## Set theory

1. Write down the following sets in tabulation Method.

i) The set A of all odd natural numbers less than 10.

$A = \{1, 3, 5, 7, 9\}$

ii) The set B of all square numbers less than 100.

$B = \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

iii) The set C of all roots of the equation $x^2 + 5x + 4 = 0$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1 \quad x = 4$

$\therefore$ The roots are 1 and 4.

$\therefore \quad C = \{1, 4\}$

(iv) The set D of all prime numbers between 10 and 20.

$D = \{11, 13, 17, 19\}$.

2) write down the following sets in set-builder form.

i) The set A of all months in a year.

$A = \{x \mid x$ is a month of a year$\}$

ii) The set B of all English alphabet.

$B = \{x / x$ is an English alphabet$\}$

(iii) $C = \{3, 6, 9, \ldots\}$

$C = \{x \mid x \in N$ and is a multiple of 3$\}$

(iv) $D = \{1, 3, 5, \ldots\}$

$D = \{x \mid x \in N$ and $x$ is odd$\}$

3. Write down the sub-sets of $A = \{a, b, c, d\}$.

Soln:

The subsets of A are $\phi$, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{b, c\}$, $\{c, d\}$, $\{a, d\}$, $\{a, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{a, b, d\}$, $\{b, c, d\}$ and $\{a, b, c, d\}$.

4) If $A = \{1, 2, 3\}$ $B = \{1, 3, 5\}$ $C = \{2, 3, 4, 6\}$

Find (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $A \cup (B \cap C)$

(v) $A - (B \cap C)$

Soln:-

(i) $A \cup B = \{1, 2, 3, 5\}$

(ii) $A \cap B = \{1, 3\}$

(iii) $A - B = \{2\}$

(iv) $A = \{1, 2, 3\}$, $B \cap C = \{3\}$

$A \cup (B \cap C) = \{1, 2, 3\}$

(v) $A - (B \cap C) = \{1, 2\}$

5) Given that $A = \{0, 1, 3, 5\}$, $B = \{1, 2, 4, 7\}$

$C = \{1, 2, 3, 5, 8\}$ P.T

(i) $(A \cap B) \cap C = A \cap (B \cap C)$

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$

(iii) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

(iv) $(A \cap B) \cup C = (A \cup C) \cap (C \cup B)$

Soln:-

i) $A \cap B = \{1\}$ $(A \cap B) \cap C = \{1\}$

$B \cap C = \{1, 2\}$ $A \cap (B \cap C) = \{1\}$

ii) $A \cup B = \{0, 1, 2, 3, 4, 5, 7\}$

$B \cup C = \{1, 2, 3, 4, 5, 7, 8\}$

$(A \cup B) \cup C = \{0, 1, 2, 3, 4, 5, 7, 8\}$

(iii) $(A \cup B) \cap C = \{1, 2, 3, 5\}$

$A \cap C = \{1, 3, 5\}$

(iv) $(A \cap B) \cup C = \{1, 2, 3, 5, 8\}$

b) Given $U = \{1, 2, 3, 4, 5, 6, 7\}$ $A = \{1, 2, 3, 4, 5\}$

$B = \{1, 3, 5, 7\}$ $C = \{2, 5, 6, 7\}$

Find

(i) $A \cup C$ (ii) $B \cap A$ (iii) $C - B$ (iv) $C' \cap A$

Soln:

$A \cup C = \{1, 2, 3, 4, 5, 6, 7\}$

$B \cap A = \{1, 3, 5\}$

$C - B = \{2, 6\}$

$C' = \{1, 3, 4\}$

$C' \cap A = \{1, 3, 4\}$

10. In a survey of 5000 persons, it was found that 2,800 read Indian Express and 2,300 read Statesman while 400 read both papers. How many read neither Indian Express nor Statesman?

Soln: Let E be the set of people Reading Indian Express and S be the set of people reading Statesman.

$n(E) = 2,800$

$n(S) = 2,300$

$n(E \cap T) = 400$

$\therefore$ Number of people reading either Indian Express or Statesman is

$n(E \cup T) = n(E) + n(T) - n(E \cap T)$

$= 2800 + 2300 - 400$

$= 4700$

$\therefore$ Number of people who read neither Indian Express nor Statesman $= 5000 - 4700$
$= 300.$

11. In a city, three daily newspapers A, B, C are published; 42% of the people in the city read A; 51% read B; 68% read C; 30% read both A and B; 28% read B and C; 36% read both A and C; 8% do not read any of the three newspapers. Find the percentage of persons who read all the three papers?

Soln:

$n(A) = 42, \quad n(B) = 51, \quad n(C) = 68$

$n(A \cap B) = 30, \quad n(B \cap C) = 28, \quad n(A \cap C) = 36$

8% do not read any of the three newspapers.

92% read atleast one of the newspapers.

$\therefore n(A \cup B \cup C) = 92$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$92 = 42 + 51 + 68 - (30 + 28 + 36) + n(A \cap B \cap C)$

$92 = 161 - 94 + n(A \cap B \cap C)$

$n(A \cap B \cap C) = 92 + 94 - 161$

$= 186 - 161$

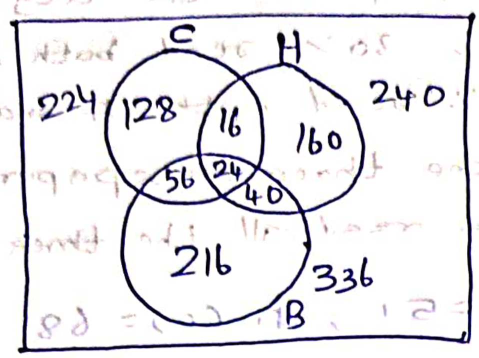
$= 25$

$\therefore$ 25% of the people read all the three newspapers.

12. Out of 880 boys in a school, 224 played cricket, 240 played hockey and 336 played Basketball; of the total 64 played both basket ball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. How many did not play any of the games and how many played only one game?

Soln:

Let C, H, B denote the set of players who played cricket, hockey and base ball respectively. We represent the above in the following venn diagram.




Number of boys who played Only cricket
$= 224 - (16+24+56) = 128$

Number of boys who played hockey only $= 240 - (16+24+40)$
$= 160$

Numbers of boys who played basketball only $= 336 - (56+24+40)$
$= 216$

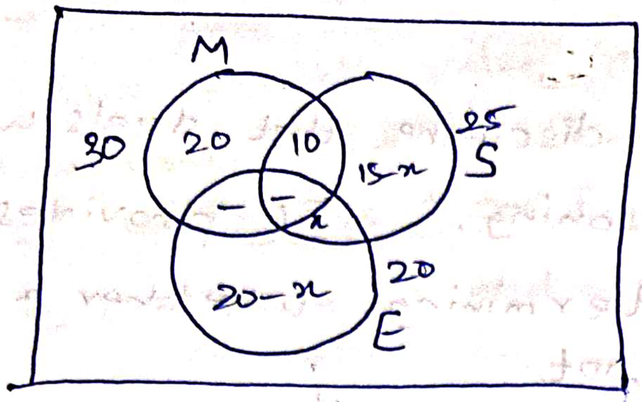
$\therefore$ Number of boys who played atleast one game.
$= 128+160+216+16+24+56+40$
$= 640$

$\therefore$ Number of boys who did not play any game $= 880 - 640 = 240$

$\therefore$ Number of boys who played only one game $= 128+160+216 = 504$.

13) Out of a group ~~of~~ 50 teachers in a High school, 30 teach Mathematics, 20 teach English and 25 teach Science. 10 teach both Mathematics and Science, and none teach Mathematics and English.
(i) How many teach science and English?
(ii) How many teach only English?

Soln:-




Total number of Teachers = 50

Let $x$ teach science and English.

Then $15-x$ teach only science

$20-x$ teach only English.

20 teach only Mathematics

$$\therefore 50 = 20 + 10 + 15 - x + x + 20 - x$$

$$x = 65 - 50$$

$$x = 15.$$

(i) Number of teachers teaching Science and English = 15.

(ii) Number of teachers teaching only English

$$= 20 - x = 20 - 15 = 5.$$

# Principle of Inclusion and Exclusion

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

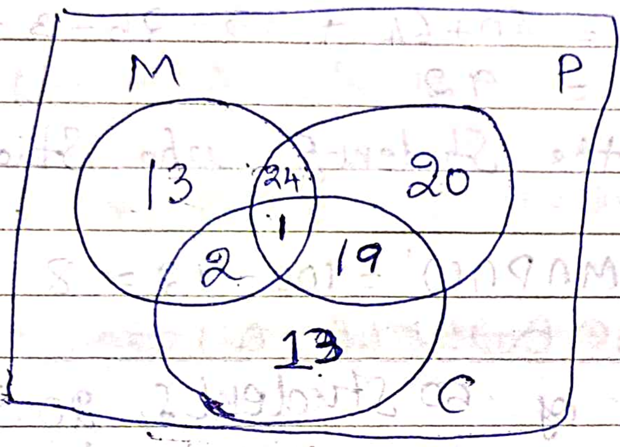
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

1) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied physics, 35 studied chemistry. 1 studied all the three subjects. 25 studied mathematics and Physics; 3 studied mathematics and Chemistry and 20 studied physics and chemistry. Find

The number of students who studied only Chemistry and the number of students who studied none of these subjects.

$n(M) = 40$ $n(P) = 64$ $n(C) = 35$
$n(M \cap P \cap C) = 1$ $n(M \cap P) = 25$ $n(M \cap C) = 3,$
$n(P \cap C) = 20.$

$24+1+19 = 44$
$64 - 44 = 20$ / $19+1+2 = 22$
$35 - 22 = 13$
$25 - 1 = 24$
$3 - 1 = 2$
$20 - 1 = 19$
$24+1+2 = 27$
$40 - 27 = 13$




The number of students studying only Mathematics and chemistry

$= n(M \cap C) - n(M \cap P \cap C)$

$= 3 - 1 = 2$

No. of students only Mathematics and Physics.

$= n(M \cap P) - n(M \cap P \cap C)$

$= 25 - 1 = 24$

No. of Students studying only Physics and chemistry.

$= n(P \cap C) - n(M \cap P \cap C)$

$= 20 - 1 = 19$

No. of Students studying only Mathematics:

$= n(M) - 24 - 1 - 2 = 13$

No. of students studying only Physics

$= n(P) - 24 - 19 - 1 = 20.$

No. of Students Studying only chemistry:
$= n(C) - 2 - 19 - 1 = 13$

$\therefore$ Total no. of Students in

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap$
$- n(M \cap C) + n(M \cap P \cap C)$
$= 40 + 64 + 35 - 25 - 3 - 20 + 1$
$= 92$

None of the Students who Studied any of the three
$= n(M \cap P \cap C)' = 100 - 92 = 8.$

2. In a class of 50 students, 20 students play foot ball and 16 students play hockey. It is found that 10 students play both the games. Use the algebra of sets to find out the number of students who play neither.

$n(F) = 20$
$n(H) = 16$
$n(F \cap H) = 10$

$n(F \cup H) = n(F) + n(H) - n(F \cap H)$
$= 20 + 16 - 10$
$= 26$

The no. of Students who play neither
i.e. $n(F \cup H)'$
$= 50 - 26 = 24$

W.E.12. *Among 50 students in a class, 26 passed in first semester and 21 passed in second semester examinations. If 17 did not pass in either semester, how many passed in both semesters?* (B.E., Apr '98, M.U.)

*Solution*

Let A, B represent respectively the sets of students passing the first semester and passing the second semester.

Then n(A) = 26 n(B) = 21.

It is given that 17 have not passed in either semester.

$\therefore$ 50 − 17 = 33 students have passed either the first semester or the second semester or both. i.e., $n(A \cup B) = 33$.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\text{i.e., } 33 = 26 + 21 - n(A \cap B)$$

$\therefore n(A \cap B) = 26 + 21 - 33 = 14$.

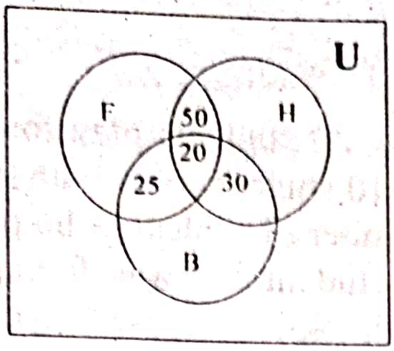
i.e., 14 students passed in both semesters.

W.E.13. *A survey of 500 television watchers produced the following information: 285 watch football games; 195 watch hockey games; 115 watch basket ball games; 45 watch football and basket ball games; 70 watch football and hockey games; 50 watch hockey and basket ball games; 50 do not watch any of the three games.*

i) *How many people in the survey watch all the three games?*

ii) *How many people watch exactly one of the three games?*

(B.E., Apr '97, Oct '98, M.U.; B.E., Nov '96, M.S.U.)




*Solution*

U is the universal set of 500 television watchers. This is represented by a rectangle. Let the three circular regions F, H and B represent the students watching football, hockey and basket ball respectively.

It is given that

a. $n(F) = 285$; $n(H) = 195$; $n(B) = 115$.
$n(F\cap B) = 45$; $n(F\cap H) = 70$; $n(H\cap B) = 50$.

b. 50 do not watch any of the three games.
i.e., $n(F\cup H\cup B)' = 50$.
$\therefore n(F\cup H\cup B) = 500 - 50 = 450$.

By the formula,

$$n(F\cup H\cup B) = n(F) + n(H) + n(B) - n(F\cap H) - n(F\cap B) - n(H\cap B) + n(F\cap H\cap B).$$

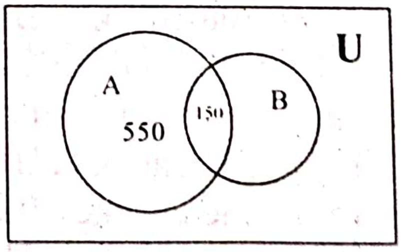
i.e., $450 = 285 + 195 + 115 - 70 - 45 - 50 + n(F\cap H\cap B)$

$n(F\cap H\cap B) = 20$. i.e., 20 people watch all the three games.

$\therefore$ Number of persons watching exactly one of the games

$$= n(F\cup H\cup B) - n(F\cap H) - n(F\cap B) - n(H\cap B) + 2\,n(F\cap H\cap B).$$
$$= 450 - (70 + 45 + 50) + 2(20) = 450 - 165 + 40 = 325.$$

W.E.14. *Show by a Venn diagram that the following data is wrong. Out of* 900 *students, it was reported that* 700 *drove cars,* 400 *rode bicycle and* 150 *both drove cars and rode bicycle.*




The basic universal set U contains 900 students. A represents the set of car driving students and B set of cylcists.

Given data: $n(A) = 700$; $n(B) = 400$ and $n(A\cap B) = 150$.

Now $n(A\cup B) = n(A) + n(B) - n(A\cap B)$ ( By Addition principle).

If the data were correct then $900 = 700 + 400 - 150$ i.e., $900 = 950$ which is absurd. Hence the data is not correct.

***Theorem 1***

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

*Proof* Suppose x is an element of A. Then $x \in B$ also ( $\because A \subseteq B$ ). Since $B \subseteq C$, we have $x \in C$.

Thus each element of A is an element of C i.e., $A \subseteq C$.

**Worked Examples**

W.E.1. *Which of the following sets are equal ?*

$\{ a, b, c \}, \{ a, b, b, c \}, \{ c, a, c, b \}, \{ b, c, a \}.$

*Solution*

All the given sets are equal to each other. The order and repetition of the elements do not change a set.

W.E.2. *Let* A = *set of all quadrilaterals*
B = *set of all parallelograms*
C = *set of all rhombuses*
D = *set of all rectangles*
E = *set of all squares*

*Find out which sets are proper subsets of others.*

*Solution*

A square has four sides. So it is also a quadrilateral. It has two pairs of opposite sides which are parallel. So it is also a parallelogram. It has four equal sides. So it is also a rhombus. It has four right angles. So it is also a rectangle. Hence $E \subseteq A$, $E \subseteq B$, $E \subseteq C$, $E \subseteq D$. i.e., E is a subset of the other four sets. But there are quadrilaterals, parallelograms, rhombuses and rectangles which are not squares. Hence E is a proper subset of the other four. In a similar manner, we find that B, C and D are proper subsets of A.

C and D are proper subsets of B.

W.E.3. *If* A *is a subset of the null set* $\phi$, *prove that* $A = \phi$.

*Solution*

It is given that $A \subset \phi$ ............(1)

Now the null set $\phi$ is a subset of every set.

So $\phi \subset A$ ............(2)

Combining (1) and (2), we set $A = \phi$.

***Theorem 3***

The union of sets is associative. i.e., If A, B and C are three sets, then $A \cup (B \cup C) = (A \cup B) \cup C$.

*Proof*

a. Let us show that $A \cup (B \cup C) \subset (A \cup B) \cup C$.

$$x \in A \cup (B \cup C)$$

$\Rightarrow x \in A$ or $x \in (B \cup C)$ by definition of union

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C.$

$\therefore A \cup (B \cup C) \subset (A \cup B) \cup C$ ........

b. Let us show that $(A \cup B) \cup C \subset A \cup (B \cup C)$.

Let $y$ be any element of the set $(A \cup B) \cup C$.

$$y \in (A \cup B) \cup C$$
$$\Rightarrow y \in (A \cup B) \text{ or } y \in C$$
$$\Rightarrow (y \in A \text{ or } y \in B) \text{ or } y \in C$$
$$\Rightarrow y \in A \text{ or } (y \in B \text{ or } y \in C)$$
$$\Rightarrow y \in A \cup (B \cup C)$$
$$\therefore (A \cup B) \cup C \subset A \cup (B \cup C)$$ ........

From (1) and (2), we have $A \cup (B \cup C) = (A \cup B) \cup C$.

***Theorem 4***

The intersection of sets is associative. i.e., If A, B and C are three sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

*Proof*

a. Let us show that $A \cap (B \cap C) \subset (A \cap B) \cap C$.

$$x \in A \cap (B \cap C)$$

$$\Rightarrow x \in A \text{ and } x \in (B \cap C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \cap C$$

$$\therefore A \cap (B \cap C) \subset (A \cap B) \cap C \qquad \text{..........(1)}$$

b. Let us show that $(A \cap B) \cap C \subset A \cap (B \cap C)$

$$y \in (A \cap B) \cap C$$

$\Rightarrow y \in (A \cap B)$ and $y \in C$.

$\Rightarrow (y \in A$ and $y \in B)$ and $y \in C$

$\Rightarrow y \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow y \in A$ and $y \in (B \cap C)$

$\Rightarrow y \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subset A \cap (B \cap C)$ ............(2)

From (1) and (2), we have $A \cap (B \cap C) = (A \cap B) \cap C$.

## DISTRIBUTIVE LAW

***Theorem 5***

For any three sets A, B, C, we have

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

***Proof***

i) Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$.

If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$, as $A \subseteq A \cup B$; $A \subseteq A \cup C$.

So $x \in (A \cup B) \cap (A \cup C)$.

If $x \in B \cap C$, then $x \in B$ and $x \in C$.

As $x \in B$, we have $x \in A \cup B$ and as $x \in C$, we have $x \in A \cup C$.

So $x \in (A \cup B) \cap (A \cup C)$.

In both cases, $x \in (A \cup B) \cap (A \cup C)$.

Thus $x \in A \cup (B \cap C) \Rightarrow x \in (A \cup B) \cap (A \cup C)$ and hence

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \qquad \text{...........}(1)$$

Conversely let $x \in (A \cup B) \cap (A \cup C)$.

Then $x \in A \cup B$ and $x \in A \cup C$.

If $x \in A$, then clearly $x \in A \cup (B \cup C)$.

If $x \notin A$, from $x \in A \cup B$ and $x \in A \cup C$, it follows that $x \in B$ and $x \in C$,

i.e., $x \in B \cap C$.

Then if $x \in (A \cup B) \cap (A \cup C)$; then $x \in A$ or $x \in B \cap C$.

i.e., $x \in A \cup (B \cap C)$.

So, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \qquad \text{...........}(2)$

From (1) and (2),

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

ii) $x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and $x \in B \cup C$.

$\Leftrightarrow x \in A$ and ( $x \in B$ or $x \in C$ )

$\Leftrightarrow$ ( $x \in A$ and $x \in B$ ) or ( $x \in A$ and $x \in C$ )

$\Leftrightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

Thus $A \cap (B \cap C) = (A \cap B) = (A \cap C)$.

## DE MORGAN'S LAWS

***Theorem 6***

For any two sets A and B,

i) $(A \cup B)' = A' \cap B'$.

*ii)* $(A \cap B)' = A' \cup B'$.

*Proof*

a. $x \in (A \cup B)' \Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$.

Thus $(A \cup B)' = A' \cap B'$.

b. $y \in (A \cap B)' \Leftrightarrow y \notin A \cap B$

$\Leftrightarrow$ either $y \notin A$ or $y \notin B$

$\Leftrightarrow$ either $y \in A'$ or $y \in B'$

$\Leftrightarrow y \in A' \cup B'$.

Thus we have $(A \cap B)' = A' \cup B'$.

*Example.* Write $A \cup (B \cap C \cap (D \cup E))$

i) without using $\cup$ and ii) without using $\cap$.

*Solution*

i) $A \cup (B \cap C \cap (D \cup E))$

$= A \cup (B \cap C \cap (D' \cap E')')$

$= (A' \cap (B \cap C \cap (D' \cap E')')')'.$

ii) $A \cup (B \cap C \cap (D \cup E))$

$= A \cup (B' \cup C' \cup (D \cup E)')'.$

## IV) PROPERTIES OF THE DIFFERENCE OPERATION

***Theorem 7***

Let A, B, C are sets and U be the universal set.

Then

a. $A' = U - A$.

b. $A - B = A \cap B'$.

c. $A - A = \phi$.

d. $A - \phi = A$.

e. $A - B = B - A$ if and only if $A = B$.

f. $A - B = A$ if and only if $A \cap B = \phi$.

g. $A - B = \phi$ if and only if $A \subseteq B$.

*Proof* We prove the properties (b), (f) and (g).

b. $x \in A - B \Leftrightarrow x \in A$ but $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B'$

$\Leftrightarrow x \in A \cap B'$

So $A - B = A \cap B'$.

f. Let $A - B = A$. Then $x \in A \Rightarrow x \in A - B \Rightarrow x \in A$ but $x \notin B$

$\Rightarrow x \notin B$.

So $x \in A \Rightarrow x \notin B$ and hence $A \cap B = \phi$.

Conversely let $A \cap B = \phi$.

Then $x \in A \Rightarrow x \notin B$ as $A \cap B = \phi$.

So $x \in A \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A - B$.

So $A \subset A - B$.

Thus $A = A - B$. (as $A - B \subset A$)

g. $A - B = \phi \Leftrightarrow A \cap B' = \phi$ ( by (b) )

$\Leftrightarrow$ if $x \in A$, then $x \notin B'$

$\Leftrightarrow$ if $x \in A$, then $x \in B$.

$\Leftrightarrow$ if $A \subseteq B$.

## v) PROPERTIES OF SYMMETRIC DIFFERENCE

We recall that the symmetric difference of the two set A and B is defined as $(A - B) \cup (B - A)$ and is denoted by $A \triangle B$.

***Theorem 8***

If A and B are sets, then

i) $A \triangle A = \phi$.

ii) $A \triangle \phi = A$.

iii) $A \triangle B = B \triangle A$.

iv) $A \triangle B = (A \cup B) - (A \cap B)$.

*Proof*

i) $A \triangle A = (A - A) \cup (A - A) = \phi \cup \phi = \phi$

ii) $A \triangle \phi = (A - \phi) \cup (\phi - A) = A \cup \phi = A$

iii) $A \triangle B = (A - B) \cup (B - A) = (B - A) \cup (A - B) = B \triangle A$.

iv) $(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)'$ as $X - Y = X \cap Y'$

$$\begin{aligned}
&= (A \cup B) \cap (A' \cup B') \\
&= ((A \cup B) \cap A') \cup ((A \cup B) \cap B') \\
&= (A \cap A') \cup (B \cap A') \cup (A \cap B') \cup (B \cap B') \\
&= (A \cap B') \cup (B \cap A') \\
&\qquad \text{as } (A \cap A') = (B \cap B') = \phi \\
&= (A - B) \cup (B - A) \\
&= A \triangle B.
\end{aligned}$$

[*Aliter* : $x \in A \triangle B$

$$\begin{aligned}
&\Leftrightarrow x \in (A - B) \cup (B - A) \\
&\Leftrightarrow (x \in A \text{ but } x \notin B) \text{ or } (x \in B \text{ but } x \notin A) \\
&\Leftrightarrow (x \in A \cup B \text{ and } x \notin B) \text{ or } (x \in A \cup B \text{ and } x \notin A) \\
&\Leftrightarrow (x \in A \cup B) \text{ and } (x \notin B \text{ or } x \notin A) \\
&\Leftrightarrow (x \in A \cup B) \text{ and } (x \notin A \cap B) \\
&\Leftrightarrow x \in (A \cup B) - (A \cap B)
\end{aligned}$$

$\therefore A \triangle B = (A \cup B) - (A \cap B)$. ]

**Exercises**

1. Write out the dual of each of the following expressions :
   i) $(A \cup B)' \cap U$.
   ii) $(A' \cup U) \cup (A \cap U)$.
   iii) $U \cap [(A \cup B') \cup U]$.
   iv) $(A \cap B) \cup (C \cap \phi) \cup (U \cap A \cap B)$.

2. Write the dual of each of the following :
   i) $(B \cap C) \cup A = (B \cup A) \cap (C \cup A)$.
   ii) $A \cap (A' \cup B) = A \cap B$.
   iii) $(A \cup \phi) \cup (U \cap A') = U$.

3. Prove the following identities; then write out the dual identities and prove them.
   i) $A \cap (A \cup B) = A$. ( Absorption Law )
   ii) $(A \cap B) \cup (A \cap B') = A$.
   iii) $(B \cup C) \cap A = (B \cap A) \cup (C \cap A)$.
   iv) $A \cup (A' \cap B) = A \cup B$.
   v) $(A \cup U) \cap (A \cap \phi) = \phi$.
   vi) $(A \cap B) \cup (B \cap C) = (A' \cap C')' \cap B$.

4. Draw a Venn diagram to illustrate
   i) $(A \cap B) \cup A = A$.
   ii) $A \cup (B \setminus A) = A$.

**Answers**

1. i) $(A \cap B)' \cup \phi$
   ii) $(A' \cap \phi) \cap (A \cup \phi)$
   iii) $\phi \cup [(A \cap B') \cap \phi]$
   iv) $(A \cup B) \cap (C \cup U) \cap (\phi \cup A \cup B)$

2. i) $(B \cup C) \cap A = (B \cap A) \cup (C \cap A)$
   ii) $A \cup (A' \cap B) = A \cup B$
   iii) $(A \cap U) \cap (\phi \cup A') = \phi$

3. i) $A \cup (A \cap B) = A$
   ii) $(A \cup B) \cap (A \cup B') = A$
   iii) $(B \cap C) \cup A = (B \cup A) \cup (C \cap A)$
   iv) $A \cap (A' \cup B) = A \cap B$
   v) $(A \cap \phi) \cup (A \cup U) = U$
   vi) $(A \cup B) \cap (B \cup C) = (A' \cup C')' \cup B$.

**Worked Examples**
The following worked examples will illustrate the use of the various theorems on set operations studied so far.

W.E.1. *Show that* $(A - B) - C = A - (B \cup C)$.
*Solution*

$$\begin{aligned}
(A - B) - C &= (A \cap B') \cap C' && \text{( Definition of difference )}\\
&= A \cap (B' \cap C') && \text{( Asso. Law for set intersection)}\\
&= A \cap (B \cup C)' && \text{( De Morgan's Law )}\\
&= A - (B \cup C).
\end{aligned}$$

W.E.2. *Show that* $A - (B - C) = (A - B) \cup (A \cap C)$.
*Solution*

$$\begin{aligned}
A - (B - C) &= A - (B \cap C') && \text{( Definition of difference )}\\
&= A \cap (B \cap C')' && \text{( Definition of difference )}\\
&= A \cap (B' \cup C) && \text{( De Morgan's Law )}\\
&= (A \cap B') \cup (A \cap C) && \text{(Distributive law for intersection)}\\
&= (A - B) \cup (A \cap C).
\end{aligned}$$

W.E. 3. *Show that* $A \cap (B - C) = (A \cap B) - (A \cap C)$
*Solution*

$$\begin{aligned}
\text{R.H.S.} &= (A \cap B) - (A \cap C)\\
&= (A \cap B) \cap (A \cap C)' && \text{( Definition of difference )}\\
&= (A \cap B) \cap (A' \cup C') && \text{( De Morgan's law )}\\
&= (A \cap B \cap A') \cup (A \cap B \cap C') && \text{( Distributive law )}\\
&= (A \cap A' \cap B) \cup (A \cap B \cap C')\\
&= (\phi \cap B) \cup (A \cap B \cap C') && (A \cap A' = \phi)\\
&= \phi \cup (A \cap B \cap C') && (\phi \cap B = \phi)\\
&= A \cap B \cap C'\\
&= A \cap (B \cap C')\\
&= A \cap (B - C) = \text{L.H.S.}
\end{aligned}$$

W.E. 4. *Show that* $A \cup (B - C) = (A \cup B) - (C - A)$
*Solution*

$$\begin{aligned}
A \cup (B - C) &= A \cup (B \cap C') && \text{( Definition of difference )}\\
&= (A \cup B) \cap (A \cup C') && \text{(Distributive law)}\\
&= (A \cup B) \cap (A' \cap C')' && \text{( De Morgan's law )}\\
&= (A \cup B) \cap (C \cap A')'
\end{aligned}$$

$$= (A \cup B) \cap (C - A)'$$
$$= (A \cup B) - (C - A).$$

Note that $A \cup (B - C) \neq (A \cup B) - (A \cup C)$. i.e., union is not distributive over difference.

W.E.5. *If* $A \cup B = A \cup C$ *and* $A \cap B = A \cap C$, *prove that* $B = C$.
(B.E. Apr '98, M. U.)

*Solution*

As it is given that $A \cup B = A \cup C$, we have

$$B \cap (A \cup B) = B \cap (A \cup C) \quad \text{........(1)}$$

L.H.S. $= B \cap (A \cup B) = B$ as $B \subseteq A \cup B$ .........(2)

R.H.S. $= B \cap (A \cup C)$

$= (B \cap A) \cup (B \cap C)$ (Distributive Law)

$= (A \cap B) \cup (B \cap C)$ (Commutative Law)

$= (A \cap C) \cup (B \cap C)$ (since $A \cap B = A \cap C$)

$= (A \cup B) \cap C$ (Distributive Law)

$= (A \cup C) \cap C$ (as $A \cup B = A \cup C$)

$= C$ (as $C$ is a subset of $A \cup C$)

Thus R.H.S. $= C$ .........(3)

From (2) and (3), $B = C$.

W.E.6. *If* $A \cup B = A \cup C$, *is* $B = C$ ? *Explain.* (B.E. Apr '97, M.U.)

*Solution*

$A \cup B = A \cup C$ need not imply $B = C$. For example, take
$A = \{1, 2, 3, 4\}$ ; $B = \{3, 5, 6\}$ ; and $C = \{1, 2, 5, 6\}$
Then $A \cup B = \{1, 2, 3, 4, 5, 6\} = A \cup C$, but $B \neq C$.
[Remark : $A \cup B = A \cup C \Leftrightarrow B - A = C - A$].

W.E.7. *If* A, B, C *are sets, prove the following*

a. $A \cup (B - A) = A \cup B$

b. $A - (B \cup C) = (A - B) \cap (A - C)$.

(B.E. Nov '96, Bharathiar Uni.)

*Solution*

a. $A \cup (B - A) = A \cup (B \cap A')$ as $B - A = B \cap A'$

$= (A \cup B) \cap (A \cup A')$ (Distributive law)

$= (A \cup B) \cap U$

$= A \cup B.$

b. $A - (B \cup C) = A \cap (B \cup C)'$

$= A \cap (B' \cap C')$

$= A \cap B' \cap C'$ .......(1)

$$(A - B) \cap (A - C) = (A \cap B') \cap (A \cap C')$$

as $A - B = A \cap B'$ & $A - C = A \cap C'$

$$(A - B) \cap (A - C) = A \cap B' \cap C' \quad \text{.......(2)}$$

From (1) and (2), we have $A - (B \cup C) = (A - B) \cap (A - C)$.

W.E.8. *If A and B are sets, prove that $A \subseteq B$ if and only if $B' \subseteq A'$.*

(M.C.A., Apr '97, Bharathiar Uni.)

*Solution*

$$\begin{aligned} A \subseteq B &\Leftrightarrow \text{if } x \in A, \text{ then } x \in B. \\ &\Leftrightarrow \text{if } x \notin A', \text{ then } x \notin B'. \\ &\Leftrightarrow \text{if } x \in B', \text{ then } x \in A'. \\ &\Leftrightarrow B' \subseteq A'. \end{aligned}$$