Unit 1 Course Outline

Number System: Decimal, Binary, Octal, Hexadecimal number system, Conversion of numbers from one number system to other, complement method of addition ,subtraction using 9's and 10's compliment method & 1's and 2's complement method.

Binary Codes: Weighted and Non-weighted code, 8421 BCD code, XS-3 code, Gray code, Binary to Gray conversion, Gray to Binary conversion

Logic Gates & Boolean Algebra: AND, OR, NOT, NAND, NOR, X-OR, X-NOR, BUFFER, Axioms and laws of Boolean algebra, D'morgans theorem, Duality, Reduction of Boolean expression.

Introduction to Digital Electronics

- Digital Electronics represents information (0, 1) with only two discrete values.
- Ideally

  “no voltage” (e.g., 0v) represents a 0 and

  “full source voltage” (e.g., 5v) represents a 1
- Realistically

  “low voltage” (e.g., <1v) represents a 0 and

  “high voltage” (e.g., >4v) represents a 1

What is a Digital System?

- Digital system
  - System that takes in digital inputs and generates digital outputs
  - Example: Computer
    - Digital inputs (letters and numbers from keyboard)
    - Digital output (new numbers or letters stored to a file or display on screen)
  - Many other digital systems exist
    - Cell phones, automobile control engines, TV set top boxes, musical instruments, DVD players, digital cameras, finger print recognition, …

Analog versus Digital

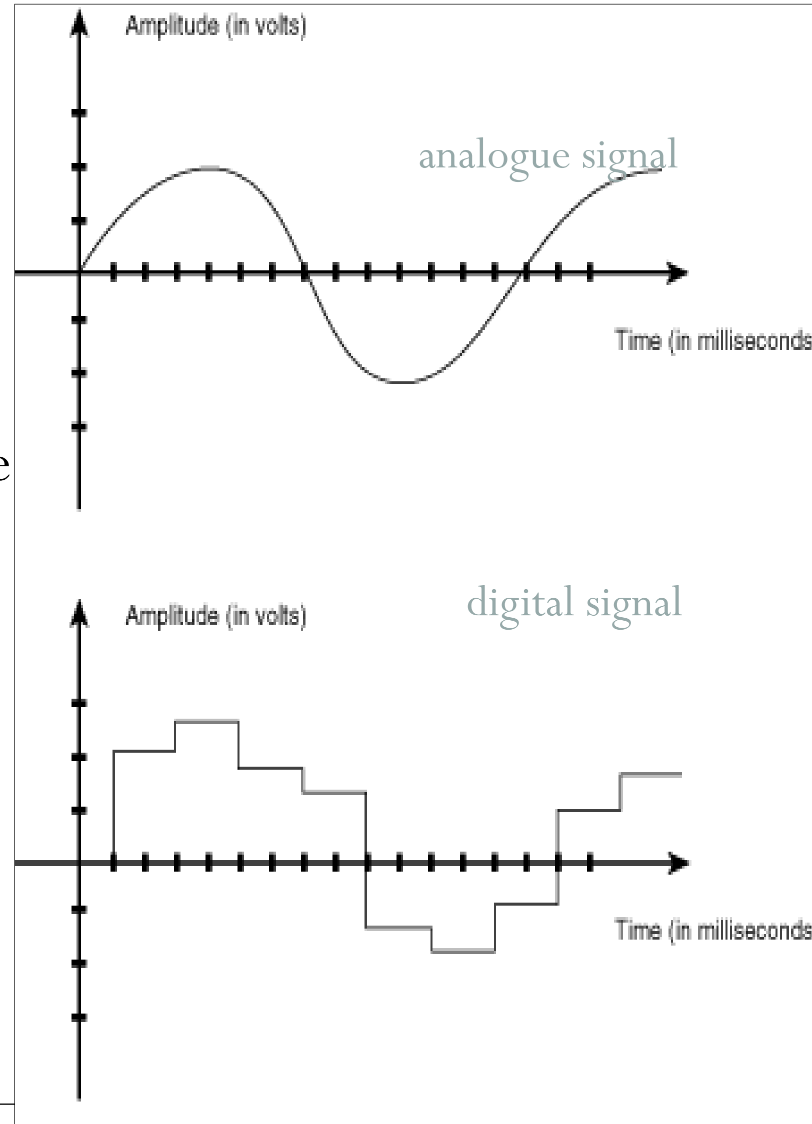
- Analog systems process time-varying signals that can take on any value across a continuous range of voltages (in electrical/electronics systems).
- Digital systems process time-varying signals that can take on only one of two discrete values of voltages (in electrical/electronics systems).
  - Discrete values are called 1 and 0 (ON and OFF, HIGH and LOW, TRUE and FALSE, etc.)

Digital vs. Analog

- Digital signal
  - Discrete
  - Signal that can have one of a finite set of possible values
- Analog signal
  - Continuous
  - Signal that can have one of an infinite number of possible values

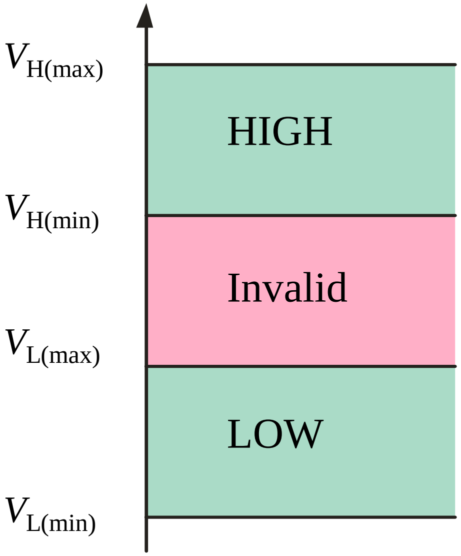
Analog versus Digital

- A digital signal, unlike continuous analogue signal, varies abruptly and changes between distinct voltage or current levels. (commonly the 0 or 1 voltage levels of a binary system)



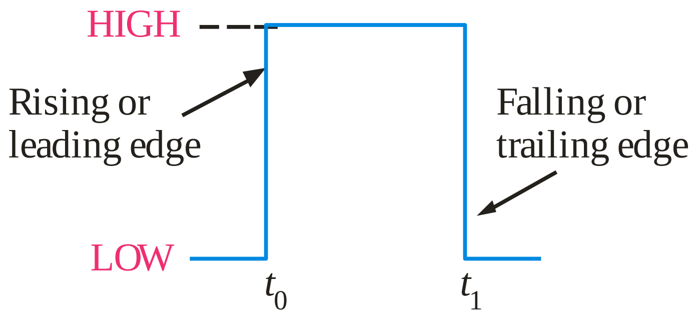
Binary Digits and Logic Levels

- Digital electronics uses circuits that have two states, which are represented by two different voltage levels called HIGH and LOW. The voltages represent numbers in the binary system
- In binary, a single number is called a ***bit*** (for ***b***inary dig***it***). A bit can have the value of either a 0 or a 1, depending on if the voltage is HIGH or LOW

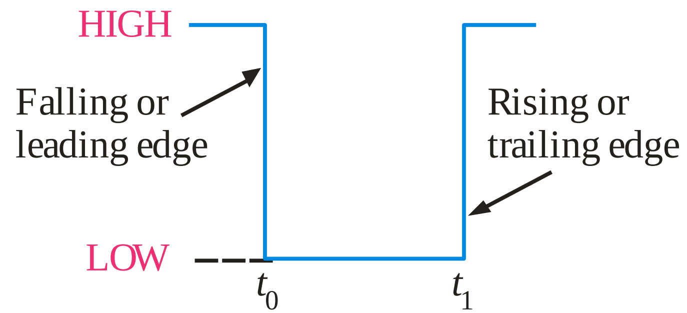


Digital Waveforms

- Digital waveforms change between the LOW and HIGH levels. A positive going pulse is one that goes from a normally LOW logic level to a HIGH level and then back again. Digital waveforms are made up of a series of pulses



(a) Positive–going pulse



(b) Negative–going pulse

Advantages of Digital Systems

1. Digital systems are generally easier to design
2. Information storage is easy
3. Accuracy is easier to maintain throughout the system
4. Operation can be programmed
5. Digital circuits are less affected by noise
6. More digital circuitry can be fabricated on IC chips

Limitation of Digital Systems

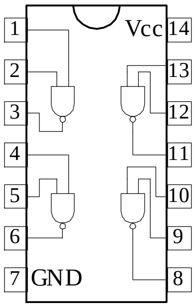
- Digital signal will not be an exact copy of the original analogue signal: sampling error
- To take advantages of digital techniques
  - Convert analog inputs to digital
  - Process the digital
  - Convert the digital outputs to analog
  - Both analog and digital technique can be employed in the same system called **hybrid system**

Integrated Circuits

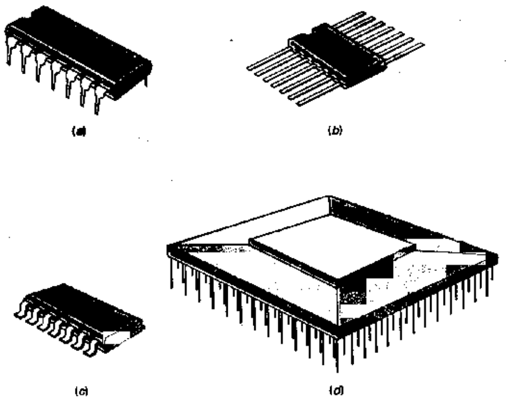
- IC is an electronic circuit, Constructed entirely on a single piece of semiconductor material called substrate ,referred as a chip.
- Classification of IC- Analog & Digital
- **Digital IC** are collection of resistors, diodes and transistor fabricated on a single chip.
- No additional component required for their operation.
- O/P – logic level 0 or 1.
- Low cost ,low power, smaller size.
- Analog ICs, such as sensors and operational amplifier, work by processing continuous signals

Integrated Circuits

- A gate is a physical implementation of a Boolean function
- Logic gates are usually embedded in Integrated Circuits ICs, sometimes referred as Chips
- According to its complexity, ICs are classified as SSI, MSI, LSI and VLSI (*s*mall, *m*edium, *l*arge & *v*ery *l*arge *s*cale *i*ntegration)
- Logic gates ICs may be CMOS type, both require a power supply of +5 volts and a 'ground' in addition to the logic inputs
- DIP type (dual-in-line package) logic ICs will be used in our experiments



Top view of a logic gate IC



Levels of Integration

- Digital IC are Categorized according to their circuit Complexity.-number of equivalent logic gates on the substrate.
- Small Scale Integration or (SSI) - Contain up to 10 transistors or a few gates within a single package such as AND, OR, NOT gates.
- Medium Scale Integration or (MSI) - between 10 and 100 transistors or tens of gates within a single package and perform digital operations such as adders, decoders, counters, flip-flops and multiplexers.
- Large Scale Integration or (LSI) - between 100 and 1,000 transistors or hundreds of gates and perform specific digital operations such as I/O chips, memory, arithmetic and logic units.

Levels of Integration (contd..)

- Very-Large Scale Integration or (VLSI) - between 1,000 and 10,000 transistors or thousands of gates and perform computational operations such as processors, large memory arrays and programmable logic devices.
- Super-Large Scale Integration or (SLSI) - between 10,000 and 100,000 transistors within a single package and perform computational operations such as microprocessor chips, micro-controllers, basic PICs and calculators.
- Ultra-Large Scale Integration or (ULSI) - more than 1 million transistors - the big boys that are used in computers CPUs, GPUs, video processors, micro-controllers, FPGAs and complex PICs.

Logic Gates

Logic Gates

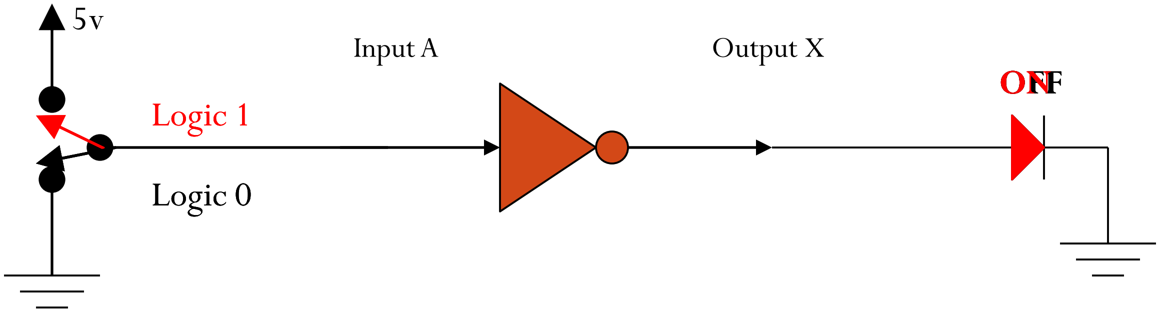
- Fundamental building block of digital System.
- To Make decisions
- It produces one output level when combination of input levels are present & different output level when other combination of input levels are present.
- O/P is dependent only state of the input at that instant
- Three basic gates –AND ,OR ,NOT

The NOT Gate (inverter):

The NOT gate is the first of the three fundamental logic gates.

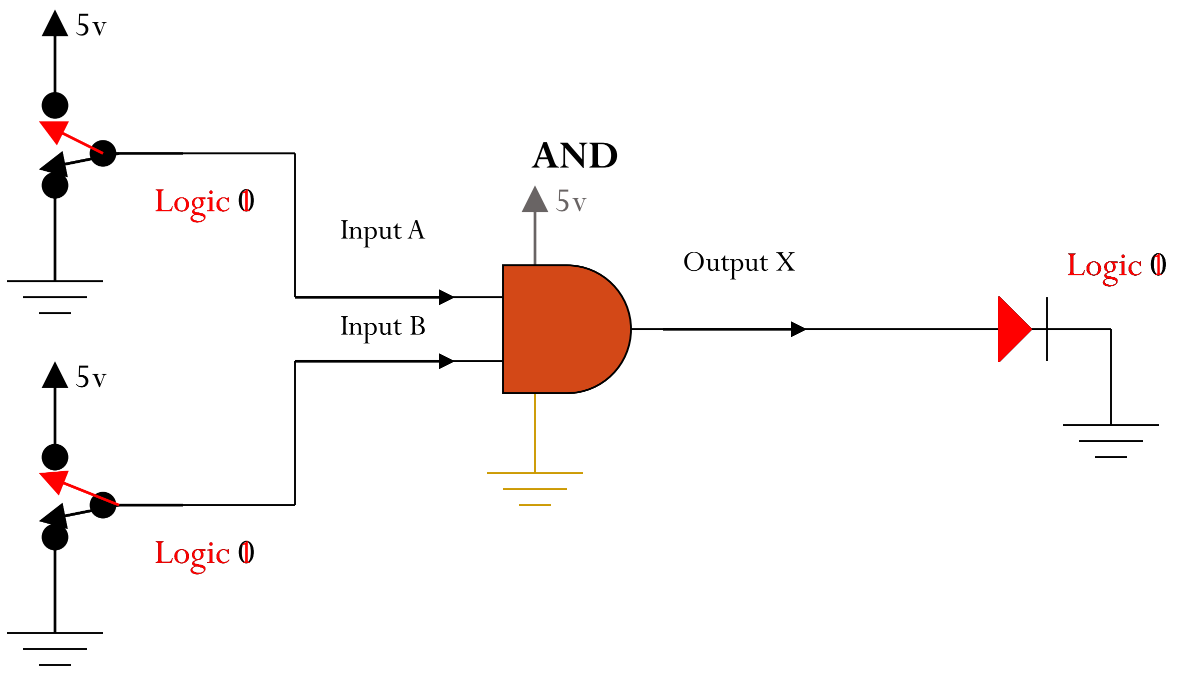
Truth Table: Is a chart that lists the input condition on the left and the gate's output response on the right. The table shows that the NOT gate responds at the output with the inverse of the signal applied to the input.

A	*X*
0	1
1	0



The AND Gate:

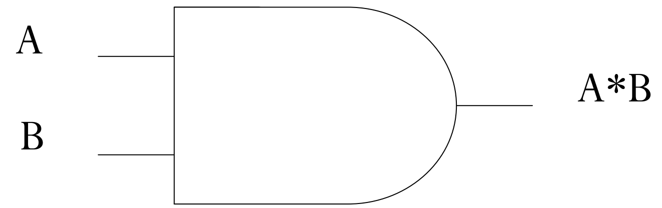
Truth Table: The table shows that the AND gate responds with a high at the output if the signal applied to the input A ***and*** B are both high.



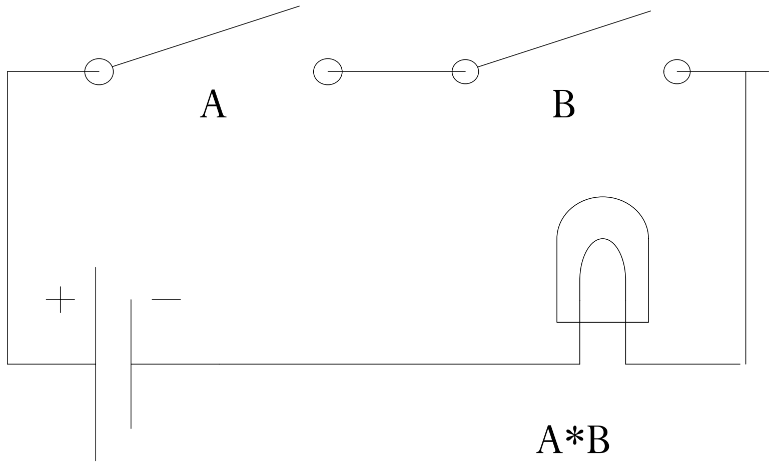
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

The AND Gate (cont..)

Logic Gate:

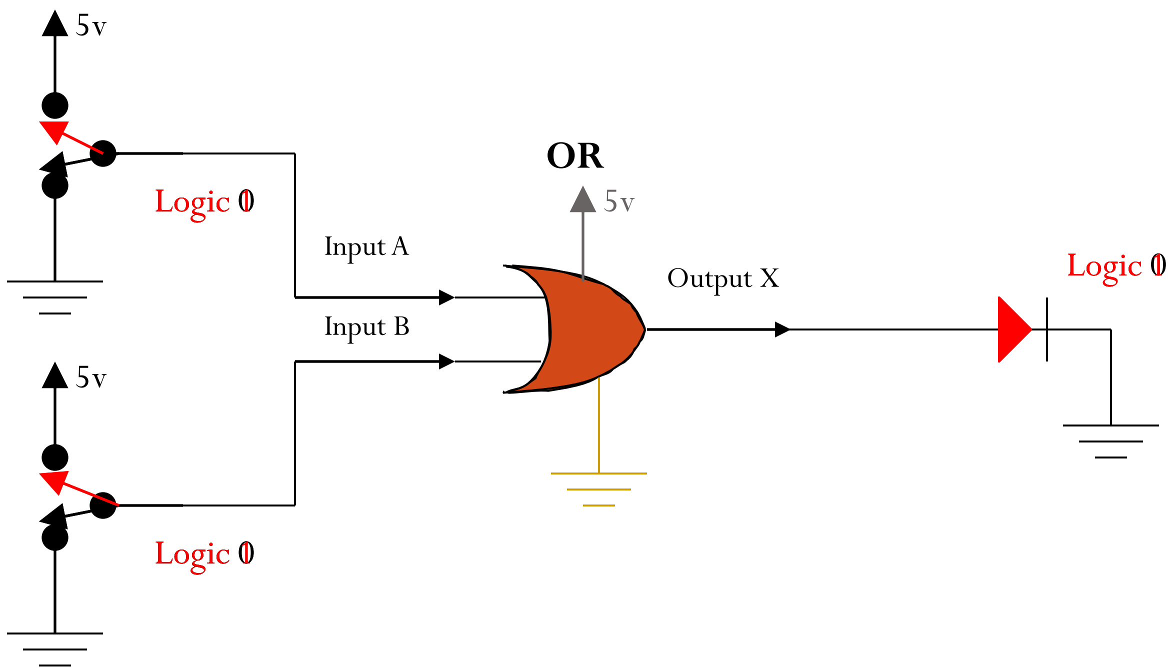


Series Circuit:



The OR Gate:

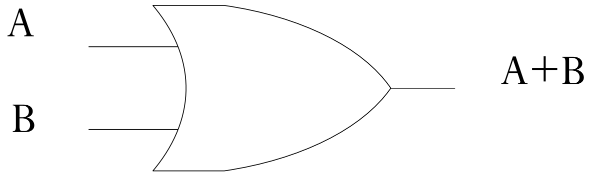
Truth Table: The table shows that the OR gate responds with a high at the output if the signal applied to the input A ***or*** B is high.



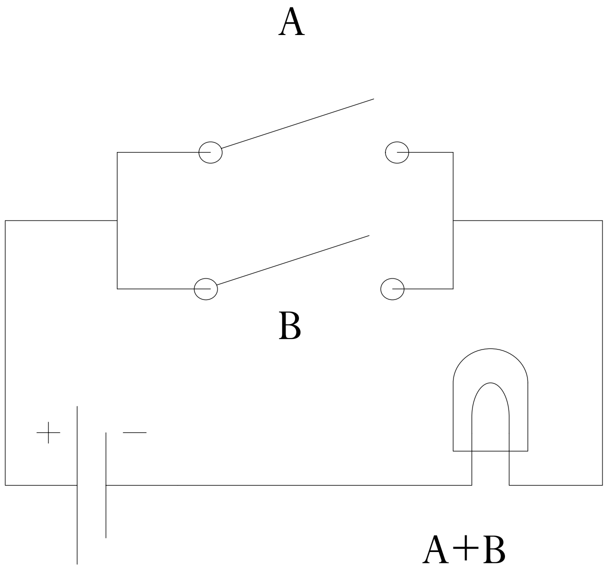
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

The OR Gate:

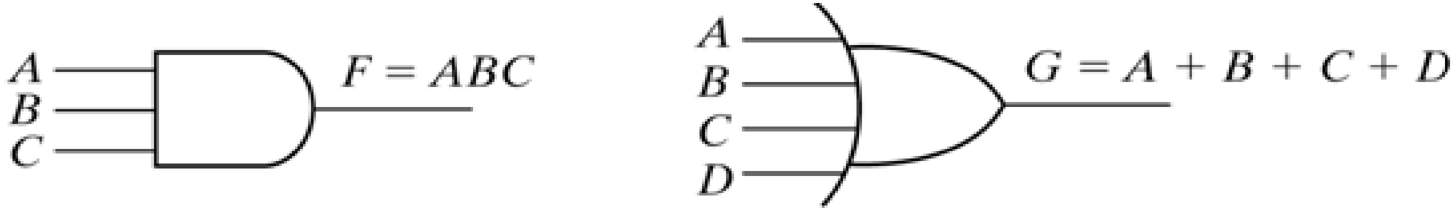
Logic Gate:



Parallel Circuit:

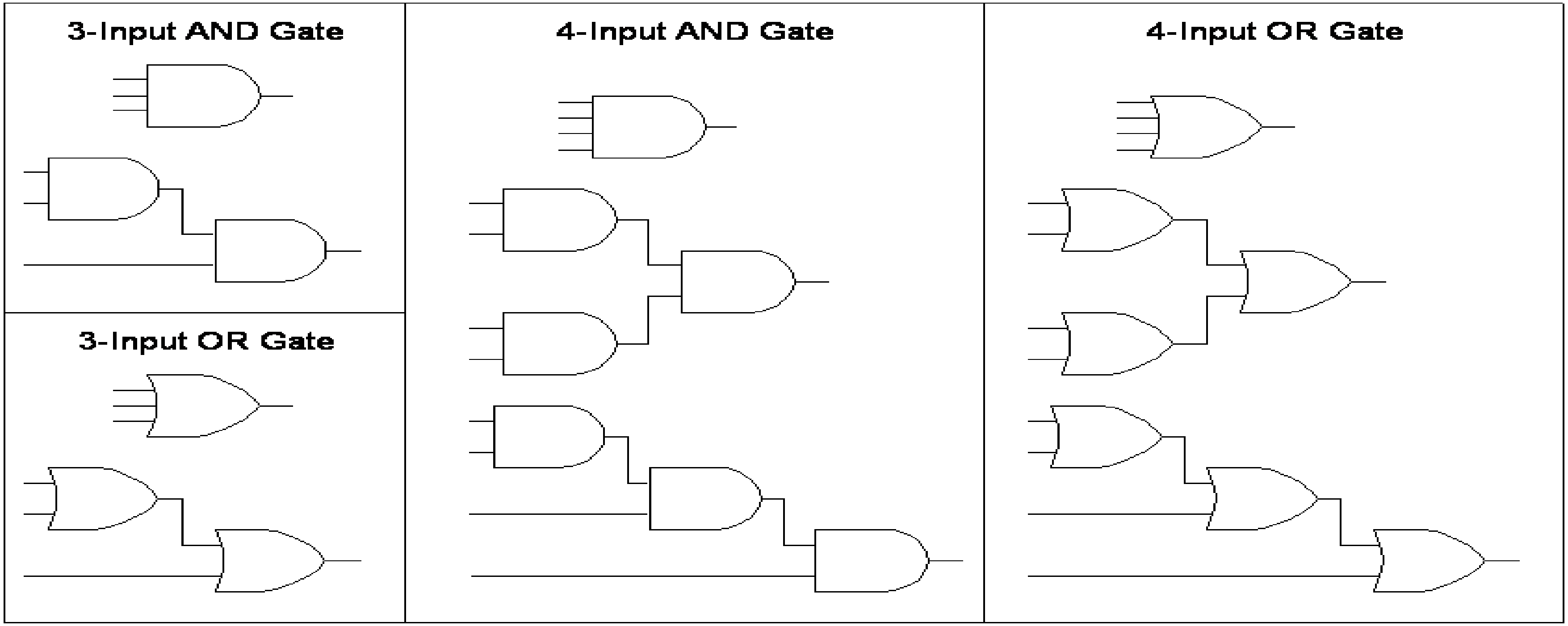


AND & OR Gate



(a) Three-input AND gate

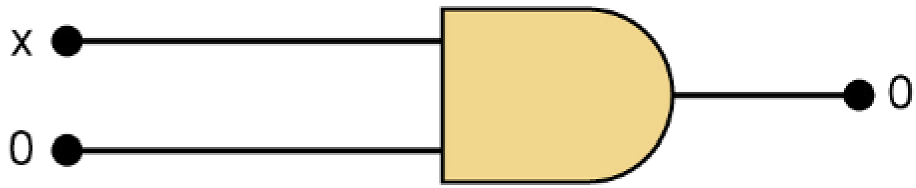
(b) Four-input OR gate



Review of Boolean algebra

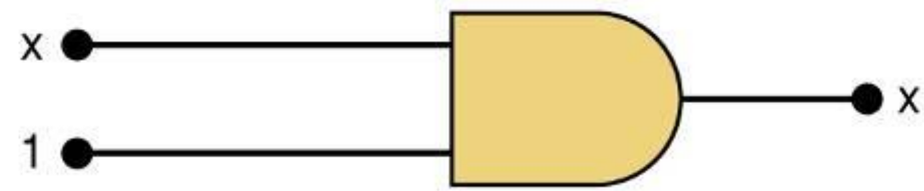
- Not is a horizontal bar above the number
  - $\overline{0} = 1$
  - $\overline{1} = 0$
- Or is a plus
  - 0+0 = 0
  - 0+1 = 1
  - 1+0 = 1
  - 1+1 = 1
- And is multiplication
  - 0*0 = 0
  - 0*1 = 0
  - 1*0 = 0
  - 1*1 = 1

Boolean Theorems (AND Laws)



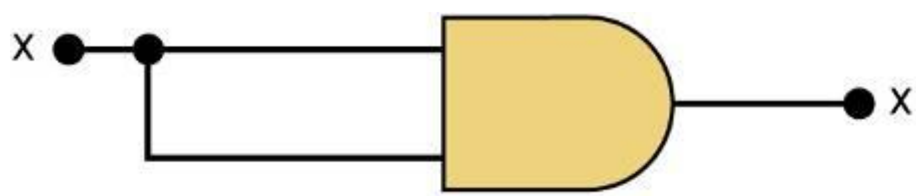
(1) $x \cdot 0 = 0$

Theorem (1) states that if any variable is ANDed with 0, the result must be 0.



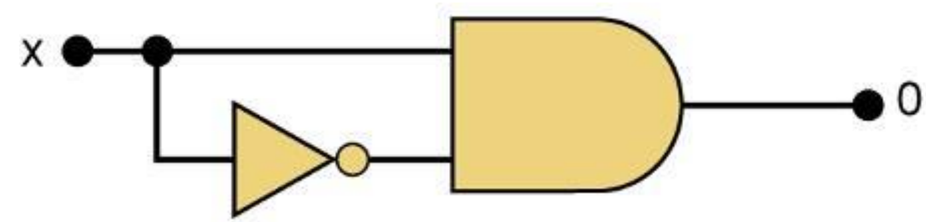
(2) $x \cdot 1 = x$

Theorem (2) is also obvious by comparison with ordinary multiplication.



(3) $x \cdot x = x$

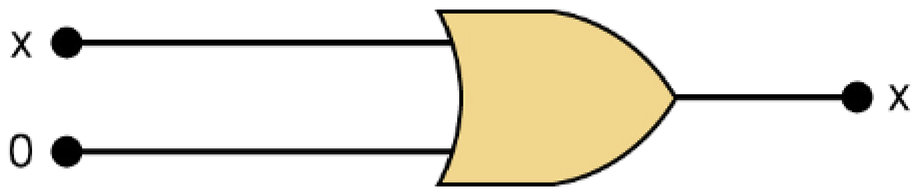
Prove Theorem (3) by trying each case.
If x = 0, then 0 • 0 = 0
If x = 1, then 1 • 1 = 1
Thus, x • x = x



(4) $x \cdot \overline{x} = 0$

Theorem (4) can be proved in the same manner.

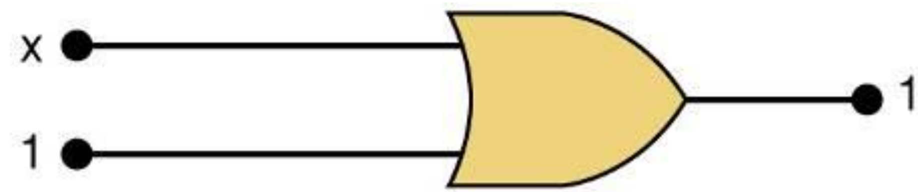
Boolean Theorems (OR Laws)



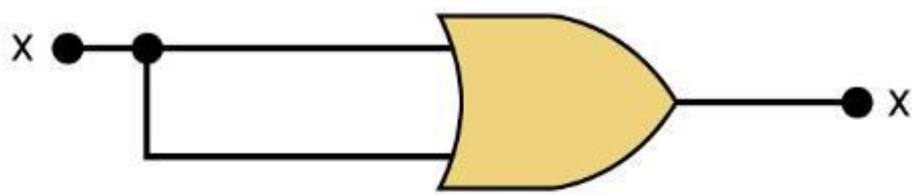
(5) $x + 0 = x$

Theorem (5) is straightforward, as 0 *added* to anything does not affect value, either in regular addition or in OR addition.

Theorem (6) states that if any variable is ORed with 1, the is always 1. Check values: 0 + 1 = 1 and 1 + 1 = 1.



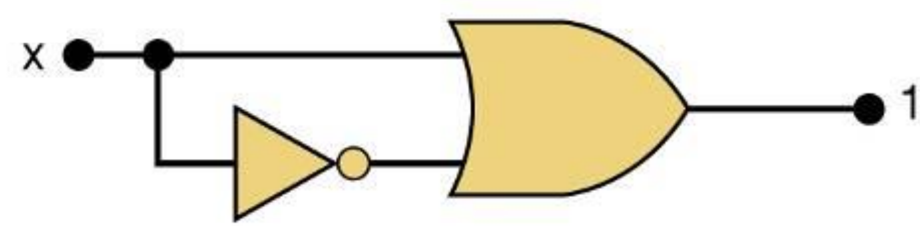
(6) $x + 1 = 1$



(7) $x + x = x$

Theorem (7) can be proved by checking for both values of x: 0 + 0 = 0 and 1 + 1 = 1.

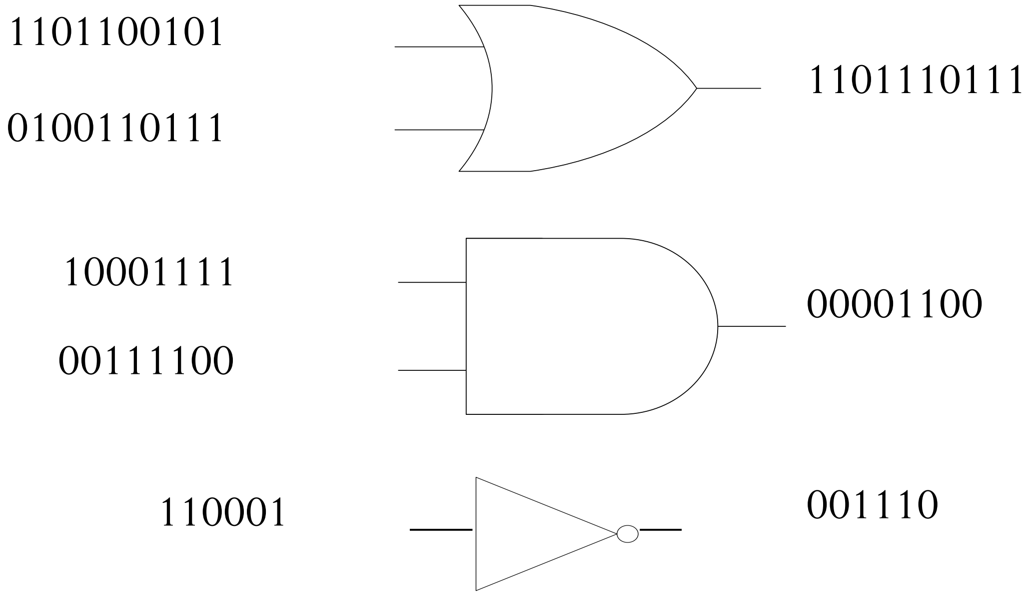
Theorem (8) can be proved similarly.



(8) $x + \overline{x} = 1$

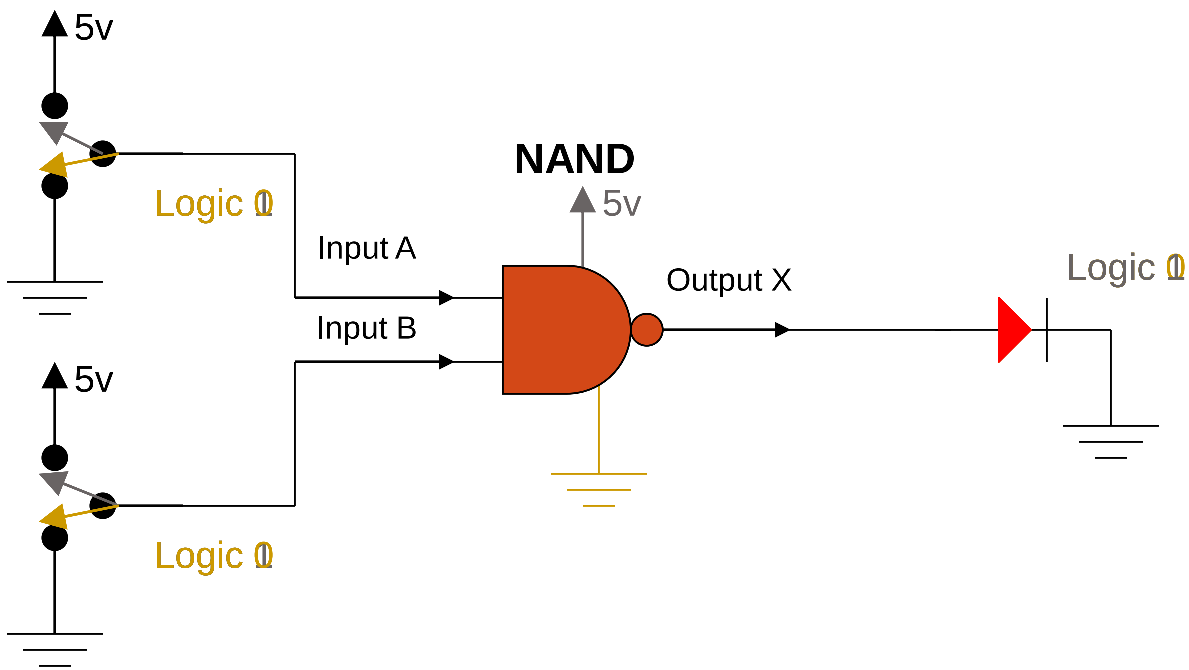
n-bit Inputs

- For convenience, it is sometimes useful to think of the logic gates processing *n*-bits at a time. This really refers to *n* instances of the logic gate, not a single logic date with *n*-inputs.



NAND Gate

The **NAND** gate is equivalent to an **AND** gate with a **NOT** gate connected to its **output**.
NAND comes from the words **N**ot **AND**



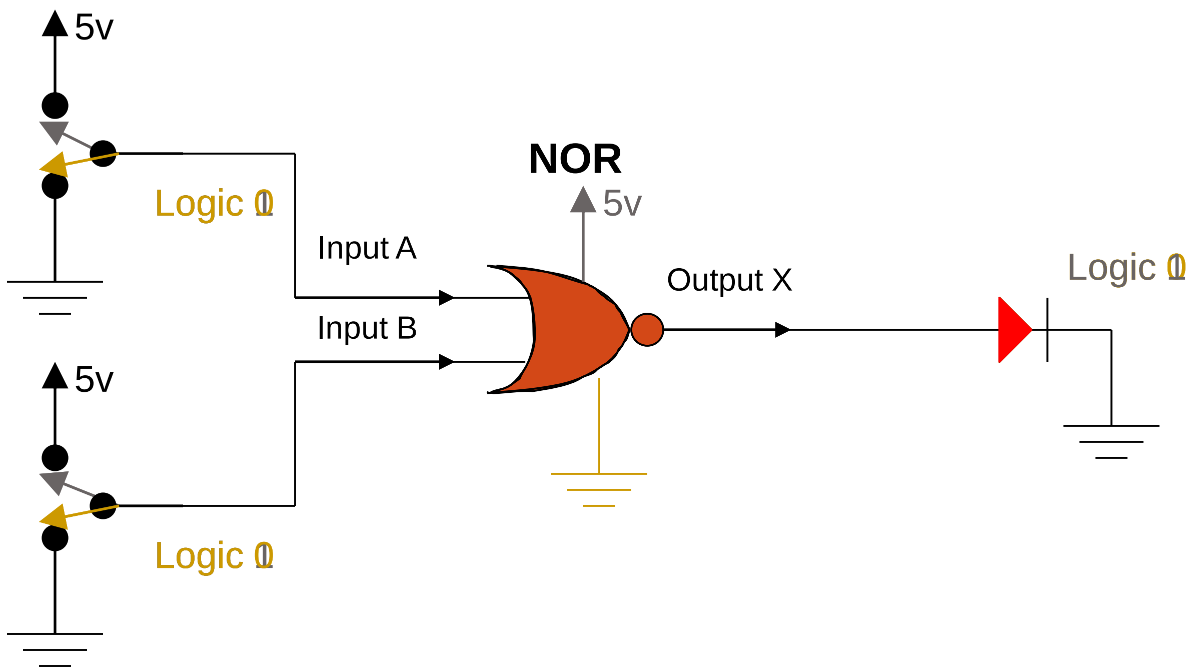
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Boolean Equation: here is the equation for the NAND gate.

$$X = \overline{A \bullet B}$$

NOR Gate

The **NOR** gate is equivalent to an **OR** gate with a **NOT** gate connected to its **output**.
NOR comes from the words **N**ot **OR.**



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

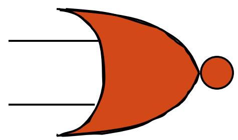
Boolean Equation: here is the equation for the NOR gate.

$$X = \overline{A + B}$$

NOR and NAND Gate Alternate Symbols:

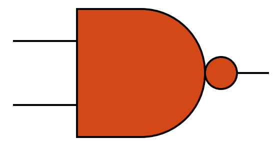
The “standard” logic symbols for the NAND and NOR gates indicates a gates response to “logic 1” at the input.

NOR



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

NAND



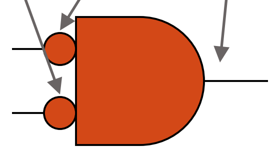
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

__Alternate NOR GATE(Bubbled AND gate)__: The bubbles at the input of the NOR gate implies that a “logic 0” at input A ***__and__*** a “logic 0” at input B are required to produce a “logic 1” at output X (NO bubble at output).

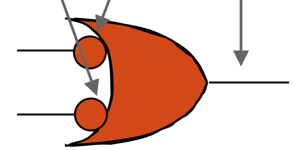
__Alternate NAND GATE (Bubbled OR gate)__: The bubbles at the input of the NAND gate implies that a “logic 0” at input A ***__or__*** a “logic 0” at input B are required to produce a “logic 1” at output X (NO bubble at output).

0 AND 0 = 1

0 OR 0 = 1



Alternate NOR

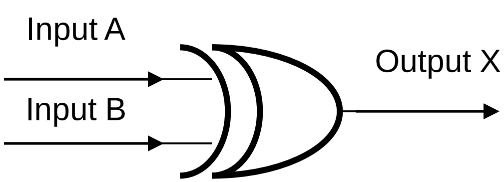


Alternate NAND

XOR Gate & XNOR Gate

The **XOR** gate is an e**x**clusive **OR** gate. It will output a logic 1 if there is an **exclusive** logic **1** at input **A** **or** **B**. **Exclusive** means: **Only one input can be high at one time**.

XOR



A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

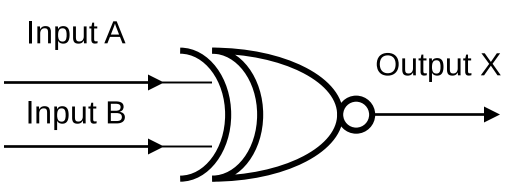
XOR Boolean Equation:

$$X = A \oplus B$$

$$X = \bar{A}B + A\bar{B}$$

The **XNOR** gate is an e**x**clusive **OR** gate with an **NOT** gate at the output. It will output a logic **0** if there is an **exclusive** logic **1** at input **A** **or** **B**.

XNOR



A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

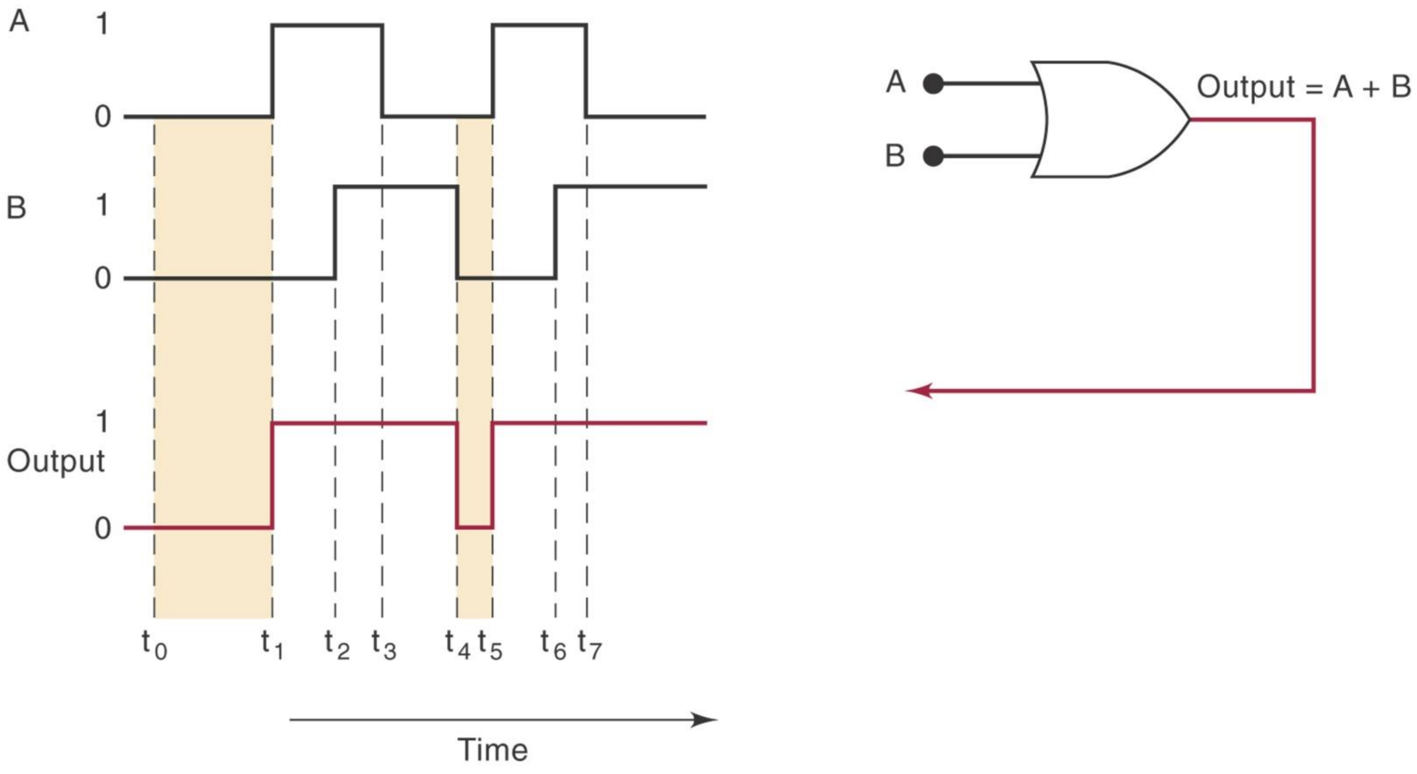
XOR Boolean Equation:

$$X = \overline{A \oplus B}$$

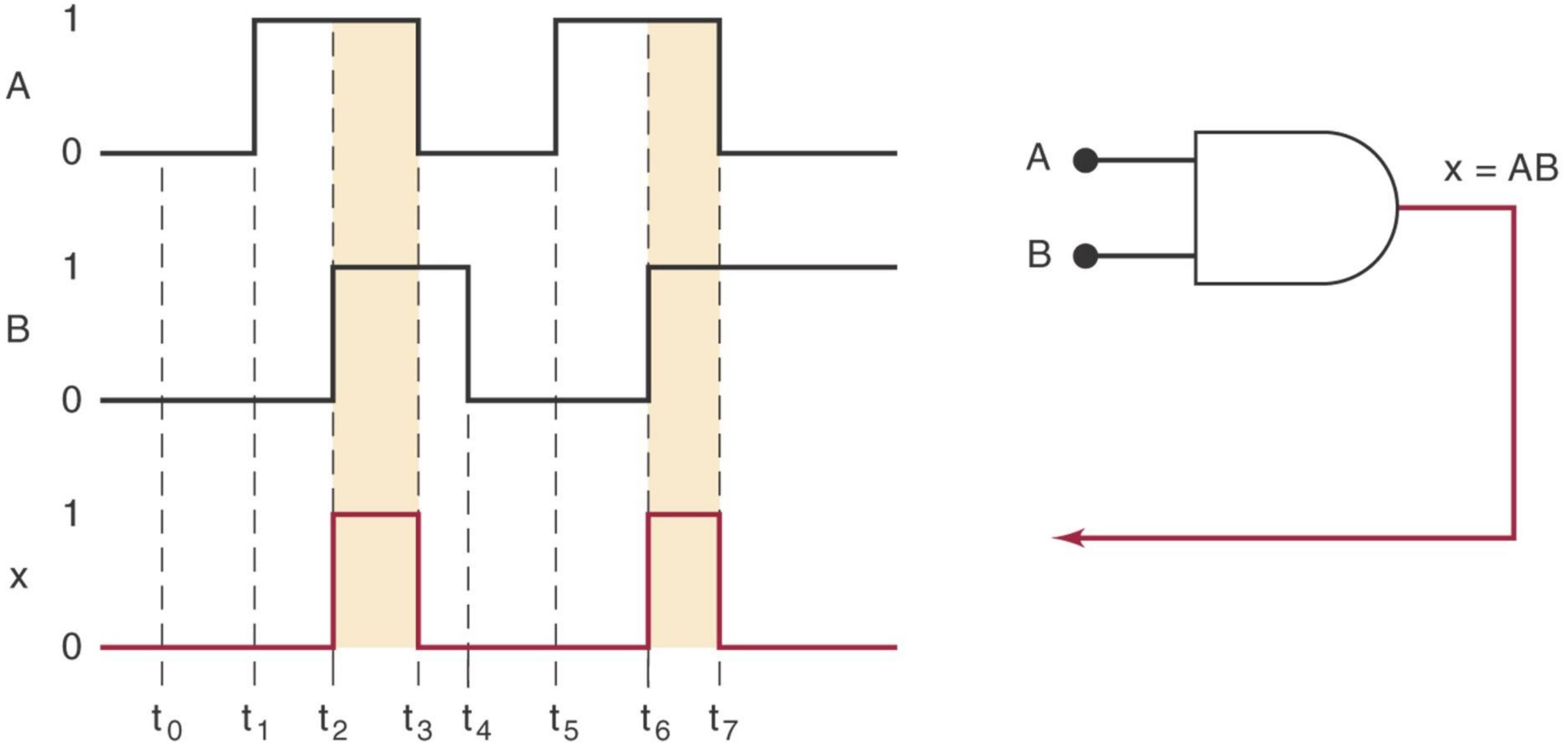
$$X = \bar{A}\bar{B} + AB$$

Pulsed operation of logic gates

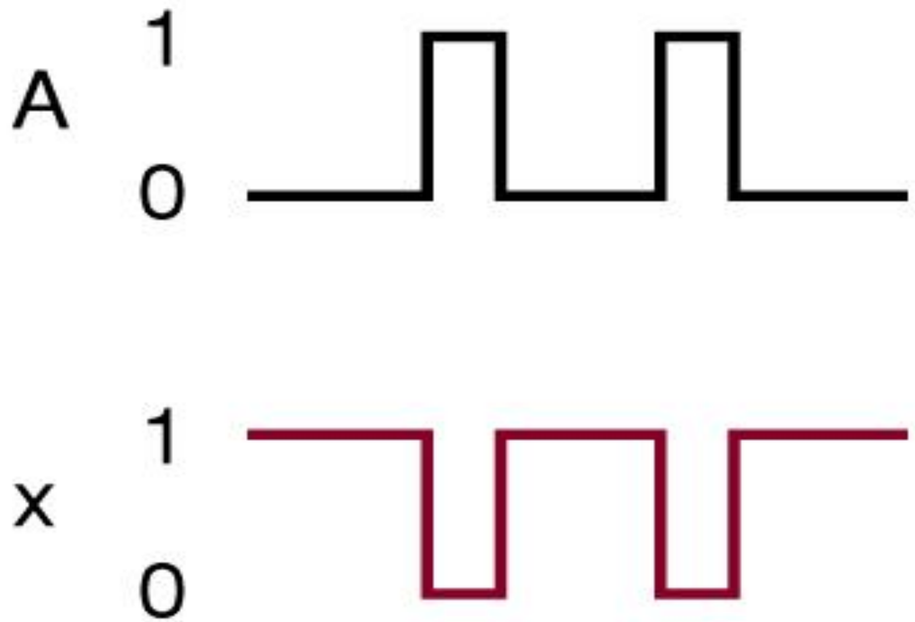
- Timing diagram (OR Gate)



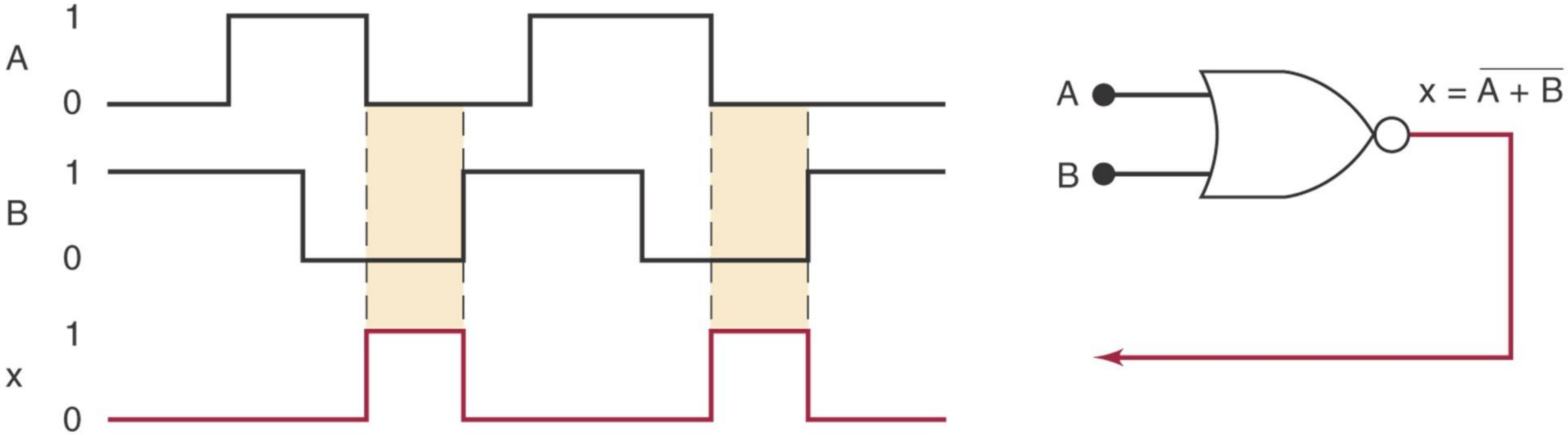
Timing Diagram for AND Gate



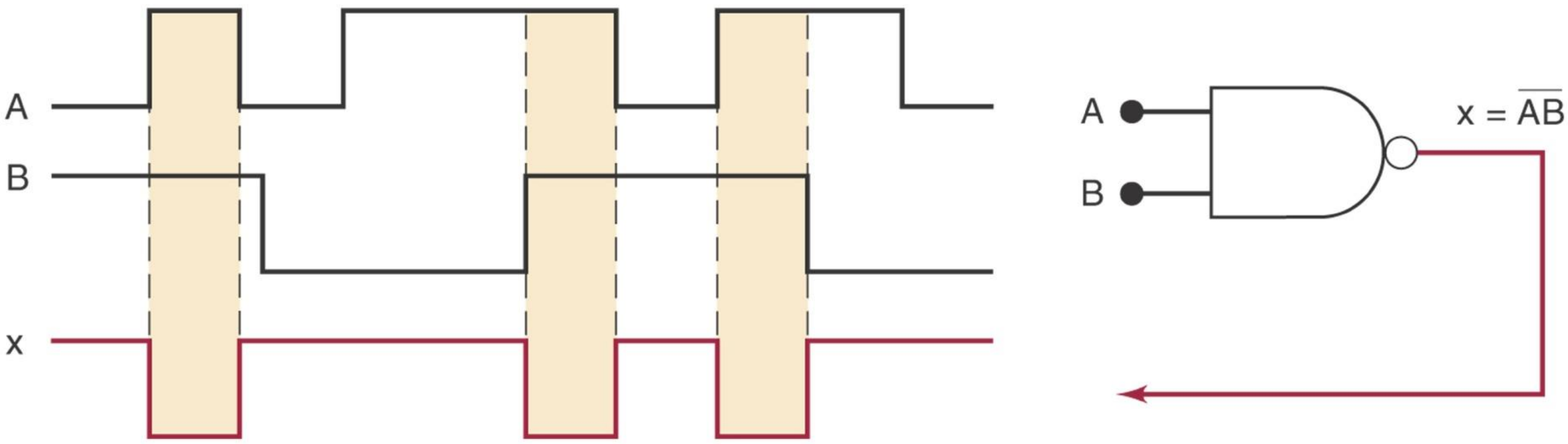
Timing Diagram for NOT Gate



Timing Diagram for NOR Gate



Timing Diagram for NAND Gate



EXAMPLE 5.29 Find the complements of the following expressions.

(a) $AB + A(B + C) + \bar{B}(B + D)$

(b) $A + B + \bar{A}\bar{B}C$

(c) $\bar{A}B + A\overline{BC} + \bar{A}BCD + \bar{A}\,\overline{BC}\bar{D}E$

(d) $ABEF + AB\bar{E}\,\bar{F} + \bar{A}\bar{B}EF$

(e) $\bar{B}\bar{C}D + \overline{(B + C + D)} + \bar{B}\bar{C}\bar{D}E$

(f) $AB + \overline{AC} + A\bar{B}C$

(g) $(A\bar{B} + A\bar{C})(BC + B\bar{C})(ABC)$

(h) $A\bar{B}C + \bar{A}BC + ABC$

(i) $(\overline{ABC})(\overline{A + B + C})$

(j) $A + \bar{B}C\,(A + B + \bar{C})$

(k) $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

Solution

To obtain the complement of an expression change the ANDs to ORs, ORs to ANDs, 0s to 1s, 1s to 0s and complement each variable. Based on this rule the complements of the above functions are as follows.

(a) $(\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{B} \cdot \bar{C}) \cdot (\bar{\bar{B}} + \bar{B}\bar{D}) = (\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{B} \cdot \bar{C}) \cdot (B + \bar{B}\bar{D})$

(b) $\bar{A} \cdot \bar{B} \cdot (\bar{\bar{A}} + \bar{\bar{B}} + \bar{C}) = (\bar{A}) \cdot (\bar{B}) \cdot (A + B + \bar{C})$

(c) $(\bar{\bar{A}} + \bar{B}) \cdot (\bar{A} + BC) \cdot (\bar{\bar{A}} + \bar{B} + \bar{C} + \bar{D}) \cdot (\bar{\bar{A}} + \overline{\overline{BC}} + \bar{\bar{D}} + \bar{E})$

$= (A + \bar{B}) \cdot (\bar{A} + BC) \cdot (A + \bar{B} + \bar{C} + \bar{D}) \cdot (A + BC + D + \bar{E})$

(d) $(\bar{A} + \bar{B} + \bar{E} + \bar{F}) \cdot (\bar{A} + \bar{B} + \bar{\bar{E}} + \bar{\bar{F}}) \cdot (\bar{\bar{A}} + \bar{\bar{B}} + \bar{E} + \bar{F})$

$= (\bar{A} + \bar{B} + \bar{E} + \bar{F}) \cdot (\bar{A} + \bar{B} + E + F) \cdot (A + B + \bar{E} + \bar{F})$

(e) $(\bar{\bar{B}} + \bar{\bar{C}} + \bar{D}) \cdot (\overline{\overline{B + C + D}}) \cdot (\bar{\bar{B}} + \bar{\bar{C}} + \bar{\bar{D}} + \bar{E})$

$= (B + C + \bar{D}) \cdot (B + C + D) \cdot (B + C + D + \bar{E})$

(f) $(\bar{A} + \bar{B}) \cdot (AC) \cdot (\bar{A} + B + \bar{C})$

(g) $(\bar{A} + \bar{\bar{B}}) \cdot (\bar{A} + \bar{\bar{C}}) + (\bar{B} + \bar{C}) \cdot (\bar{B} + \bar{\bar{C}}) + (\bar{A} + \bar{B} + \bar{C})$

$= (\bar{A} + B) \cdot (\bar{A} + C) + (\bar{B} + \bar{C}) \cdot (\bar{B} + C) + (\bar{A} + \bar{B} + \bar{C})$

(h) $(\bar{A} + \bar{\bar{B}} + \bar{C}) \cdot (\bar{\bar{A}} + \bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{C})$

(i) $\overline{\overline{(ABC)}} + \overline{\overline{(A + B + C)}} = (ABC) + (A + B + C)$

(j) $\bar{A} \cdot (\bar{\bar{B}} + \bar{C}) + (\bar{A}\bar{B}\bar{\bar{C}}) = \bar{A} \cdot (B + \bar{C}) + (\bar{A}\bar{B}C)$

Boolean Algebra

Axioms and laws of Boolean Algebra

- Set of logical expression that we accept without proof and upon which useful theorems can built.
- Three basic logic operations (AND ,OR,NOT)
- $0 \cdot 0 = 0$
- $1 + 1 = 1$
- $1 \cdot 1 = 1$
- $0 + 0 = 0$
- $0 \cdot 1 = 1 \cdot 0 = 0$
- $1 + 0 = 0 + 1 = 1$

Complementation Laws

- if A = 0, then A’ = 1
- if A = 1, then A’ = 0
- 0’=1
- 1’=0
- (A’)’ = A

Commutative Laws

Law1: $A+B=B+A$

$A+B+C=B+C+A=C+A+B=B+A+C$

Law2: $A.B=B.A$

$A.B.C=B.C.A=C.A.B=B.A.C$

Associative Laws

Law 1 : $(A+B)+C=A+(B+C)$

$A+(B+C+D) = (A+B+C)+D = (A+B)+(C+D)$

Law 2: $(A.B)\ C = A(B.C)$

$A(BCD)=(ABC)D=(AB)(CD)$

Distributive Laws

Law 1: $A(B+C)=AB+AC$

$ABC(D+E)=ABCD+ABEF$

$AB(CD+EF)=ABCD+ABEF$

Law 2: $A+BC=(A+B)(A+C)$

Law 3: $A+A'B=(A+B)$

Proof by Truth Table

Law 2:A+BC=(A+B) (A+C)

a	*b*	*c*	*b*c*	*a+(b*c)*	*a+b*	*a+c*	*(a+b)*(a+c)*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*1*	*0*	*0*	*0*	*1*	*0*
0	*1*	*0*	*0*	*0*	*1*	*0*	*0*
0	*1*	*1*	*1*	*1*	*1*	*1*	*1*
1	*0*	*0*	*0*	*1*	*1*	*1*	*1*
1	*0*	*1*	*0*	*1*	*1*	*1*	*1*
1	*1*	*0*	*0*	*1*	*1*	*1*	*1*
1	*1*	*1*	*1*	*1*	*1*	*1*	*1*

Idempotence Laws

Law 1: $A.A = A$

Law2 : $A + A = A$

Absorption Laws

Law1 : $A+A.B=A$

Law 2: $A(A+B)=A$

Included Factor Theorem

Theorem 1 : $AB + A'C + BC = AB + A'C$

Theorem 2 : $(A+B)(A'+C)(B+C) = (A+B)(A'+C)$

Transposition Theorem

Theorem : $AB + A'C = (A+C)(A'+B)$

DeMorgan's Theorems

DeMorgan's Theorems are two additional simplification techniques that can be used to simplify Boolean expressions. Again, the simpler the Boolean expression, the simpler the resulting logic.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

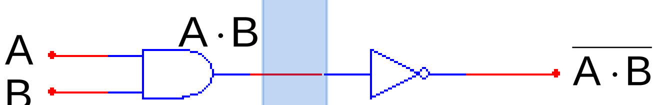
(A+B+C)' = ((A+B)+C)' = ((A+B)')•C' = A'•B'•C'
(A•B•C•....•X)' = A' + B' + C' + ...+ X'

DeMorgan's Theorem #1

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

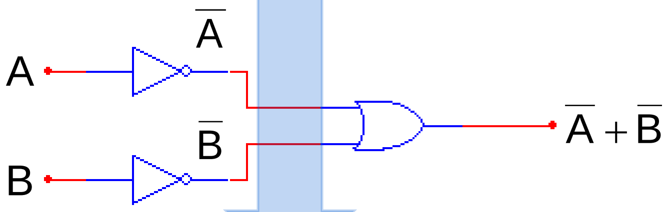
Proof

$$\overline{A \cdot B}$$



A	B	$A \cdot B$	$\overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

$$\overline{A} + \overline{B}$$



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

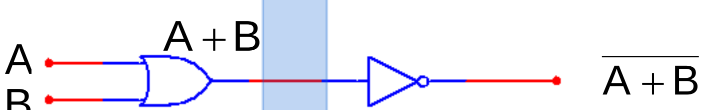
The truth-tables are equal; therefore,
the Boolean equations must be equal.

DeMorgan's Theorem #2

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

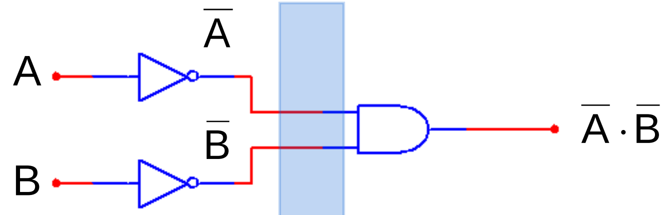
Proof

$$\overline{A+B}$$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

$$\overline{A} \cdot \overline{B}$$



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

The truth-tables are equal; therefore,
the Boolean equations must be equal.

DeMorgan Shortcut

BREAK THE LINE, CHANGE THE SIGN

Break the *LINE* over the two variables,
and change the *SIGN* directly under the line.

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

For Theorem #14A, break the line, and change the AND function to an OR function. Be sure to keep the lines over the variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

For Theorem #14B, break the line, and change the OR function to an AND function. Be sure to keep the lines over the variables.

DeMorgan's: Example #1

Example

Simplify the following Boolean expression and note the Boolean or DeMorgan's theorem used at each step.

$$F_1 = \overline{\overline{(X \cdot \overline{Y})} \cdot (\overline{Y} + Z)}$$

Solution

$$F_1 = \overline{\overline{(X \cdot \overline{Y})} \cdot (\overline{Y} + Z)}$$

$$F_1 = \overline{\overline{(X \cdot \overline{Y})}} + \overline{(\overline{Y} + Z)}$$

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$

$$F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z})$$

$$\boxed{F_1 = X\overline{Y} + Y\overline{Z}}$$

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + \overline{\overline{(XY)}}$$

$$F_2 = \overline{(\overline{X} + Z)} + (XY)$$

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$

$$F_2 = (X\overline{Z}) + (XY)$$

$$F_2 = X\overline{Z} + XY$$

DeMorgan's: Example

$$\overline{\overline{A + BC} + \overline{A\overline{B}}}$$

Breaking longest bar

$$\overline{\overline{(A + BC)}}\ (\overline{\overline{A\overline{B}}})$$

Applying identity $\overline{\overline{A}} = A$ wherever double bars of equal length are found

$$(A + BC)(A\overline{B})$$

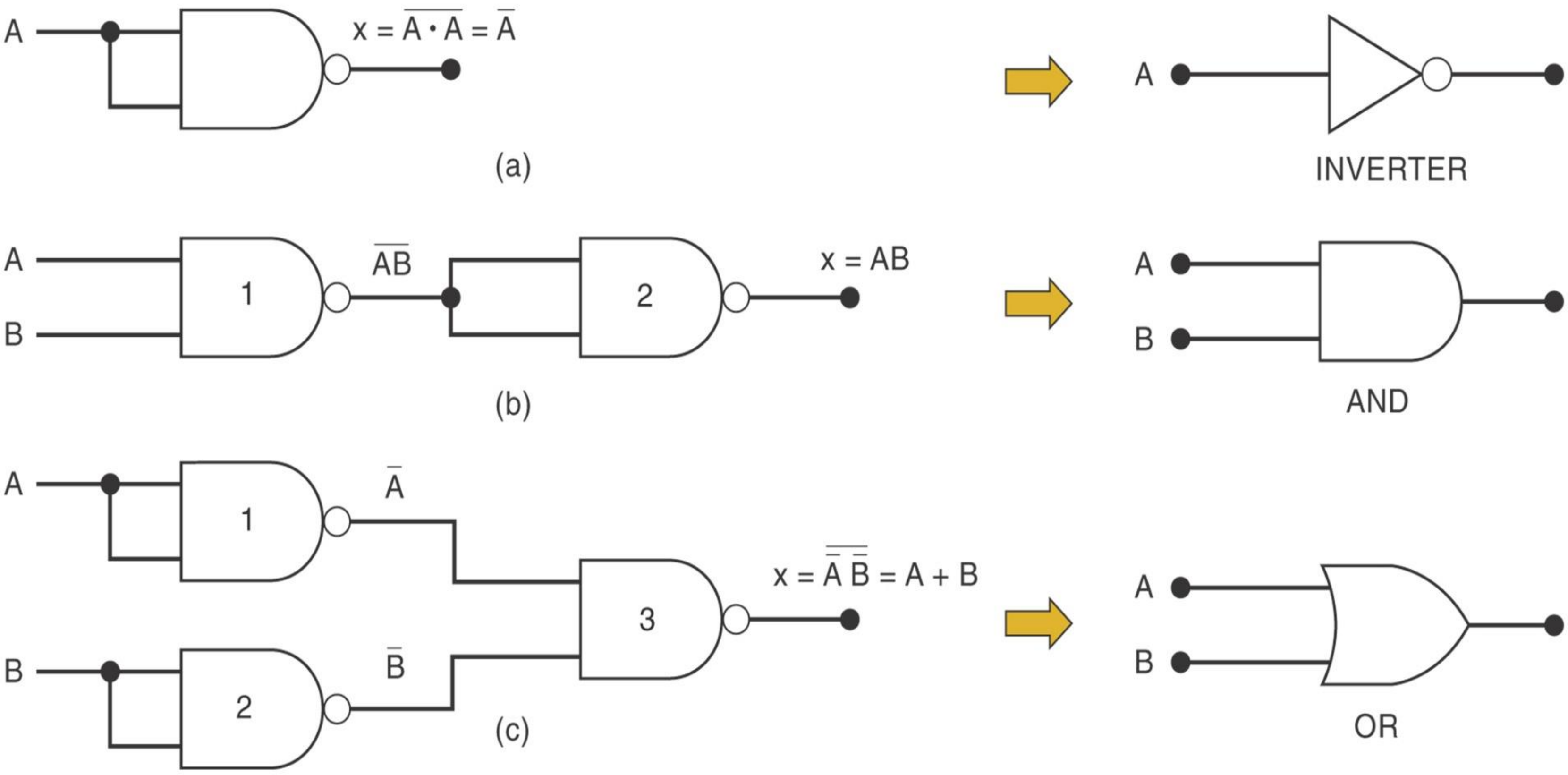
Distributive property

$$AA\overline{B} + BCA\overline{B}$$

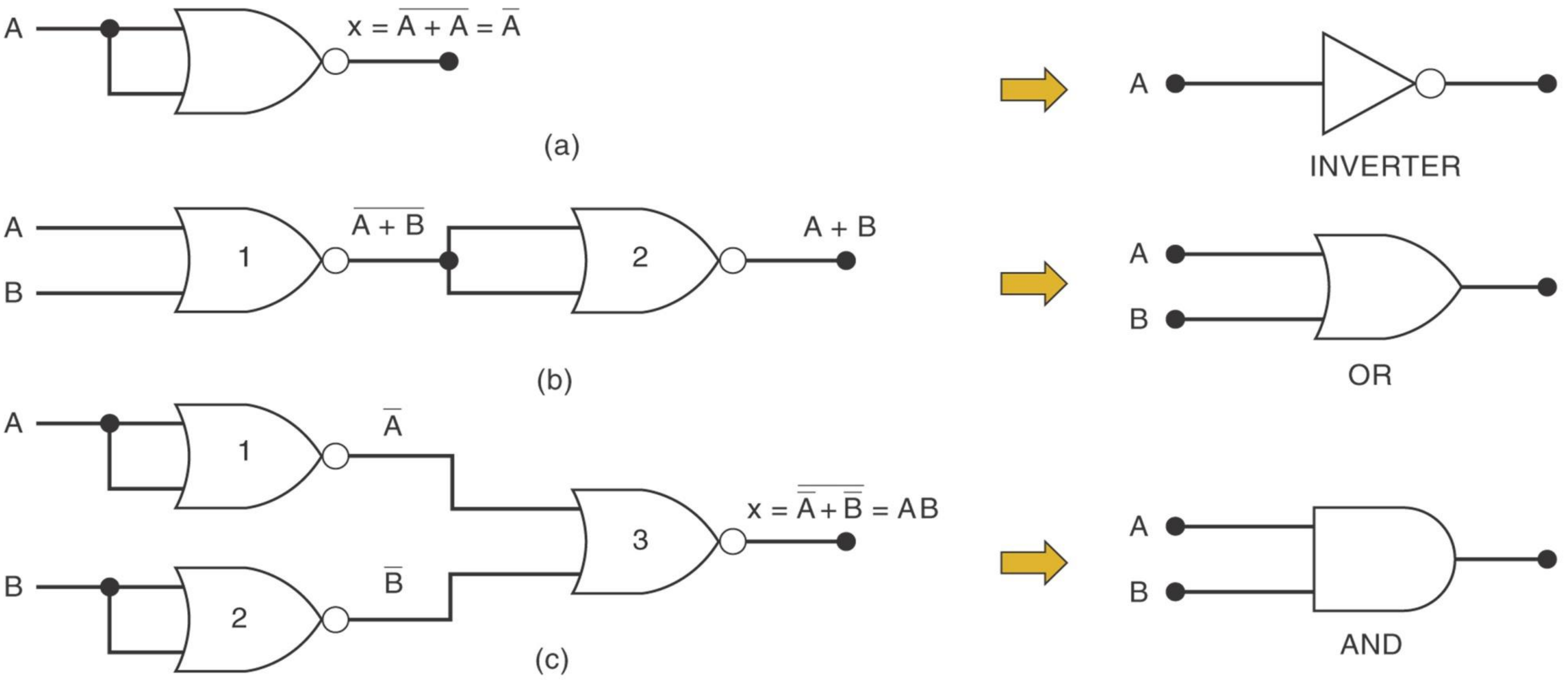
Applying identity $AA = A$ to left term; applying identity $A\overline{A} = 0$ to B and $\overline{B}$ in right term

$$A\overline{B} + 0$$

Universality of NAND Gates



Universality of NOR Gates



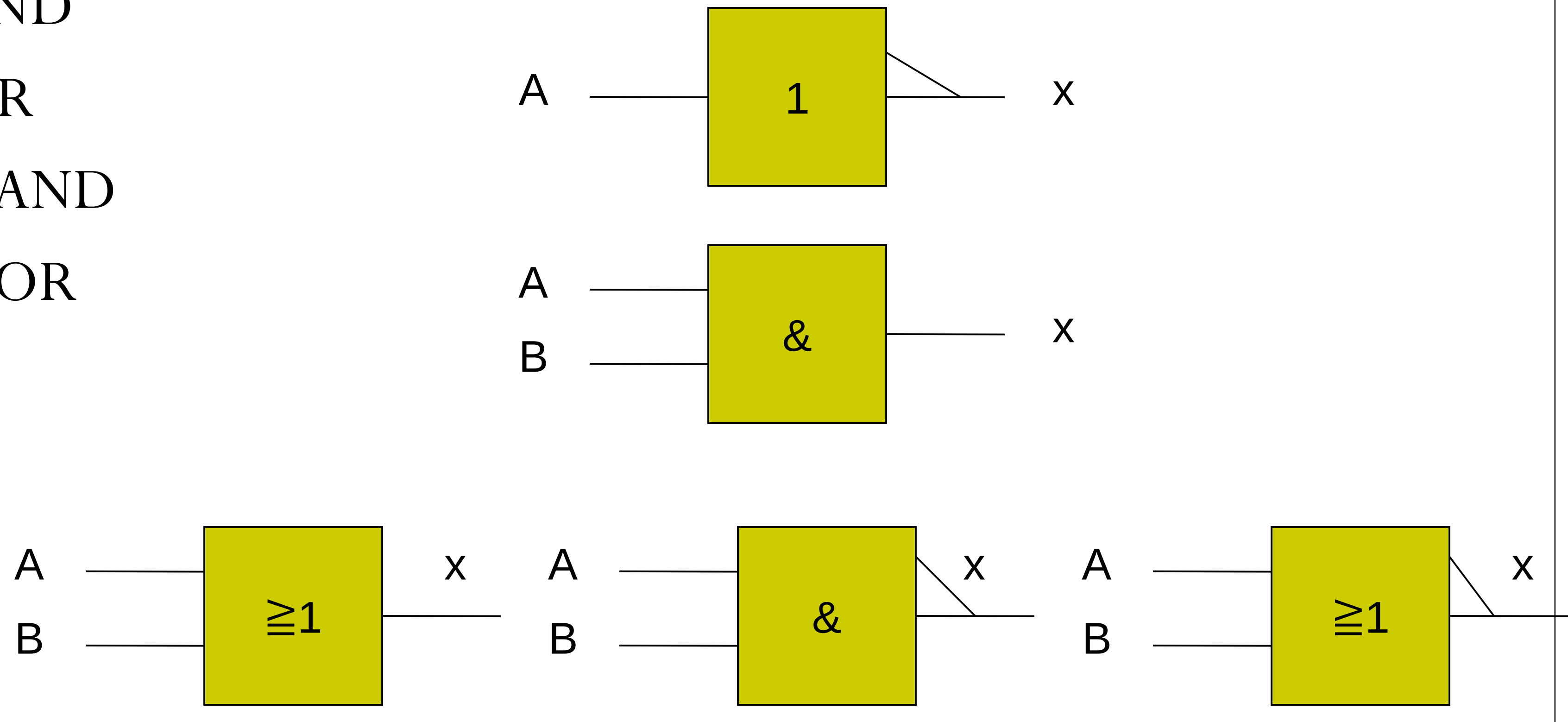
Alternate Logic Symbols

- Step 1: Invert each input and output of the standard symbol
- Change the operation symbol from AND to OR, or from OR to AND.
- Examples: AND, OR, NAND, NOR, INV

Alternate Logic-Gate Representation (Active Low Notation)

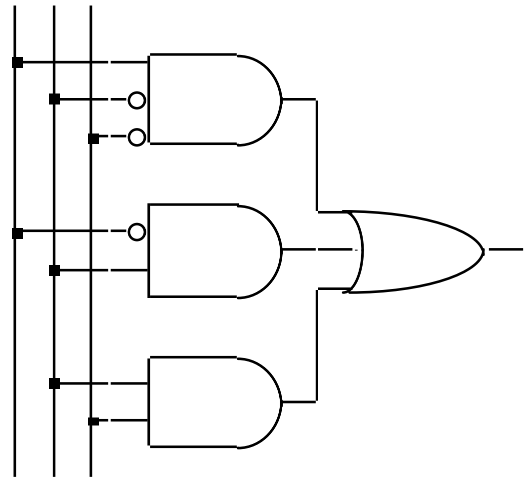
IEEE Standard Logic Symbols

- NOT
- AND
- OR
- NAND
- NOR

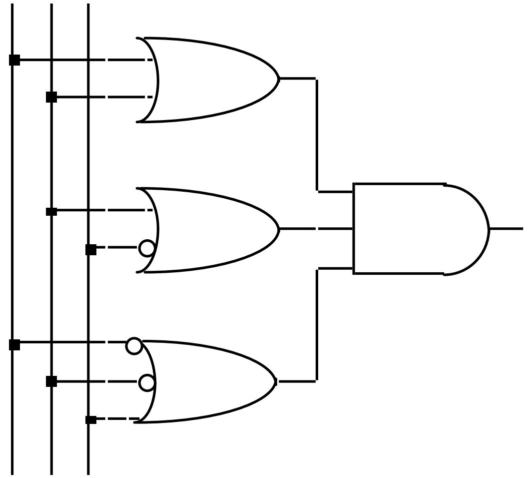


SOP & POS form

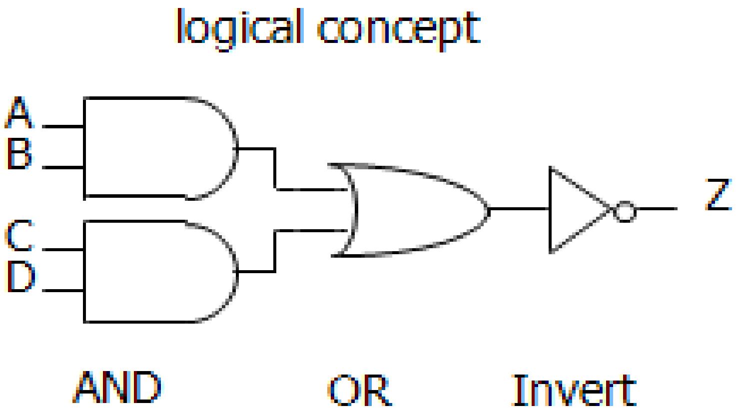
- Sum-of-products
  - AND gates to form product terms (minterms)
  - OR gate to form sum



- Product-of-sums
  - OR gates to form sum terms (maxterms)
  - AND gates to form product



AND-OR-Invert Gates



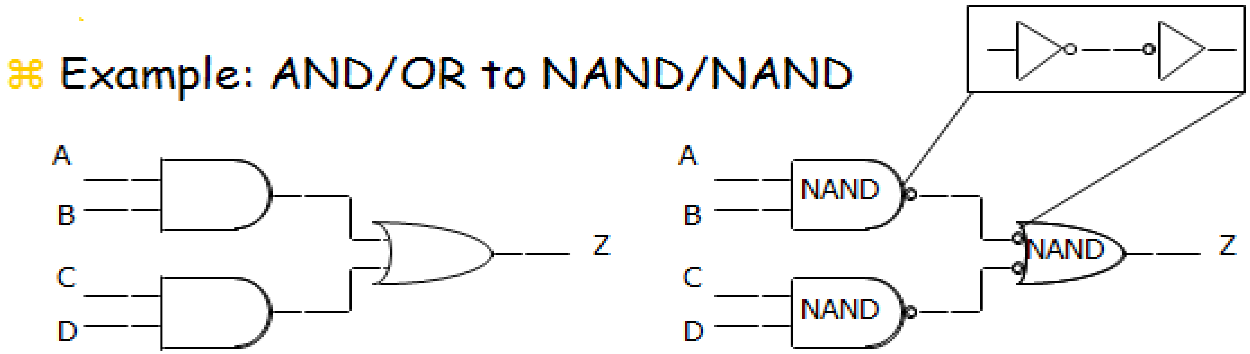
Converting AOI logic to NAND/NOR logic

- NAND

1) Add a circle (bubble) at output of each AND gate
2) Add a circle (bubble) at input of all OR gate.

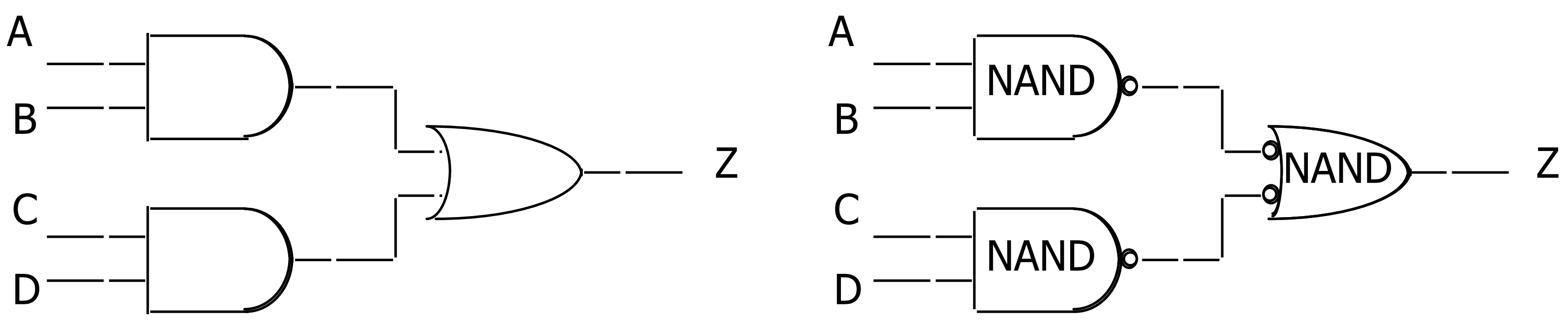
- NOR

1) Add a circle at output of each OR gate
2) Add a circle at input of all AND gate



Conversion Between Forms (cont'd)

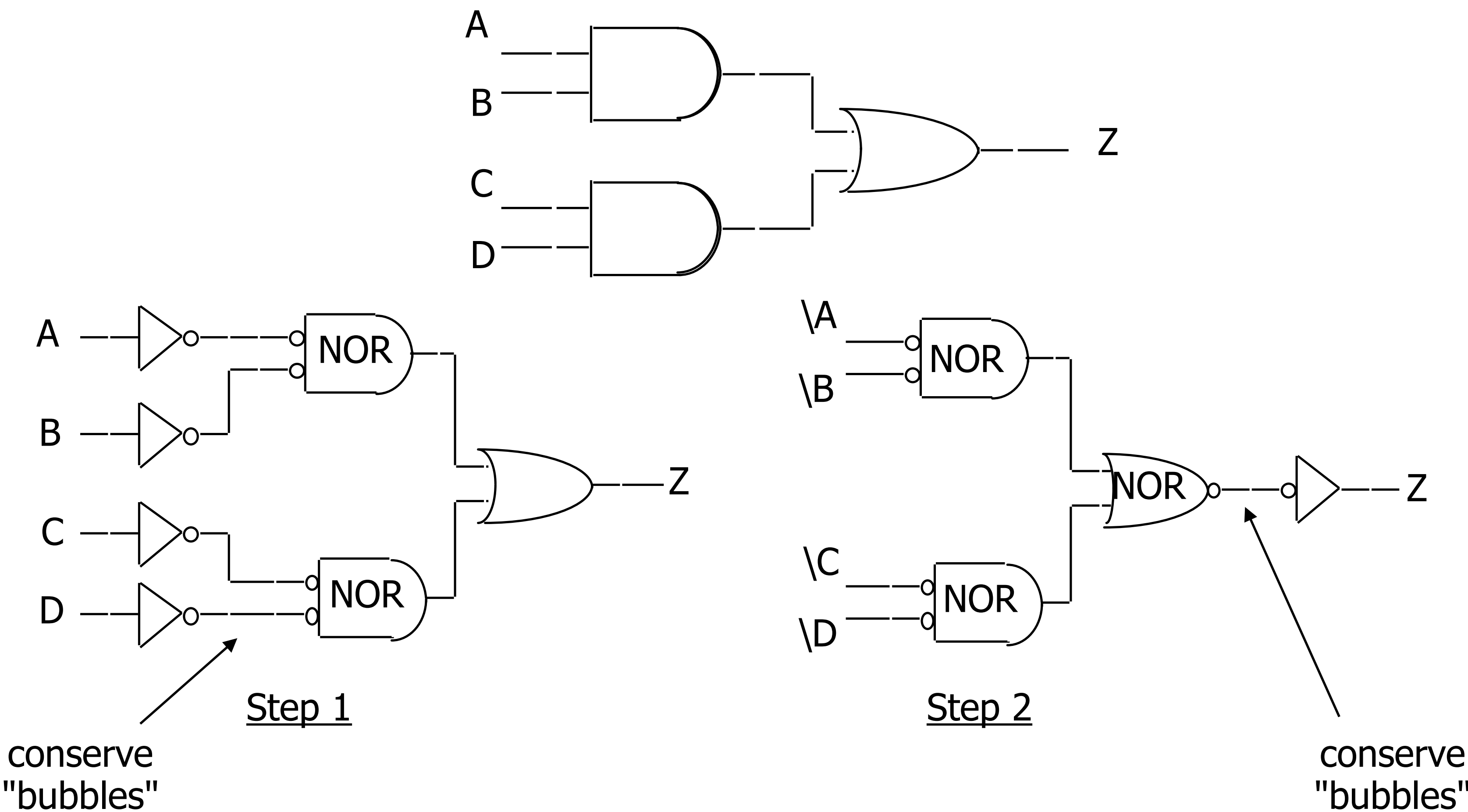
- Example: verify equivalence of two forms



$$Z = [\,(A \cdot B)' \cdot (C \cdot D)'\,]'$$
$$= [\,(A' + B') \cdot (C' + D')\,]'$$
$$= [\,(A' + B')' + (C' + D')'\,]$$
$$= (A \cdot B) + (C \cdot D) \checkmark$$

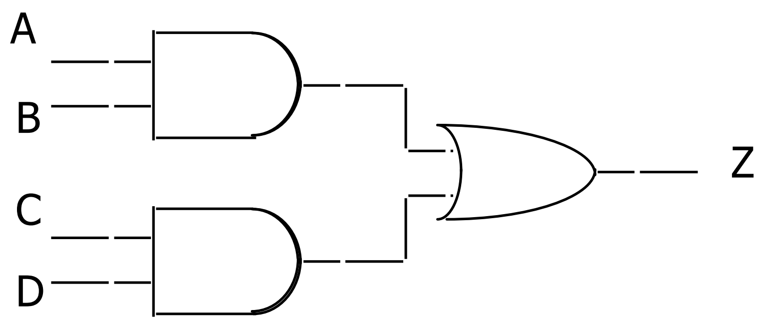
Conversion Between Forms (cont'd)

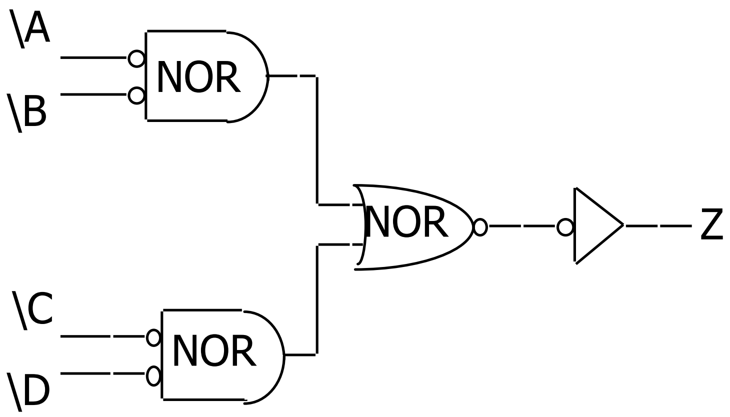
- Example: map AND/OR network to NOR/NOR network



Conversion Between Forms (cont'd)

- Example: verify equivalence of two forms





$$Z = \{\ [\ (A' + B')' + (C' + D')'\ ]'\ \}'$$

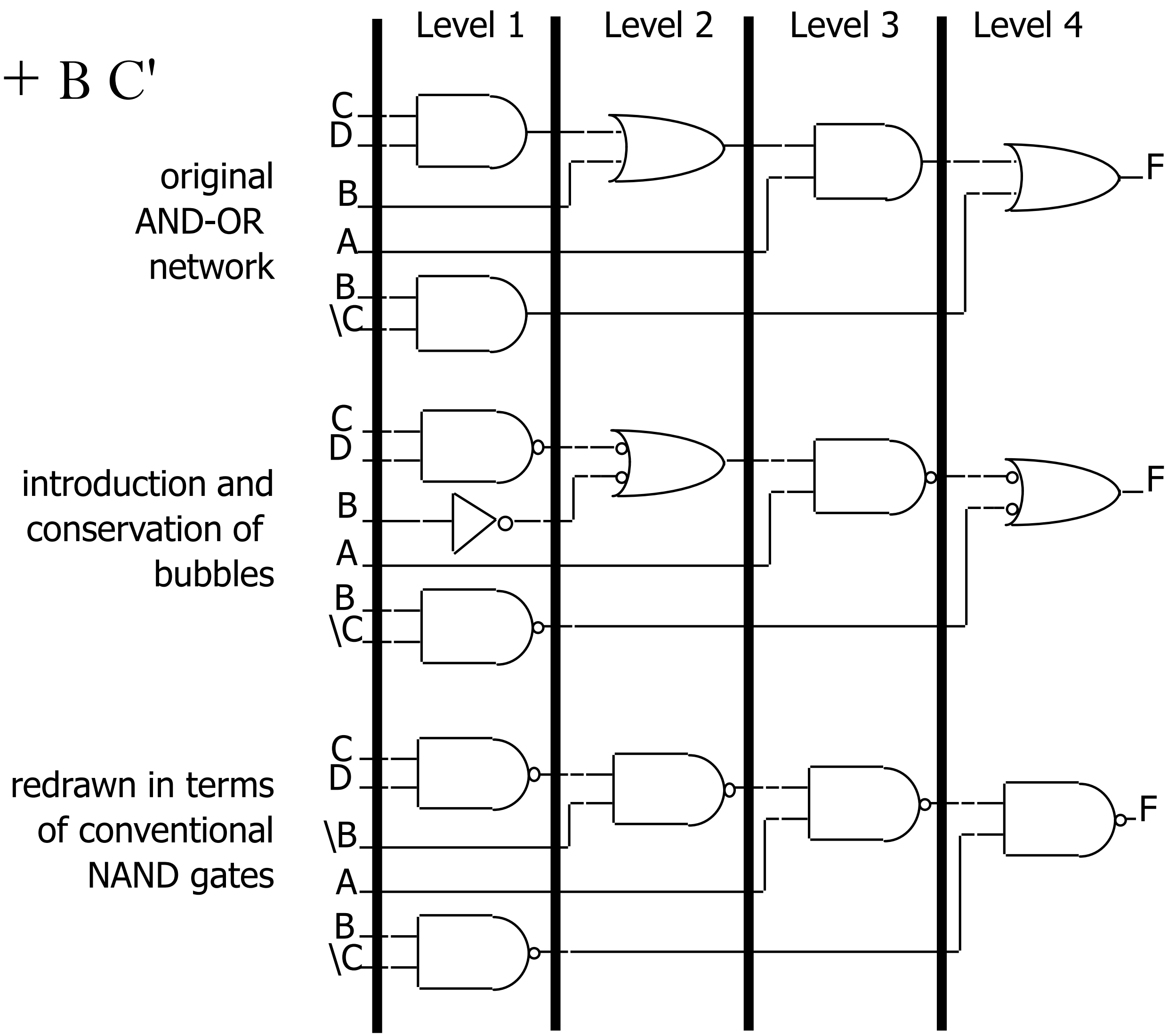
$$= \{\ (A' + B') \cdot (C' + D')\ \}'$$

$$= (A' + B')' + (C' + D')'$$

$$= (A \cdot B) + (C \cdot D)\ \checkmark$$

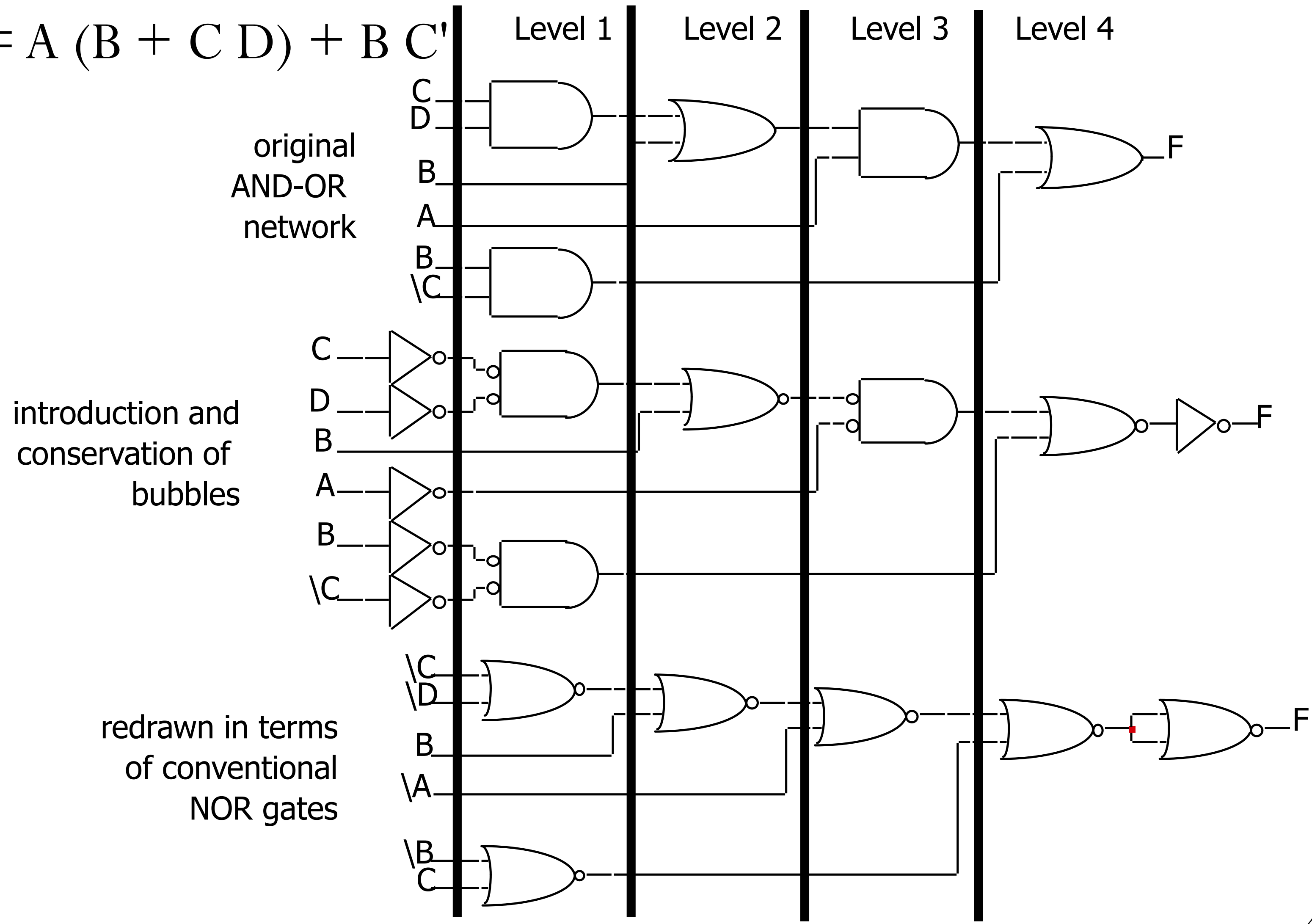
Conversion of Multi-level Logic to NAND Gates

- F = A (B + C D) + B C'

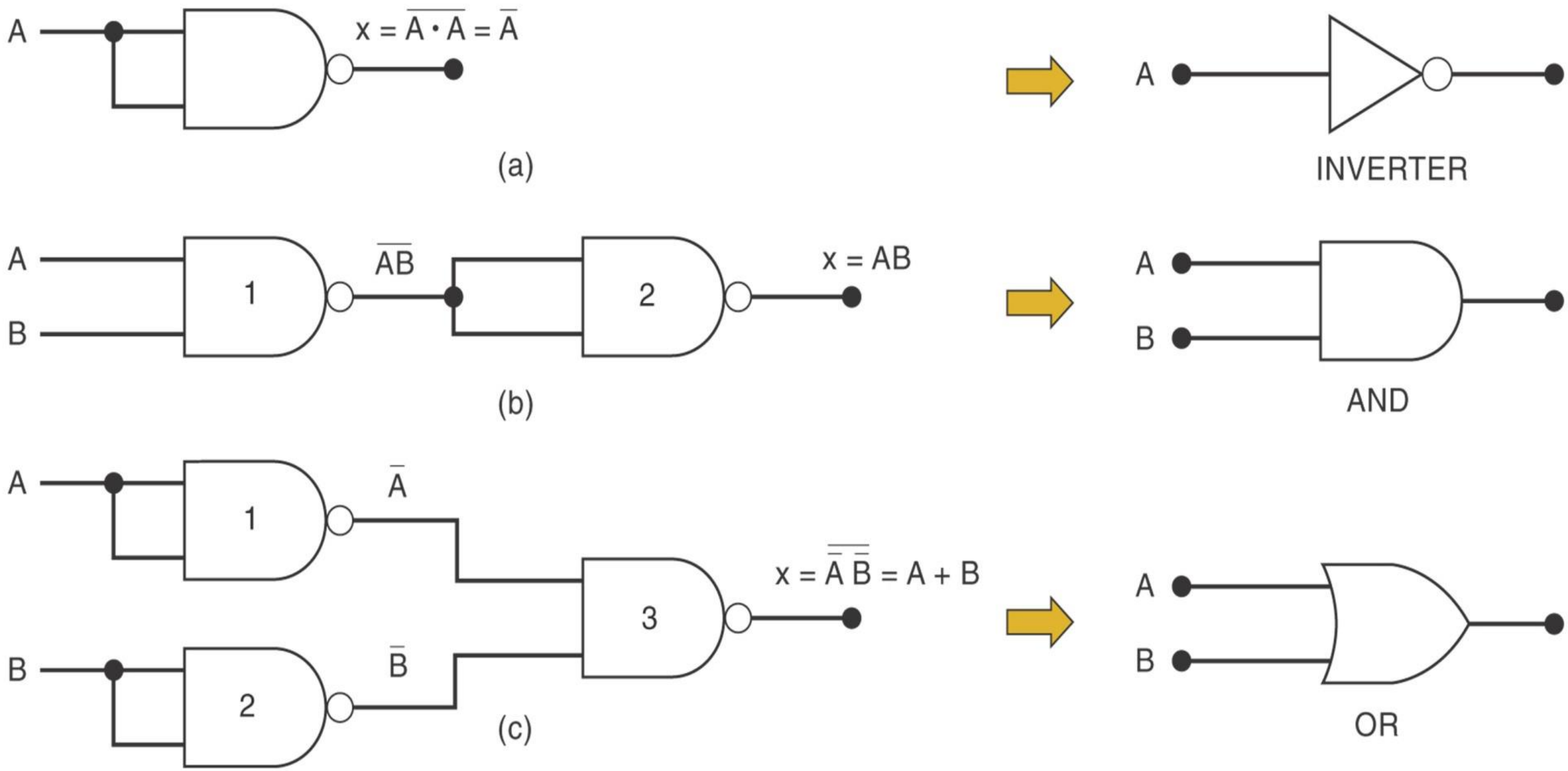


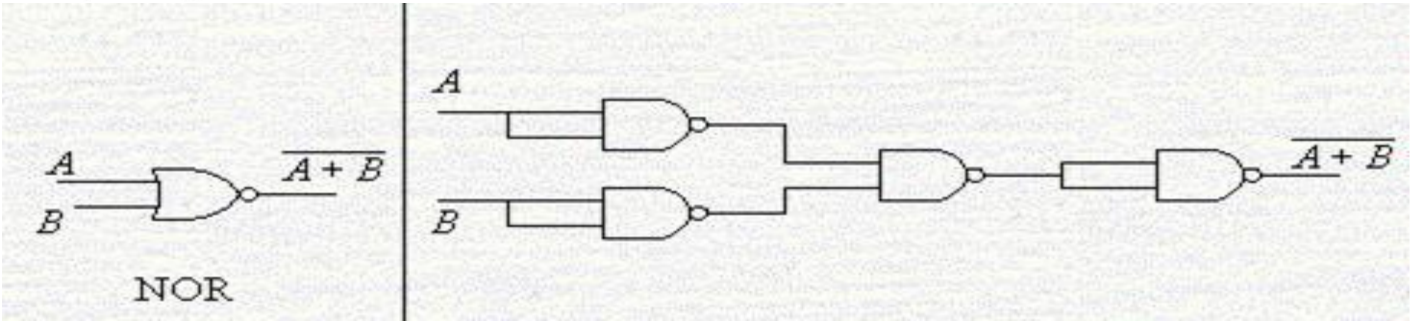
Conversion of Multi-level Logic to NORs

- $F = A (B + C D) + B C'$

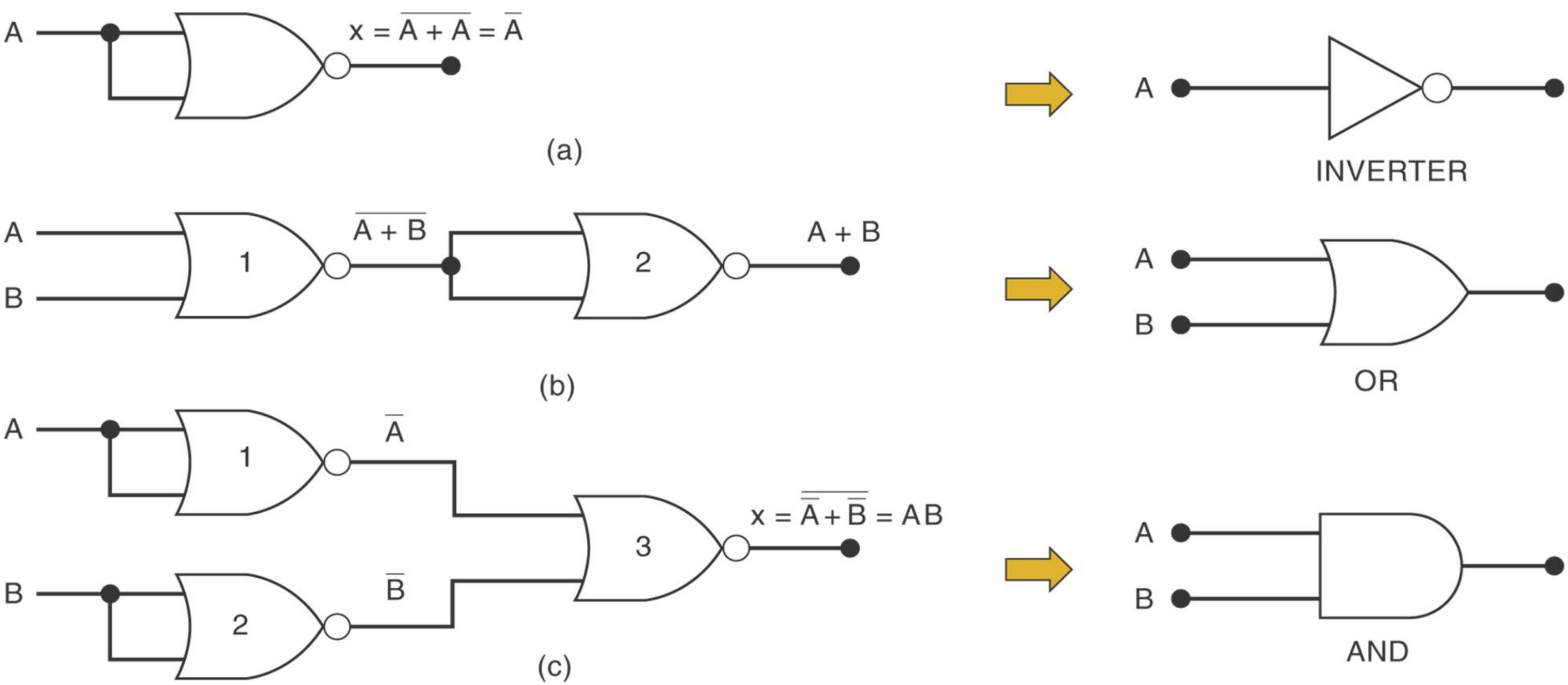


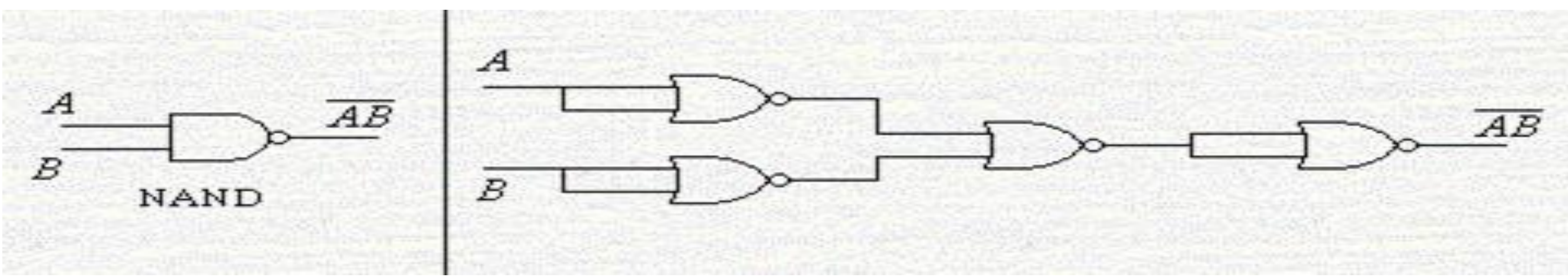
Universality of NAND Gates





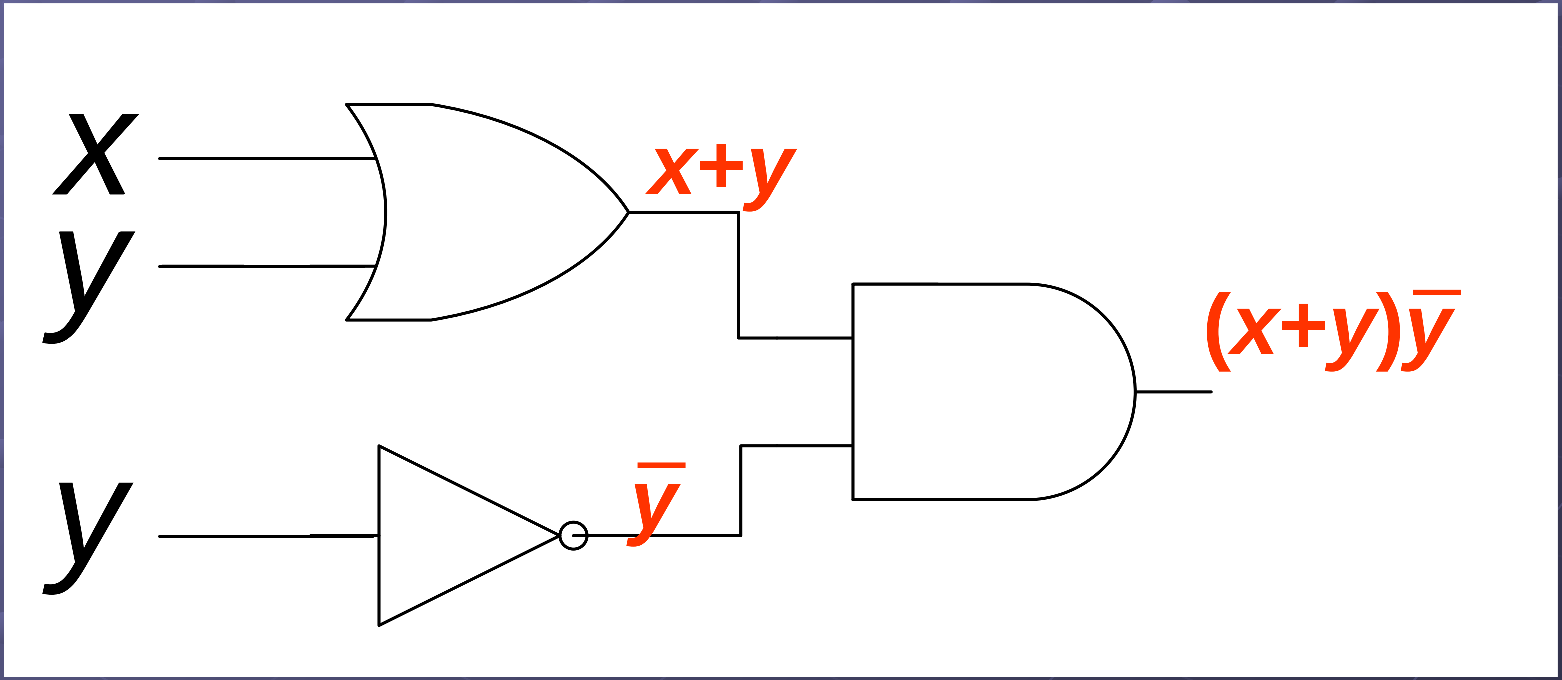
Universality of NOR Gates





Logic Diagrams

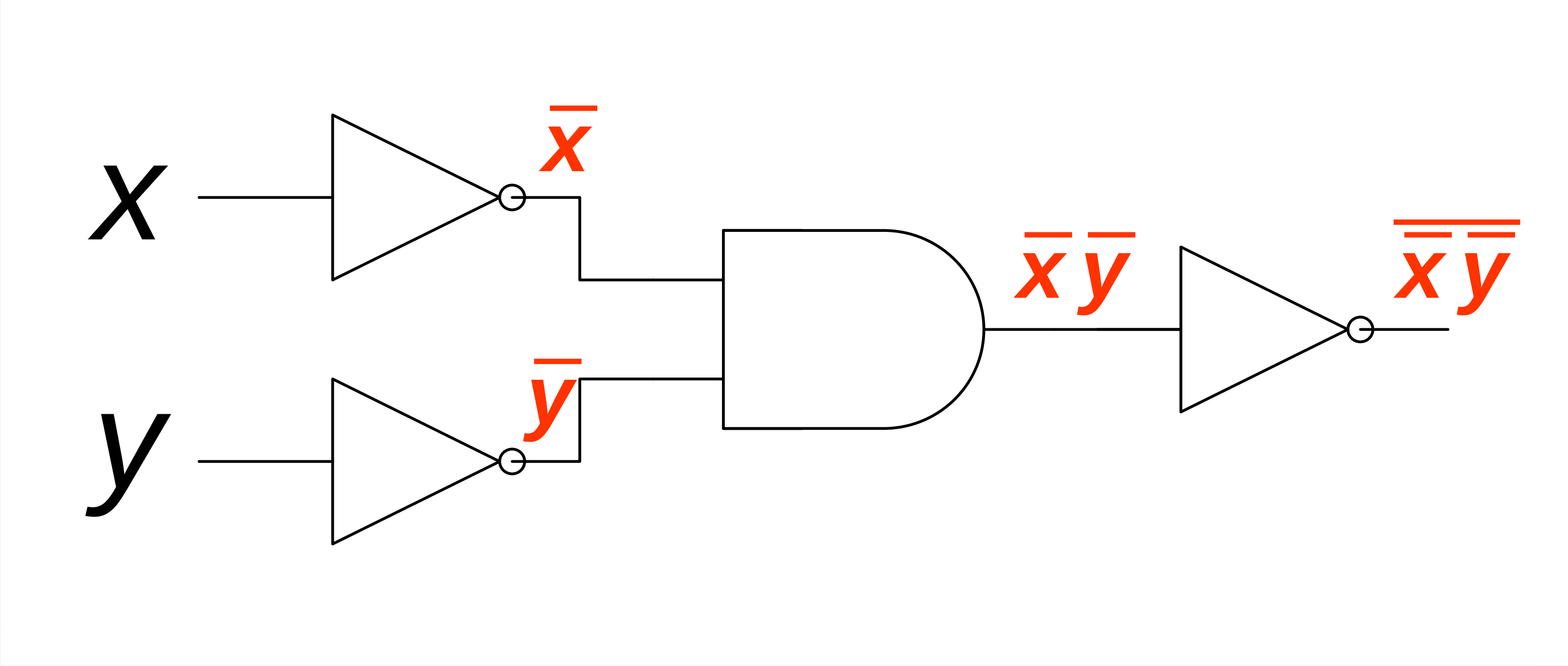
- Find the output of the following circuit



- Answer: $(x+y)\overline{y}$

Logic Diagrams (cont…)

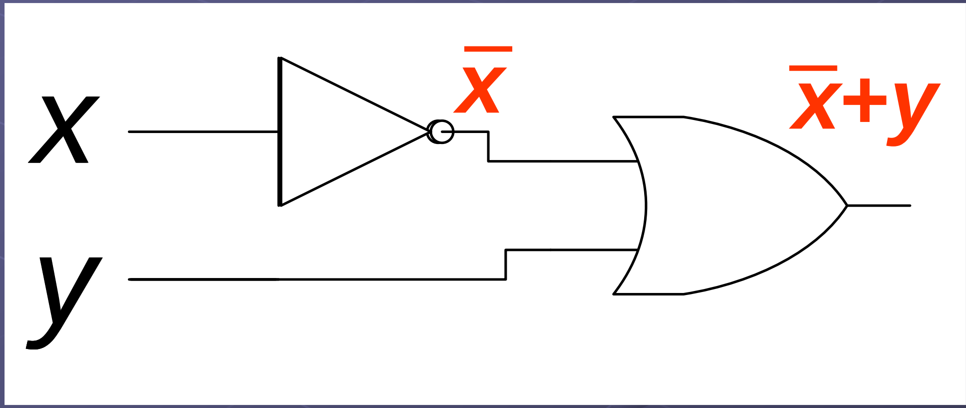
- Find the output of the following circuit



- Answer: $\overline{\overline{x}\overline{y}}$

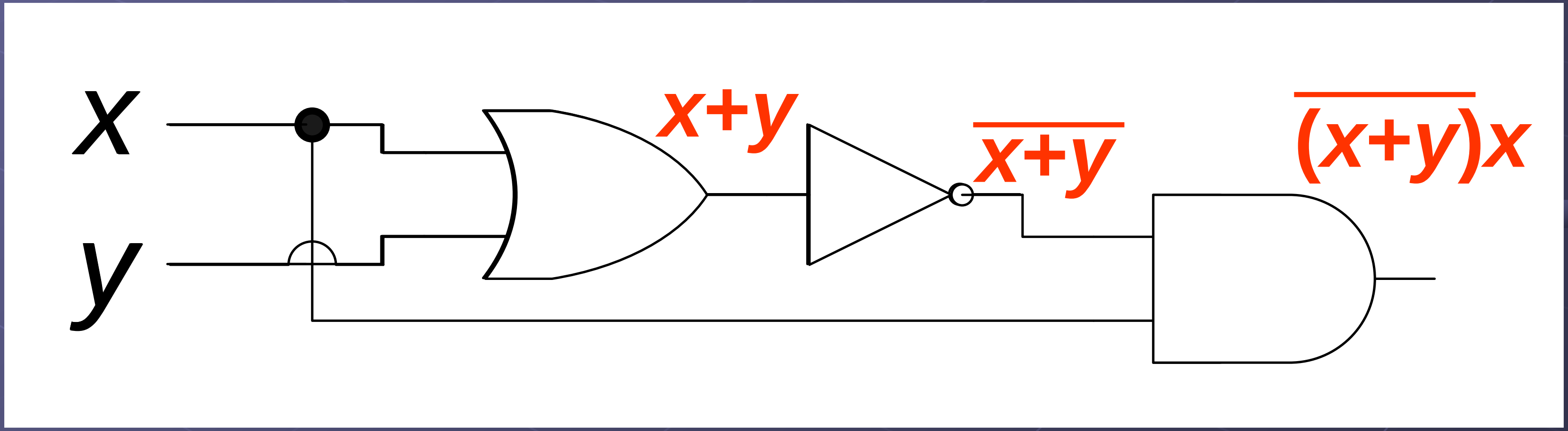
- Write the circuits for the following Boolean algebraic expressions

a) $\overline{x}+y$



- Write the circuits for the following Boolean algebraic expressions

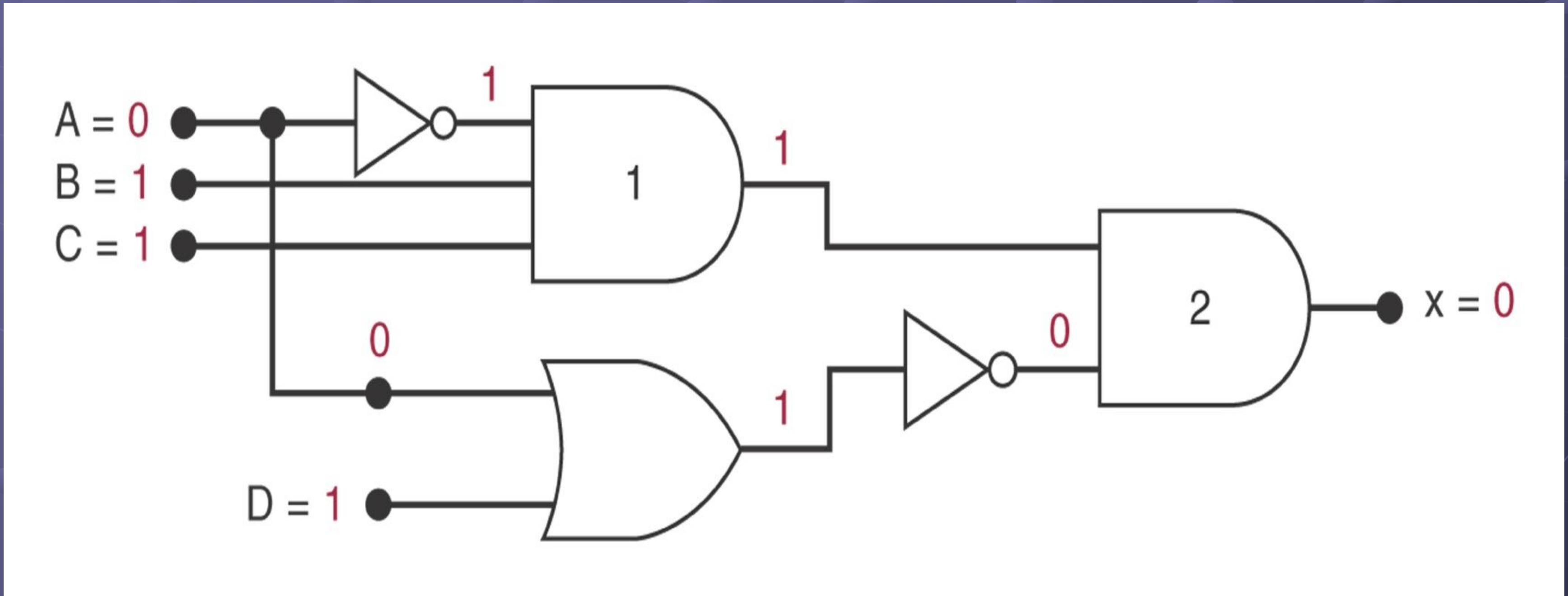
b) $\overline{(x+y)}x$

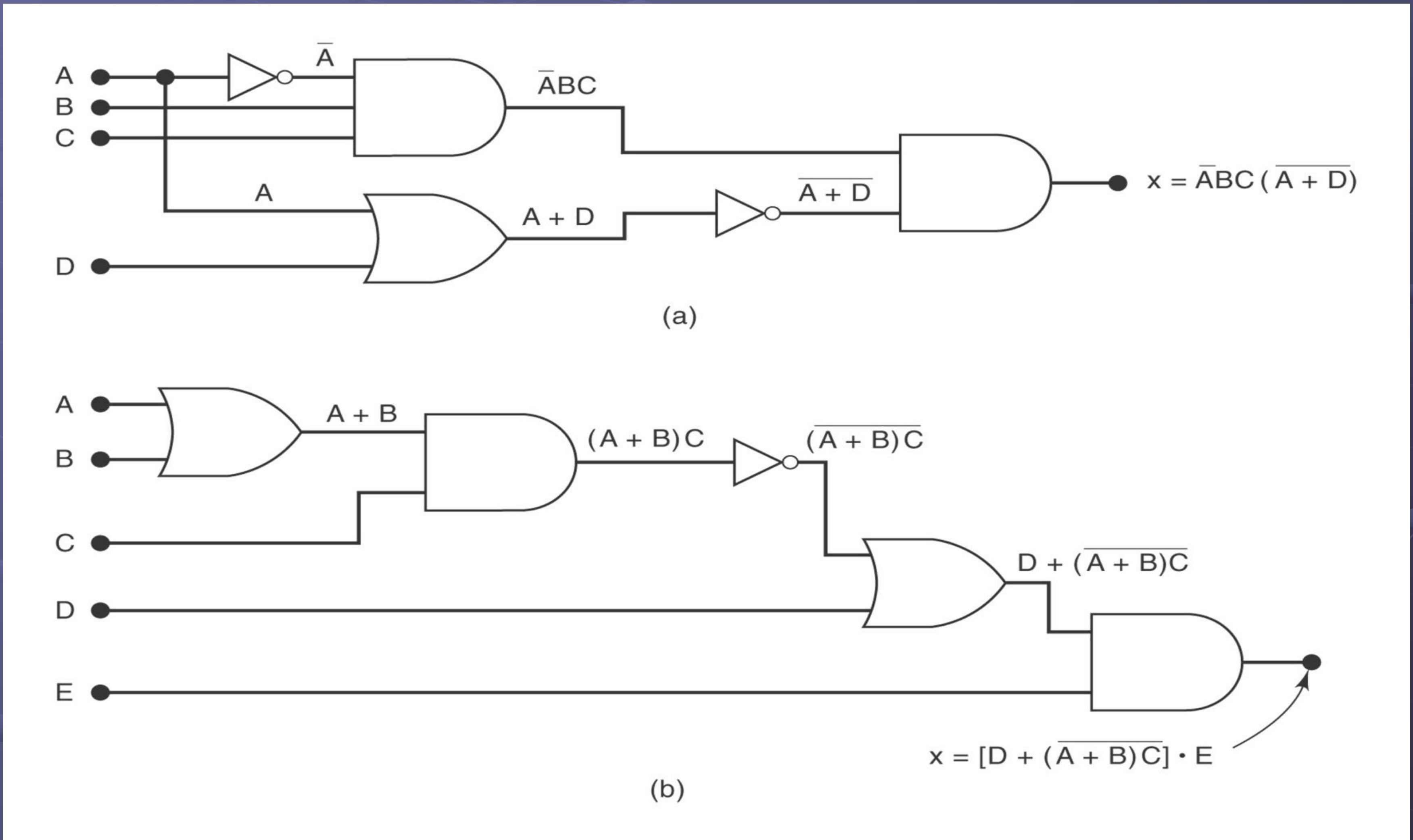


Evaluating Logic-Circuit Outputs

- $x = ABC(A+D)$

- Determine the output x given A=0, B=1, C=1, D=1.
- Can also determine output level from a diagram


A = 0
B = 1
C = 1
1
1
1
2
x = 0
0
0
1
D = 1


A
B
C
D
$\overline{A}$
$\overline{A}BC$
A
A + D
$\overline{A + D}$
$x = \overline{A}BC(\overline{A + D})$
(a)
A
B
C
D
E
A + B
(A + B)C
$\overline{(A + B)C}$
$D + (\overline{A + B)C}$
$x = [D + (\overline{A + B)C}] \cdot E$
(b)

EXAMPLE 5.31 Simplify the following Boolean expressions to a minimum number of literals.

(a) $\bar{x}\,\bar{y} + xy + \bar{x}y$

(b) $x\,\bar{y} + \bar{y}\,\bar{z} + \bar{x}\,\bar{z}$

(c) $(x + y)(x + \bar{y})$

(d) $\bar{x}y + xy + x\,\bar{z} + x\,\bar{y}\,\bar{z}$

(e) $(A + B)(\bar{A} + C)(\bar{B} + D)(C\bar{D})$

(f) $\bar{A}\bar{C} + ABC + A\bar{C}$ to three literals

(g) $\overline{(\bar{x}\,\bar{y} + z)} + z + xy + wz$ to three literals

(h) $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD)$ to one literal.

(i) $(\bar{A} + C)(\bar{A} + \bar{C})(\bar{A} + B + \bar{C}D)$ to one literal

(j) $AB + A(B + C) + \bar{B}(B + D)$

(k) $A + B + \bar{A}\bar{B}C$

(l) $\bar{A}B + \bar{A}B\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}E$

(m) $ABEF + AB\overline{EF} + \overline{AB}EF$

(n) $ABC\bar{D} + A + AB\bar{D} + (\bar{D})(\bar{A}\bar{B}\bar{C})$

(o) $x[y + z(\overline{xy + xz})]$

(p) $\bar{x}\,\bar{z} + \bar{y}\,\bar{z} + y\,\bar{z} + xyz$

Solution

The simplication of the above Boolean expressions is as follows:

(a) $\bar{x}\,\bar{y} + xy + \bar{x}y = \bar{x}\,\bar{y} + y(x + \bar{x}) = \bar{x}\,\bar{y} + y = (y + \bar{y})(y + \bar{x}) = \bar{x} + y$

(b) $x\,\bar{y} + \bar{y}\,\bar{z} + \bar{x}\,\bar{z} = x\,\bar{y} + \bar{x}\,\bar{z} + \bar{y}\,\bar{z}(x + \bar{x}) = x\,\bar{y} + \bar{x}\,\bar{z} + \bar{y}\,\bar{z}x + \bar{y}\,\bar{z}\,\bar{x}$

$= x\,\bar{y}(1 + \bar{z}) + \bar{x}\,\bar{z}(1 + \bar{y}) = x\,\bar{y} + \bar{x}\,\bar{z}$

(c) $(x + y)(x + \bar{y}) = xx + xy + x\,\bar{y} + y\,\bar{y} = x + xy + x\,\bar{y} + 0 = x(1 + y + \bar{y}) + 0 = x$

(d) $\bar{x}y + xy + x\,\bar{z} + x\,\bar{y}\,\bar{z} = y(x + \bar{x}) + x\,\bar{z}(1 + \bar{y}) = y + x\,\bar{z}$

(e) $(A + B)(\bar{A} + C)(\bar{B} + D)(C\bar{D}) = (A\bar{A} + AC + \bar{A}B + BC)(\bar{B}C\bar{D} + DC\bar{D})$

$= (AC + \bar{A}B + BC)\bar{B}C\bar{D} = A\bar{B}C\bar{D}$

(f) $\bar{A}\bar{C} + ABC + A\bar{C} = \bar{C}(\bar{A} + A) + ABC = \bar{C} + ABC = (\bar{C} + C)(\bar{C} + AB) = \bar{C} + AB$

(g) $\overline{(\bar{x}\,\bar{y} + z)} + z + xy + wz = \overline{\bar{x}\,\bar{y}} \cdot \bar{z} + z(1 + w) + xy = (x + y)\,\bar{z} + z + xy$

$= (z + \bar{z})(z + x + y) + xy$

$= x + y + z + xy = x + y + z$

(h) $\bar{A}B(\bar{D}+\bar{C}D)+B(A+\bar{A}CD)=\bar{A}B\bar{D}+\bar{A}B\bar{C}D+AB+\bar{A}BCD$

$=\bar{A}BD(C+\bar{C})+\bar{A}B\bar{D}+AB=\bar{A}BD+\bar{A}B\bar{D}+AB$

$=\bar{A}B(D+\bar{D})+AB=AB+\bar{A}B=B(A+\bar{A})=B$

(i) $(\bar{A}+C)(\bar{A}+\bar{C})(\bar{A}+B+\bar{C}D)=(\bar{A}+\bar{A}C+\bar{A}\bar{C}+C\bar{C})(\bar{A}+B+\bar{C}D)$

$=\bar{A}(1+C+\bar{C})(\bar{A}+B+\bar{C}D)=\bar{A}(\bar{A}+B+\bar{C}D)$

$=\bar{A}+\bar{A}B+\bar{A}\bar{C}D=\bar{A}(1+B+\bar{C}\bar{D})=\bar{A}$

(j) $AB+A(B+C)+\bar{B}(B+D)=AB+AB+AC+\bar{B}B+\bar{B}D$

$=AB+AC+\bar{B}D=A(B+C)+\bar{B}D$

(k) $A+B+\bar{A}\bar{B}C=(A+\bar{A})(A+\bar{B}C)+B=A+B+\bar{B}C=A+(B+\bar{B})(B+C)=A+B+C$

(l) $\bar{A}B+\bar{A}B\bar{C}+\bar{A}BCD+\bar{A}B\bar{C}\bar{D}E=\bar{A}B(1+\bar{C}+CD+\bar{C}\bar{D}E)=\bar{A}B$

(m) $ABEF+AB\overline{EF}+\overline{AB}EF=AB(EF+\overline{EF})+\overline{AB}EF=AB+\overline{AB}EF$

$=(AB+\overline{AB})(AB+EF)=AB+EF$

(n) $ABC\bar{D}+A+AB\bar{D}+(\bar{D})(\bar{A}\bar{B}\bar{C})=ABC\bar{D}+A+AB\bar{D}+(\bar{A}\bar{B}\bar{C}\bar{D})$

$=A(1+B\bar{D}+BC\bar{D})+\bar{A}\bar{B}\bar{C}\bar{D}=A+\bar{A}\bar{B}\bar{C}\bar{D}$

$=(A+\bar{A})(A+\bar{B}\bar{C}\bar{D})=A+\bar{B}\bar{C}\bar{D}$

(o) $x[y+z(\overline{xy+xz})]=x[y+z(\overline{xy}\cdot\overline{xz})]=xy+xz\cdot\overline{xy}\cdot\overline{xz}=xy+0$

$=xy$

(p) $\bar{x}\bar{z}+\bar{y}\bar{z}+y\bar{z}+xyz=\bar{x}\bar{z}+\bar{z}(y+\bar{y})+xyz=\bar{x}\bar{z}+\bar{z}+xyz$

$=\bar{z}(1+\bar{x})+xyz=\bar{z}+xyz=(\bar{z}+z)(\bar{z}+xy)=\bar{z}+xy$

Binary System :-

Decimal Number - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Base or radix is ten(10)

(1)

greatest positional weight out of all digits depends MSD [most significant digit] or MSB [most significant bit]

[left most bit]

[righ most bit - lowest positional weight depends LSD least significant digit - least significant bit] → right most.

→ 1 2 3
MSB . LSB

Binary Number System has Base 2

Nine's complement method.

→ 9's complement number is obtained by subtracting each digit of that decimal number by 9

→ 10's complement number is obtained by adding 1 to 9's complement.

If there is a carry it indicates that the answer is +ve.

Add this carry to the LSB

Decimal digit.

0 → 9
1 → 8
2 → 7
3 → 6
4 → 5
5 → 4
6 → 3
7 → 2
8 → 1
9 → 0

find 9's complement.

(1) 3456

$$\begin{array}{r} 9999 \\ -\,3456 \\ \hline 6543 \end{array}$$

(2) 782.54

$$\begin{array}{r} 999.99 \\ 782.54 \\ \hline 217.45 \end{array}$$

$$\begin{array}{r} 99 \\ 01 \\ \hline 98 \end{array}$$

(3) 4526.075

$$\begin{array}{r} 9999.999 \\ -\,4526.075 \\ \hline 5473.924 \end{array}$$

$$\begin{array}{r} 12 \\ -\;\; 7 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ -\;\; 1 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 12 \\ +\;\; 8 \\ \hline 10 \end{array}$$

12
+ 98

12

* find 10's complement.

(1) 4069

$$
\begin{array}{r}
9999 \\
-4069 \\
\hline
5930
\end{array}
\leftarrow 9\text{'s complement}
$$

$$
\begin{array}{r}
5930 \\
+\quad 1 \\
\hline
5931
\end{array}
\leftarrow 10\text{'s complement}
$$

(2) 1056.74

$$
\begin{array}{r}
9999.99 \\
-1056.74 \\
\hline
8943.25
\end{array}
\leftarrow 9\text{'s c}
$$

$$
\begin{array}{r}
8943.25 \\
+\quad 1 \\
\hline
8943.26
\end{array}
\leftarrow 10\text{'s c}
$$

* subsract the following numbers using 9's complement m

(4) 745.81 - 436.62

$$
\begin{array}{r}
999.99 \\
-436.62 \\
\hline
563.37
\end{array}
$$

$$
\begin{array}{r}
745.81 \\
+563.37 \\
\hline
1\ 309.18 \\
+\quad 1 \\
\hline
309.19
\end{array}
$$

309.19 answer +ve

find the 9's comp of 2nd digit N and then add to 1

If carry is there then answer is +ve

(5) 436.62 - 745.81

$$
\begin{array}{r}
999.99 \\
-745.81 \\
\hline
254.18
\end{array}
$$

$$
\begin{array}{r}
436.62 \\
+254.18 \\
\hline
690.80
\end{array}
$$

$$
\begin{array}{r}
999.99 \\
-690.80 \\
\hline
\boxed{-309.19}
\end{array}
$$

← Answer.

→ But if there is no carry, answer is -
→ So final answer 9's complement of
→ Add -ve sign

→ Arithmetic operation in digital system easier using 10's complement method.

→ 2928.54 − 0416.73

 9999.99
 0416.73
 ———
 9583.26
 1
 ———
 9583.27 ← 10's cop.

~~2928.54~~
~~9583.27~~
~~2511.81~~

 2928.54
 9583.27
 ———
 (1) 2511.81
 1
 ———
~~12511.82~~ ← Answer.

Ignore carry.

→ find 10's complement of second number, then add to first number

The carry indicating that answer is +ve.

416.73 − 2928.54

 9999.99
 −2928.54
 ———
 7071.45
 + 1
 ———
 7071.46 ← 10's cop

(1) 0416.73
 7071.46
 ———
 7488.19 No carry
 ↓
(2) 9999.99
 7488.19
 ———
 2511.80
 1
 ———
$\boxed{-2511.81}$ ← Answer.

* Binary Number System:-

→ Base of it ← 2 (Two).

→ Binary Number consist of sequence of bit, each bit is either 0 or 1

→ Convert 10101_2 to decimal.

MSB LSB

1 0 1 0 1

2^4 2^3 2^2 2^1 2^0

 16 8 4 2 1
 1 0 1 0 1
 ———
 16 4 1

$(d_n \times 2^n) + (d_{n-1} \times 2^{n-1})$
$+ (d_1 \times 2^1 + d_0 \times 2^0)$
$+ (d_{-1} 2^{-1} + d_{-2} 2^{-2})$

$2^0 \times 1 + 2^1 \times 0 + 2^2 \times 1 + 2^3 \times 0 + 2^4 \times 1$

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 2^4 + 0 + 2^2 + 0 + 2^0$

$= 16 + 4 + 1$

$\boxed{= 21_{10}}$

(2) convert 11011.101_2 to decimal

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 . \quad 2^{-1} \quad 2^{-2} \quad 2^{-3}$

$1 \quad 1 \quad 0 \quad 1 \quad 1 . \quad 1 \quad 0 \quad 1$

$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25 \quad \frac{1}{8} = 0.125$

$= 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 + 2^{-1} \times 0 + 2^{-2} \times 0$

$= 2^4 + 2^3 + 2^1 + 2^0 + 0.5 + 0 + 0.125$

$= 16 + 8 + 2 + 1 + 0.5 + 0 + 0.125$

$= (27.625)_{10}$

(3) convert 1001011 to decimal

$1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

(1) 110110 1 → $2^1 \; 2^0$

$= 2^6 + 2^3 + 2^1 + 2^0$

$= 64 + 8 + 2 + 1$

$= (75)_{10}$

* Decimal to Binary [double dabble method]

- 1 168.(8A)5)

(1) $(105.15)_{10}$

		Reminder
2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1

0.15 × 2 = 0.30

(2) 52

		Reminder
2	52	
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

(110100)

last ... 1st

Fraction 0.15

$0.15 \times 2 = 0.30$
$0.30 \times 2 = 0.60$
$0.60 \times 2 = 1.20$
$0.20 \times 2 = 0.40$
$0.40 \times 2 = 0.80$
$0.80 \times 2 = 1.60$

(001001)

$(1001.001001)_2$

0.75

$0.75 \times 2 = 1.50$
$0.50 \times 2 = 1.00$

$s = (0.11)_2$

163.875

2	163	Reminder
2	81	→ 1
2	40	→ 1
2	20	→ 0
2	10	→ 0
2	5	→ 0
2	2	→ 1
2	1	→ 0
	0	→ 1

$0.875 \times 2 = 1.750$
$0.750 \times 2 = 1.500$
$0.500 \times 2 = 1.000$

$(10100011.111)_2$

Binary addition = 0+0 = 0, 0+1 = 1, 1+0 = 1, 1+1 = (1)0 carry

1+1+1 = (1) carry (1). Ans.

```
  1 1
  1 0 1 0 .
    1 1 1
  ---------
  0 0 0 1 (Ans.
```

(2)

```
   1 1 1   1 1
   1 1 0 1 . 1 0 1
     1 1 1 . 0 1 1
   ---------------
 (1) 0 1 0 1 . 0 0 0 (Ans.
```

Binary substraction:-

0-0=0, 1-1=0, 1-0=1, 0-1=1 with

(1) Substract 10_2 from 1000_2

```
   01
  1000
-   10
  ----
  0110
```

(2) Substract 111.111_2 from 1010.01_2

```
  1010.010
-  111.111
  --------
  0010.011   Ans.
```

(3) multiplication

```
     1101
      110
    -----
     0000
    1101
   1101
  -------
  1001110
```

```
  1000
-   10
  ----
  0110
```

Extra Example

(1) Binary to Decimal

(1) 1 1 0 1 1 0 1

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$2^6 \times 1 + 2^5 \times 1 + 0 + 2^3 \times 1 + 2^2 \times 1 + 0 + 2^0 \times 1$

$= 64 + 32 + 8 + 4 + 0 + 1$

$= 109$

(2) 110111 0 . 011

$\frac{1}{2^6}\ \frac{1}{2^5}\ \cancel{\frac{0}{2^4}}\ \frac{1}{2^3}\ \frac{1}{2^2}\ \frac{1}{2^1}\ \cancel{\frac{0}{2^0}}\ .\ \cancel{\frac{0}{2^{-1}}}\ \frac{1}{2^{-2}}\ \frac{1}{2^{-3}}$

$64 + 32 + 8 + 4 + 2 . + \frac{1 \times 1}{4} + \frac{1 \times 1}{8}$

$110 . + 0.25 + 0.125$

$110 + 0.375$

110.375

(3) 1011 → 11

(4) 1101.11 → 13.75

Page
Date

Decimal to Binary

① 37

2	37	
2	18	1
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

Ans: $(100101)_2$

②

2	28	
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

Ans. $(11100)_2$

(197.56)

2	197	
2	98	1
2	49	0
2	24	1
2	12	0
2	6	0
2	3	0
2	1	1
	0	1

Ans - (11000101.100)

$0.56 \times 2 = 1.12$

$0.12 \times 2 = 0.24$

$0.24 \times 2 = 0.48$

$0.48 \times 2 = 0.96$

(4) (205.05)

2	205	
2	102	1
2	51	0
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	1	1

2	205	
2	102	1
2	51	0
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

4

$0.05 \times 2 = 0.1$
$0.1 \times 2 = 0.2$
$0.2 \times 2 = 0.4$
$0.4 \times 2 = 0.8$
$0.8 \times 2 = 1.6$

(11001101.00001)

	Carry	Sum
1+1 =	1	0
1+1+1		1
1+1+1+1		
1+1+1+1+1		
1+1+1+1+1+1		

①

Binary Addition

1)
```
  1  1  1
  1  0  1  0
     1  1  1
--------------
1 0  0  0  1
```

②
```
 1 1  1 1    1 1
 1 1 0 1 . 1 0 1
   1 1 1 . 0 1 1
------------------
1 0 1 0 1 . 0 0 0
```

	sum	carry
1+1 =	0	1
1+1+1	+1 / 1	1
1+1+1+1	+1 / 0	1+1 = 0, next 1

next to next 1

③
```
  1    1  1  1
  1    1  0  1  1
+      1  1  0  1
-------------------
1  0  1  0  0  0
```

④
```
1  1 . 0  1  1
   1  1  0  1
   1  0  0  1
   1  1  1  1
----------------
1  1  0  0  0  0
→ 1 1 0 0 0 0
```

⑤
```
   1    1 1
 1 0 1 1 1 . 1 0 1
1 1 0 1 1 1 . 0 1 0
----------------------
1 0 0 1 1 1 0 . 1 1 1
```

4 2 1
1 0 1

1+1	0	1
1+1+1	1	1
1+1+1+1	0 +1	+1 / 10
1+1+1+1+1	1	10
1+1+1+1+1+1	+1 / 0	1 / 11

4 2 1
1 0 0
1 0 1

4 2 1
1 1 0

```
     1      1
1 1  1      1       1
     1  0 1 0  .  1 1
     1 1 0 1   .  1 0
     1 0 0 1   .  1 1
     1 1 1 1   .  1 1
   ----------------------
   1 1 0 0 0 1 .  1 1
```

* Binary Multiplication

(1)

```
          1 1 0 1
            1 1 0
    ------------------
    1     0 0 0 0
        1 1 0 1
      1 1 0 1
    ------------------
    1 0 0 1 1 1 0
```

	carry	sum
1+1	1	0
	+	1
	1.	
1+1+1	1.	1
	+	1
1+1+1+1+1	1 0	0
		1
	+	
1+1+1+1+1+1+1	1 0	0
		1

1 1 0 1

1 0 1

1 1

1 1 0 1

0 0 0 0 0

1 0 1 0 0

1 0 0 0 0 1

1 1 0 1

x 1 0 1

1 1 1

1 1 0 1

0 0 0 0 0

1 1 0 1 0 0

1 0 0 0 0 0 1

3

1 1 0 0 1

x 1 0

0 0 0 0 0

1 1 0 0 1

1 1 0 0 1 0

1 1 0 1 1 . 1 1

1 0 1 1 . 1 0

---

1 1 1

1 0 0 0 0 . 0 0

1 1 0 1 1 1

1 1 1 0 1 1 1

0 0 0 0 0 0

~~1 1 0 1 1 1~~

1 1 0 1 1 1

---

1 0 0 1 0 1 1 1 . 1 . 0 . 1 0

1 0 0 1 0 1 1 1 . 1 0 1 0

(5)

1 0 1 1 0 . 0

1 0 . 1

1

1 0 1 1 0 0

0 0 0 0 0 0

1 0 1 1 0 0

---

1 1 0 1 1 1 . 0 . 0

1011.101 X 101.01 = 1111101.00001

* Binary Subtraction

1)

$$\begin{array}{r} \overset{0}{1}\,\overset{1}{\cancel{0}}\,0\,0 \\ -\quad 1\,0 \\ \hline 0\,1\,1\,0 \end{array}$$

②

$$\begin{array}{r} \overset{0}{1}\,\overset{1}{\cancel{0}}\,\overset{0}{\cancel{1}}\,\overset{1}{\cancel{0}}.\overset{1}{\cancel{0}}\,\overset{0}{\cancel{1}}\,0 \\ 1\,1\,1.\,1\,1\,1 \\ \hline 0\,0\,1\,0.\,0\,1\,1 \end{array}$$

③

$$\begin{array}{r} \overset{0}{1}\,\overset{1}{\cancel{0}}\,1\,1 \\ -\quad 1\,0\,1 \\ \hline 0\,1\,1\,0 \end{array}$$

④

$$\begin{array}{r} 1\,\overset{1}{\cancel{0}}\,\overset{0}{1}\,\overset{0}{1}\,0 \\ -\quad 1\,0\,1\,1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \overset{0}{1}\,0\,\overset{0}{1}\,\overset{0}{1}\,0 \\ -\quad 1\,0\,1\,1 \\ \hline 0\,1\,0\,\cancel{0}\,1 \end{array}$$

0 1 0 1

$-$ 0 1 0 1 1

$$
\begin{array}{r}
1\,1\,0\,0\,.\,1\,0 \\
-\quad 1\,1\,1\,.\,0\,1 \\
\hline
0\,1\,0\,1\,.\,0\,1
\end{array}
$$

(6)

$$
\begin{array}{r}
1\,0\,0\,0\,1\,.\,0\,1 \\
-\quad 1\,1\,1\,1\,.\,1\,1 \\
\hline
0\,0\,0\,0\,1\,.\,1\,0
\end{array}
$$

	32	16	8	4	2	1	
0	1	0	1	0	0	1	= +41
1	1	0	1	0	0	1	= -41

unsigned and signed Number

unsigned Binary → positive number

m bit : 8 bit : $2^m - 1$

$= 2^8 - 1$

$= 256 - 1$

0 to 255 = 255

Signed Number required an arithmetic sign

MSB → in binary system used to represent signed bit

If signed bit is zero → +ve

one → -ve

	421		421
+3 :	0011	-3	1011
+7 :	0111		1111

Octal Number System

→ It is used by minicomputers.
→ Its base or radix is 8.
→ It has 8 independent symbols 0, 1, 2, 3, 4, 5, 6 and 7
→ since its base = 8 = 2^3
→ every 3 bit of binary can be represented by octal digit.
∴ An octal = 1/3 of Binary

→ Application :- expressing large Numbers.
It represents
(1) actual numerical data
(2) location in memory
(3) Instruction code.

* Octal to Binary Conversion

(1) $(367.52)_8$ to Binary

3 | 6 | 7 . 5 2
011 | 110 | 111 . 101 010
$(011111011.101010)_2$

* Binary to octal conversion

① 110101101010

110 101 101 010
6 5 . 5 2
$(65.52)_8$

(8) convert $(10|101|111|001.0111)_2$ to octal

before zero | After zero

010 101 111 001 . 011 100

2 5 7 1 . 3 4$)_8$

Que -	Ans
463 -	307
2056 -	1070.
2057.64 -	1071.85
6534.04 -	3420.06

ε50

* Octal to Decimal conversion.

(1) convert $(4057.06)_8$ to decimal.

$4057.06 = 4\times8^3 + 0\times8^2 + 5\times8^1 + 7\times8^0$

$+ 0\times8^{-1} + 6\times8^{-2}$

$= 2048 + 0 + 40 + 7 + 0 + 0.0937$

$= (2095.0937)_{10}$

* Decimal to octal conversion

(1) convert ~~3798~~ $(378.93)_{10}$ to octal

8	378	
8	47	2
8	5	7
	0	5

$(572)_8$

(2) $0.93\times8 = 7.44$

$0.44\times8 = 3.52$

$0.52\times8 = 4.16$

$0.16\times8 = 1.28$

7341

$(572.7341)_8$

Decimal to octal Conversion	
Que.	Ans
287 -	437
3956 -	7564
420.6 -	644.463
8476.47 -	20434.360

Q. $(937)_{10}$ to octal

937/8 : 7/441

Convert $(5497)_{10}$ to Binary

$$\begin{array}{r|lr} 8 & 5497 & \\ 8 & 687 & 1 \\ 8 & 85 & 7 \\ 8 & 10 & 5 \\ 8 & 1 & 2 \\ & 0 & 1 \end{array}$$

$(12571)_8$

$=(001010101111001)_2$

Octal Addition

(1) By decimal method.

2) Add the digit in each column in decimal convert this Sum into octal.

327.54
665.37

$7+4=(11)_{10}=(13)_8$
$=$ Sum $=3$ carry $=1$

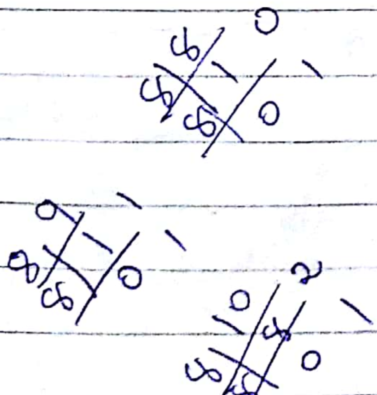
$5+3+1=(9)_{10}=(11)_8$
$=$ Sum $=1$, carry $=1$

$7+1+5=13=(15)_8$ $=$ Sum $=5$
carry $=1$

$6+2+1=(9)=(11)_8$ (Sum $=1$, carry $=1$)

$6+3+1=(10)=(12)_8$
Sum $=2$
Carry $=1$

$$\begin{array}{r} 1\ \ 1\ \ \ \ \ 1\ \ \ \ \ \\ 327.54 \\ 665.37 \\ \hline 1215.13 \end{array}$$



* octal subtraction.

```
       20
 1 7 8
 8 0 8
 20 . 14
- 16 . 47
-----------
(01 . 45)8
```

$8+4-7 = 12-7 = 5$

$8-4 = 4$

$7-6 = 1$

Octal multiplication :-

```
 5 4 2 3
 2 7 0 3 . 5
x          6
-------------
1 7 6 6 . 5 6
```

(1) $6\times5 = (30)_{10} = (36)_8$

[Sum = 6, carry = 3]

(2) $6\times3 = (18)_{10} = (22)_8 + (3)_8$

$= (25)_8$

(3) $6\times6 = (36)_{10} = (44)_8$

$= +(2)_8$

$= (46)_8$

(4) $7\times6 = (42)_{10} = (52)_8$

$= (52)_8 + (4)_8$

$= (56)_8$

(5) $6\times2 = 12 = (14)_8 + (5)_8$

$= (19)_8$

$8+4-7$

```
 8 8
 1 8 0
 20 . 14
 16 . 47
       5
```

$46\times4 = 230$

$267\times5 = 1623$

~~20.50×2025~~

```
 1 8 0
 20 . 14
 16 . 47
       5
```

```
 1 8 0 14
 20 . 14
 16 . 47
```

find 7's and 8's complement of following numbers

(a) 4015 (b) 2057.106

$$\begin{array}{r} 7777 \\ -4015 \\ \hline 3762 \end{array} \quad 7's$$

$$\begin{array}{r} + \quad 1 \\ \hline 3763 \end{array} \quad 8's.$$

$$\begin{array}{r} 7777.777 \\ 2057.106 \\ \hline 5720.671 \end{array} \leftarrow 7's$$

$$\begin{array}{r} + \quad 1 \\ \hline 5720.672 \end{array} \quad 8's$$

* Subtract $(0236.43)_8$ from $(5407.65)_8$ using 7's comp method.

$$\begin{array}{r} 7\ 777.77 \\ 0\ 236.43 \\ \hline 7\ 541.34 \end{array}$$

$$\begin{array}{r} 5407.65 \\ 7541.34 \\ \hline (1)\ 5171.21 \\ 1 \\ \hline 5171.22 \end{array}$$

carry +ve

* Subtract 5407.65 from 236.43 using 8's comp method.

8's = 7777.77

$$\begin{array}{r} 5407.65 \\ \hline 2350.12 \\ + \quad 1 \\ \hline 2350.13 \end{array} \leftarrow 8's$$

$$\begin{array}{r} 0236.043 \\ 5407.65 \\ \hline 5666.080 \end{array}$$

$$\begin{array}{r} 0236.43 \\ +\ 2350.13 \\ \hline 2606.56 \end{array}$$

no carry Ans: -ve.

8's comp =

2606.56

$$\begin{array}{r} 7777.77 \\ 2606.56 \\ \hline 5171.21 \\ 1 \\ \hline -5171.22 \end{array}$$

8410a

* convert following octal to decimal

(a) 463

463

$= \underset{8^2}{4} \quad \underset{8^1}{6} \quad \underset{8^0}{3}$

$= 4 \times 8^2 + 6 \times 8^1 + 3 \times 8^0$

$= 4 \times 64 + 6 \times 8 + 3$

$= 256 + 48 + 3$

$= (115)_{10}$

$(307)_{10}$

$$\begin{array}{r} 64 \\ 48 \\ \hline 112 \\ 3 \\ \hline 115 \end{array} \qquad \begin{array}{r} 64 \\ 4 \\ \hline 256 \end{array} \qquad \begin{array}{r} 256 \\ 48 \\ \hline 304 \\ 3 \\ \hline 307 \end{array}$$

Decimal to octal

2057.64

8	2057	
8	257	1
8	32	1
8	4	0
	0	4

4011.

$0.64 \times 8 = 5.12$

$0.12 \times 8 = 0.96$

$0.96 \times 8 = 7.68$

$(4011.507)_{10}$.

Addition:-

$$\begin{array}{r} 1247 \\ +\ 2053 \\ \hline 3322 \end{array}$$

$$\begin{array}{r} 273.56 \\ 425.07 \\ \hline 720.65 \end{array}$$

$$\begin{array}{r} 173 \\ 265 \\ \hline 460 \end{array}$$

0

8^{-}

87

Subsraction

@
$$\begin{array}{r} 64 \\ -37 \\ \hline 25 \end{array}$$

8+4-7

(b) 462-175
$$\begin{array}{r} 462 \\ 175 \\ \hline 265 \end{array}$$

8+2-5
8+5-7 = 13-7
= 6

(c)
$$\begin{array}{r} 175.6 \\ -47.7 \\ \hline 125.7 \end{array}$$

8+6-7 =
8+4-7 = 11-7
= 5

(d)
$$\begin{array}{r} 3006.05 \\ -2657.16 \\ \hline 0126.67 \end{array}$$

8+5-6 = 13-6
8+5-7 = 13-7 = 6

(e)
$$\begin{array}{r} 20.14 \\ -16.47 \\ \hline 01.45 \end{array}$$

8+4-7 = 10-7
8+0-4 = 8-4 = 4

$$\begin{array}{r} 3006.05 \\ 2657.16 \\ \hline 0126.67 \end{array}$$

8+5-6 = 13-6
8+5-7 = 13-7
= 6

$$\begin{array}{r} 3006.05 \\ -2657.16 \\ \hline 67 \end{array}$$

8+5-6
8+5-7

Octal Table

Decimal Number	Octal Number
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11
10	12
11	13
12	14
13	15
14	16
15	17
16	20
17	21
18	22
19	23
20	24
21	25
22	26
23	27
24	30
25	31

*Octal Addition

$$\begin{array}{r} {}^{1\,1}\\ 173 \\ +265 \\ \hline 460 \end{array}$$

$5+3=8)_{10}=10$

$7+6+1=(14)=16$

$$\begin{array}{r} 1247 \\ +2053 \\ \hline 3322 \end{array}$$

$10=12$

$10=12$

$$\begin{array}{r} 25.76 \\ +16.57 \\ \hline 44.55 \end{array}$$

$13-15$

$13-15$

$12=14$

$$\begin{array}{r} 273.56 \\ +425.07 \\ \hline 720.65 \end{array}$$

$13-15$

$8-10$

$9-11$

Subtraction

$$\begin{array}{r} 3006.05 \\ -2657.16 \\ \hline 126.67 \end{array}$$

$8+5-6=$

$8+5-7=13-7=6$

Octal subtraction

$$\begin{array}{r} 64 \\ -37 \\ \hline 25 \end{array}$$

$8+4-7=5$

(2)

$$\begin{array}{r} 462 \\ -175 \\ \hline 265 \end{array}$$

$8+2-5$

$8+5-7$

(3)

$$\begin{array}{r} 175.6 \\ -47.7 \\ \hline 125.7 \end{array}$$

$8+6-7$

$8+4-7$

(4)

$$\begin{array}{r} 3006.05 \\ -2657.16 \\ \hline 0126.67 \end{array}$$

$8+5-6$

$8+5-7=6$

126.67

$$\begin{array}{r} 20.14 \\ -16.47 \\ \hline 1.45 \end{array}$$

$8+4-7=$

③

Hexadecimal Number system

(1) Convert $(1011011011)_2$ to Hexadecimal.

1011 011011

0010 1101 1011

(2 D B.$)_{16}$

= $(2DB)_{16}$

(2) Convert 01011111011.011111

00 10111111011 . 011111

0010 1111 1011 . 0111 1100

2 F B . 7 C

=> $(2FB.7C)_{16}$.

* Binary to Hexadecimal

0 → 0
9 → 9
10 → A
11 → B
12 → C
13 → D
14 → E
15 → F

* Hexadecimal to Binary :-

(1) convert $(4BAC)_{16}$ to Binary

4 B A C
0100 1011 1010 1100

=> $(0100101110101100)_2$

(2) Convert $(3A9.E.B0D)_{16}$ to Binary

3 A 9 E . B 0 D
0011 1010 1001 1110 . 1011 0000 1101

* Hexadecimal to Decimal.

(1) convert $(5C7)_{16}$ to decimal.

$$5C7 = 5\times16^2 + C\times16^1 + 7\times16^0$$

$16^2\ 16^1\ 16^0$

$$= 5\times16^2 + 12\times16 + 7\times16^0$$

$$= 1280 + 192 + 7$$

$$= (1479)_{10}.$$

(2) convert $(A0F9.0EB)_{16}$ to Decimal.

A 0 F 9 . 0 E B

$16^3\ 16^2\ 16^1\ 16^0\ 16^{-1}\ 16^{-2}\ 16^{-3}$

$$= 10\times16^3 + 0\times16^2 + 15\times16^1 + 9\times16^0 + 0\times16^{-1} + 14\times16^{-2} + 11\times16^{-3}$$

$$= 40960 + 0 + 240 + 9 + 0 + 0.0546 + 0.002$$

$$= (41209.0572)_{10}.$$

Ex

→ (1) AB6 → 2742

(2) 2EB7 → 11959

(3) A08F. EA → 41103.879

(4) 8F47. AB → 36423.629

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* Decimal to Hexadecimal.

(1) Convert $(2598.675)_{10}$ to Hexadecimal

		Reminder
16	2598	
16	162	6
16	10	2
	0	10 → A

2598 = A26

10 → A
11 → B
12 → C
13 → D
14 → E
15 → F

0.675 × 16 = 10.8 A
0.800 × 16 = 12.8 C
0.800 × 16 = 12.8 C
0.800 × 16 = 12.10 C

$(A26.ACCC)_{16}$

(2) Convert $(49056)_{10}$ to Binary.

→ This number is very large. So it is very tedious to convert directly to the Binary.

16	49056	0		
16	3066	0	0	0000
16	191	10	A	1010
16	11	15	F	1111
	0	11	E	1011

$= (0AFE)_{16}$

$= (0000\ 1010\ 1111\ 1011)_2$

→ (4) 452 → 1C4
(5) 4796 → 12BC
~~(b) 10080 → 0A~~
~~(d) ...~~
(c) 1248.56 → 4E0.8F5
(d) 8957.75 → 22FD.C

* Octal to Hexadecimal

(1) convert $(756.603)_8$ to Hexadecimal.

Octal	7	5	6	.	6	0	3
each place to Binary	111	101	110	.	110	000	011
each of four for Hexa.	0001	1110	1110	.	1100	0001	1000
	1	E	E	.	C	1	8

$\Rightarrow (1EE.C18)_{16}$.

* Hexadecimal to octal:-

(1) convert $(B9F.AE)_{16}$ to octal.

Hex. Number	B	9	F	.	A	E			
Binary of Hex	1011	1001	1111	.	1010	1110			
making group of 3	101	110	011	111	.	101	011	100	
Octal	5	6	3	7	.	5	3	1	

Ans $\Rightarrow (5637.531)_8$.

* Hexadecimal Addition

Ex →

Q.15 (1) Convert following octal Number to Hexadecimal

(a) 256 → AE

(b) 2035 → 41D

(c) 1762.46 → 3F2.98

~~(d) BC79.AE →~~

(d) 6254.263 → CAC.598

* Convert following Hexadecimal number to octal.

(a) 2AB → 1253

(b) 42FD → 41375

(c) 4F7.A6 → 2367.52

(d) BC70.0E → 136160.034

* Hexadecimal Addition

① Add $(2A7C.30D)_{16}$ and $(8D9.E8B)_{16}$

	1	1	1	1			1	
	2	A	7	C	.	3	0	D
		8	D	9	.	E	8	B
	3	3	5	6	.	1	9	8

$D+B = 13_{10}+11_{10} = (24)_{10} = (18)_{16}$ (Sum = 8, carry = 1)

$0+8+1 = (8)_{10}+(1)_{10} = (9)_{10} = (9)_{16}$

$E+3 = (14+3)_{10} = (17)_{10} = (11)_{16}$ (Sum = 1, carry = 1)

$1+C+9 = 1_{10}+12_{10}+9_{10} = (22)_{10} = (16)_{16}$ (Sum = 6 carry = 1)

$1+7+D = 1_{10}+7_{10}+(13)_{10} = (21)_{10} = (15)_{16}$ (Sum = 5, carry = 1)

$1+A+8 = 1_{10}+10_{10}+8_{10} = (19)_{10} = (13)_{16}$ (Sum = 3, carry = 1)

⇒ Ans $= (3356.198)_{16}$.

16 | 17 1

16 | 1 1

 0

=) Add $(3BCA \cdot 5078)_{16} + (9EBD \cdot 97F3)_{16} + (5FB.$

3 / B / C A . 5 0 7 8
9 E / B / D . 9 7 F / 3
5 / F / B / . E 2

. 2

3	B	C	A	.	5	0	7	8
9	E	B	D	.	9	7	F	3
	5	F	B	.	E	2	C	
E	D	8	3	.	C	B	2	B

$8 + 3 = (11)_{10} = B_{16}$ (sum = B, carry = 0

$7 + F + C = 7_{10} + 15_{10} + 12_{10} = 34_{10} = (22)_{16}$ (sum = 2, C =

$0 + 7 + 2 + 2 = (11)_{10} = B_{16}$ S = B C = 0.

$5 + 9 + E = 5_{10} + 9_{10} + 14_{10} = 28_{10} = 1C_{16}$

S = C C = 1

$A + D + B + 1 = 10_{10} + 13_{10} + 11_{10} + 1_{10} = (35)_{10} (23)_{16}$

(sum = 3 carry = 2

$C + B + F + 2 = 12_{10} + 11_{10} + 15_{10} + 2_{10} = (40)_{10} = (28)_{16}$

(S = 8 C = 2

$B + E + 5 + 2 = 11_{10} + 14_{10} + 5_{10} + 2_{10} = (32)_{10}$

$= (20)_{16}$ (S = 0 C =

$3 + 9 + 2 = (14)_{10} = E_{16}$ (sum = E, C = 0)

Hexadecimal Addition

(1) AC6 + B59 → 161F
(2) A0FC + B75F → 1585B
(3) E0F3.5D + 49E6.F7 → 12ADA.54
(4) ABC7.54 + 26F3.AB + DAC9.6F → 1AD84.6E

Hexadecimal Subtraction

(1) BC5 − A2B → 19A
(2) F27 − B9E → 389
(3) CDF7.52 − AB5.8 → C341.D2
(4) 67F2.6E − 4A0E.A9 → 1DE3.C5

Hexadecimal multiplication

(1) 28A × B → 1BEE
(2) 5A9B × 7 → 27A3D
(3) 5032.6E × 6 → 1E12E.94
(4) 92.5 × B.3 → 664.DF

* Hexadecimal Substraction:

(1) Substract $7806.3B_{16}$ from B08E.A1

$$\begin{array}{r} A\ 15\ 16\ \ \ 9\ 16 \\ B\ 0\ 8\ E.\ A\ 1 \\ -\ 7\ 8\ 0\ 6.\ 3\ B \\ \hline 3\ 7\ B\ 8.\ 6\ 6 \end{array}$$

(1) $16+1-B = 17-B = 17-11 = 6$

$16+8-D = 24-13 = 11 = B$

$15+0-8 = 7$

Substract $04AB.6B_{16}$ from $(5074.56)_{16}$ using 15's complement method.

$$\begin{array}{r} FFFF.FF \\ -\ 04AB.6B \\ \hline FB54.94 \end{array} \qquad \begin{array}{r} 5074.56 \\ +\ FB54.94 \\ \hline (1)4BC8.EA \\ 1 \\ \hline 4BC8.EB \end{array}$$

carry indicate that result is +ve.

* substract 5070.56 from 4AB.68 using 16's complement method

FFFF.FF
507 0.56
AF82.A9
1
AF82.AA

04AB.68
AF82.AA
B42E.12

No carry 16's comp of

FFFF.FF
− B42E.11
4BD1.EE
+ 1
4BD1.EF

* subtract 48₁₆ - 26₁₆ using 1's complement

① 2 6
0010 0110
1's 1101 1001
Hexadecimal D 9

48
+D9
21
+ 1
22

② 45 - 74
7 4
0111 0100
1's 1000 1011
Hex. 8 B

45
+8B
D0
No carry 2's

1's 1101 0000
0010 1111

11+5 = 16 = 10

A − 10
B − 11
C − 12
D − 13

10 A
11 B
12 C
13 D
14 E
15 F

Hexadecimal multipliation:

9A8 By 6 16

3 3
9 A 8
6
F F 0
8

$$
\begin{array}{r|l}
16 & 48 \\
\hline
16 & 3\ 0 \\
\hline
 & 0\ 3
\end{array}
$$

3 6
5
0 1

$$
\begin{array}{r|l}
16 & 60 \\
\hline
16 & 3\ C \\
\hline
 & 0\ 3
\end{array}
$$

3C
3
3F

3C
5
47

8x6 = 48 = 30
Ax6 = 10x6 = 60 = ~~6A8 = 183~~ = 3C+3 = 3F
6x9 = 12 = C = C+3 = F
~~8xB = 8x11 = 88 = 58~~
~~Ax6 = 10x6 = 60 = 3C+5~~

3 3
9A8
x 6
FFF0

* Octal Substraction

(1)

$$\begin{array}{r} \overset{5}{\cancel{6}}4 \\ -37 \\ \hline 25 \end{array}$$

$8+4-7 = 12-7 = 5$

(2)

$$\begin{array}{r} \overset{3}{\cancel{4}}\overset{8}{\overset{5}{\cancel{6}}}\overset{8}{2} \\ -175 \\ \hline 265 \end{array}$$

$8+2-5 = 10-5 = 5$

$8+5-7 = 13-7$
$= 6$

(3)

$$\begin{array}{r} 1\overset{6}{\cancel{7}}\overset{8}{\overset{4}{\cancel{5}}}.\overset{8}{6} \\ -47.7 \\ \hline 125.7 \end{array}$$

$8+6-7 = 14-7 =$

$8+4-7 = 12-7 = 5$

(4)

$$\begin{array}{r} 20.14 \\ -16.47 \\ \hline 01.45 \end{array}$$

$8+4-7 = 12-7$

(5)

$$\begin{array}{r} 3006.05 \\ -2657.16 \\ \hline 0127.67 \end{array}$$

$8+5-6 = 13-6 = 7$

$8+6-7 = 14-7 = 7$