UNIT 13
TESTS OF SIGNIFICANCE

This unit focuses on Inferential Statistics—the techniques used to draw conclusions about a large population based on data collected from a small sample. This is done primarily through Hypothesis Testing.

1. INTRODUCTION

A hypothesis is an unproven statement or proposition about a factor or phenomenon that the researcher is interested in. Hypothesis Testing is a statistical procedure used to determine whether there is enough evidence in a sample to support a belief (hypothesis) about the entire population.

We start with an assumption (e.g., "Our new ad campaign increased sales") and use statistics to see if the data we collected proves that assumption is true, or if the change was just due to random chance.

Key Concepts

- **Null Hypothesis (H0):** A statement of no difference or no effect. The researcher always attempts to reject the null hypothesis. (e.g., H0: There is no difference in sales between cities A and B.)
- **Alternative Hypothesis (H1):** A statement that reflects the researcher's belief, suggesting that some difference or effect exists. (e.g., H1: Sales in city A are higher than in city B.)
- **Level of Significance (α):** The probability of Type I error (rejecting the null hypothesis when it is actually true). Typically set at 0.05 (or 5%).
- **P-Value:** The probability of obtaining a test result at least as extreme as the one observed, assuming the null hypothesis is true. As per the example above, if P-Value $<\alpha$ (e.g., 0.05), we reject H0.
- **Type I Error (α):** Rejecting the null hypothesis (H0) when it is actually true (a false positive).
  - *Example:* Concluding that the new ad campaign worked, when in reality it didn't.
- **Type II Error (β):** Failing to reject the null hypothesis (H0) when it is actually false (a false negative).
  - *Example:* Concluding that the new ad campaign failed, when in reality it worked.

2. METHODS OF HYPOTHESIS TESTING

The general procedure involves five main steps:

1. **Formulate** H0 **and** H1**:** State the null and alternative hypotheses clearly.
2. **Select Appropriate Test:** Choose the correct statistical test (e.g., t-test, Z-test, Chi-Square) based on the scale of measurement and the number of samples.
3. **Choose Significance Level (α):** Determine the threshold for rejecting H0 (usually 0.05).

4. **Calculate Test Statistic:** Compute the statistical value (e.g., t or Z score) using the sample data.

5. **Make Decision:** Compare the calculated P-Value to α (or the test statistic to the critical value) to decide whether to reject or fail to reject H0.

3. PARAMETRIC TESTS

A parametric test is a statistical test that makes specific assumptions about the population data it's analyzing, such as assuming the data comes from a normal distribution and has known parameters (like the population mean or variance). These tests require continuous data, use the mean as the measure of central tendency, and are considered more statistically powerful than non-parametric tests, which lack these distributional assumptions.

A. Z-Test (One Sample)

- **Objective:** To test if the population mean (μ) is significantly different from a hypothesized value (μ_0).

- **Use Case:** When the sample size is large ($n \geq 30$) or when the population standard deviation (σ) is known.

- *Example:* Testing if the average annual spend of our customers (μ) is greater than the industry benchmark ($\mu_0 = \$500$).

B. t-Test (One Sample)

- **Objective:** Same as Z-test, but used when the population standard deviation (σ) is unknown and the sample size is small ($n < 30$). The t-distribution is used instead of the normal distribution.

- *Example:* Testing if a sample of 25 respondents gives an average attitude score different from the neutral point of 4.0.

C. Independent Samples t-Test (Two Samples)

- **Objective:** To determine if the means of two unrelated groups are significantly different.

- **Assumptions:** The variance of the two groups are assumed to be equal (or tested for equality).

- *Example:* Is the average satisfaction score of Male customers significantly different from the average satisfaction score of Female customers?

D. Paired Samples t-Test (Two Samples)

- **Objective:** To determine if the means of two related groups are significantly different. This is used for "before and after" measurements on the same sample.

- *Example:* Did the average willingness-to-buy score significantly increase after the same group of consumers viewed the new product demonstration?

4. CHI-SQUARE (X2) ANALYSIS

Chi-Square analysis is a Non-Parametric Test used for nominal or ordinal data. A nonparametric test is a type of statistical test that does not assume data follows a specific probability distribution, such as a normal distribution, making it a "distribution-free" method. It's used when data is ordinal, ranked, categorical, or continuous but does not meet the distributional assumptions of parametric tests, often due to a small sample size or a non-normal distribution.

The Chi-Square ($\chi 2$) test is used to test the Goodness of Fit (one-way analysis) and the Association or independence (two-way analysis) between two categorical variables.

The test determines if the observed pattern of frequencies (what we actually found in the sample) is significantly different from the pattern we would *expect* if there were no relationship between the variables.

A. Chi-Square Goodness of Fit (One-Way)

- **Objective:** To determine if the observed frequency distribution for a single categorical variable matches a theoretically expected distribution.
- *Example:* We expect 25% of our customers to prefer Brand A, 25% Brand B, 25% Brand C, and 25% Brand D. Does the actual survey data fit this equal expectation?

B. Chi-Square Test of Association (Two-Way)

- **Objective:** To test whether there is a statistically significant association between two categorical variables in a cross-tabulation table.
- **Null Hypothesis (**H0**):** The two variables are independent (no association).
- *Example:* Is there an association between **Region** (North, South, East, West) and **Product Purchased** (Product X, Product Y)? If the $\chi 2$ is significant, we conclude the relationship is not random.