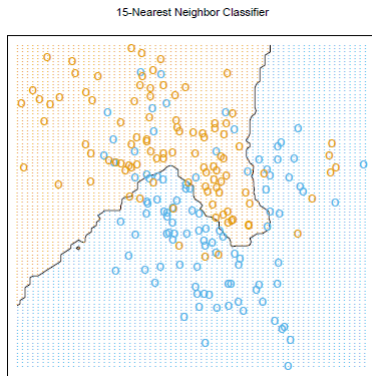
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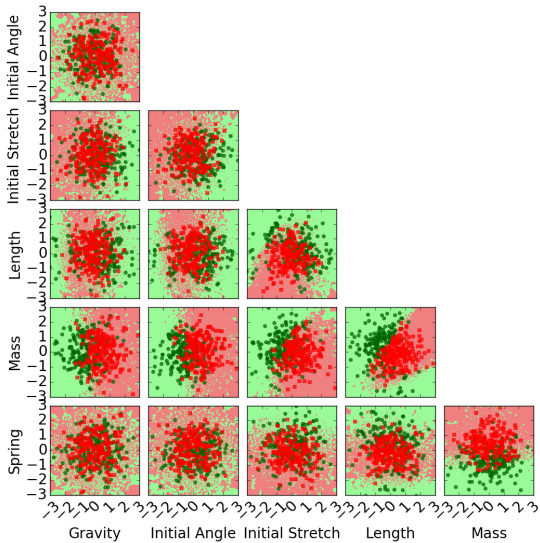


K-Nearest Neighbors

What is the k-nearest neighbor method?



<https://web.stanford.edu/hastie/Papers/ESLII.pdf>



Accuracy

Parameters	Accuracy
Mass vs Length	84.374%
Mass vs Initial Stretch	79.264%
Mass vs Initial Angle	74.108%
Mass vs Spring	73.521%
Mass vs Gravity	73.196%
Length vs Initial Stretch	65.593%
Length vs Spring	59.245%
Length vs Gravity	59.159%
Initial Stretch vs Spring	59.155%
Initial Angle vs Length	59.144%
Initial Angle vs Initial Stretch	59.129%
Initial Stretch vs Gravity	57.850%
Initial Angle vs Gravity	52.774%
Initial Angle vs Spring	52.759%
Spring vs Gravity	52.049%