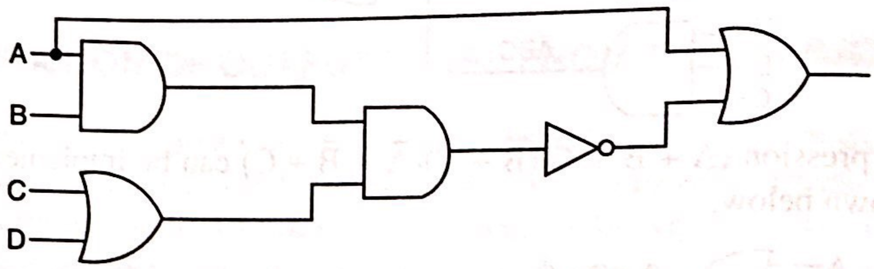
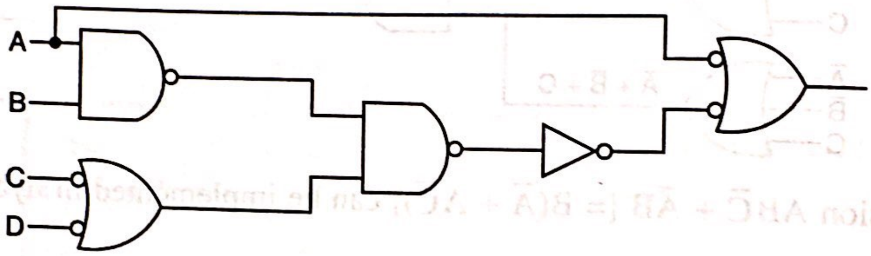
**EXAMPLE 5.24** Convert the following AOI logic circuit to (a) NAND logic, and (b) NOR logic.

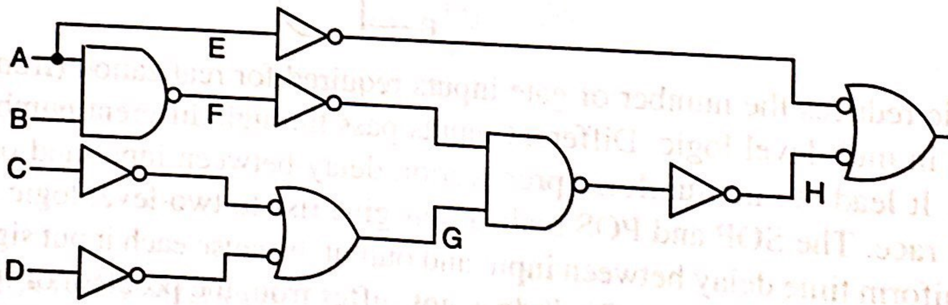



***Solution***

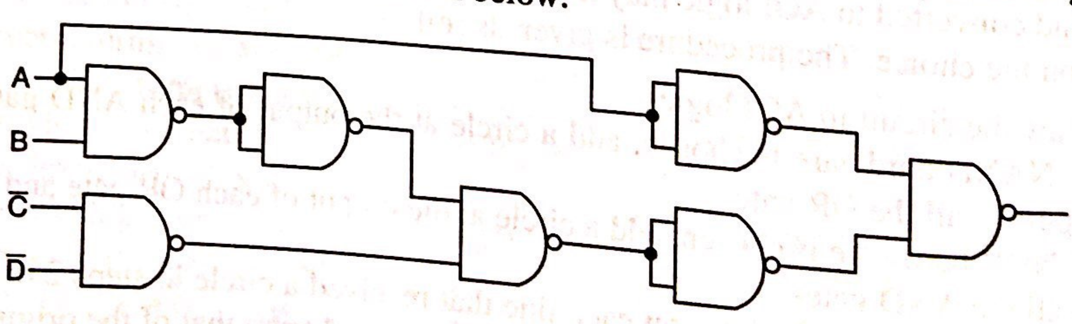
(a) NAND logic: Put a circle at the output of each AND gate and at the inputs to all OR gates as shown below.



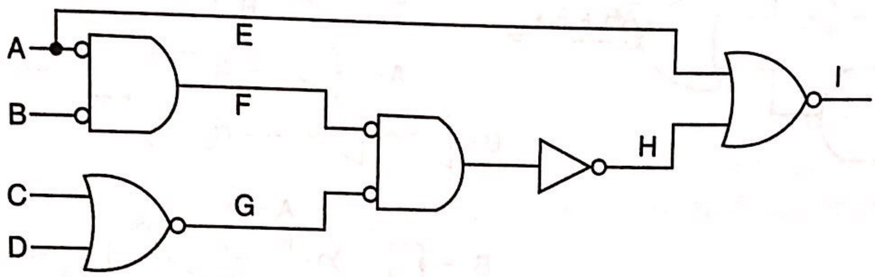

Add an inverter to each of the lines E, F, C, D that received only one circle in the previous step as shown below so that the polarity of these lines remains unchanged. Inverters in lines C and D can be removed, if C and D are replaced by $\overline{C}$ and $\overline{D}$. Line H received two circles. So, no change is required.



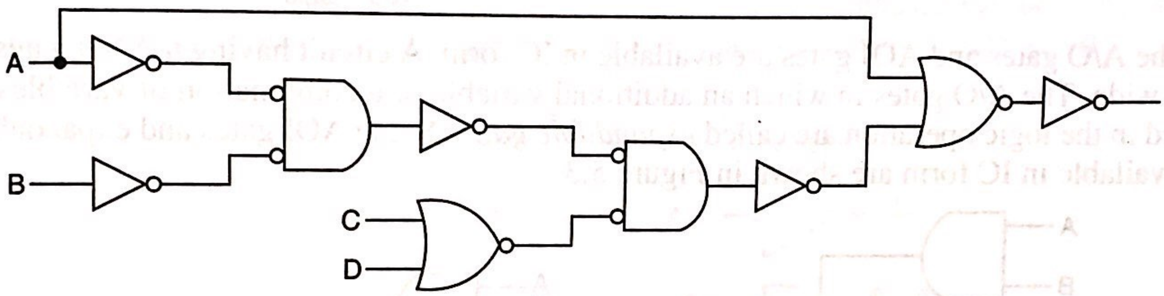

Replace bubbled OR gates and NOT gates by NAND gates. Using only NAND gates, the logic circuit can now be drawn as shown below.



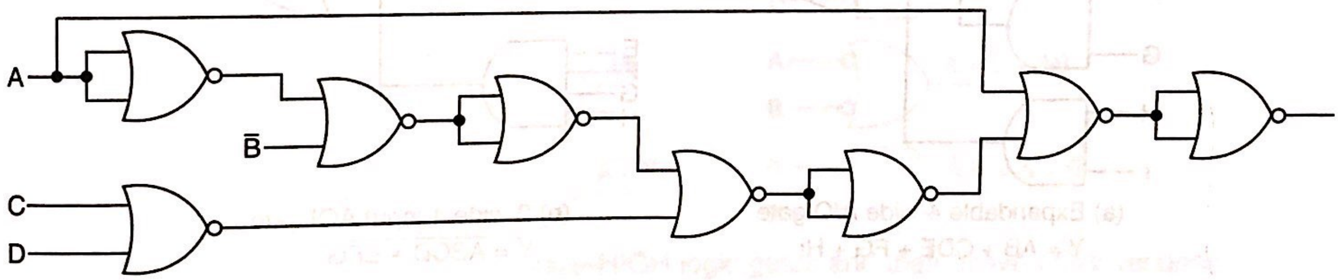
(b) NOR logic: Put a circle at the output of each OR gate and at the inputs to all AND gates as shown below.




Add an inverter in each of the lines A, B, F, and I that received only one circle in the previous step, so that the polarity of these lines remains unchanged. Line G received two circles. So, no change is required.




Replace bubbled AND gates and NOT gates by NOR gates. Using only NOR gates, the logic circuit can now be drawn as shown below. Note that the inverter in line B has been removed assuming that $\overline{B}$ is available.



## Minterm:-

→ Binary variable may appear in normal form (x) so its complement form (x')

→ Suppose 2 variable are there so $2^n$ combination

$2^2 = 4$. $xy$, $x'y'$, $x'y$, $xy'$

→ So AND product in SOP (sum of products) is called minterm.

| x | y | | | |
|---|---|---|---|---|
| 0 | 0 | → $x'y'$ | $m_0$ | |
| 0 | 1 | → $x'y$ | $m_1$ | |
| 1 | 0 | → $xy'$ | $m_2$ | ← for minterm |
| 1 | 1 | → $xy$ | $m_3$ | |

0 Depends ← complement variable ($x'$)
1 represent ← simple variable ($x$)

$f_1 = x'y'z + xy'z' + xyz$
$= 001 + 100 + 111$
$= m_1 + m_4 + m_7$.

Any Boolean fn can be expressed as sum of minterm.

$f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$.
maxterm.
$= (000)$

maxterm ← OR term in POS.

Any Boolean fn can be expressed as product of maxterm

$= (000)\ (010)\ (011)\ (101)\ (110)$
$= M_0\ \ M_2\ \ M_3\ \ M_5\ \ M_6$.

* minterms and maxterms are related.

$(x'y'z')'$ $m_0$ $x+y+z$ $M_0$

$\boxed{m_0' = M_0.}$ ← complement with each other.

(2) $A+B'C = A(B+\bar{B})$

Express the Boolean f $F = A+B'C$ in a minterms.

$= AB + A\bar{B} \quad (C+\bar{C})$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

$\bar{B}C = \bar{B}C(A+\bar{A})$

$= \bar{B}CA + \bar{A}\bar{B}C$

$$\begin{array}{ccccccccccc} & & 421 & & 421 & & 421 & & 421 & & 421 \\ A+B'C & = & ABC & + & AB\bar{C} & + & A\bar{B}C & + & A\bar{B}\bar{C} & + & \bar{A}\bar{B}C \end{array}$$

$= 111 + 110 + 101 + 100 + 001$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$= m_1 + m_4 + m_5 + m_6 + m_7$

$\Sigma m(1, 4, 5, 6, 7)$

→ Boolean $f^n$ may be expressed algebraically from a given truth Table.

find the minterms and maxterms for F described by following truth Table.

→ The canonical sum of products form of a logic $f^n$ can be obtained by using following procedure.

# chapter-6

## The Karnaugh map (K-map).

K map ← systematic method of simplifying Boolean expression

→ graphical representation of Boolean expression.

$2^n$ cells → $2^2 = 4$. cell ← Two variable K map

$2^3 = 6$. Three variable Kmap.

$2^4 = 8$ four variable Kmap.

→ Product Term in SOP format. ← minterm

sum term in POS format ← maxterm.

→ Each minterm is → AND operation of all variable in th (variable may be normal or complemented)

→ min term → complemented variable represent by 0

non complemented variable represnt by 1

→ for maxterm opposite from minterm.

* Boolean expression to SOP form [expansion].

multiply missing variable with sum of its complement. (min term in SOP form).

for ~~max~~ min term.

(1)

$\bar{A}+\bar{B} = \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$ ← 1st step. (multiplying missing variable with sum of its complement)

$= \bar{A}B + \bar{A}\bar{B} + \bar{B}A + \bar{A}\bar{B}$

$= \bar{A}B + \bar{A}\bar{B} + A\bar{B}$.

$= 01 + 00 + 10$. (← 4 2 1)

$= m_1 + m_0 + m_2$

$\boxed{= \Sigma m(0,1,2)}$ ← in SOP
min term

this Two variable kmap ← $2^n$ ← $2^2$ = 4 cells.
minterm

So $m_3$ ← is missing in SOP form.

maxterm $M_3$ will be present in POS form.

max term
$\boxed{= \Pi M_3}$ ← POS.

$\bar{A}+\bar{B} = x\bar{A} + x\bar{B}$ [Put x in each variable.

$= x0 + \bar{B}x$

$= x0 + 0x$ } Put complemented variable = 0.

$= x0 + 0x$

$= 10 + 00$

$+ 01 + 00$ make all possible combination

$= 10 + 01 + 00$

$= \Sigma m(0,1,2)$

Ex→2 $A + B\bar{C} + AB\bar{D} + ABCD$

$A(B+\bar{B}) = AB + A\bar{B}$

$= A(B+\bar{B})(C+\bar{C})(D+\bar{D})$

$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

$\rightarrow B\bar{C} = B\bar{C}(A+\bar{A})(D+\bar{D})$

$= AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$.

$\rightarrow AB\bar{D} = AB\bar{D}(C+\bar{C})$

$= ABC\bar{D} + AB\bar{C}\bar{D}$

$A + B\bar{C} + AB\bar{D} + ABCD = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$.

$2^4 = 16$.

$= \Sigma m(4, 5, 8, 9, 10, 11, 12, 13, 14, 15)$ ← min term in SOP form.

$= \Pi m(0, 1, 2, 3, 6, 7)$. ← max term in POS form

$A + B\bar{C} + AB\bar{D} + ABCD$

$= AXXX$

$= 1000 + 1001 + 1010 + 1100 + 1101 + 1110 + 1111$

$= m8 + m9 + m10 + m11 + m12 + m13 + m14 + m15$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

$B\bar{C} = XB\bar{C}X = X10X$.

$= 0100 + 0101 + 1100 + 1101$

$= m4 + m5 + m12 + m13$

$AB\bar{D} = ABX\bar{D} = 11X0 = 1100 + 1110$

$= m12 + m14$.

$\Sigma m = (4, 5, 8, 9, 10, 11, 12, 13, 14, 15.)$

sion of a Boolean Expression to POS form



1) $A(\bar{B}+A)B$ to maxterm and minterm.

$A = A + B\bar{B} = (A+B)(A+\bar{B})$ — adding product of each terms and its complement

$B: B + A\bar{A} = (A+B)(\bar{A}+B)$

$\rightarrow A(\bar{B}+A)B = \underline{(A+B)}(A+\bar{B})(\bar{B}+A)\underline{(A+B)}(\bar{A}+B)$

$= (A+B)(\bar{A}+B)(A+\bar{B})$

$= 00 \quad (01)(10) \quad = \Pi M(0,1,2) \leftarrow$ maxterm in pos

$A(\bar{B}+A)B: A \times (\bar{B}+A) \times B$

$= 0 \times (\bar{B}+A) \times 0$

$= 00 \quad (01) \quad (10)$

$\rightarrow$ Expand $A(\bar{A}+B)(\bar{A}+B+\bar{C})$ to max term and minterm

$A = \underline{A + B\bar{B}} + C\bar{C} = (A+B)(A+\bar{B}) + C\bar{C}$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$

$(A+\bar{B}+\bar{C})$ ~~$(\bar{A}+B+\bar{C})$~~

$\bar{A}+B: (\bar{A}+B) + C\bar{C} = (\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$A(\bar{A}+B)(\bar{A}+B+\bar{C}) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$

$(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$A = 0XX = (000)(001)(010)(011)$

$= m_0 m_1 m_2 m_3$

$\bar{A}+B = 10X = 100, 101$

$= m_4, m_5$

$\bar{A}+B+\bar{C} = 101 = m_5$

$\Pi M(0,1,2,3,4,5)$

$= (000)(001)(010)(011)$

$(100)(101)$

$= \Pi M(0,1,2,3,4,5) \leftarrow$ maxterm in pos

min term $\Sigma m(6,7) \leftarrow$ min term in SOP)

$\rightarrow$ Write down the algebric form for minterm

| $m_0$ | $m_5$ | $m_9$ | $m_{14}$ |
|---|---|---|---|
| | 8421 | 8421 | 8421 |
| | 0101 | 1001 | 1110 |
| $\bar{A}\bar{B}\bar{C}\bar{D}$ | $\bar{A}B\bar{C}D$ | $A\bar{B}\bar{C}D$ | $ABC\bar{D}$ |

| $M_3$ | $M_9$ | $M_{11}$ | $M_{14}$ |
|---|---|---|---|
| 8421 | 8421 | 8421 | 8421 |
| 0011 | 1001 | 1011 | 1110 |
| $(A+B+\bar{C}+\bar{D})$ | $(\bar{A}+B+C+\bar{D})$ | $(\bar{A}+B+\bar{C}+\bar{D})$ | $(\bar{A}+\bar{B}+\bar{C}+D)$ |

max term :- suppose there are two binary variable a & b and associate with OR operation. then it is called max term

$a+b$, $\bar{a}+\bar{b}$, $\bar{a}+b$, $\bar{b}+a$

$F = xy + x'z$

$= (xy + zz') + (x'z + yy')$

$xy \quad (z\bar{z})$

$(x' + y + zz') \quad (x + z + \underline{yy'}) \quad (y + z + x\underline{x'})$

$(x' + y) \quad (x + z) \quad (y + z)$

(100) (

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3) $A\bar{C} + B$

$= A\bar{C}(B+\bar{B}) + B\,(A+\bar{A})\,(C+\bar{C})$

$A\bar{C}(B+\bar{B}) = AB\bar{C} + A\bar{B}\bar{C}$

$B(A+\bar{A})(C+\bar{C}) = (AC + A\bar{C} + \bar{A}C + \bar{A}\bar{C})\,B$

$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$.

$A\bar{C} + B = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C}$.

$= 111 + 110 + 011 + 010 + 100$

$= m_7 + m_6 + m_3 + m_2 + m_4$

4) $AD + ABC$.

$AD(B+\bar{B})\,(C+\bar{C})$

$= AD\,(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C})$

$= \underline{ABCD} + AB\bar{C}D + \underline{A\bar{B}CD} + A\bar{B}\bar{C}D$.

$ABC(D+\bar{D}) = \underline{ABCD} + ABC\bar{D}$

$AD + ABC = ABCD + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D + ABC\bar{D}$

$= 1111 + 1101 + 1011 + 1001 + 1110$

$= m_{15} + m_{13} + m_{11} + m_9 + m_{14}$

maxterm.

5

$F(A,B,C) = (A+B')(B+C)(A+C)$ obtain the product of sum / max-term form following th

$= (A+B'+CC')(AA'+B+C)(A+C+BB')$

$= (A+B'+C)(A+B+C')(B+C+A)(B+C+A')$

$(A+C+B)(A+C+B')$

$= (A+B'+C)(A+B+C')(A+B+C)(A'+B+C)$

$= \overset{421}{(010)}\ \overset{421}{(001)}\ \overset{421}{(000)}\ \overset{401}{(100)}$

$= \Pi(2,1,0,4)$

$= \Pi(0,1,2,4)$

mostly used distributive property

$F(A,B,C) = (A+B'C)$

$= (A+B')(A+C)$

$= (A+B'+CC')(A+C+BB')$

$= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)$

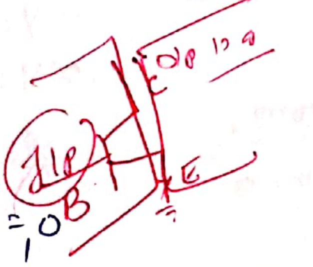
$= (A+B'+C)(A+B'+C')(A+B+C)$

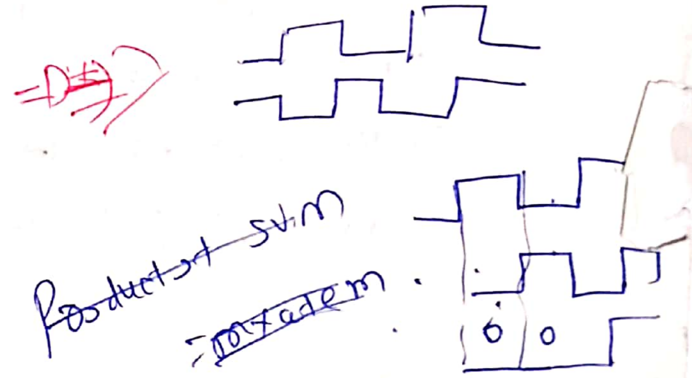
$= \overset{421}{(010)}\ \overset{421}{(011)}\ \overset{420}{(000)}$

$= \Pi(2,3,0)$

-nlariable

open collec
o/p





→ if complement = 0B
minterm

→ A product term which contain variables of the function either complemented or uncomplemented for is called minterm.

→ n variable to have $2^n$ minterm.

→ It is denoted as $m_0, m_1, m_2$.

Product of sum
=maxterm.

Sum of product
=minterm.

$m_0 = \bar{A}\bar{B}\bar{C}$

$m_1 = \bar{A}\bar{B}C$

$m_2 = \bar{A}B\bar{C}$

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→ Express the following $f^n$ as sum of minterm and product of max-term.

$F(A,B,C,D) = \bar{B}D + \bar{A}D + BD$

$= \bar{B}D + BD + \bar{A}D$

$= (\bar{B}+B)D + \bar{A}D$

$= \bar{A}D + D.$

$= D(1+\bar{A})$

$= D.$

$D(A+\bar{A})(B+\bar{B})(C+\bar{C})$

$\bar{B}D + \bar{A}D + BD = X\bar{B}XD + \bar{A}XXD + XBXD$

$= X0X1 + 0XX1 + X1X1$

$= 0001,$
$0011,$
$1001,$
$1011,$

(b) $(xy+z)(xz+y) = (x+z)(y+z)(y+x)(y+z)$

$= (x+y)(y+z)(x+z).$

$= (x+y+zz')(y+z+xx')(x+z+yy')$

$= (x+y+z)(x+y+z')(x+y+z)(x'+y+z)(x+z+y)(x+z+y')$

$= (x+y+z)(x+y+z')(x'+y+z)(x+z+y')$

$= (\overset{421}{000})(\overset{421}{001})(\overset{421}{100})(\overset{421}{010})$

$= \Pi m(0,1,4,2)$

$= \Pi m(0,1,2,4)$ ← max term.

$= \Sigma m(3,5,6,7)$ ← minterm.

$xyz = 111$

$(xyz)' = x'+y'+z' = 000$

$AB + C$

$AB + \bar{A}C$ . find product of maxterm 

$= (AB + \bar{A})(AB + C)$

$= (A + \bar{A})(\bar{A} + B)(A + C)(B + C)$

$= (\bar{A} + B)(B + C)(A + C)$

$= (\bar{A} + B + CC')(B + C + AA')(A + C + BB')$

$= (A' + B + C)(A' + B + C')(A' + B + C)(A + B + C)(A + B + C)(A + B' + C).$

$F = \underset{000}{(A+B+C)}\ \underset{010}{(A+B'+C)}\ \underset{100}{(A'+B+C)}\ \underset{101}{(A'+B+C')}$

$= \Pi M(0, 2, 4, 5)$

→ kmap (Karnaugh map)

→ used to simplify Boolean expression

1) Algebric 2) Graphical. method ← kmap

→ K map is $f^n$ of n variable.

2 variables ← kmap ← $2^2$ terms or $2^2$ cells are there

→ two 2 Variable kmap ← $2^n = 2^2 = 4$ cells, 4 minterms.

→ Each cells represent unique minterm

4 2 1
0 1

| minterm | I/P | O/P |
|---|---|---|
| 0 | A B 0 0 | 1 |
| 1 | $\bar{A}B = 01$ | 0 |
| 2 | $A\bar{B} = 10$ | 1 |
| 3 | $AB = 11$ | 1 |

← presence or absence of minterm in particular expression.

$= \bar{A}\bar{B} + A\bar{B} + AB$   This is only two SOP.

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $\bar{A}\bar{B}$ (0) | $\bar{A}B$ (1) |
| 1 | $A\bar{B}$ (2) | $AB$ (3) |

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1. |

$\Sigma m(0, 2, 3)$.

$f = \bar{A}B + A\hat{B}$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

$x'y + xy' + xy$

| x \ y | 0 | 1 |
|---|---|---|
| 0 | | 1 |
| 1 | 1 | 1 |

expand $A(\bar{B}+A)B$ to maxterm and minterm

$- A(\bar{B}+A)B$

$A+B\bar{B} = (A+B)(A+\bar{B})$

$(\bar{B}+A) = A+\bar{B}$

$B = \cancel{B+(A\bar{A})} = B+A\bar{A}$

$= (\bar{A}+B)(A+B)$

$(A+B)(A+\bar{B})(\bar{A}+B)$

$(\overset{0}{0}\overset{1}{0})(\overset{0}{0}\overset{1}{1})(\overset{0}{1}\overset{1}{0})$

$\Pi m(0,1,2)$

$\Sigma m(3)$

$A - AX - 0X = (00)(01)$

$(\bar{B}+A) = (10)$

$B - XB - X0 = (10)(00)$

$(00)(01)(10) \quad \Pi m(0,1,2)$

Scale :

Expand $A(\bar{A}+B)(\bar{A}+B+\bar{C})$.

$A \rightarrow \quad Axx \rightarrow 0xx$

$\rightarrow$ 0 0 0
0 1 0
0 1 1
0 0 1

$\bar{A}+B+X \rightarrow$ 1 0 X $= (101)\ (100)$

$\bar{A}+B+\bar{C} \rightarrow$ 1 0 1

$\begin{array}{c}421\\(001)\end{array}$ $\begin{array}{c}421\\(010)\end{array}$ $\begin{array}{c}421\\(011)\end{array}$ $\begin{array}{c}421\\(001)\end{array}$ $\begin{array}{c}421\\(101)\end{array}$ $\begin{array}{c}421\\(100)\end{array}$

$\Pi m\,(1, 2, 3, 1, 5, 4)$

$\Pi m\ 0,1,2,3,4,5)$

$\Sigma m\,(6, 7, 8)$

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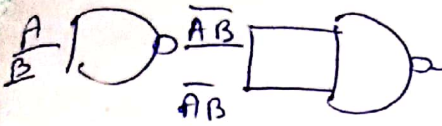
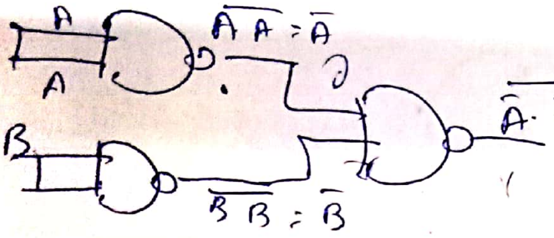
# NAND Gate & NOR gate as Universal gate.


$\overline{A \cdot A} = \bar{A}$ NOT gate



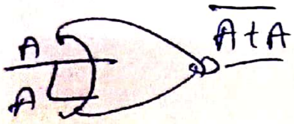

$\overline{\overline{AB} \cdot \overline{AB}} = \overline{\overline{AB}} = AB$ AND gate.



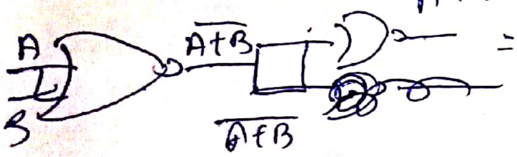

$\overline{\bar{A} \cdot \bar{B}} = A + B$ : OR gate

---

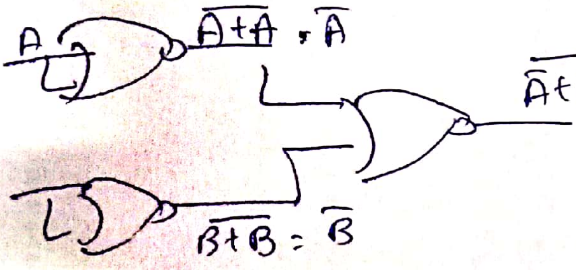
NOR



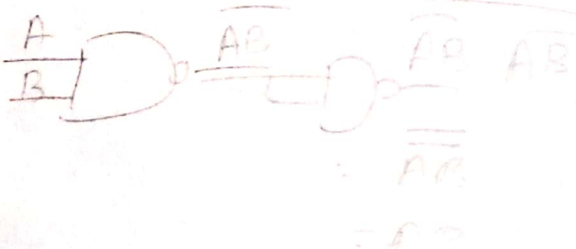

$\overline{A+A} = \bar{A}$ NOT gate



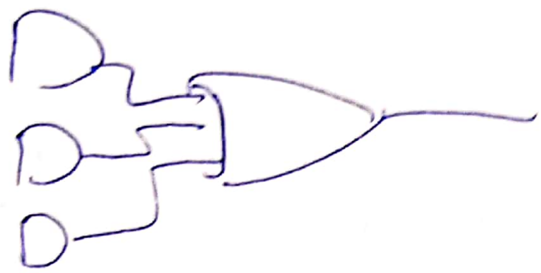

$\overline{\overline{A+B} + \overline{A+B}} = \overline{\overline{A+B}} = A + B$ → OR gate



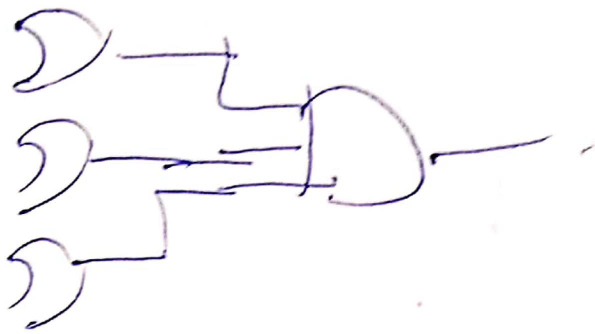

$\overline{\bar{A} + \bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = AB$ → AND gate.



SOP ← sum of product



pos →



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Kmap - Karnaugh map. **Two Variable** (1)

→ Boolean fn can be simplified by two ways
1) Algebraic method 2) Graphical method (Kmap)

→ K-map → fn of n variables

2 variable Kmap → $2^n = 2^2 = 4$ minterm or 4 squares.
$2^n = 2^3 = 8$ minterm or 8 squares
$2^n = 2^4 = 16$ minter or 16 squares

→ Each squares represents minterm.

* 2 Variable K-map

| minterm | I/P combination | O/P |
|---|---|---|
| 0 | 0 0 = $\bar{A}\bar{B}$ | 1 |
| 1 | 0 1 = $\bar{A}B$ | 0 |
| 2 | 1 0 = $AB$ | 1 |
| 3 | 1 1 = $AB$ | 1 |

→ O/P = 1 indicates that that I/P combination or minterm are present in Boolean Expression

| | B=0 | B=1 |
|---|---|---|
| A=0 | $\bar{A}\bar{B}$ | $\bar{A}B$ |
| A=1 | $A\bar{B}$ | $AB$ |

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 $m_0$ | 1 $m_1$ |
| 1 | 2 $m_2$ | 3 $m_3$ |

→ for example $F = \bar{A}\bar{B} + A\bar{B} + AB$ or $\Sigma m = (0, 1, 3)$ Reduce the expression

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | |
| 1 | 1 | 1 |

Kmap or Reduce expression. $\bar{A}B + A\bar{B}$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | | 1 |
| 1 | 1 | |

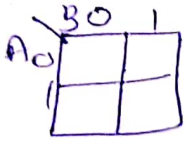
→ map or Reduce $x'y + xy' + xy$

| x \ y | 0 | 1 |
|---|---|---|
| 0 | | 1 |
| 1 | 1 | 1 |

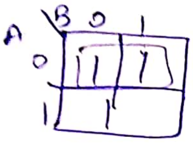
## * Minimize SOP Expression :-

Ex-1 $\bar{A}\bar{B} + \bar{A}B$.

Step-1. $2^n : 2^2 =$ 4 square → It is 2 variable K map



Step-2. Enter 1 in the cells corresponding minterm presents in the expression

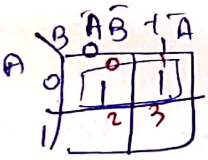


Step 3 Two squares are said to be adjacent if their minterm are differ in only one variable.

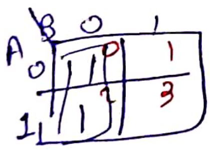
Step-4. or. After putting 1, consider only those variable which remain constant throught the square. then write it

Step-5 The variable which remain constant 1, write non complemented variable

Step-6. The variable which remain constant 0 write complemented variable.

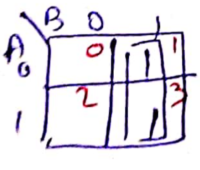
→  $\bar{A}\bar{B} + \bar{A}B$ Ans: $\bar{A}$. $\Sigma m(0,1)$

→ We have to look forward for adjant square having 1's
→ that means minterms are adjacent to each other.
→ combine them to large square.
→ Two squares are said to be adjacent, if their minterm are differ in only one variable.

Ex-2. 

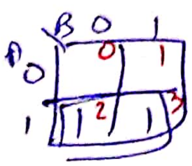
$\bar{A}\bar{B} + A\bar{B}$

Ans : $\bar{B}$ $\Sigma m(0,2)$

Ex-3 

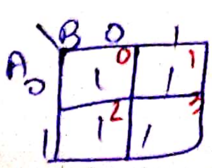
$\Sigma m(1,3)$ $\bar{A}B + AB$.

Ans: B.

Ex-4 

$A\bar{B} + AB$

Ans: A. $\Sigma m(2,3)$

Ex-5 

$AB + \bar{A}B + A\bar{B} + AB$ $\Sigma m(0,1,2,3)$

= 1

no variable remain constant.

Kmap - Karnaugh map. Two Variable (1)



Ex 6 Reduce the expression $F = \Sigma m(0,1,3)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | | 3 |

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | | 1 |

$= \bar{A} + B$. or

| A | B | X |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$\bar{A}\bar{B} + \bar{A}B + AB$

$\bar{A}(B+\bar{B}) + AB$

$\bar{A} + AB$

$(A+\bar{A})(\bar{A}+B)$

$= \bar{A}+B$.

Ex 7. $F = \Sigma m(0,2,3)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | |

$= \cancel{\bar{A} + \bar{B}}$.

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | |
| 1 | 1 | 1 |

$= A + \bar{B}$.

$\bar{A}\bar{B} + A\bar{B} + AB$

$\bar{B}(A+\bar{A}) + AB$

$(\bar{B}+A)(B+\bar{B})$

$= A+\bar{B}$.

Ex 8 $F = \Sigma m(0,1,2)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | |

$= \bar{A} + \bar{B}$.

$\bar{A}\bar{B} + \bar{A}B + A\bar{B}$

$\bar{B}(A+\bar{A}) + \bar{A}B$

$= (\bar{B} + \bar{A}B)$ $\quad (\bar{B}+\bar{A})(B+\bar{B})$.

* Kmap for pos expression :- (mapping of pos expression).

- Each sum term in standard Pos is called maxterm.
- Sop for two variable has four possible conditions

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $A+B$ | $A+\bar{B}$ |
| 1 | $\bar{A}+B$ | $\bar{A}+\bar{B}$ |

$A+B$, $\bar{A}+B$, $A+\bar{B}$, $\bar{A}+\bar{B}$

0 0   1 0   0 1   1 1

- maxterm obtained by non complemented variable ← 0, complemented variable ← 1.

→ mapping of pos ← minimization of. pos.

0 are placed in the squares corresponding to maxterm are present in Boolean expressions.

P/M $(A+B)$ $(\bar{A}+B)$ $(\bar{A}+B)$
$\quad 0\,0 \quad\quad 1\,0 \quad\quad 1\,1$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | 0 |

Ex→2 $(A+B)$ $(A+\bar{B})$ $(\bar{A}+\bar{B})$
$\quad 0\,0 \quad\quad 0\,1 \quad\quad 1\,1$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | | 0 |

$\Pi M(0,1,3)$

$= \cancel{A\,\bar{A}\bar{B}} = A\bar{B}.$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $A+B$ | $A+\bar{B}$ |
| 1 | $\bar{A}+B$ | $\bar{A}+\bar{B}$ |

Ex→3

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | | |

$A/0$

$(A+B)\ (A+\bar{B})$

Ans $= \bar{A}$ $\cdot$ $\Pi M(0,1)$

Ex→4

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | | 0 |

$(A+B)\ (\bar{A}+\bar{B})$

$= \bar{B}.$ $\quad \Pi M(1,3)$ $\quad \frac{B}{1}$

Ex→5

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |

$(A+B)\ (\bar{A}+B)$ $\quad \Pi M(0,2)$ $\quad B=0.$

$= B.$

Ex→6

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | 0 |

$(\bar{A}+B)\ (\bar{A}+\bar{B})$

$= \bar{A}$ $\quad \Pi M(2,3)$

Value = 1
complemented variable
Value = 0
non complemented variable

Ex→7

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

$(A+B)\ (\bar{A}+B)\ (A+\bar{B})\ (\bar{A}+\bar{B})$.

$= 0.$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $m_0$ | $m_1$ |
| 1 | $m_2$ | $m_3$ |

$\Pi(0,1,2)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | ● |

$= \cancel{AB+B} = AB$

# Three Variable K-map.

## SOP form

| | |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

Eight possible condition

| A | B | C | SOP form | POS form |
|---|---|---|---|---|
| 0 | 0 | 0 | $= \bar{A}\bar{B}\bar{C}$ | $A+B+C$ |
| 0 | 0 | 1 | $= \bar{A}\bar{B}C$ | $A+B+\bar{C}$ |
| 0 | 1 | 0 | $= \bar{A}B\bar{C}$ | $A+\bar{B}+C$ |
| 0 | 1 | 1 | $= \bar{A}BC$ | $A+\bar{B}+\bar{C}$ |
| 1 | 0 | 0 | $= A\bar{B}\bar{C}$ | $\bar{A}+B+C$ |
| 1 | 0 | 1 | $= A\bar{B}C$ | $\bar{A}+B+\bar{C}$ |
| 1 | 1 | 0 | $= AB\bar{C}$ | $\bar{A}+\bar{B}+C$ |
| 1 | 1 | 1 | $= ABC$ | $\bar{A}+\bar{B}+\bar{C}$ |

POS form.

Minterm SOP form

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $\bar{A}\bar{B}\bar{C}$ (0) | $\bar{A}\bar{B}C$ (1) | $\bar{A}BC$ (3) | $\bar{A}B\bar{C}$ (2) |
| 1 | $A\bar{B}\bar{C}$ (4) | $A\bar{B}C$ (5) | $ABC$ (7) | $AB\bar{C}$ (6) |

Maxterm in POS form.

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $A+B+C$ | $A+B+\bar{C}$ | $A+\bar{B}+\bar{C}$ | $A+\bar{B}+C$ |
| 1 | $\bar{A}+B+C$ | $\bar{A}+B+\bar{C}$ | $\bar{A}+\bar{B}+\bar{C}$ | $\bar{A}+\bar{B}+C$ |

* Map the Expression of ABI

1) Step-I – look at the 1s present on map which means minterms are present in Boolean Expression.

2) Find the minterm adjacent to each other, in order to combine them into larger square.

3) Some of minterms have many adjacency. Always start with the minterm with least number of adjacencies. Always try to form large square as possible.

4) Next consider minterm with next least number of adjacencies.

Examples: → 1) $\bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | (3) | 1 (2) |
| 1 | 1 (4) | (5) | (7) | (6) |

$= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

$\Sigma m(0,1,2,4)$

Consider only those variable which remain constant

2) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$ (1 0 1)

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | 1 (3) | (2) |
| 1 | (4) | 1 (5) | (7) | (6) |

$: \bar{A}\bar{B} + \bar{B}C + \bar{A}C$

$\Sigma m(0,1,3,5)$

3) $\bar A\bar B\bar C + \bar A\bar B C + \bar A B\bar C + A\bar B\bar C + A\bar B C + AB\bar C$

$\quad 010 \qquad 101 \qquad 110$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | (3) | 1 (2) |
| 1 | 1 (4) | 1 (5) | (7) | 1 (6) |

$= \bar B + \bar C$

$\Sigma m(0,1,2,4,5,6)$

4) $\bar A\bar B\bar C + \bar A\bar B C + \bar A B C + A\bar B\bar C + A\bar B C + ABC$.

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | 1 (3) | (2) |
| 1 | 1 (4) | 1 (5) | 1 (7) | (6) |

$= \bar B + C$.

000
001
100
101

$\Sigma m(0,1,3,4,5,7)$

5) $\cancel{\bar A\bar B\bar C} + \cancel{\bar A\bar B C} + \cancel{\bar A B\bar C} + \cancel{\bar A B\bar C} = \bar A\bar B\bar C + \bar A\bar B C + \bar A BC + \bar A B\bar C$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | 1 (3) | 1 (2) |
| 1 | (4) | (5) | (7) | (6) |

$= \bar A$ $\quad \Sigma m(0,1,2,3)$

6) All ← Ans = 1

7) $\Sigma m(0,2,3,4,5,6)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 (0) | (1) | 1 (3) | 1 (2) |
| 1 | 1 (4) | 1 (5) | (7) | 1 (6) |

$\bar C + \cancel{\bar A B} + \cancel{B\bar C}$.

$= \bar C + \bar A B + A\bar B$

8) $\Sigma m(1,2,4,6,7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | 1 |
| 1 | 1 | | 1 | 1 |

$= \bar A\bar B C + A\bar C + AB + B\bar C$

POS form ← Product of sum.



$\Pi m(0, 2, 3, 5, 6, 7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 0 | 0 | 0 |
| 1 | | 0 | 0 | 0 |

Ex → $(A+B+\bar{C})\ (A+\bar{B}+\bar{C})\ (A+\bar{B}+C)$
$(\bar{A}+B+\bar{C})\ (\bar{A}+\bar{B}+\bar{C})\ (\bar{A}+\bar{B}+C)$

$= (\bar{B})(\bar{C})$

Put complemented variable if its value remains constant 1

complemented value = 1
Non complemented = 0

Ex → 2

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | | 0 |
| 1 | 0 | | 0 | 0 |

Ex → $(A+\bar{B}+C)\ (\bar{A}+\bar{B}+C)\ (\bar{A}+\bar{B}+\bar{C})$
$(\bar{A}+B+C)$. $\Pi m(2, 4, 6, 7)$

$= (\bar{B}+C)\ (\bar{A}+\bar{B})\ (\bar{A}+C)$
B=1 C=0 A=1 B=1 A=1 C=0

$\Pi m(2, 4, 6, 7)$

Ex → 3 $\Pi m(0, 1, 2, 3, 4, 7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | | 0 | |

$= (A)\ (\bar{B}+\bar{C})\ (B+C)$.

Ex → 4 $\Pi m(0, 3, 5)$

| A \ BC | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 0 | 0 | | 0 | |
| 1 | | 0 | | |

$= (A+B+C)\ (A+\bar{B}+C)\ (\bar{A}+B+\bar{C})$.

# Four Variable K-map.

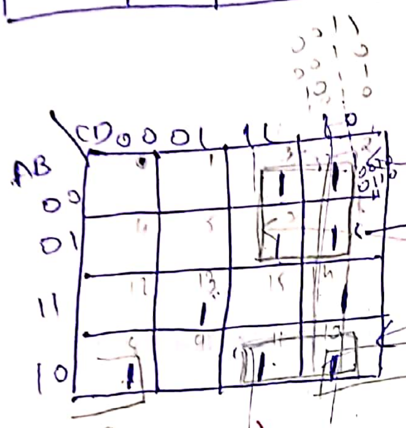
$2^4 = 16$ possible conditions

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 $A'B'C'D'$ | 1 $A'B'C'D$ | 3 $A'B'CD$ | 2 $A'B'CD'$ |
| 01 | 4 $A'BC'D'$ | 5 $A'BC'D$ | 7 $A'BCD$ | 6 $A'BCD'$ |
| 11 | 12 $ABC'D'$ | 13 $ABC'D$ | 15 $ABCD$ | 14 $ABCD'$ |
| 10 | 8 $AB'C'D'$ | 9 $AB'C'D$ | 11 $AB'CD$ | 10 $AB'CD'$ |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $A+B+C+D$ | $A+B+C+D'$ | $A+B+C'+D'$ | $A+B+C'+D$ |
| 01 | $A+B'+C+D$ | $A+B'+C+D'$ | $A+B'+C'+D'$ | $A+B'+C'+D$ |
| 11 | $A'+B'+C+D$ | $A'+B'+C+D'$ | $A'+B'+C'+D'$ | $A'+B'+C'+D$ |
| 10 | $A'+B+C+D$ | $A'+B+C+D'$ | $A'+B+C'+D'$ | $A'+B+C'+D$ |

SOP
Sum of product Kmap

Ex-1



→ Start with least number of adjacencies.

$C\bar{D} + \bar{A}C + \bar{B}C + A\bar{B}\bar{D} + AB\bar{C}D.$

$\Sigma m(2, 3, 6, 7, 8, 10, 11, 13, 14)$

$\Sigma m(2, 3, 6, 7, 8, 10, 11, 13, 14)$

8421
1010

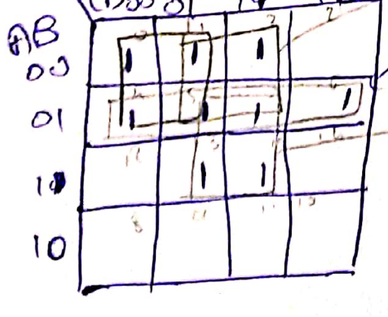
Ex-2 $\Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$

$\bar{A}D + A\bar{C} + \bar{B}\bar{D}$

$\Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 10)$

$\bar{A}\bar{B} + \bar{A}D + A\bar{C}$

Ex-3 $\Sigma m(0, 1, 3, 4, 5, 6, 7, 13, 15)$



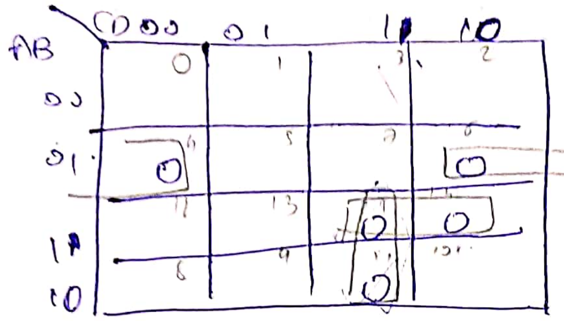
$\bar{A}B + \bar{A}D + BD + \bar{A}\bar{C}$

1, 1, 2, 3, 1, 4, 6, 1, 3, 7, 1, 60

pos ← Product of Sum.

Ex→1 $\Pi m (4, 6, 11, 14, 15)$

11




$= (\bar{A} + \bar{B} + \bar{C}) \; (\bar{A} + \bar{C} + \bar{D})$

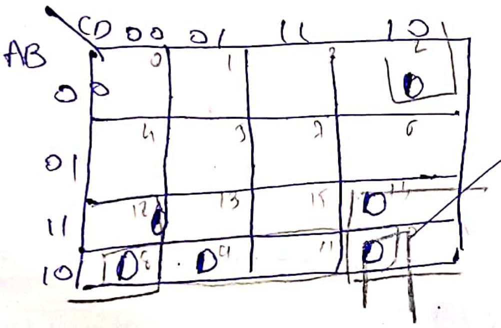
$(A + \bar{B} + D)$

Ex→2 $\Pi m (2, 8, 9, 10, 12, 14)$

1000
1

1 1 0 0
1 0 0 0
1 1 1 0
1 0 1 0

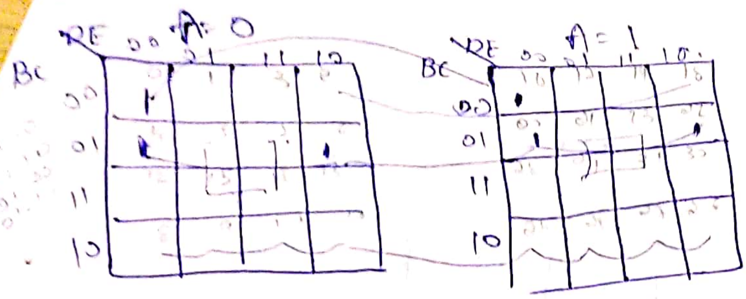
0 0 1 0
1 0 1 0




$(\bar{A} + B) \; (\bar{A} + D) \; (B + \bar{C} + D)$

# Five Variable K-map.





$2^5 = 32$ possible conditions

$m0, m16 \leftarrow \bar{B}\bar{C}\bar{D}\bar{E}$ $\quad M0, M16 = B + C + D + E$.

$m2, m18 \leftarrow \bar{B}\bar{C}D\bar{E}$

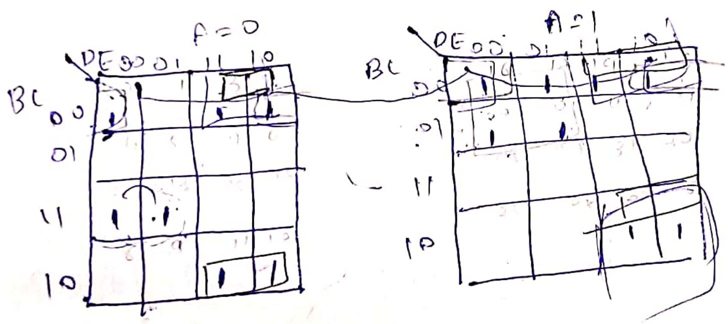
$m4, m6, m20, m22 \leftarrow \bar{B}C\bar{E}$

$m5, m7, m13, m15, m21, m23, m29, m31 - CE$

$m8, m9, m10, m11, m24, m25, m26, m27 = B\bar{C}$.

* $\Sigma m(0, 2, 3, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27)$

SOP form



$m12, m13 \leftarrow \bar{A}BC\bar{D}$

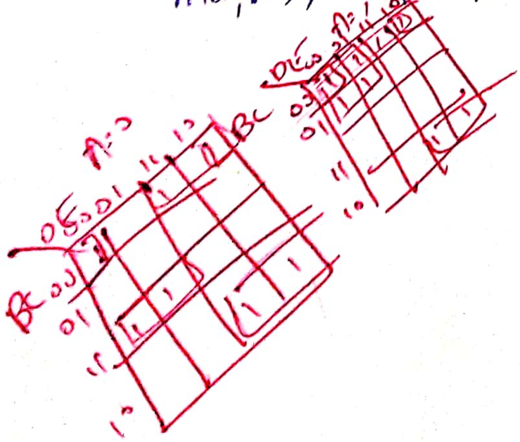
$2^5 = 32$

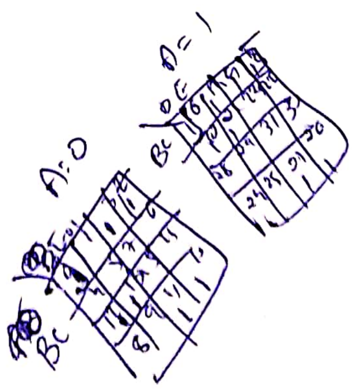
$m0, m2, m16, m18 \leftarrow \bar{B}\bar{C}\bar{E}$

$m16, m17, m20, m21 \leftarrow$

$m20, m21, m16, m17 \leftarrow A\bar{B}\bar{D}$.

$m2, m3, m18, m19, m10, m11, m26, m27 \rightarrow \bar{C}D$.

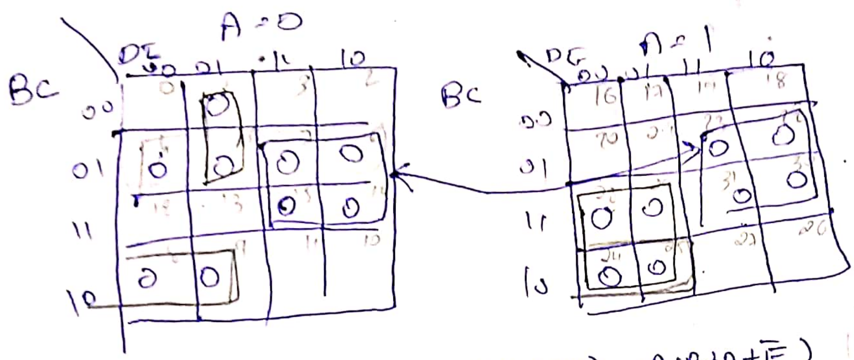




BC

POS form:-

$\Pi M(1, 4, 5, 6, 7, 8, 9, 1\ldots, 15, 22, 23, 24, 25, 28, 29, 30, 31)$




$m_1, m_5 \leftarrow (\cancel{\bar{A} + B + D + E})$ $(A+B+D+\bar{E})$

$m_4, m_5, m_6, m_7 \leftarrow A + B + \bar{C}$ ✓

$m_8, m_9, m_{24}, m_{25} \leftarrow \bar{B} + C + D)$

$m_{28}, m_{29}, m_{24}, m_{25} \leftarrow \bar{A} + \bar{B} + D.$

$m_{30}, m_{31}, m_{29}, m_{28}$ X ← Don't do that.

$\left.\begin{matrix} m_6, m_7, m_{14}, m_{15} \\ m_{22}, m_{23}, m_{30}, m_{3} \end{matrix}\right\} \leftarrow \bar{C} + \bar{D}.$

✓

$\Sigma m(0, 1, 2, 3, 8, 9, 16, 17, 20, 21, 24, 25, 28, 29, 30, 31)$

$F = A\bar{D} + \bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + ABC$

# Don't care condition.

While dealing with BCD [Binary coded Decimal] numbers encoded as four bits. We may not care about any codes above the BCD range of (0 to 9).

-> 4 bit binary codes for the Hexadecimal numbers (Ah, Bh, Ch, Eh, Fh) are not valid BCD codes.

-> So we would not normally care to fill in those codes because those code will never exits so it is called Don't care condition. (1010, 1011, 1100, 1101, 1110, 1111)

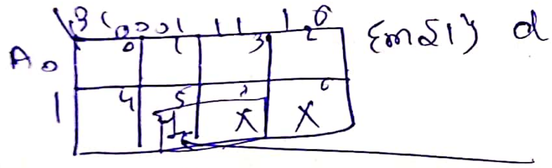
-> Don't care in K-map may be either 1's or 0's. As long as we don't care

-> When forming group of cells, Treat the don't care cells as either 0 or 1 or ignore it.

-> This is helpful if it is allow us to form a larger group.

-> SOP form.

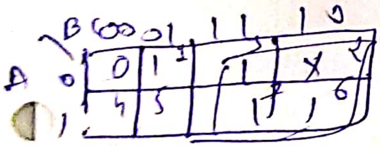
$\Sigma m(5)$ $d(6,7)$



$= AC$

if you will remove although don't care

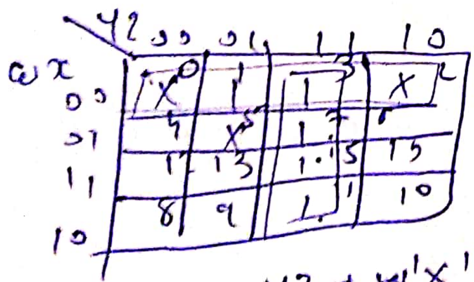
Ans: $A\bar{B}C$



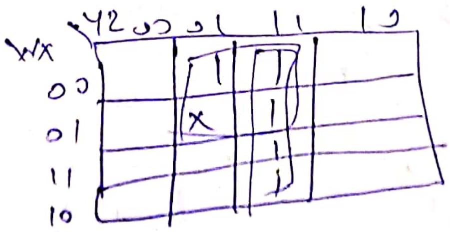
Ans = $B$

$\Sigma m(5,6,7)$ $d(2)$

-> $F(w,x,y,z) = \Sigma(1,3,7,11,15)$, $dc(w,x,y,z) = \Sigma(0,2,5)$



$F = yz + w'x'$

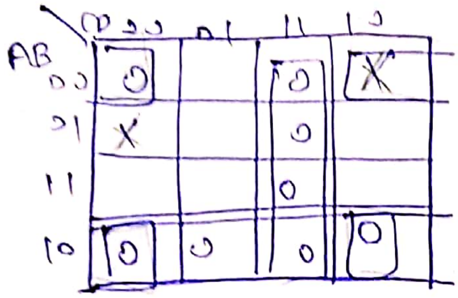


$= yz + w'z$

pos form

po

$\Pi m\,(0,3,7,8,9,10,11,12) \cdot d(2,4)$



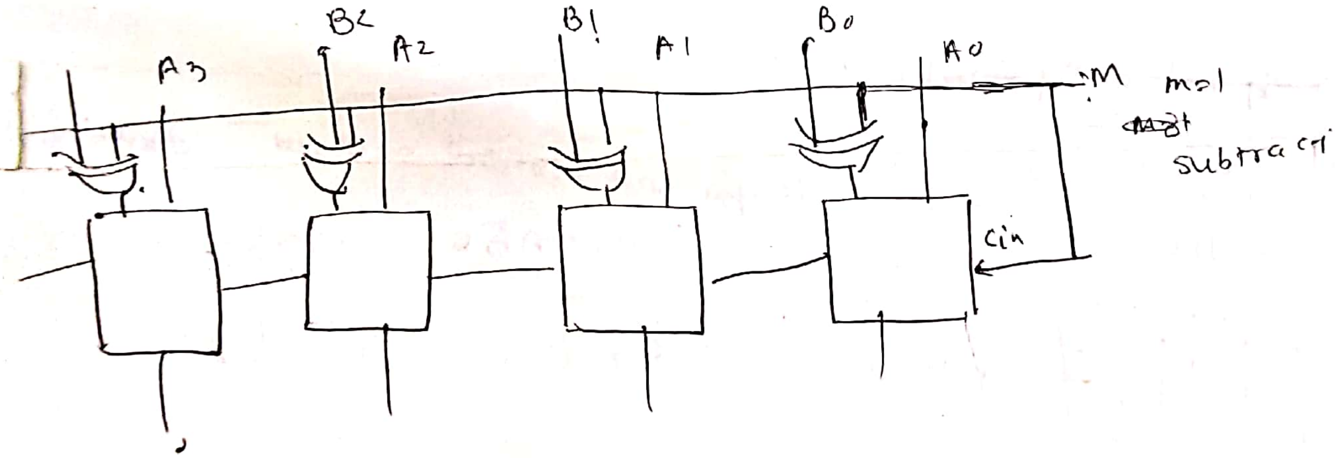

26 →

$(\bar{C}+\bar{D})\,(\bar{A}+B)\,(B+D)$

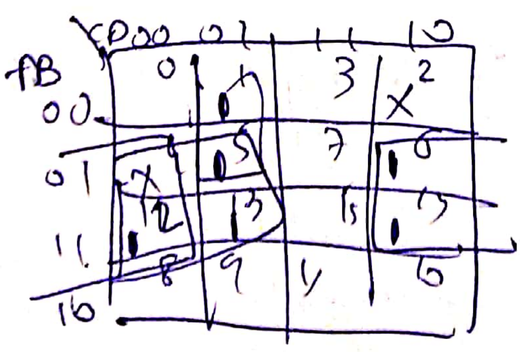
Binary adder subtractor

m = 0 adder

M = 1 Subtractor




$\Sigma m\,(1,5,6,12,13,14) + d(2,4)$




$B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$