Friis Transmission Equation

Group Meeting 10/10/13

Friis Equation Origins

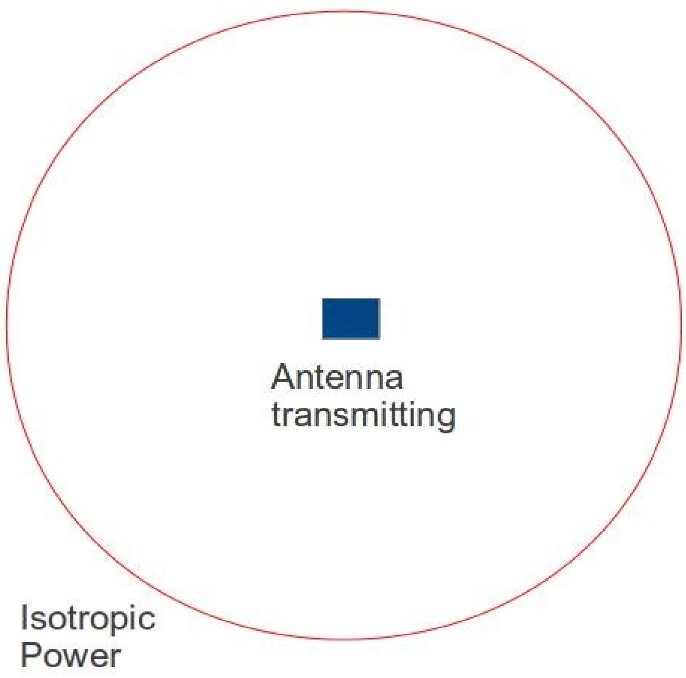
- Derived in 1945 by Bell Labs worker Harald T. Friss
- Gives the amount of power an antenna received under ideal conditions from another antenna
  - Antennas must be in far field
  - Antennas are in unobstructed free space
  - Bandwidth is narrow enough that a single wavelength can be assumed
  - Antennas are correctly aligned and polarized

Simple Form of Friis Equation

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

- P_r : Power at the receiving antenna
- P_t : output power of transmitting antenna
- G_t, G_r: gain of the transmitting and receiving antenna, respectively
- λ: wavelength
- R: distance between the antenna

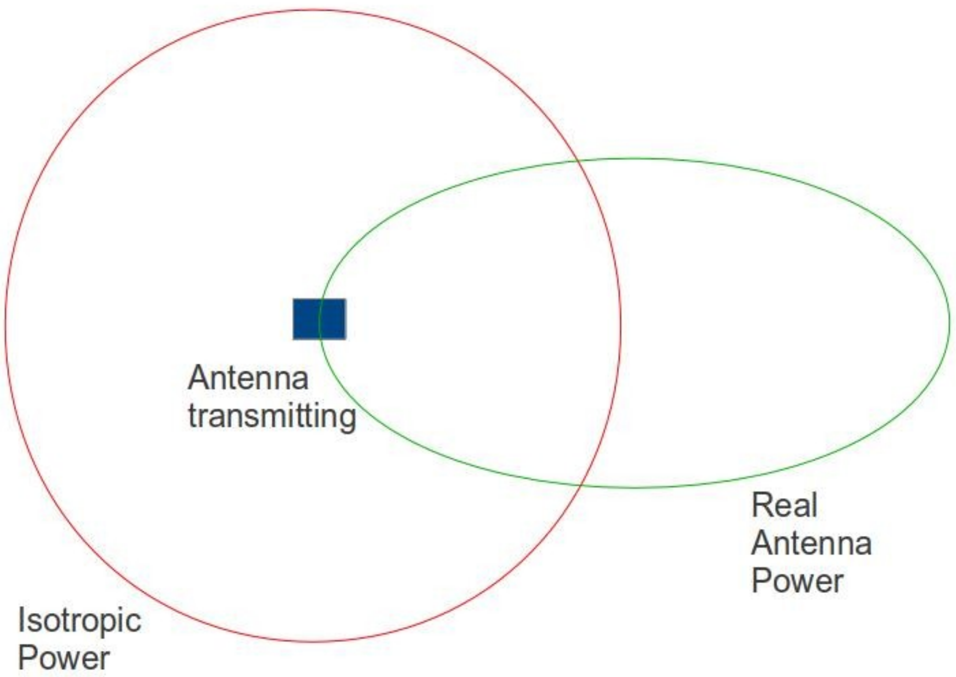
Derivation of equation



- Power from isotropic antenna falls off as r^2
- Power density (p) would be:

$$p = \frac{P_t}{4\pi R^2}$$

Derivation of equation



Multiplying by gain of the transmitting antenna gives a real antenna pattern

$$p = \frac{P_t}{4\pi R^2} G_t$$

- If receiving antenna has an effective aperture of A_{eff} the power received by this antenna (P_r) is

$$P_r = p\, A_{eff}$$

thus:

$$P_r = \frac{P_t}{4\pi R^2} G_t A_{eff}$$

- The effective aperture of an antenna can be written as

$$A_e = \frac{\lambda^2}{4\pi} G$$

plugging in:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Modifications to Friis equation (Complicated Form)

$$\frac{P_r}{P_t} = G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r) \left(\frac{\lambda}{4\pi R}\right)^2 (1-|\Gamma_t|^2)(1-|\Gamma_r|^2) |\boldsymbol{a}_t \cdot \boldsymbol{a}_r^*|^2 e^{-\alpha R}$$

- G_t , G_r : modifications to gain of antennas in which the antennas “see” each other.
- Γ_t and Γ_t are the reflection coefficients of the antennas
- $\mathbf{a}_t$ and $\mathbf{a}_r$ are the polarization vectors of the antennas
- α is the absorption coefficient of the medium