Fluid Mechanics [CEC-104]

Drag and Lift

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Course content

Evaluation Procedure

- **CWS: 15% [attd-10; Quiz-5]**
- **PRS: 20%**
- **Mid-term: 25%**
- **End-term: 40%**

Suggested Books

S. No.	Name of Books / Authors	Published by	Edition	Year of Publication/Reprint
1.	Som, S.K. and Biswas, G., "**Fluid Mechanics and Fluid Machines**".	McGraw Hill Education Private Limited, New Delhi	Third	2017
2.	Garde, R.J. and Mirajgaoker, A. G., "**Engineering Fluid Mechanics**".	Nem Chand & Bros.	Third	1988
3.	Fox, R.W. and McDonald, A.T., "**Introduction to Fluid Mechanics**".	John Wiley & Sons	Eighth	2011
4.	Asawa, G. L., "**Fluid Flow in Pipes and Channels**".	CBS Publishers and Distributors PVT LTD	First	2008
5.	Schlichting, H. and Gersten, K., "**Boundary Layer Theory**".	Springer	Ninth	2004
6.	Streeter, V. L. and Benjamin, W. E., "**Fluid Mechanics**".	McGraw Hill Education Private Limited, New Delhi	Eighth	1985

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NAME OF DEPARTMENT: Civil Engineering

Subject Code: CEC-104 **Course Title:** Fluid Mechanics

L-T-P: 3-0-2/2 **Credits:** 03 **Subject Area:** PCC

Course Outlines: To introduce fundamentals of stagnant, flowing fluids and flow through different conduits.

S. No.	Contents	Contact Hours
1.	**Introduction:** Fluid properties, types of fluids, continuum principle	**3**
2.	**Principles of Fluid statics:** Basic equations, manometers, hydrostatic forces on submerged surfaces, buoyancy.	**7**
3.	**Kinematics of Flow:** Visualization of flow, types of flow, streamline, path line, streamline, the principle of conservation of mass, velocity, acceleration, velocity potential and stream functions, vorticity, circulation.	**4**
4.	**Fluid dynamics:** Control volume approach, Euler's equation, Bernoulli's equation and its applications, Reynolds transport theorem, momentum and angular momentum equation and their applications.	**7**
5.	**Dimensional Analysis and Similitude:** Dimensional homogeneity, Buckingham's pi theorem, dimensional numbers, similitude.	**3**
6.	**Boundary Layer Theory:** the concept of the boundary layer, laminar and turbulent boundary layers, boundary layer thickness, von Karman integral equation, laminar sublayer, hydro-dynamically smooth and rough boundaries, separation of flow and its control, cavitation.	**6**
7.	**Laminar and Turbulent Flow through Pipes:** Laminar flows through pipes, turbulent flow, Reynolds equations, Prandtl's mixing length theory, velocity distribution over a flat plate and in pipe section, Darcy-Weisbach equation, friction factor, Moody diagram, minor losses, pipe networks, Venturi meter, orifice meter, water hammer, surge tanks	**9**
8.	**Drag and Lift:** Skin-friction and form drag, drag on sphere, cylinder, and flat plate, Karman vortex shedding, generation of lift around a cylinder, lifting vanes.	**3**
	Total	**42**

Forces on submerged bodies

Introduction

- When a fluid is flowing over a stationary body, a force is exerted by the fluid on the body.
- Similarly, when a body is moving in a stationary fluid, a force is exerted by the fluid on the body.
- Also when the body and fluid both are moving at **different** velocities, a force is exerted by the fluid on the body.

Relative motion

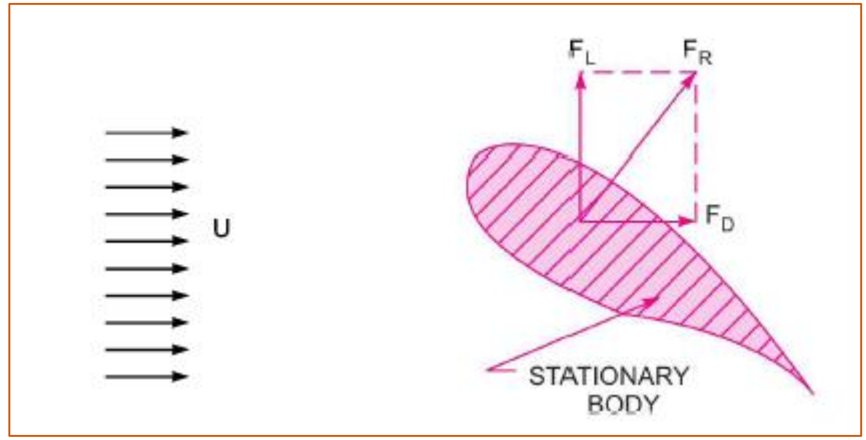
Examples

1. Flow of air over buildings
2. Flow of water over bridges
3. Submarines, ships, airplanes and automobiles moving through water or air

F by flowing fluid on stationary body

Real fluid flowing at a uniform velocity U

Force on a stationary body



The total force (F_R) exerted by the fluid on the body is

- perpendicular to the surface of the body.

Thus the total force is inclined to the direction of motion.

F_R can be resolved in two components, one in the direction of motion and other perpendicular to the direction of motion.

Drag | Lift

F by flowing fluid on stationary body

Drag [F_D].

Component of F_R in direction of motion is called 'drag'. Thus drag is the force exerted by the fluid in the direction of motion.

Lift [F]

Component of F_R perpendicular to the direction of motion is known as 'lift'. Thus lift is the force exerted by the fluid in

Special cases

If the axis of the body is parallel to the direction of fluid flow, $\boldsymbol{F} = \mathbf{0}$

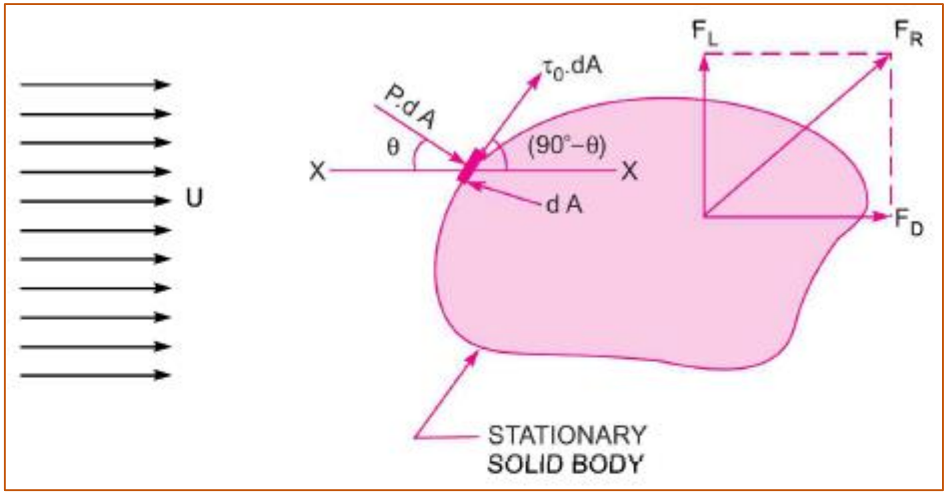
If the fluid is assumed ideal and the body is symmetrical such as a sphere or cylinder, $\boldsymbol{F} = \mathbf{0}$ & $\boldsymbol{F_D} = \mathbf{0}$

Expression for drag and lift

Arbitrary shaped solid body placed in a real fluid, flowing with a uniform velocity U in a horizontal direction.

Consider a small elemental area dA on the surface of the body.

Drag and lift



Forces acting on dA are:

1. Pressure force $(= p * dA)$, acting perpendicular to the surface.
2. Shear force $(= \tau_0 * dA)$, acting tangential to the surface.

Let θ = Angle made by pressure force with horizontal direction.

Expression for drag and lift

In direction of fluid motion

(a) Drag Force (F_D): The drag force on dA

= Force due to pressure + Force due to shear stress

$= pdA\cos\theta + \tau_0 * dA\cos(90° - \theta) = pdA\cos\theta + \tau_0\, dA\sin\theta$

Total drag, $F_D = \boxed{\int p\cos\theta\, dA} + \boxed{\int \tau_0 \sin\theta\, dA}$

Profile Drag

pressure drag or form drag

friction/skin/viscous/shear drag

Perpendicular to the direction of motion

(b) Lift Force (F): The lift force on dA

= Force due to pressure + Force due to shear stress

$= -pdA\sin\theta + \tau_0 dA\sin\,(90° - \theta) = -pdA\sin\theta + \tau_0 dA\cos\theta$

p –ve downwards

Total lift, $F_L = \int \tau_0 \cos\theta\, dA - \int p\, sin\theta\, dA$

Expression for drag and lift

The drag and lift for a body moving in a fluid of density ρ, at a uniform velocity U are calculated mathematically, as

$$F_D = C_D A \frac{\rho U^2}{2} \qquad F_L = C_L A \frac{\rho U^2}{2}$$

derived by the method of dimensional analysis.

C_D: Co-efficient of drag, C_L: Co-efficient of lift,

A = projected area of the body perpendicular to the direction of flow Or Largest projected area of the immersed body.

Then resultant force on the body, $F_R^2 = \sqrt{F_D^2 + F_L^2}$

Dimensional Analysis of drag and lift

Force exerted by a fluid on a supersonic plane

$$F = \rho L^2 U^2 \emptyset \left[\frac{\mu}{\rho U L}, \frac{K}{\rho U^2} \right] \qquad ...(1)$$

Force exerted by a fluid on a partially submerged body

$$F = \rho L^2 U^2 \emptyset \left[\frac{\mu}{\rho U L}, \frac{Lg}{U^2} \right] \qquad ...(2)$$

Combining (1) & (2) [for general expression]

Force exerted by a fluid on a body $F = \rho L^2 U^2 \emptyset \left[\frac{\mu}{\rho U L}, \frac{Lg}{U^2}, \frac{K}{\rho U^2} \right]$

L = Length of body, μ = Viscosity of fluid, F = Force exerted, U = Velocity of body,

ρ = Density of fluid, k = Bulk modulus of fluid, g = Acceleration due to gravity.

Dimensional Analysis of drag and lift

- For completely sub-merged body, force exerted by fluid on body due to gravitational effect is negligible → non-dimensional term containing 'g', i.e., $\frac{Lg}{U^2}$ is neglected.
- If $V_{body} \approx V_{sound}$, effect due to compressibility is to be considered. But if $\frac{V_{body}}{V_{sound}} < 0.3$, force exerted by fluid on body due to compressibility ~ negligible.

Hence the non-dimensional term containing K can be neglected. Then the force exerted by fluid on the body is given as

$$F = \rho L^2 U^2 \emptyset \left[\frac{\mu}{\rho U L}\right] = \rho L^2 U^2 \emptyset \left[\frac{\rho U L}{\mu}\right] = \rho L^2 U^2 \emptyset \,[R_e]$$

Now $\boldsymbol{F}$ is the total force exerted by the fluid on the body. It has two components, drag force and lift force.

Dimensional Analysis of drag and lift

The two components of F are expressed

Recap

$$\boldsymbol{F_D} = \frac{\rho L^2 U^2}{2} * C_D$$

[C_D is a function of R_e, and is called co-efficient of drag]

$$= C_D\, A\, \frac{\rho\, U^2}{2}$$

$$L^2 = area = A$$

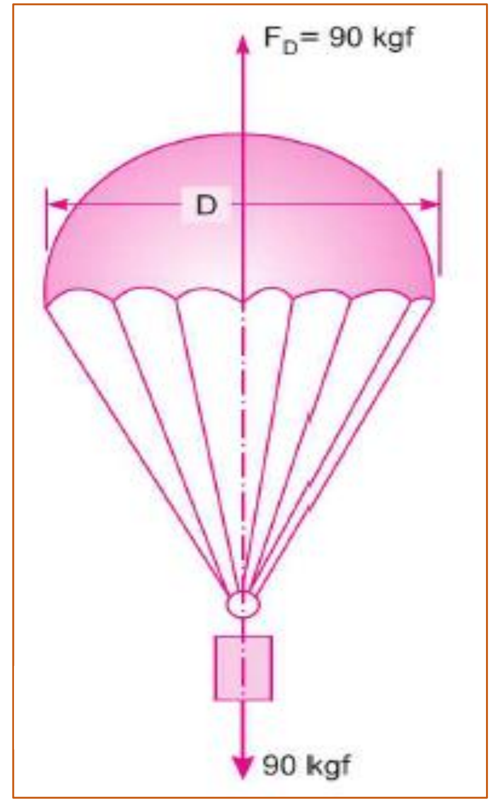
$$\boldsymbol{F_L} = \frac{\rho L^2 U^2}{2} * C_L$$

$$= C_L\, A\, \frac{\rho\, U^2}{2}$$

[C_L is a function of R_e, and is called co-efficient of lift]

Problem

A man weighing $90\,kgf$ descends to the ground from an aeroplane with the help of a parachute against the resistance of air. The velocity with which the parachute, which is hemispherical in shape, comes down is $20\,m/s$. Find the diameter of the parachute. Assume $C_D = 0.5$ and density of air = $1.25\,kg/m^3$.



Solution

Weight of man, $W = 90 \text{ kgf} = 90 \times 9.81 \text{ N} = 882.9 \text{ N}$ $(\because \quad 1 \text{ kgf} = 9.81 \text{ N})$

Velocity of parachute, $U = 20 \text{ m/s}$

Co-efficient of drag, $C_D = 0.5$

Density of air, $\rho = 1.25 \text{ kg/m}^3$

Let the diameter of parachute $= D$

$\therefore$ Area, $A = \dfrac{\pi}{4} D^2 \text{ m}^2.$

When the parachute with the man comes down with a uniform velocity, $U = 20$ m/s, the drag resistance will be equal to the weight of man, neglecting the weight of parachute. And projected area of the hemispherical parachute will be equal to $\dfrac{\pi}{4} D^2$.

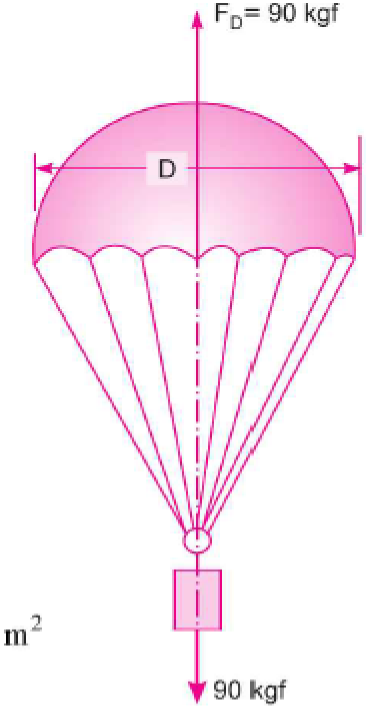
$\therefore$ Drag, $F_D = 90 \text{ kgf} = 90 \times 9.81 = 882.9 \text{ N}$

Using equation (14.3), $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$

$$\therefore \quad 882.9 = 0.5 \times \frac{\pi}{4} D^2 \times \frac{1.25 \times 20^2}{2}$$

$$\therefore \quad D^2 = \frac{882.9 \times 4 \times 2.0}{0.5 \times \pi \times 1.25 \times 20 \times 20} = 8.9946 \text{ m}^2$$

or $D = \sqrt{8.9946} =$ **2.999 m. Ans.**



More practical examples

Pressure Drag and Friction Drag

Recap

Total drag, $F_D = \int p \cos\theta \, dA + \int \tau_0 \sin\theta \, dA$

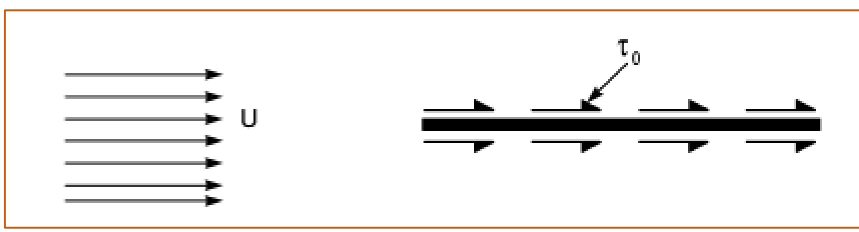
Relative contribution of the pressure drag and friction drag to F_D depends on;

(i) Shape of the immersed body,

(ii) Position of the body immersed in the fluid, and

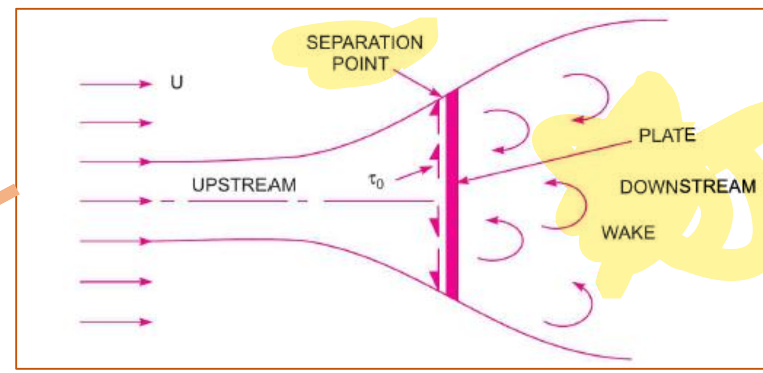
(iii) Fluid characteristics.

Consider below two cases:



Flat plate parallel to flow

F_D = friction drag (or shear drag)



Flat plate perpendicular to flow

F_D = pressure drag [due to pressure difference between the upstream and downstream sides of the plate]

Pressure Drag and Friction Drag

Stream-lined/well-shaped Body

- Whose surface coincides with stream-lines, when placed in a flow.
- Separation of flow will take place only at the trailing edge.
- Though the boundary layer will start at the leading edge, will become turbulent from laminar, yet it does not separate up to the rearmost part of the body.
- Behind a stream-lined body, the wake formation zone will be very small and consequently, the pressure drag (Δp) will be very small.

Then the total drag on the stream-lined body will be **due to friction (shear) only**

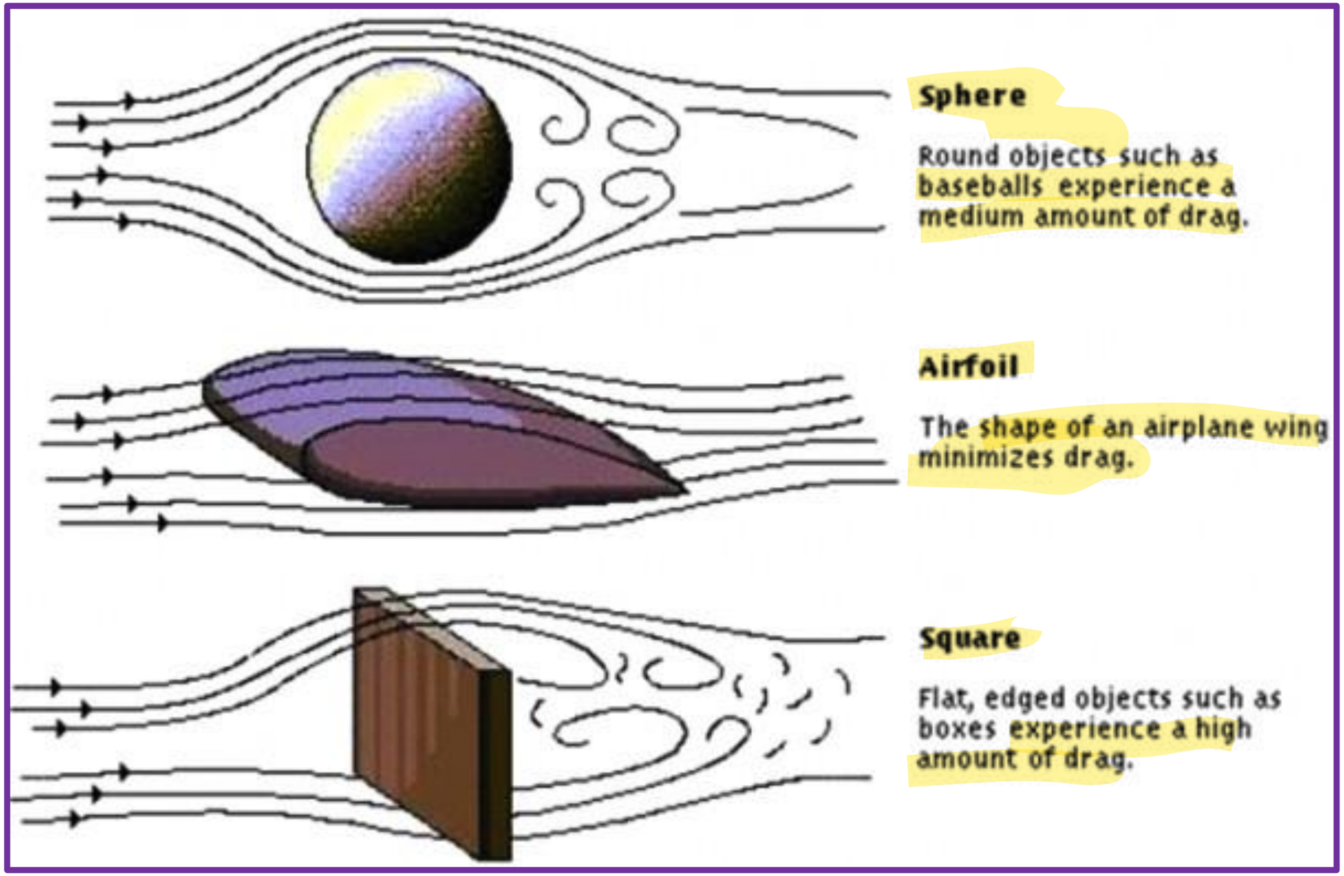
A body may be stream-lined:

1. at low velocities
2. when placed in a particular position in the flow

Drag variation with body shape

Can we keep on elongating the body to shift the point of separation as far as possible [thus minimum wake]??

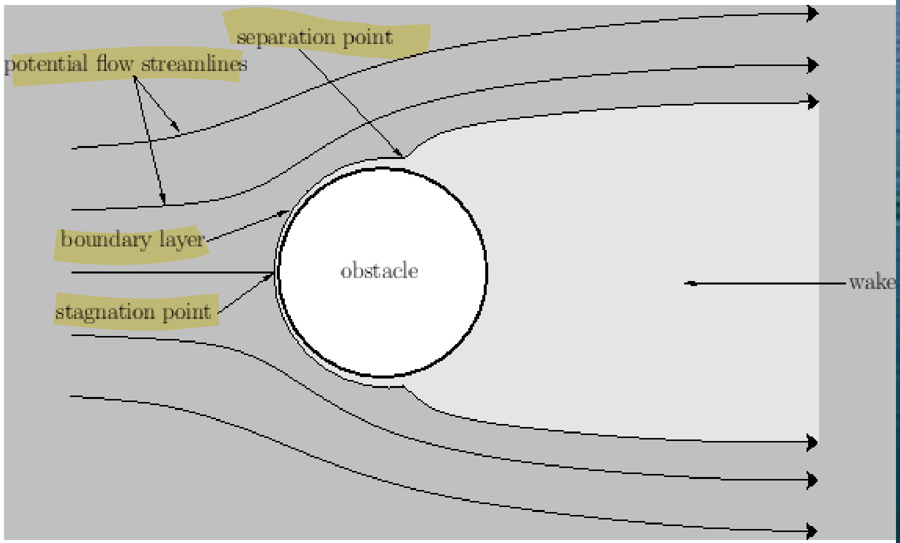
Optimum:
Total Dag is the minimum



Near the plate edges: High velocity & Low pressure

Streamlines will be closely spaced

Wake: Practical Example



Pressure Drag and Friction Drag

Bluff Body

- Whose surface does not coincide with streamlines, when placed in a flow.
- Flow is separated from the surface of the body much ahead of its trailing edge with the result of a very large wake formation zone.
- Drag due to pressure will be very large as compared to the drag due to friction on the body.

Thus the bodies of such a shape in which the **pressure drag is very large as compared to friction drag** are called bluff bodies.

Drag on a Sphere

Consider the flow of a real fluid past a sphere.

U = Velocity of the flow of fluid over sphere,
D = Diameter of sphere,
ρ = Mass density of fluid, and
μ = Viscosity of fluid.

If R_e of the flow is very small (i.e., $R_e = \frac{\rho U D}{\mu} < 0.2$), the viscous forces are much more dominant and hence important than the inertial forces.

G.G. Stokes, developed a mathematical equation for total drag on a sphere immersed in a flowing fluid for which R_e is up to 0.1 or 0.2, so that inertia forces may be assumed negligible.

Drag on a Sphere contd…

According to his solution, total drag is

$$F_D = 3\pi\mu DU$$

Simplifying Navier Stokes eq. of motion in x-direction

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Stokes further observed that 2/3rd of F_D is contributed by skin friction

Skin friction drag $\quad F_{D_f} = \frac{2}{3}\, F_D = 2\pi\mu DU$

Pressure drag $\quad F_{D_p} = \frac{1}{3}\, F_D = \pi\mu DU$

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1. C_D for sphere when $R_e < 0.2$

Total drag $\quad F_D = C_D\, A\, \frac{\rho\, U^2}{2} \qquad \left[A = \frac{\pi}{4} D^2\right]$

For sphere, $\quad F_D = 3\pi\mu DU$

Drag on a Sphere contd…

Equating the two eqs.

CWS??

$$\boxed{C_D = \frac{24}{R_e}}$$

$$[R_e = \frac{\rho U D}{\mu}]$$

Stoke's Law

2. C_D for sphere when $0.2 < R_e < 5$ [inertial forces increase]

$$C_D = \frac{24}{R_e}\left[1 + \frac{3}{16\, R_e}\right]$$

Oseen Formula

3. C_D for sphere when $5 < R_e < 1000$ $\quad C_D = 0.4$
4. C_D for sphere when $10^3 < R_e < 10^5$ $\quad C_D = 0.5$
5. C_D for sphere when $R_e > 10^5$ $\quad C_D = 0.2$

Think, why decreasing??

Terminal Velocity of a body

Maximum constant velocity of a falling body (e.g., sphere or a composite body: parachute+man) with which it will be travelling.

When the body is allowed to fall ($V_{initial} = 0$), the V_{body} ↑ due to g.

With ↑ of V_{body}, the drag force; opposing the motion of body also ↑ .

Equilibrium stage is reached: upward drag force = weight of the body

The net external force acting on the body will be zero and the body will be travelling at constant speed [terminal velocity]

For fluid case; forces acting on the body at this stage will be:

1. Weight of body (W), acting downward,
2. Drag force (F_D), acting vertically upward, and
3. Buoyant force (F_B), acting vertically up.

$$\boxed{W = F_D + F_B}$$

Problem-Solution

Determine the velocity of fall of rain drop of a $30 * 10^{-3}\ cm$ diameter, $\rho = 0.0012\ gm/cc$ and $\nu = 0.15\ cm^2/s$.

Diameter of rain drops, $D = 30 \times 10^{-3}$ cm

Density of rain drops, $\rho = 0.0012$ gm/cm^3

Kinematic viscosity, $\nu = 0.15$ cm^2/s

Using the relation, $\nu = \dfrac{\mu}{\rho}$ or $0.15 = \dfrac{\mu}{.0012}$

$\therefore$ $\mu = 0.15 \times .0012 = 0.00018\ \dfrac{\text{gm}}{\text{cm sec}}$

Now weight of rain drop $= \rho \times g \times$ Volume of rain drop

$= \rho \times g \times \dfrac{\pi}{6} D^3$ ($\because$ Rain drop is a sphere)

Drag force, F_D, on rain drop is given by equation (14.8) as $F_D = 3\pi\mu DU$

When rain drop is falling with a uniform velocity U, the drag force must be equal to the weight of rain drop. Hence equating these two values, we get

Weight of rain drop = Drag force

or $\rho \times g \times \dfrac{\pi}{6} D^3 = 3\pi\mu DU$ or $U = \dfrac{\rho \times g \times \dfrac{\pi}{6} \times D^3}{3\pi\mu D} = \dfrac{\rho g D^2}{18\mu}$

$$= \frac{0.0012 \times 981 \times \left(30 \times 10^{-3}\right)^2}{18 \times .00018} = \textbf{0.327 cm/s. Ans.}$$

Let us check for Reynolds number, R_e

$$R_e = \frac{\rho UD}{\mu} = \frac{UD}{\nu} = \frac{0.327 \times 30 \times 10^{-3}}{0.15} = 0.0654$$

As the Reynolds number is less than 0.2, the expression $F_D = 3\pi\mu DU$ is valid.

Drag on a Cylinder

Consider a real fluid flowing over a circular cylinder (D, L), when cylinder is placed in fluid such that its length is perpendicular to the direction of flow.

If $R_e \left(\frac{Ud}{v} < 0.2\right)$, the inertia force is negligibly small as compared to viscous force and hence the flow pattern about the cylinder will be symmetrical.

- As R_e is ↑, inertia forces ↑ and hence they must be taken into consideration for analysis of flow over cylinder.
- With ↑ of R_e, the flow pattern becomes unsymmetrical with respect to an axis perpendicular to the direction of flow.
- The drag force, i.e., the force exerted by the flowing fluid on the cylinder in the direction of flow depends upon R_e of the flow.

Drag on a Cylinder Contd…

From experiments, it has been observed that:

(i) When $R_e < 1$, the drag force is directly proportional to velocity and hence the drag co-efficient (C_D) is inversely proportional to R_e.

(ii) With ↑ R_e from $1\ to\ 2000$, C_D ↓ and reaches minimum $(0.95\,; R_e = 2000)$.

(iii) With the further ↑ of the R_e from $2000\ to\ 3 * 10^4$, the C_D ↑ and attains maximum value of 1.2 at $R_e = 3 * 10^4$.

(iv) C_D ↓ if the R_e is ↑ from $3 * 10^4$ to $3 * 10^5$. At $R_e = 3 * 10^5$, the value of $C_D = 0.3$.

(v) If R_e is ↑ beyond $3 * 10^5$, C_D ↑ and it becomes equal to 0.7 in the end.

Development of Lift on a Circular Cylinder

Development of lift on a circular cylinder

When a body is placed in a fluid in such a way that its axis is parallel to the direction of fluid flow and **body is symmetrical**, the resultant force acting on the body is in the direction of flow.

There is no force component on the body perpendicular to the direction of flow: lift will be zero.

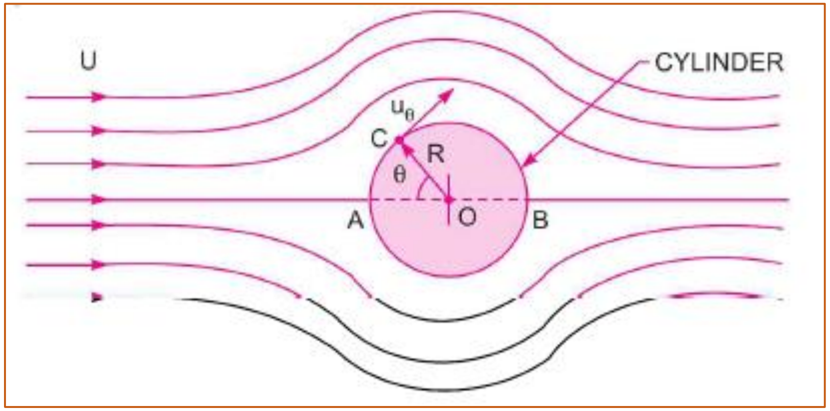
The lift will be acting on the body when the axis of the symmetrical body is inclined to the direction of flow or body is unsymmetrical.

- In the case of circular cylinder, the body is symmetrical and the axis is parallel to the direction of flow when cylinder is stationary. Hence the lift will be zero.
- If the cylinder is rotated, the axis of the cylinder is no longer parallel to the direction of flow and hence lift will be acting on the rotating cylinder.

Flow of Ideal Fluid over Stationary Cylinder

Consider ideal fluid flow over a stationary cylinder

U = Free stream velocity of fluid
R = Radius of the cylinder
θ = Angle made by any point say C on the circumference of the cylinder with the direction of flow.



Flow of ideal fluid over stationary cylinder

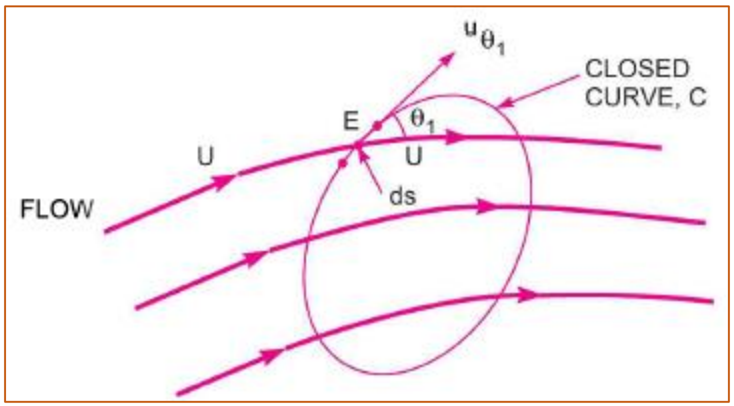
- Flow pattern will be symmetrical.
- Velocity at any point C on the surface of cylinder; $u_\theta = 2U \sin\theta$

The velocity distribution over upper half and lower half of cylinder from the axis AB of the cylinder are identical and hence the pressure distributions will also be same. Hence the lift acting on the cylinder will be zero.

Flow Pattern Around the Cylinder when a Constant Circulation is Imparted to the Cylinder

Circulation

flow along a closed curve.



Circulation

Mathematically

Product of velocity component along the curve at any point and the length of the small element containing that point is integrated around the curve.

- Fluid flowing with a free stream velocity U.
- Within the fluid consider a closed curve.
- Let E is any point on the closed curve and dS is a small length of the closed curve containing point E.

Let

θ_1 = Angle made by the tangent at E with the direction of flow,

u_{θ_1} = Tangential component of free stream velocity at E $(= U \cos \theta_1)$

Contd...

By definition, circulation [gamma] along the closed curve

$$\Gamma = \oint velocity\ component\ along\ curve * length\ of\ element$$

$$\Gamma = \oint U\cos\theta_1 \ * dS \qquad ...(1)$$

Circulation for the Flow-field in a Free-Vortex

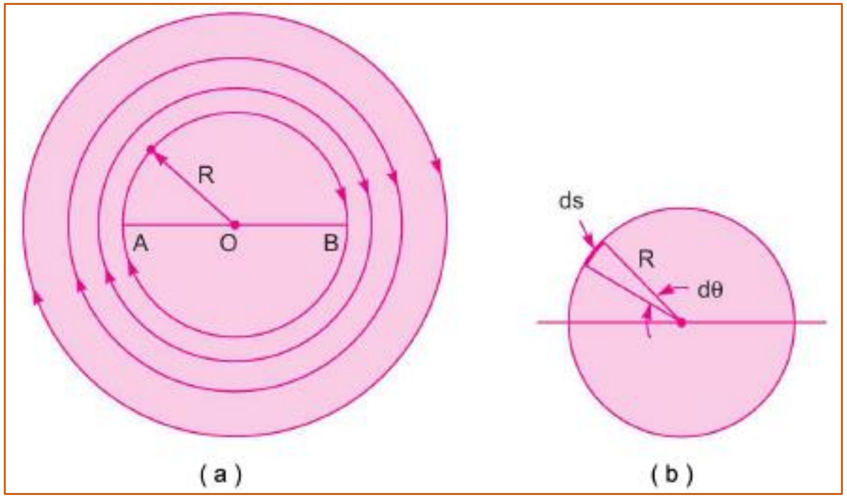
Equation for the free vortex flow

$$u_{\theta_1} * r = constant\ (say\ k) \qquad ...(2)$$

where u_{θ_1}: velocity of fluid in a free-vortex flow

r = Radius, where velocity is u_{θ_1}

The flow-pattern for a free-vortex flow consists of streamlines which are series of concentric circles.



Streamlines for free vortex

Circulation for the Flow-field in a Free-Vortex

For FV flow, stream velocity at any point on a circle of radius R = tangential velocity at that point → angle between the streamlines and tangent on the stream is zero.

Length of the element ds; $ds = Rd\theta$

$U = u_{\theta_1}$; $\cos\theta_1 = 1$; $ds = Rd\theta$

Use in eq. (1) to get the circulation for a free vortex as

$$\Gamma = \oint u_{\theta_1} * 1 * Rd\theta$$

From eq. (2), for a radius R, we have,

$$u_{\theta_1} * R = K$$

$$\Gamma = \oint Kd\theta = 2\pi K = 2\pi u_{\theta_1} * R$$

$$u_{\theta_1} = \frac{\Gamma}{2\pi R}$$

$$\oint d\theta = 2\pi$$

$$K = u_{\theta_1} * R$$

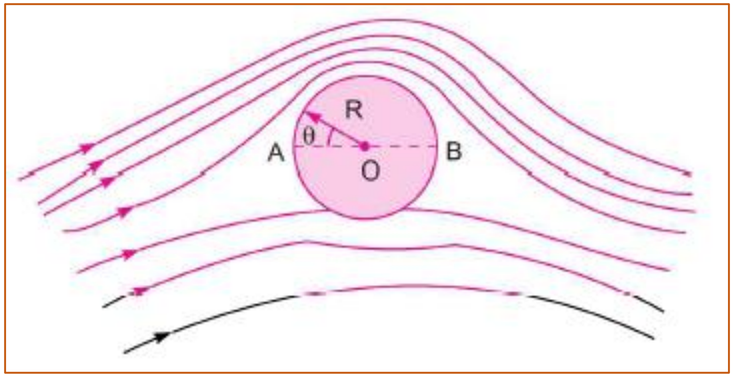
Flow over Cylinder due to Constant Circulation

The flow pattern over a cylinder to which a constant circulation (Γ) is imparted is obtained by combining the flow patterns.

The resultant flow pattern is shown in Fig. below.

The velocity at any point on the surface of the cylinder is

$$\boxed{u = u_\theta + u_{\theta_1} = 2U\,sin\theta + \frac{\Gamma}{2\pi R}}$$



Flow pattern over a rotating cylinder

Flow over Cylinder due to Constant Circulation contd…

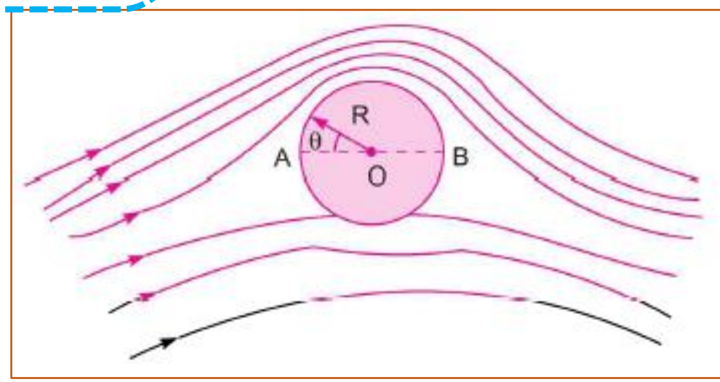
For upper half portion of cylinder	For lower half portion of cylinder
$0° < \theta < 180°$	$180° < \theta < 360°$
$2U \sin\theta > 0$	$2U \sin\theta < 0$

component of velocity

$$V_{upper\ half} > V_{lower\ half}$$

Bernoulli's theorem: velocity ↓, pressure ↑

$$P_{lower\ half} > P_{upper\ half}$$



Flow pattern over a rotating cylinder

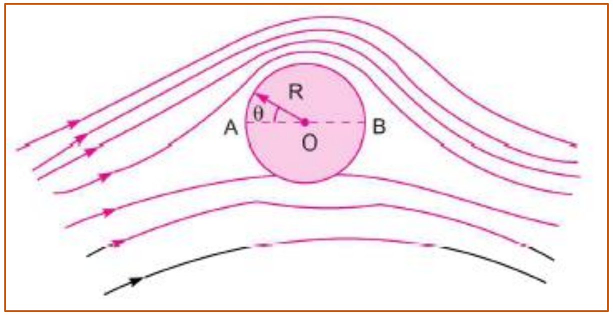
Pressure difference on the two portions → a force [lift] will be acting of the cylinder in a direction ⊥ to the direction of flow.

By rotating a cylinder at constant velocity in a uniform flow field, a lift force can be developed

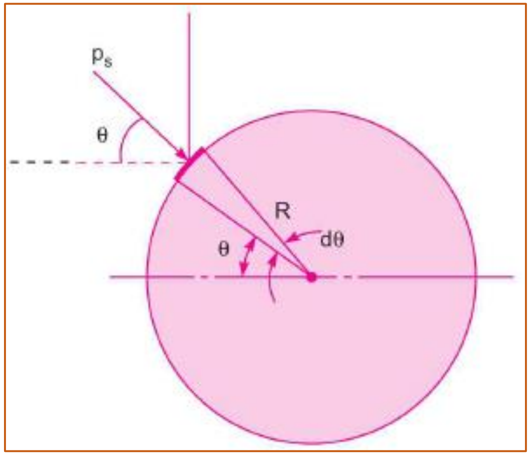
Expression for Lift Force Acting on Rotating Cylinder

- Cylinder is rotating in a uniform flow field. **Resultant flow pattern→Fig.**

Consider a small length of the element on the surface of the cylinder



p_s = Pressure on the surface of the element
ds = Length of element
R = Radius of cylinder
$d\theta$ = Angle made by length ds at centre of cylinder
p = Pressure of the fluid far away from the cylinder
U = Velocity of fluid far away from the cylinder
u_s = Velocity of fluid on the surface of the cylinder



Lift on a rotating cylinder

Apply Bernoulli's equation to a point far away from cylinder and to a point lying on the surface of cylinder such that both the points are on the same horizontal line.

CWS??

Expression for Lift Force Acting on Rotating Cylinder

$$\frac{P}{\rho g}+\frac{U^2}{2g}=\frac{p_s}{\rho g}+\frac{u_s^2}{2g}$$

$$\frac{p_s}{\rho g}=\frac{P}{\rho g}+\frac{U^2}{2g}\left[1-\frac{u_s^2}{U^2}\right] \qquad \text{...(1)}$$

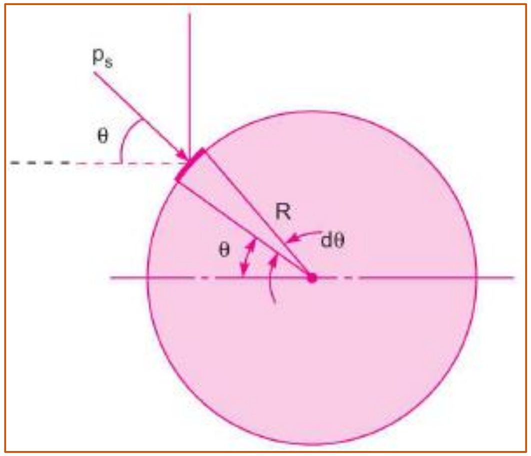
Velocity on the surface of cylinder

$$u_s=u=2U\sin\theta+\frac{\Gamma}{2\pi R}$$

Put in eq. (1) and get

CWS??

$$\boxed{p_s=p+\frac{\rho g U^2}{2g}\left[1-4\,sin^2\theta-\frac{\Gamma^2}{4\pi^2R^2U^2}-\frac{4\,sin\theta\;\Gamma}{U*2\pi R}\right]} \qquad \text{...(2)}$$



Lift on a rotating cylinder

Expression for Lift Force Acting on Rotating Cylinder

Lift force acting on small length ds on element (dF_L), due to pressure p_s

= Component of p_s, ⊥ to flow × Area of element

$= (-p_s \sin\theta) * (ds * L)$

-ve is taken, as $p_s \sin\theta$ is acting ↓

L = length of the cylinder

$$ds = R * d\theta$$

$$dF_L = -p_s \sin\theta * Rd\theta * L \quad \text{...(3)}$$

Total lift force: integrate over the centre surface of cylinder

$$F_L = \int_0^{2\pi} -p_s \,\mathrm{R}\,\mathrm{L} \sin\theta \; d\theta$$

Put p_s from eq. (2) and get **CWS??**

$$F_L = -RL \int_0^{2\pi} p \sin\theta + \frac{\rho g U^2}{2g}(\sin\theta - 4\, sin^3\theta - \frac{\Gamma \sin\theta}{4\,\pi^2 R^2 U^2} - \frac{4\, sin^2\theta\; \Gamma}{U * 2\pi R})]d\theta$$

Expression for Lift Force Acting on Rotating Cylinder

$$\int_0^{2\pi} \sin\theta \, d\theta = \int_0^{2\pi} sin^3\theta \, d\theta = 0$$

$$F_L = -RL \int_0^{2\pi} \frac{\rho g U^2}{2g} \left(-\frac{4 sin^2\theta \, \Gamma}{U * 2\pi R} \right) d\theta$$

$$= RL \frac{\rho g U^2}{2g} \frac{4\Gamma}{U * 2\pi R} \int_0^{2\pi} sin^2\theta \;\, d\theta = \frac{L}{g} \frac{\rho g U \Gamma}{\pi} \int_0^{2\pi} sin^2\theta \;\, d\theta$$

$$\boxed{\int_0^{2\pi} sin^2\theta \;\, d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{2\pi} = \left(\frac{2\pi}{2} - \frac{\sin 4\pi}{4} \right) = \pi}$$

$$\boxed{F_L = \frac{L}{g} \frac{\rho g}{\pi} \, U\Gamma * \pi = \rho L U \Gamma}$$

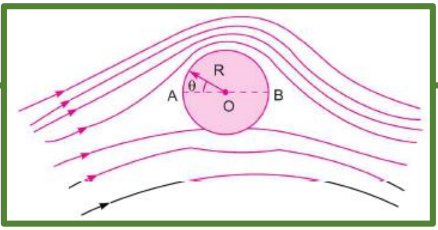
Kutta-Joukowski equation

Drag Force Acting on a Rotating Cylinder

Fig.: Resultant flow pattern for a rotating cylinder in a uniform flow field.

It is symmetrical about the vertical axis of the cylinder.

Velocity (& pressure) distribution is symmetrical



There will be no drag on the cylinder

Expression for Lift Co-efficient for Rotating Cylinder

Recap

$$F_L = C_L A \frac{\rho U^2}{2} \quad \text{...(1)}$$

C: Lift co-efficient, A = Projected area $[2RL]$

U = Free stream velocity or uniform velocity of flow.

Expression for Lift Co-efficient for Rotating Cylinder

Recap

For a rotating cylinder, the lift force is given by

$$F_L = \rho L U \Gamma \quad \text{...(2)}$$

From eqs. (1) & (2)

$$\rho L U \Gamma = C_L * 2RL * \frac{\rho U^2}{2} \quad \Rightarrow \quad C_L = \frac{\rho L U \Gamma}{RL\rho U^2} = \frac{\Gamma}{RU} \quad \text{...(3)}$$

Also,

$$u_{\theta_1} = \frac{\Gamma}{2\pi R} \quad \Rightarrow \quad \frac{\Gamma}{R} = 2\pi u_{\theta_1}$$

From eq. (3)

$$C_L = \frac{2\pi u_{\theta_1}}{U}$$

u_{θ_1}: velocity of rotation of the cylinder in the tangential direction

Location of Stagnation Points for a Rotating Cylinder in a Uniform Flow-field

Stagnation points: points on the surface of the cylinder, where velocity is zero. For a rotating cylinder, the resultant velocity is

Recap

$$u = 2U \sin\theta + \frac{\Gamma}{2\pi R}$$

For stagnation point, $u = 0$

$$\sin\theta = -\frac{\Gamma}{4\pi U R} \qquad \text{...(1)}$$

Solution of eq. (1): stagnation points' locations.

As $\sin\theta$ is negative; $180° < \theta < 360°$. The two values of θ are such that $180° < \theta_1 < 270°$ and $270° < \theta_2 < 360°$.

For a single stagnation point, $\theta = 270^{\circ}$ → From eq. (1) $\boxed{\Gamma = 4\pi U R}$

Magnus Effect

When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon of the **lift force produced by a rotating cylinder in a uniform flow** is known as Magnus Effect. This fact was investigated by a German physicist H.G. Magnus.

Magnus Effect:

https://www.youtube.com/watch?v=3jiXhnue8HU

Problem

The air having a velocity of $40\ m/s$ is flowing over a cylinder of diameter $1.5\ m$ and length $10\ m$, when the axis of the cylinder is perpendicular to the air stream. The cylinder is rotated about its axis and a lift of $6867\ N$ per metre length of the cylinder is developed. Find the speed of rotation and location of the stagnation points. The density of air is given as $1.25\ kg/m^3$.

Solution

Velocity of air,	$U = 40$ m/s
Diameter of cylinder,	$D = 1.5$ m
Length of the cylinder,	$L = 10$ m
Lift/metre length,	$\dfrac{F_L}{L} = 6867$ N
Density of air,	$\rho = 1.25$ kg/m^2

From equation (14.16), we have $F_L = \rho L U \Gamma$ or $\dfrac{F_L}{L} = \rho U \Gamma$

$$\therefore \qquad 6867 = 1.25 \times 40 \times \Gamma$$

$$\therefore \qquad \Gamma = \frac{6867}{1.25 \times 40} = 137.36 \text{ m}^2\text{/s.}$$

Let the speed of rotation corresponding to circulation 137.36 = u_θ. Using equation (14.14),

$$u_\theta = \frac{\Gamma}{2\pi R} = \frac{137.36}{2\pi \times \frac{D}{2}} = \frac{137.36 \times 2}{2\pi \times 1.5} = 29.15 = \frac{\pi D N}{60}$$

$$\therefore \qquad N = \frac{60 \times 29.15}{\pi \times D} = \frac{60 \times 29.15}{\pi \times 1.5} = \mathbf{371.15 \text{ r.p.m. Ans.}}$$

Position of stagnation points are given by equation (14.19)

$$\sin\theta = -\frac{\Gamma}{4\pi U R} = -\frac{137.36}{4\pi \times 40 \times \frac{D}{2}} = -\frac{137.36 \times 2}{4\pi \times 53 \times 1.5}$$

$$= -0.3643 = -\sin 21.36°$$

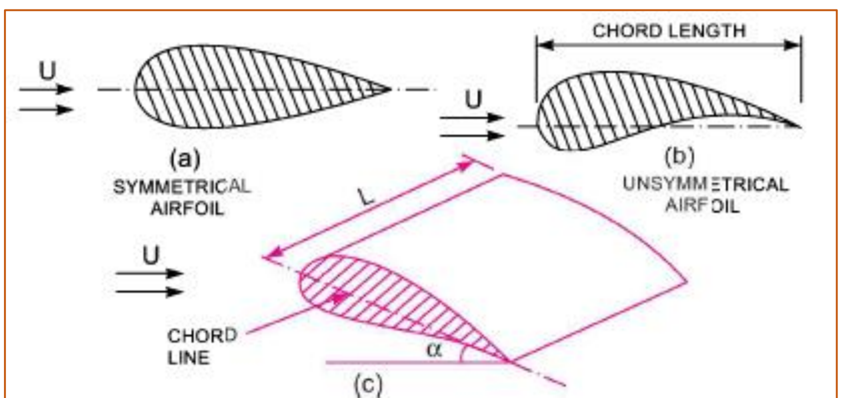
$$= \sin(180° + 21.36°) \text{ and } \sin[360° - 21.36°]$$

$$= \sin 201.36° \text{ and } \sin 338.64°$$

$$\therefore \qquad \theta = \mathbf{201.36° \text{ and } 338.64°. \text{ Ans.}}$$

Development of lift on an airfoil

Fig. below shows the two shapes of the airfoils, which are stream-lines bodies which may be symmetrical or unsymmetrical in shapes. The airfoil is characterized by its chord length C, angle of attack α (which is the angle between the direction of the fluid flowing and chord line) and span L of the airfoil. The lift on the airfoil is due to negative pressure created on the upper part of the airfoil. The drag force on the airfoil is always small due to the design of the shape of the body, which is stream-lined.



Shapes of airfoils

Development of lift on an airfoil

From the theoretical analysis, the circulation Γ developed on the airfoil so that the stream-line at the trailing edge of the airfoil is tangential to the airfoil, is given as

$$\Gamma = \pi C U \sin\alpha$$

Lift force is given by

$$F_L = \rho L U \Gamma = \pi \rho C U^2 L \sin\alpha$$

Recap

Also,

$$F_L = C_L * A * \frac{\rho U^2}{2} = C_L * (C * L) * \frac{\rho U^2}{2}$$

Equate above two

$$C_L = 2\pi \sin\alpha$$

Observation: co-efficient of lift depends upon the angle of attack

Steady-state of a Flying Object (aeroplane):

- Weight = lift force
- Engine thrust = drag force

$$W = C_L \frac{\rho A U^2}{2}$$

Problem

A jet plane which weighs $29430\ N$ and has a wing area of $20\ m^2$ flies at a velocity of $250\ km/hr$. When the engine delivers $7357.5\ kW$. 65% of the power is used to overcome the drag resistance of the wing. Calculate the co-efficient of lift and co-efficient of drag for the wing. Take density of air equal to $1.21\ kg/m^3$.

Solution

Weight of plane, $W = 29430$ N

Wing area, $A = 20\ \text{m}^2$

Velocity of plane, $U = 250$ km/hr $= \dfrac{250 \times 1000}{60 \times 60}$ m/s $= 69.44$ m/s

Power delivered by engine $= 7357.5$ kW

Power required to overcome drag resistance

$= 65\%$ of $7357.5 = 0.65 \times 7357.5 = 4782.375$ kW.

Density of air, $\rho = 1.21\ \text{kg/m}^3$

Now, weight of plane $= \text{Lift force} = C_L \times A \times \dfrac{\rho U^2}{2}$

$$\therefore \quad 29430 = C_L \times 20 \times 1.21 \times \frac{69.44^2}{2}$$

$$\therefore \quad C_L = \frac{29430 \times 2}{20 \times 1.21 \times 69.44^2} = \mathbf{0.5046.\ Ans.}$$

Let $F_D = \text{Drag force}$

Power required to overcome drag resistance $= \dfrac{F_D \times U}{1000}$ kW

$$\therefore \quad 4782.375 = \frac{F_D \times 69.44}{1000}$$

$$\therefore \quad F_D = \frac{4782.375 \times 1000}{69.44} = 68870.6\ \text{N}$$

Now drag force, $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$

$$\therefore \quad 68870.6 = C_D \times 20 \times \frac{1.21 \times 69.44^2}{2}$$

$$\therefore \quad C_D = \frac{68870.6 \times 2}{20 \times 1.21 \times 69.44^2} = \mathbf{1.18.\ Ans.}$$

Thank you ☺