Channel Hydraulics [CEN-205]

Rapidly Varied Flow-2

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Course content

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	Total	**42**

Evaluation Procedure

- **CWS: 15%**

[attd-10; Quiz-5]

- **PRS: 20%**
- **Mid-term: 25%**
- **End-term: 40%**

S. No.	Name of Books / Authors	Year of Publication
1.	Chow, V.T., "Open Channel Hydraulics", McGraw Hill.	1959
2.	Subramanya, K., "Flow in Open Channels", Tata McGraw-Hill.	1997
3.	Ranga Raju, K.G., "Flow through Open Channels", Tata McGraw-Hill.	2003
4.	Chanson, H., "The Hydraulics of Open Channel Flow: An Introduction", Elsevier Scientific.	2004
5.	Chaudhry, M.H., "Open Channel Flow", Prentice-Hall, New Jersey, USA	1993

Open channel flow

Introduction

Rapidly varied flows (RVF)

- class of flows with high curvatures → presence of non-hydrostatic pressure distribution zones in a major part of flow.
- Local phenomenon in the sense that friction plays a minor role.
- E.g., hydraulic jump

A few basic and important RVF flow types

(i) Sharp-crested weirs
(ii) Overflow spillways
(iii) Broad-crested weirs
(iv) End depths
(v) Sluice gates
(vi) Culverts

Many of the RVFs studied here are used for flow measurement purposes

Sharp-crested Weir

Weir

a structure built across a channel to raise the level of water, with the water flowing over it.

Sharp-crested weir (notch or a thin plate weir)

- If the water surface, while passing over the weir, separates at the upstream end and the separated surface jumps clear off its thickness.
- Extensively used as a fairly precise flow-measuring device in laboratories, industries, and irrigation practice.
- SC weirs used in practice are usually vertical metal plates with an accurately-machined upstream edge of thickness < 2 mm and a bevel of angle $> 45°$ on the downstream face edge.

Most common: Rectangular and triangular geometric shapes

(a) Rectangular Weir

Upper nappe [UN]
The water surface of the stream curves rapidly at the upstream of the weir and plunges down in a parabolic trajectory on the downstream.

Lower nappe [LN]
At the weir crest, the flow separates to have a free surface which initially jumps up to a level higher than the weir crest before plunging down.

Weir extending to the full channel width

LN encloses air-contained space at atm

Flow proceeds; some air entrained by moving water

pressure in the air pocket < atm

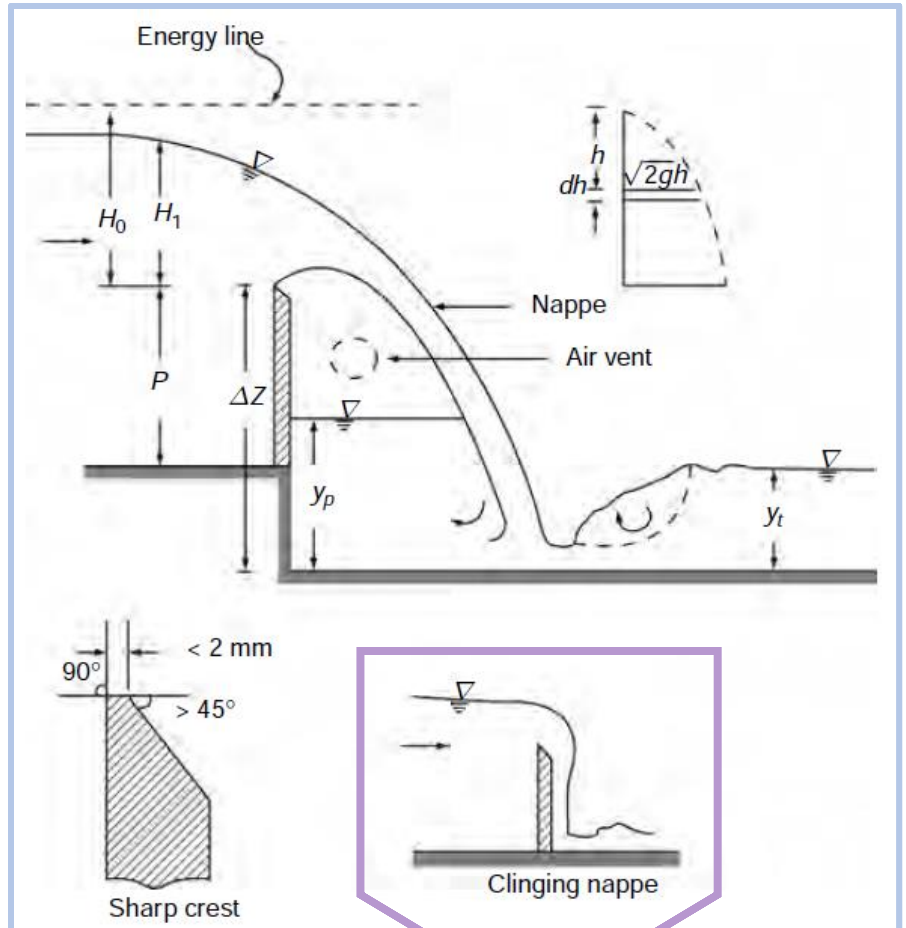
Nappe surfaces to be depressed.

This change is a progressive phenomenon.

provision of air vents

Weir flow as above assumes at tailwater level far below the crest and is termed ***free flow***

Definition sketch



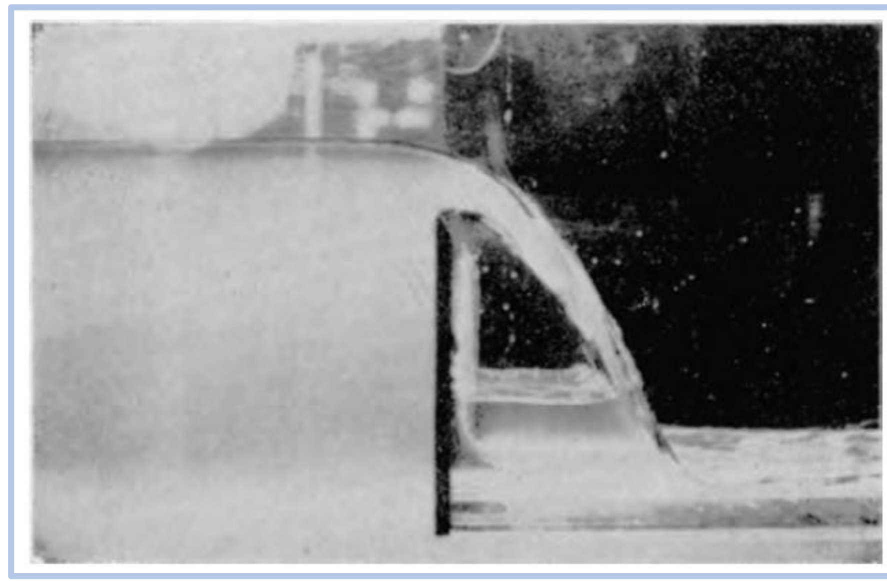
A limiting case of the air pocket completely evacuated: *clinging nappe*

(b) Discharge Equation

It is usual to derive the discharge equation for free flow over a sharp-crested weir by

- considering an ideal undeflected jet
- to apply a coefficient of contraction to account for the deflection due to the action of gravity.

Non-aerated nappe (Courtesy: M G Bos)



Fully-aerated nappe (Courtesy: M G Bos)

(b) Discharge Equation contd...

For a rectangular weir of length L spanning full-width B of a rectangular channel ($i.e., L = B$), the ideal Q through an elemental strip of thickness dh at a depth h below the energy line is given by

$$dQ_i = L\sqrt{2gh}\,dh \qquad \text{...(1)}$$

Thus the ideal discharge

$$Q_i = L\sqrt{2g}\int_{\frac{V_0^2}{2g}}^{H_1+\frac{V_0^2}{2g}} \sqrt{h}\,dh \qquad \text{...(2)}$$

and the actual discharge

$$\boxed{Q = \boxed{C_c} Q_i} \qquad \text{...(3)}$$

coefficient of contraction

$$Q = \frac{2}{3}C_c\sqrt{2g}L\left[\left(H_1 + \frac{V_0^2}{2g}\right)^{\frac{3}{2}} - \left(\frac{V_0^2}{2g}\right)^{\frac{3}{2}}\right] \qquad \text{...(4)}$$

Inconvenient to use

(b) Discharge Equation contd…

In terms of H_1: depth of flow upstream of weir measured above the weir crest

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H_1^{3/2} \qquad \text{...(5)}$$

where C_d is coefficient of discharge [takes into account the velocity of approach as]

$$C_d = C_c \left[\left(1 + \frac{V_0^2}{2gH_1}\right)^{\frac{3}{2}} - \left(\frac{V_0^2}{2gH_1}\right)^{\frac{3}{2}} \right] \qquad \text{...(6)}$$

In ideal fluid flow, $C_d = f(H_1/P)$. In real fluid flow C_d, should in general be a function of R_e and W_e, AND weir height factor H_1/P. If R_e is sufficiently large and if H_1 is sufficiently high [surface tension effects: negligible; $C_d = f(H_1/P)$

Variation of C_d, for rectangular sharp-crested weirs is given by Rehbock formula

$$\boxed{C_d = 0.611 + 0.08\ {H_1}/{P}} \qquad \text{...(7)}$$

which is valid for $H_1/P \leq 5$

(C) Sills

For very small values of P relative to H_1, i.e., for ${}^{H_1}/_{P} > 20$, the weir acts as a sill placed al the end of a horizontal channel.

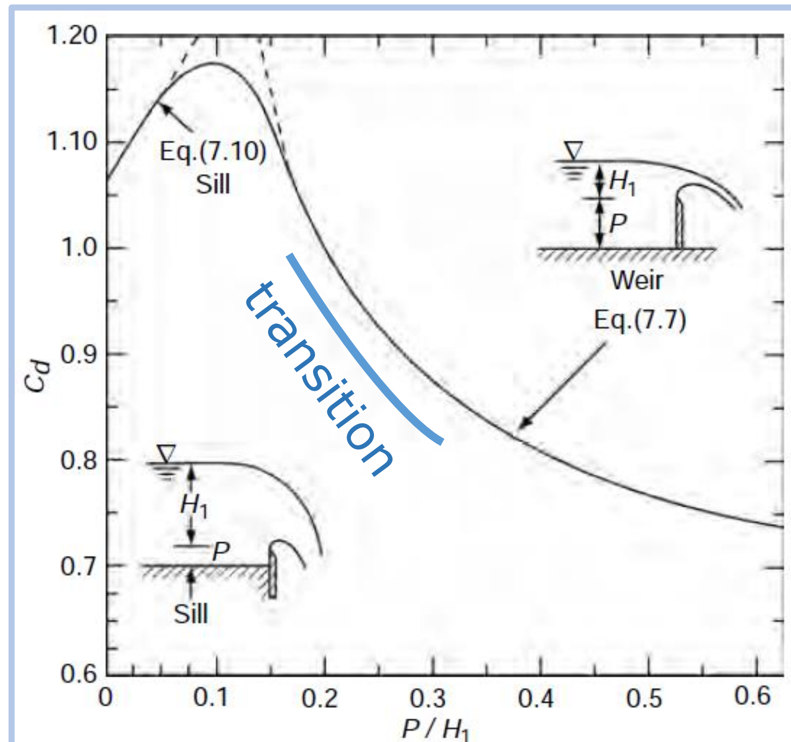
Assuming that the critical depth y_c occurs at the sill:

i.e. $$Q = B\sqrt{g}\,(H_1 + P)^{3/2} = \frac{2}{3} C_d \sqrt{2g}\, L\, H_1^{3/2} \qquad \ldots(9)$$

and $$L = B$$

From Eq. 9

$$\boxed{C_d = 1.06\left(1 + \frac{P}{H_1}\right)^{3/2}} \qquad \ldots(10)$$



Variation of C_d for weirs and sills

(C) Sills contd...

In the intermediate region of weirs and sills (i.e. $20 > {}^{H_1}/_{P} > 5$), C_d values have a smooth transition from Eq. 7 to Eq. 10. [figure]

Effect of liquid properties on C_d

- Except at very low heads, i.e. $H_1 < \sim 2$ cm, for the flow of water in rectangular channels, the effects of R_e and W_e on C_d are insignificant → Eqs. 7 & 10 can be used for estimating discharges for practical purposes.
- The head H_1 is to be measured upstream of the weir surface at a distance of $\sim 4\, H_1$ from the weir crest.
- If weirs are installed for metering purposes, the relevant standard specifications (e.g. International Standards: ISO: 1438, 1979, Thin-plate weirs) must be followed in weir settings.

(d) Submergence

Submerged flow

When the tailwater level is above the weir crest.

Submergence ratio $({}^{H_2}/_{H_1})$

H_2 = downstream water-surface elevation measured above the weir crest.

The discharge over the weir Q_s depends upon the submergence ratio:

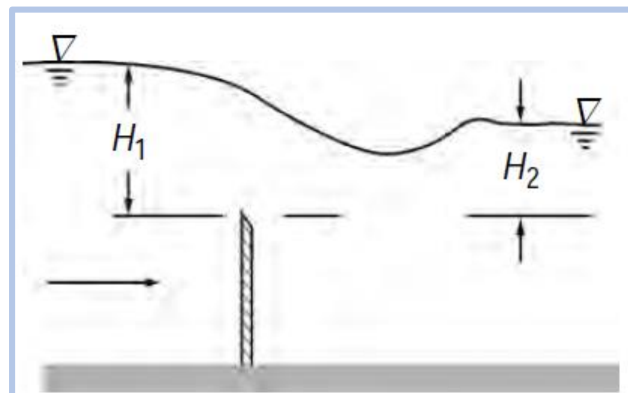
Villemonte formula

$$Q_s = Q_1[1 - \left(\frac{H_2}{H_1}\right)^n]^{0.385} \quad \text{...(11)}$$

where Q_1 =free-flow discharge under head H_1, n=exponent of head in the head-discharge relationship $Q = KH_n$.For a rectangular weir, n = 1.5.

Modular limit or submergence limit

The minimum value of $\frac{H_2}{H_1}$ at which the discharge under a given head H_1 deviates by 1% from the value determined by the free-flow equation.



Submerged sharp-crested weir

(d) Submergence contd…

- In sharp-crested rectangular weirs the modular limit is negative, i.e. the submergence effect is felt even before the tailwater reaches the crest elevation.
- Thus to ensure free-flow it is usual to specify the tailwater surface to be at least 8 cm below the weir crest for small weirs. This minimum distance will have to be larger for large weirs to account for fluctuations of the water level immediately downstream of the weir due to any wave action.

(e) Aeration Need of Rectangular Weir

The rate of air supply (Q_a in m^3/s) required to completely meet the aeration need is given by

$$\frac{Q_a}{Q} = \frac{0.1}{\left({y_p}/{H_1}\right)^{3/2}} \qquad \text{...(12)}$$

in which Q = water discharge and y_p= water-pool depth on downstream of weir plate. If a submerged hydraulic jump takes place, y_p can be estimated by the tailwater depth. On the other hand, for the case of a free jump occurring on downstream, y_p can be estimated by

$$y_p = \Delta Z \left[\frac{Q_2}{L^2 g\,(\Delta Z)^3}\right]^{0.22} \qquad \text{...(13)}$$

where ΔZ = difference in elevation b/w weir crest and the downstream floor. To cause air flow into the air pocket through an air vent, a pressure difference b/w the ambient atmosphere and the air pocket is needed. Assuming a maximum permissible negative pressure in the pocket (say 2 cm of water column), the size of the air vent can be designed by using the usual Darcy-Weisbach pipe flow equation.

Example-Solution

A $2\,m$ wide rectangular channel has a discharge of $0.35\ m^3/s$.

Find the height of a rectangular weir spanning the full width of the channel that can be used to pass this discharge while maintaining an upstream depth of $0.85\ m$.

Solution A trial-and-error procedure is required to solve for P. Assuming $C_d =$ 0.640, by Eq. 7.5

$$H_1^{3/2} = 0.350 \Big/ \left(\frac{2}{3} \times 0.640 \times \sqrt{19.62} \times 2.0 \right) = 0.0926$$

$$H_1 = 0.205 \text{ m and } P = 0.850 - 0.205 = 0.645\text{m}$$

$$H_1/P = 0.318 \text{ m and } C_d = 0.611 + (0.08 \times 0.318) = 0.636$$

2nd iteration: Using the above value of C_d

$$H_1^{3/2} = \frac{0.0926}{0.636} \times 0.640 = 0.09318$$

$$H_1 = 0.206 \text{ m}, \ P = 0.644 \text{ m}, \ H_1/P = 0.320$$

and $$C_d = 0.637$$

Accepting the value of C_d the final values are $H_1 = 0.206$ m and $P = 0.644$ m. The height of the required weir is therefore $P = 0.644$ m.

Example-Solution

A $2.5\,m$ wide rectangular channel has a rectangular weir spanning the full width of the channel. The weir height is $0.75\,m$ measured from the bottom of the channel. What discharge is indicated when this weir is working under submerged mode with depths of flow, measured above the bed channel, of $1.75\,m$ and $1.25\,m$ on the upstream and downstream of the weir, respectively ?

Solution Weir height $P = 0.75$ m,

$$H_1 = 1.75 - 0.75 = 1.00 \text{ m and } H_2 = 1.25 - 0.75 = 0.5 \text{ m}$$

$$H_1 / P = 1.0/0.75 = 1.333 \text{ and } H_2/H_1 = 0.5/1.0 = 0.50$$

$$C_d = 0.611 + 0.08\,(1.333) = 0.718$$

$$Q_f = \frac{2}{3} C_d \sqrt{2g} L (H_1)^{3/2} = \frac{2}{3} \times 0.718 \times \sqrt{19.62} \times 2.5 \times (1.0)^{3/2} = 5.30 \text{ m}^3/\text{s}$$

By Villemonte equation for submerged weir flow, and noting that for rectangular weir flow $n = 1.5$

$$Q_s = Q_f \left[1 - \left(\frac{H_2}{H_1}\right)^{1.5}\right]^{0.385} = 4.24 \times \left[1 - (0.5)^{1.5}\right]^{0.385} = 4.48 \text{ m}^3/\text{s}.$$

Non-Rectangular Weirs

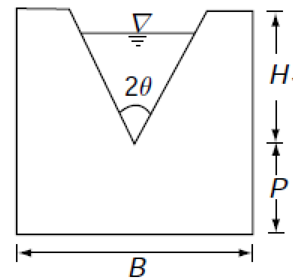
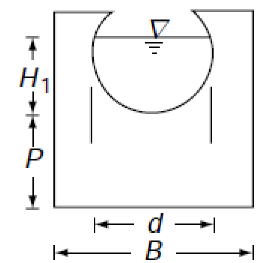
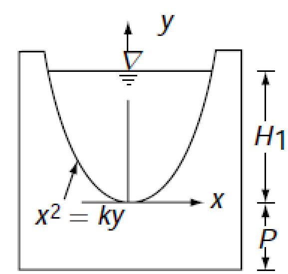
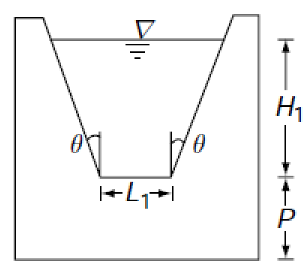
- Sharp-crested weirs of various shapes are adopted to meet specific requirements based on accuracy, range and head-discharge relationships.

- General form of head-discharge relationship for a weir can be expressed as $Q = KH_1^n$, where K and n are coefficients. n depends upon the weir shape and K depends upon the weir shape and its setting. [Table]

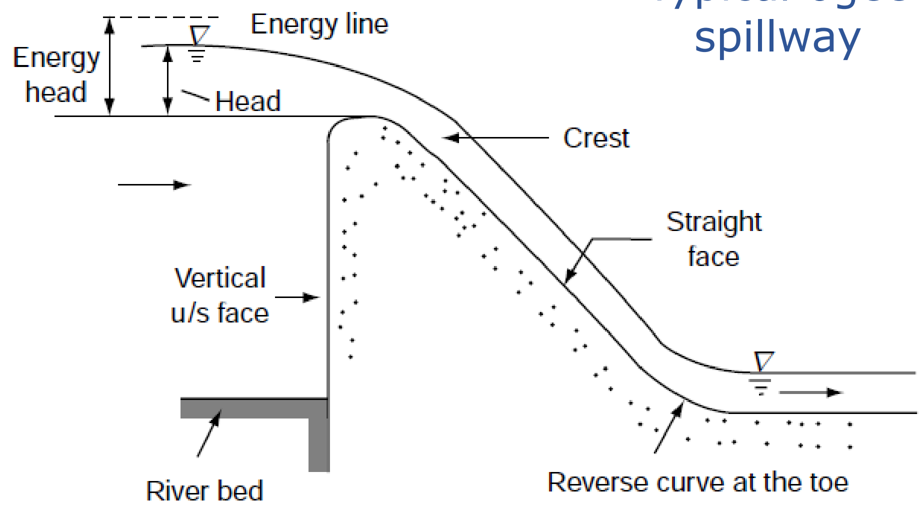
- A variety of sharp-crested weir shapes have been designed to give specific head discharge relationships and are described in the literature.

- A type of weir for which the discharge varies linearly with head, known as **Sutro Weir**, finds use in flow measurement of small discharges and in automatic control of flow, sampling and dosing through float operated devices.

Discharge Relationships for some Commonly Used Non-Rectangular Thin-Plate Weirs

Shape		*Discharge*
Triangular		$Q = \frac{8}{15} C_d \sqrt{2g} \tan\theta \, H_1^{5/2}$ $C_d = fn(\theta)$ For $2\theta = 90^{\circ}, C_d = 0.58$
Circular		$Q = C_d \phi d^{2.5}$, $\phi = f(H_1/d)$, $C_d = f(H_1/d)$
Parabolic		$Q = \frac{1}{4} \pi C_d \sqrt{k} \sqrt{2g} H_1^{2}$
Trapezoidal		$Q = \frac{2}{3} C_d \sqrt{2g} H_1^{3/2} \left(L_1 + \frac{4}{5} H_1 \tan\theta\right)$

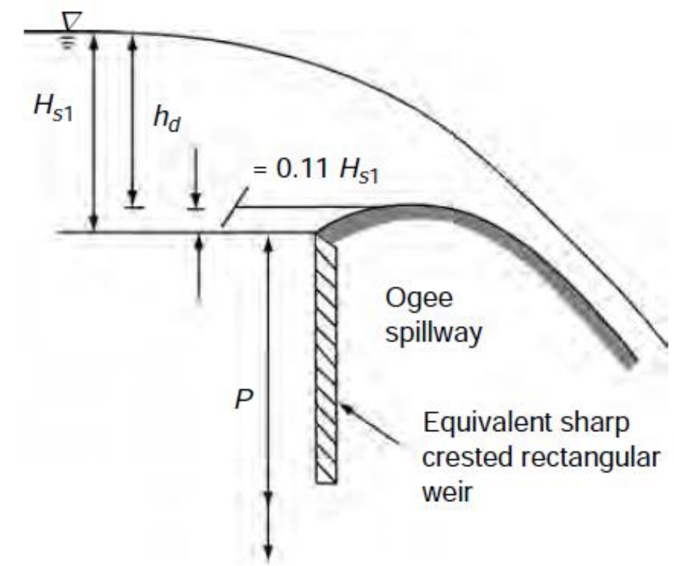
Ogee Spillway



Typical ogee spillway

Overflow spillway

- A control weir having an ogee (S-shaped) overflow profile.
- the most extensively used spillway to safely pass the flood flow out of a reservoir.



Lower nappe as a spillway profile

- Crest profile is chosen to provide a high C_d without causing dangerous cavitation conditions and vibrations.
- Profile is made to conform to lower nappe emanating from a well-ventilated sharp-crested rectangular weir: for design head, a high C_d, and atm pressure on weir.

Smaller heads: small trajectory; +ve pressure; low C_d.
Higher heads: large trajectory; -ve pressure; hig C_d.

Ogee Spillway contd...

For a high spillway $\left({}^{H_{s1}}/_{P}\right) \cong 0$, it is found experimentally that the spillway apex is about $0.11\, H_{s1}$ above the equivalent sharp-crested weir crest. The design head for the spillway is then $h_d = 0.89\, H_{s1}$.

Considering discharge eq. with suffix '*s*' for an equivalent sharp-crested weir

$$\boxed{q = \frac{2}{3} C_{ds} \sqrt{2g}\, H_{s1}^{3/2}}$$

For sharp-crested weir

and

$$\boxed{q = \frac{2}{3} C_{d1} \sqrt{2g}\, h_d^{3/2}}$$

For overflow spillway

Here, $C_{dv} = 1.19\, C_{ds}$, i.e. the ogee spillway discharge coefficients are numerically about 20% higher than the corresponding sharp-crested weir coefficients

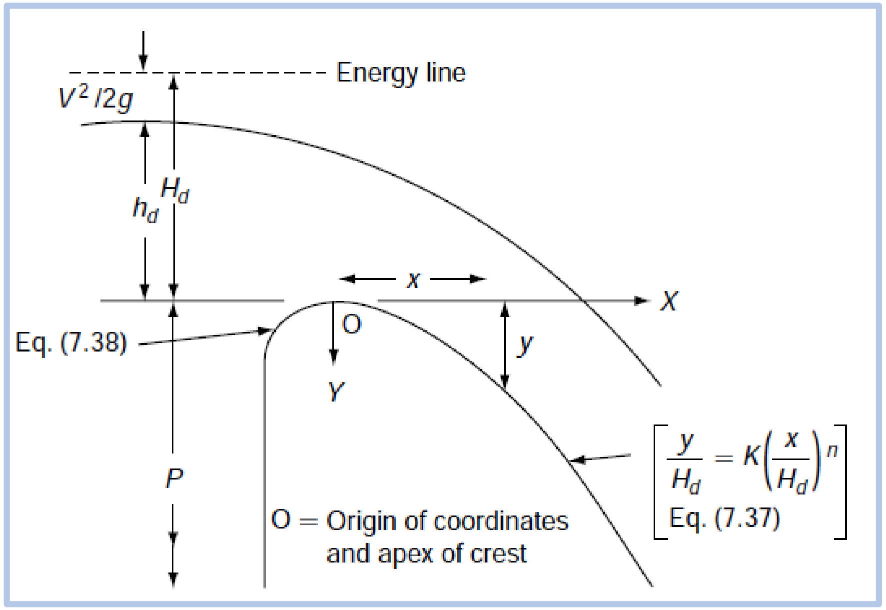
Uncontrolled Ogee Crest

If there are no crest gates over them, such spillways are designated as uncontrolled spillways.

Considering a typical overflow spillway crest, the profile of the crest downstream of the apex can be expressed as

$$\frac{y}{H_d} = K\left(\frac{x}{H_d}\right)^n$$

where, x and y are coordinates of the downstream curve of the spillway [origin of coordinates on the apex], H_d= design energy head, i.e. design head measured above the crest to the energy line. K and n are constants [depend upon the inclination of the upstream face and on $V_{appraoch}$].



Elements of a spillway crest

Uncontrolled Ogee Crest contd…

For low velocities of approach, typical values of K and n are

Upstream face	*K*	*n*
Vertical	*0.500*	*1.850*
1 Horizontal : 1/3 vertical	*0.517*	*1.836*
1 Horizontal : 1 vertical	*0.534*	*1.776*

Crest profile upstream of apex is usually given by a series of compound curves.

Cassidy reported the equation for the upstream portion of a vertical faced spillway as

$$\frac{y}{H_d} = 0.724\left(\frac{x}{H_d} + 0.27\right)^{1.85} - 0.432\left(\frac{x}{H_d} + 0.27\right)^{0.625} + 0.126 \qquad (7.38)$$

This is valid for the region $0 \geq \frac{x}{H_d} \geq -0.27$ and $0 \leq \frac{y}{H_d} \leq 0.126$. Same coordinate system as for the downstream profile (Eq. 7.37) is used for Eq. 7.38 also.

Uncontrolled Ogee Crest contd…

Since the hydraulic characteristics of the approach channel vary from one spillway to another, it is found desirable to allow explicitly for the effect of the velocity of approach in various estimations related to the overflow spillway. With this in view, the expression for the design discharge Q_d over an ogee spillway at the design head is written as

$$Q_d = \frac{2}{3} C_{d0} \sqrt{2g}\, L_e H_d^{3/2} \tag{7.39}$$

in which H_d = design-energy head (i.e., head inclusive of the velocity of approach head), C_{d0} = coefficient of discharge at the design head and L_e = effective length of the spillway.

Uncontrolled Ogee Crest contd…

If H_0 = any energy head over the ogee spillway, the corresponding discharge Q can be expressed as

$$Q = \frac{2}{3} C_0 \sqrt{2g}\, L_e H_0^{3/2} \qquad (7.40)$$

where C_0 = coefficient of discharge at the head H_0. In general, C_0 will be different from C_{d0}. If $H_0/H_d > 1$ then $C_0/C_{d0} > 1$. If on the other hand, $H_0/H_d < 1$, then $C_0/C_{d0} < 1$. By definition, if $H_0/H_d = 1$, $C_0/C_{d0} = 1$.

The discharge coefficients C_0 and C_{d0} are both functions of P/H_0 and P/H_d, respectively, and of the slope of the upstream face.

Example

An overflow spillway is to be designed to pass a discharge of $2000\, m^3/s$ of flood flow at an upstream water-surface elevation of $200\, m$. The crest length is $75\, m$ and the elevation of the average stream bed is $165\, m$. Determine the design head and profile of the spillway.

Solution

Solution A trial-and-error method is adopted to determine the crest elevation.

Discharge per unit width $q_d = \dfrac{2000}{75} = 26.67$ m³/s/m. Assume $C_{d0} = 0.736$.

By Eq. 7.39 $q_d = \dfrac{2}{3} C_{d0} \sqrt{2g} \left(H_d\right)^{3/2}$

$$26.27 = \frac{2}{3}(0.736)\sqrt{19.62}\left(H_d\right)^{3/2}$$

$$H_d = 5.32 \text{ m}$$

Velocity of approach $V_a = \dfrac{q}{P + h_0} = \dfrac{26.67}{(200.00 - 165.00)}$

$$= 0.762 \text{ m/s}$$

$$h_a = \frac{V_a^2}{2g} = 0.0296 \approx 0.03 \text{ m}$$

Elevation of energy line $= 200.03$ m

Crest elevation $= 200.03 - 5.32 = 195.71$m

$$P = 195.71 - 165.00 = 30.71\text{m}$$

$$P/H_d = \frac{30.71}{5.32} = 5.77$$

For this value of P/H_d from Fig. 7.13, $C_{d0} = 0.738$.

2nd iteration

$$\left(H_d\right)^{3/2} = \frac{26.67}{(2/3)(0.738)\sqrt{19.62}}, \quad H_d = 5.31\text{m}$$

$h_a \approx 0.03$. Elevation of energy line $= 200.03$ m

Solution

Crest elevation $200.03 - 5.31 = 194.72$ m

$$P = 194.72 - 165.00 = 29.72 \text{ m}$$

$P/H_d = 5.60$. For this P/H_d, from Fig. 7.13, $C_{d0} = 0.738$. Hence no more iterations are required.

Design energy head $H_d = 5.31$m

and crest elevation $= 194.72$ m

The downstream profile of the crest is calculated by Eq. 7.37, which for the present case is

$$\frac{y}{5.31} = 0.50\left(\frac{x}{5.31}\right)^{1.85}$$

The upstream profile is calculated by Eq. 7.38 which, for the range $0 \leq -\times \leq 1.434$, is given as

$$\frac{y}{5.31} = 0.724\left(\frac{x}{5.31} + 0.270\right)^{1.85} - 0.432\left(\frac{x}{5.31} + 0.270\right)^{0.625} + 0.126$$

The apex of the crest at elevation 194.72 m is the origin of coordinates of the above two profile equations.

Spillway with Crest Gates

- When spillways are provided with crest gates, they have to operate as uncontrolled spillways under high flood conditions and with partial gate openings at lower flows.
- At partial gate openings, the water issues out of the gate opening as an orifice flow and the trajectory is a parabola. If the ogee is shaped by Eq. 7.37 the orifice flow, being of a flatter trajectory curve, will cause negative pressures on the spillway crest.
- These negative pressures can be minimized if the gate sill is placed downstream of the apex of the crest. In this case, the orifice flow will be directed downwards at the initial point itself, causing less difference between the ogee profile and the orifice trajectory.

If the trajectory of the orifice flow with the gate sill located at the apex of the crest is adopted for the spillway profile, the coefficient of discharge at the full gate opening will be less than that of an equivalent uncontrolled overflow spillway.

The discharge from each bay of a gated ogee spillway is calculated from the following large orifice equation.

$$Q = \frac{2}{3}\sqrt{2g}C_g L_b \left(H_0^{\frac{3}{2}} - H_1^{\frac{3}{2}}\right)$$

where C_g=coefficient of the gated spillway, L_b=effective length of the bay after allowing for two end contractions, H_0=energy head above the spillway crest and H_1=energy head above the bottom edge of the gate. The coefficient of discharge C_g depends upon the geometry of the gate, gate installation, interference of adjacent gates and flow conditions.

Broad-crested Weir

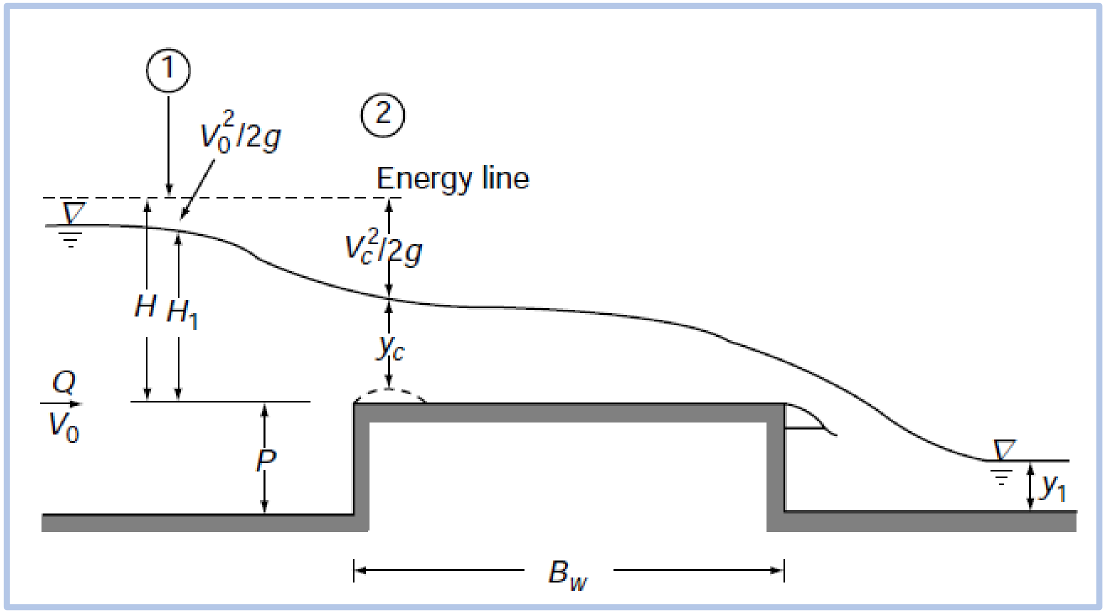
- Weirs with a finite crest width in the direction of flow are called broad-crested weirs [weirs with finite crest width]
- Extensive applications as control structures and flow-measuring devices.

Salient flow characteristics of rectangular, horizontal broad-crested weir

$$\boxed{H = y_c + \frac{V_c^2}{2g} = \frac{3}{2} y_c}$$

$$V_c = \sqrt{g y_c} \qquad \&$$

$$y_c = \frac{2}{3} H$$



Definition sketch of a broad-crested weir

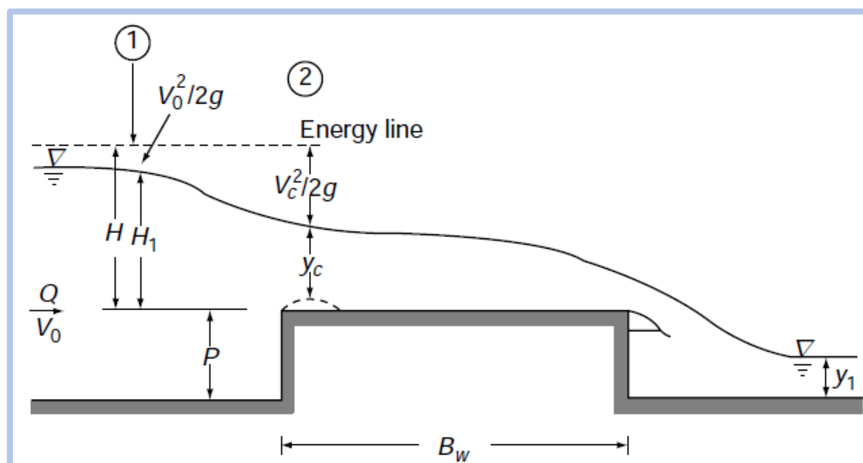
Broad-crested Weir contd…

This weir has a sharp upstream corner which causes the flow to separate and then reattach enclosing a separation bubble.

If the width B_w of the weir is sufficiently long, the curvature of the stream lines will be small and the hydrostatic pressure distribution will prevail over most of its width. The weir will act like an inlet with subcritical flow upstream of the weir and supercritical flow over it.

A critical-depth control section will occur at the upstream end-probably at a location where the bubble thickness is maximum.

Assuming no loss of energy between Sections 1 and 2, and further assuming the depth of flow at Section 2 to be critical,



Broad-crested Weir contd…

The ideal discharge per unit width of the weir is

$$\boxed{q_t = V_c y_c = \frac{2}{3}\sqrt{\left(\frac{2}{3}g\right)}H^{3/2} = 1.705\ H^{3/2}}$$

To account for the energy losses and the depth at Section 2 being not strictly equal to the critical depth, the coefficient of discharge C_{d1} is introduced in Eq. 7.45 to get an equation for the actual discharge q as

$$q = C_{d1}q_t = 1.705\ C_{d1}H^{3/2}$$

and $Q = qL$, where L = length of the weir

Broad-crested Weir contd…

In broad-crested weirs, L = length of the weir measured in a transverse direction to the flow and B_w = width of the weir measured in the longitudinal directon. Thus B_w is measured at right angles to L. In suppressed weirs $L = B = width\ of\ the\ channel$ [terminology: caution].

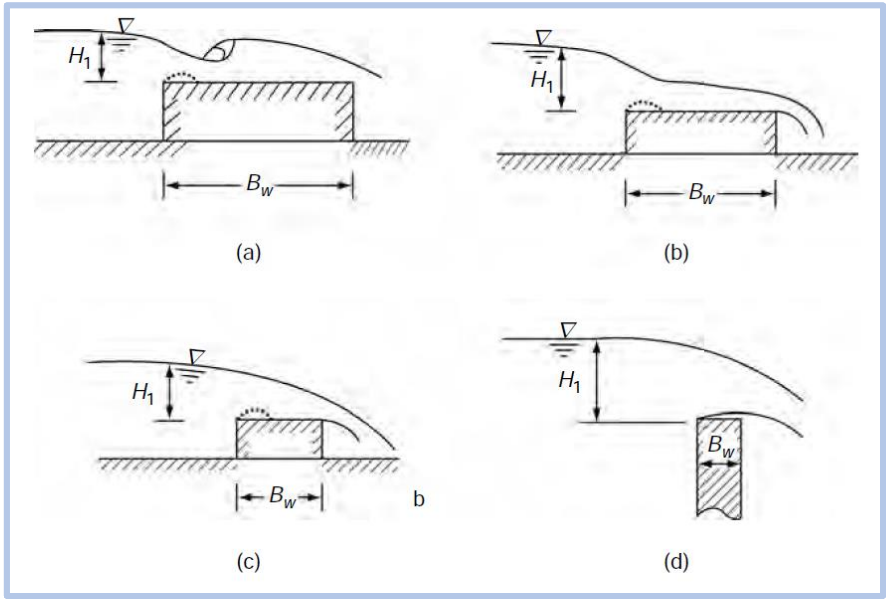
Since Eq. 7.46 is rather inconvenient to use as it contains the energy head H, an alternate form of the discharge equation commonly in use is

$$\boxed{Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H_1^{3/2}}$$

where H_1 = height of the water-surface elevation above weir surface measured sufficiently upstream of the weir face and C_d = coefficient of discharge.

If upstream end is rounded, separation bubble will not exist and instead, a boundary layer will grow over weir with critical-depth control point shifting towards this downstream end. Flow over most parts of its crest will be subcritical. Considerable flow resistance from the upstream face to critical flow section exists, influencing value of C_d.

Classification



(a) Long-crested weir $(H_1/B_w \leq 0.1)$

(b) Broad-crested weir $(0.1 \leq H_1/B_w \leq 0.35)$

(c) Narrow-crested weir $(0.35 \leq H_1/B_w \leq 1.5)$

(d) Sharp-crested weir $(H_1/B_w \geq 1.5)$

Example

A broad-crested weir with an upstream square corner and spanning the full width of a rectangular canal of $2\,m$ width is planned. The proposed crest length is $2.5\,m$ and the crest elevation is $1.2\,m$ above the bed.

Calculate the water-surface elevation upstream of the weir when the discharge is

(a) $2\,m^3/s$ and

(b) $3.5\,m^3/s$

Solution

Solution (a) $Q = 2.0\ \text{m}^3/\text{s}$

Assume the weir to function in the broad-crested weir mode and hence assume $C_d = 0.525$ as a first guess. From Eq. 7.47

$$2.0 = \frac{2}{3} \times 0.525 \times \sqrt{19.62} \times 2.0 \times H_1^{3/2}$$

$$H_1^{3/2} = 0.645 \text{ and } H_1 = 0.747 \text{ m}$$

$$\frac{H_1}{B_w} = 0.299. \text{ The assumption is OK.}$$

By Eq. 7.52 $$C_d = 0.028(0.299) + 0.521 = 0.529$$

Substituting this C_d value in Eq. 7.47

$$H^{3/2} = 0.640.\ H_1 = 0.743 \text{ m and from Eq. 7.52}$$

$$C_d = 0.529$$

Hence the water-surface elevation above the bed=1.943 m.

Solution

(b) $Q = 3.25$ m³/s

Since Q is higher than in case (a), it is likely that $H_1/B_W > 0.35$. Hence assuming the weir to function in the narrow-crested weir mode, the calculations are started by assuming $C_d = 0.55$.

Ist iteration $C_d = 0.55$

From Eq. 7.28, $3.50 = \frac{2}{3} \times 0.55 \times \sqrt{19.62} \times 2.0 \times H_1^{3/2}$

$$H_1^{3/2} = 1.077, \quad H_1 = 1.05 \text{ m}, \quad \frac{H_1}{B_w} = 0.42$$

The weir flow is in the transition region between the broad-crested and narrow-crested weir modes. Hence, by Eq. 7.53

$$C_d = 0.120 \times (0.42) + 0.492 = 0.534$$

2nd iteration Using $C_d = 0.534$ in Eq. 7.47

$$H_1^{3/2} = 1.109, \quad H_1 = 1.071 \text{ m}, \quad \frac{H_1}{B_w} = 0.429$$

From Eq. 7.53, $C_d = 0.543$

3rd iteration $H_1^{3/2} = 1.091, \ H_1 = 1.060 \text{ m}, \ \frac{H_1}{B_w} = 0.424$

$$C_d = 0.543$$

Hence, $H_1 = 1.060$ and the water-surface elevation above the bed is 2.260 m.

Example

Show that for a triangular broad crested weir flowing free the discharge equation can be expressed as

$$\boxed{Q = \frac{16}{25} C_{d1}\ tan\theta\ \sqrt{\frac{2g}{5}} H^{5/2}}$$

where H = energy head measured from the vertex of the weir, θ = semi-apex angle and C_{d1} = coefficient of discharge.

Solution

Solution $H = y_c + \frac{V_c^2}{2g} = 1.25\,y_c$

$$\text{or}\ \ y_c = \frac{4}{5}H$$

$$A = my_c^2 = \frac{16}{25}\tan\theta\ H^2\ \ \text{where } \theta = \text{semi vertex angle.}$$

$$F = \frac{V\sqrt{2}}{\sqrt{gy_c}} = 1\ \ \text{or}\ \ V = \sqrt{\frac{2g}{5}}H^{1/2}$$

$$Q = C_{d1}VA$$

$$Q = \frac{16}{25}C_{d1}\tan\theta\sqrt{\frac{2g}{5}}H^{5/2}$$

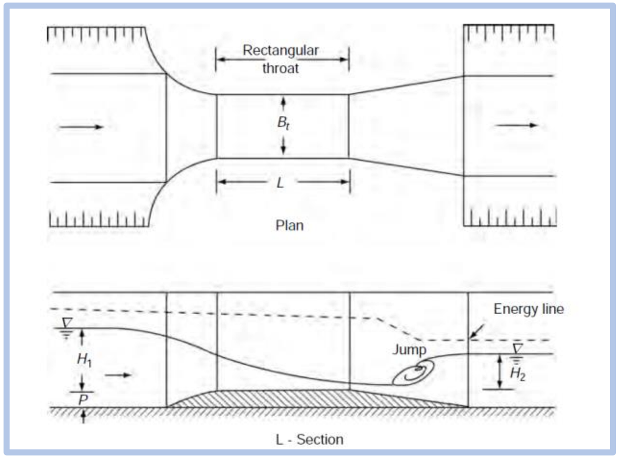
Critical-depth flumes

Critical-depth flumes

- Flow-measuring devices in which a control section is achieved through the creation of a critical-flow section by a predominant width constriction.
- In practice, these are like broad-crested weirs but with a major change that these are essentially flow-measuring devices and cannot be used for flow-regulation purposes.
- A typical critical-depth flume consists of a constricted portion called the *throat* and a diverging section. Sometimes a hump is also provided to assist in the formation of critical flow in the throat.

Parshall flume is a type of critical-depth flume popular in the USA.

Standing-wave flume [throated flume]



Thank you ☺