Homework 2

Due Friday, February 20, 2015

1. Let $2 = P_1, P_2, ...P_n$ denote the first n consecutive prime numbers. For $1 \leq i \leq n-1$, $n \geq 2$, prove the following statement, or provide a counterexample of it failing: $\frac{P_i + P_{i+1}}{2}$ is always prime.

2. Let P be a prime greater than 3. Show that if P and $P+2$ are prime, then their average is divisible by 6.

3. Prove that if $n \in \mathbb{Z}$, $3|n^2$ if and only if $3|n$ (you must prove both directions!).

4. Give an example of 10 consecutive integers all of which are composite.

5. Let $a, b, c \in \mathbb{Z}$. Prove if $(a, b) = 1$, and if $c|a$, then $(c, b) = 1$.

6. Prove that $\sqrt{10}$ is irrational.

7. Let p be a prime greater than 5. Prove that $p^2 + 2$ is never prime.

8. Find the gcd(a, b) using either the Euclidean Algorithm or Extended Euclidean Algorithm. Express the gcd(a,b) as a linear combination of a and b. Show your work!

   (a) $a = 147$, $b = 321$

   (b) $a = 100$, $b = 55$.

9. (a) Find the smallest positive integer k so that $1111x + 693y = 10004 + k$ has an integer solution.

   (b) A person buys 48 eggs, some of which are medium size, and the rest are jumbo (not necessarily in dozens). The total price is \$5.68. Suppose the jumbo eggs cost 10 cents more than the medium eggs, and also assume more jumbo eggs were bought than medium size eggs. What are all possible combinations of the number of medium and jumbo sized eggs purchased?

10. Use congruences to determine the remainder when 4^{44} is divided by 19

11. Prove $4^n \equiv 1 + 3n \pmod 9$ for any integer $n \geq 1$.

Suggested Exercises (Rosen, Sixth Edition)

Section 3.2
#3, 12(a,b,c)

Section 3.3

#1,3, 4, 7, 13, 24-26

Section 3.4
#1, 3, 5

Section 3.5
2, 6-8, 10

Section 3.7
#1, 3, 5, 7, 9, 15(b)

Section 4.1
#3, 4, 5 8(a,b), 10-12, 16, 17, 27, 28, 30, 34, 41