Multiple Linear Regression by Hand (Step-by-Step)

Suppose we have the following dataset with one response variable y and two predictor variables X_1 and X_2:

y	X_1	X_2
140	60	22
155	62	25
159	67	24
179	70	20
192	71	15
200	72	14
212	75	14
215	78	11

The estimated linear regression equation/Hypothesis is:

$$\underline{\hat{y} = b_0 + b_1 * x_1 + b_2 * x_2}$$

Where

b_0 is: $y - b_1X_1 - b_2X_2$

for MLR, b_1 is: $[(\Sigma x_2^2)(\Sigma x_1 y) - (\Sigma x_1 x_2)(\Sigma x_2 y)] / [(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2]$

for MLR The formula to calculate b_2 is: $[(\Sigma x_1^2)(\Sigma x_2 y) - (\Sigma x_1 x_2)(\Sigma x_1 y)] / [(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2]$

Please note: in b1and b2 the Σx_1^2 ,Σx_2^2, $\Sigma x_1 y$,$\Sigma x_2 y$,$\Sigma x_1 x_2$ are Regression Sums and calculated from the following formulas :

- $\Sigma x_1^2 = \Sigma \mathbf{X_1^2} - (\mathbf{\Sigma X_1})^2 / n$
- $\Sigma x_2^2 = \Sigma \mathbf{X_2^2} - (\mathbf{\Sigma X_2})^2 / n$
- $\Sigma x_1 y = \Sigma \mathbf{X_1 y} - (\mathbf{\Sigma X_1 \Sigma y}) / n$
- $\Sigma x_2 y = \Sigma \mathbf{X_2 y} - (\mathbf{\Sigma X_2 \Sigma y}) / n$
- $\Sigma x_1 x_2 = \Sigma \mathbf{X_1 X_2} - (\mathbf{\Sigma X_1 \Sigma X_2}) / n$

Use the following steps to fit a multiple linear regression model to this dataset.

Step 1: Calculate X_1^2, X_2^2, X_1y, X_2y and X_1X_2 and their cumulative sum.

	y	X_1	X_2
	140	60	22
	155	62	25
	159	67	24
	179	70	20
	192	71	15
	200	72	14
	212	75	14
	215	78	11
Mean	181.5	69.375	18.125
Sum	1452	555	145

	X_1^2	X_2^2	X_1y	X_2y	X_1X_2
	3600	484	8400	3080	1320
	3844	625	9610	3875	1550
	4489	576	10653	3816	1608
	4900	400	12530	3580	1400
	5041	225	13632	2880	1065
	5184	196	14400	2800	1008
	5625	196	15900	2968	1050
	6084	121	16770	2365	858
Sum	38767	2823	101895	25364	9859

Step 2: Calculate Regression Sums.

Next, make the following regression sum calculations:

- $\Sigma x_1^2 = \Sigma X_1^2 – (\Sigma X_1)^2 / n = 38{,}767 – (555)^2 / 8 = \mathbf{263.875}$
- $\Sigma x_2^2 = \Sigma X_2^2 – (\Sigma X_2)^2 / n = 2{,}823 – (145)^2 / 8 = \mathbf{194.875}$
- $\Sigma x_1y = \Sigma X_1y – (\Sigma X_1\Sigma y) / n = 101{,}895 – (555*1{,}452) / 8 = \mathbf{1{,}162.5}$
- $\Sigma x_2y = \Sigma X_2y – (\Sigma X_2\Sigma y) / n = 25{,}364 – (145*1{,}452) / 8 = \mathbf{-953.5}$
- $\Sigma x_1x_2 = \Sigma X_1X_2 – (\Sigma X_1\Sigma X_2) / n = 9{,}859 – (555*145) / 8 = \mathbf{-200.375}$

	y	X_1	X_2
	140	60	22
	155	62	25
	159	67	24
	179	70	20
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	200	72	14
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	215	78	11
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	X_1^2	X_2^2	X_1y	X_2y	X_1X_2
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	5184	196	14400	2800	1008
	5625	196	15900	2968	1050
	6084	121	16770	2365	858
Sum	38767	2823	101895	25364	9859
Reg Sums	263.875	194.875	1162.5	-953.5	-200.375

Step 3: Calculate b_0, b_1, and b_2.

The formula to calculate b_1 is: $[(\Sigma x_2^2)(\Sigma x_1 y) - (\Sigma x_1 x_2)(\Sigma x_2 y)] / [(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2]$

Thus, $\mathbf{b_1} = [(194.875)(1162.5) - (-200.375)(-953.5)] / [(263.875)(194.875) - (-200.375)^2] = \mathbf{3.148}$

The formula to calculate b_2 is: $[(\Sigma x_1^2)(\Sigma x_2 y) - (\Sigma x_1 x_2)(\Sigma x_1 y)] / [(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2]$

Thus, $\mathbf{b_2} = [(263.875)(-953.5) - (-200.375)(1152.5)] / [(263.875)(194.875) - (-200.375)^2] = \mathbf{-1.656}$

The formula to calculate b_0 is: $y - b_1X_1 - b_2X_2$

Thus, $\mathbf{b_0} = 181.5 - 3.148(69.375) - (-1.656)(18.125) = \mathbf{-6.867}$

Step 5: Place b_0, b_1, and b_2 in the estimated linear regression equation.

The estimated linear regression equation is: $\hat{y} = b_0 + b_1{*}x_1 + b_2{*}x_2$

In our example, it is $\mathbf{\hat{y} = -6.867 + 3.148x_1 - 1.656x_2}$

How to Interpret a Multiple Linear Regression Equation

Here is how to interpret this estimated linear regression equation: $\hat{y} = -6.867 + 3.148x_1 - 1.656x_2$

b_0 = -6.867. When both predictor variables are equal to zero, the mean value for y is -6.867.

b_1 = 3.148. A one unit increase in x_1 is associated with a 3.148 unit increase in y, on average, assuming x_2 is held constant.

b_2 = -1.656. A one unit increase in x_2 is associated with a 1.656 unit decrease in y, on average, assuming x_1 is held constant.