Tutorial 3

1\. Let R_1 & R_2 be two equivalence relations on A.

Consider $R_1 \cap R_2$.

$\because$ R_1 is reflexive, $(a, a) \in R_1 \ \forall a \in A$.

$\because$ R_2 is reflexive, $(a, a) \in R_2 \ \forall a \in A$.

$\therefore$ $R_1 \cap R_2$ will contain $(a, a) \ \forall a \in A$.

$\therefore$ $R_1 \cap R_2$ is reflexive.

Let $(a, b) \in R_1 \cap R_2$.

$\Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$

$\Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ $\quad \because$ R_1 and R_2 are symmetric.

$\Rightarrow (b, a) \in R_1 \cap R_2$

$\Rightarrow R_1 \cap R_2$ is symmetric.

Let (a, b) & $(b, c) \in R_1 \cap R_2$

$\Rightarrow (a, b)$ & $(b, c) \in R_1$ and (a, b) & $(b, c) \in R_2$

$\Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2$ $\quad \because$ R_1 & R_2 are Transitive...

$\Rightarrow (a, c) \in R_1 \cap R_2$

$\Rightarrow R_1 \cap R_2$ is Transitive.

2\. Consider $A = \{1, 2, 3\}$ $\quad R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$R_2 = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

Both R_1 & R_2 are Equivalent.

$R_1 \cup R_2 = \{(1,1) (2,2) (3,3) (1,2) (2,1) (2,3) (3,2)\}$

For transitive property, $(1,2)$ & $(2,3) \rightarrow (1,3)$

$(3,2)$ & $(2,1) \rightarrow (3,1)$

Since these elements are not contained in $R_1 \cup R_2$, The union of two equivalent relations is not necessarily equivalent

3\) $A = \{1, 2, 3, 4\}$. $\quad A \times A = \left\{ \begin{matrix} 11 & \dots & 14 \\ 21 & & 24 \\ 31 & & 34 \\ 41 & \dots & 44 \end{matrix} \right\}$

$(a, b) \, R \, (c, d)$ if $ad = bc$

Ans $[(1,1)]_R = \{(1,1), (2,2), (3,3), (4,4)\}$

$[(1,2)]_R = \{(1,2), (2,4)\}$

$[(1,3)]_R = \{(1,3)\}$

$[(1,4)]_R = \{(1,4)\}$

$d = \frac{3}{2}c$

$[(2,1)]_R = \{(2,1),(4,2)\}$ $[(2,3)] = [(2,3)]$

$[(3,1)]_R = \{(3,1)\}$ $[(3,2)]_R = \{(3,2)\}$ $[(3,4)] = \{(3,4)\}$

$[(4,1)]_R = \{(4,1)\}$ $\cancel{[(4,2)]_R = \{(4,2)\}}$ $[(4,3)]_R = \{(4,3)\}$

4. $A = \{1,2,3,4\}$ R on $A \times A$ $(a,b) R (c,d)$ if $a+d = b+c$

Let $(a,b) \in A \times A$.

$\cancel{a+b \ne a+b}$

$a+b = a+b$

$a+b = b+a$

$\therefore (a,b) R (a,b)$

$\therefore R$ is reflexive

$1+2 = 2+1$

Let $(a,b) R (c,d)$

$a+d = b+c$

$\therefore -c-b = -d-a$

$\therefore c+b = d+a$

$\therefore (c,d) R (a,b)$

R is Symmetric

Let $(a,b) R (c,d)$ & $(c,d) R (e,f)$

$\Rightarrow a+d = b+c \quad \cdots ①$ $c+f = d+e \quad \cdots ②$

$\Rightarrow ① + ② \Rightarrow a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$

$a+f = b+e$

$(a,b) R (e,f)$

$\therefore R$ is Transitive.

$\therefore R$ is Equivalent.

$\cancel{[(1,1)]_R = \{(1,1)\}}$ $\cancel{[(2,2)]_R = \{(2,2),(1,3),(3,1)\}}$

$\cancel{[(3,3)]_R = \{(3,3),(2,4),(4,2)\}}$

$\cancel{[(4,4)]_R = \{(4,4)\}}$

$[(1,1)]_R = \{(1,1),(2,2),(3,3),(4,4)\}$

$[(1,2)]_R = \{(1,2),(2,3),(3,4)\}$

$[(1,3)]_R = \{(1,3),(2,4)\}$

$[(1,4)]_R = \{(1,4)\}$

$1+4 = 2+3$ $1+2 = 2+1$

$[(2,1)]_R = \{(2,1),(3,2),(4,3)\}$

$[(3,1)]_R = \{(3,1),(4,2)\}$

$[(4,1)]_R = \{(4,1)\}$

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5) ~~Let~~ R ~~R~~ = a+b is even.

Let $a \in \mathbb{Z}$

$a + a = 2a$ = Even Number.

$\therefore (a,a) \in R$ ~~and~~ $\therefore R$ is Reflexive.

Let $(a,b) \in R$.

$\Rightarrow a+b =$ ~~Even~~ $2k$ k is an integer

$\Rightarrow b+a = 2k$

$\Rightarrow (b,a) \in R$.

$\Rightarrow R$ is symmetric.

Let (a,b) & $(b,c) \in R$.

$\Rightarrow a+b=2k$ & $b+c=2j$

$\Rightarrow a+b+b+c = 2k+2j$

$\Rightarrow a+c = 2(k+j-b)$ = Even

$\Rightarrow$ ~~a+c~~ $(a,c) \in R$

$\Rightarrow R$ is Transitive.

6) ~~Let~~ aRb ~~iff~~ iff $(2a+5b)\%7 = 0$

Let $a \in \mathbb{Z}$

$\Rightarrow 2a+5a = 7a$ is divisible by 7. $\therefore (a,a) \in R \ \forall a \in \mathbb{Z}$

$\Rightarrow$ ~~2a~~ R is Reflexive

Let $(a,b) \in R$.

$2a+5b = 7k$

Add $2b+5a$ on both sides.

$7a+7b = 7k +$ ~~2b+5b~~ $2b+5a$

$\underbrace{7(a+b-k)}_{\text{Multiple of 7}} = 2b+5a$

$\therefore (b,a) \in R \therefore R$ is Symmetric

Let (a,b) & $(b,c) \in R$

$\Rightarrow 2a+5b = 7m$ & $2b+5c = 7n$

$\Rightarrow 2a+7b+5c = 7m+7n$

$\Rightarrow 2a+5c = \underbrace{7(m+n-b)}_{\text{Multiple of 7}}$

$\Rightarrow (a,c) \in R$

$\Rightarrow R$ is Transitive