Homework 1

Due Friday, February 6, 2015

For all induction proofs, we assume n is a positive integer.

1. Prove the following by induction:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

2. Prove the following by induction, or provide a counterexample:

   (a) $2^n > n$ for $n > 1$.

   (b) $2^n > n^2$ for $n > 4$.

3. We derived the geometric series formula from scratch in class:

$$\sum_{i=0}^{n} ar^i = \frac{a(1-r^{n+1})}{1-r}.$$

   Verify the formula by induction.

4. Problem 13, Section 1.3 (**See example 1.24 in text**).

5. If f_n denotes the n-th Fibonacci number, prove that

$$f_2 + f_4 + ... + f_{2n} = f_{2n+1} - 1.$$

6. Without the use of induction, show that

$$f_{n-2}^2 = f_{n+1}^2 - 4f_nf_{n-1}.$$

   Hint: Begin with $f_{n-2} = f_n - f_{n-1}$, and manipulate this equation accordingly.

Suggested Exercises (Rosen, Sixth Edition)

Section 1.1
#1-8, 11-13

Section 1.3
#3, 6, 7, 10, 11, 14, 20, 24

Section 1.4

#3-6, 13-15, 18, 19

Section 1.5
10-15, 20, 21, 23, 25, 37

Section 3.1
#3, 6, 13, 14