Time complexity for CPU: O($n^2$)

The CPU implementation shows a time complexity of O($n^2$), which is evident from the data. As the input size increases by a factor of 10, the runtime increases approximately by a factor of 100. This quadratic growth becomes especially noticeable with larger inputs. For instance, the runtime jumps from 0.001184s for size 1000 to 11.331399s for size 100000. Beyond size 1000000, the algorithm takes too long to run efficiently. This is consistent with the expectation for an O($n^2$) algorithm, where each element needs to be compared or processed with every other element in some way, leading to quadratic behavior.

Time complexity for naïve approach: O(n)

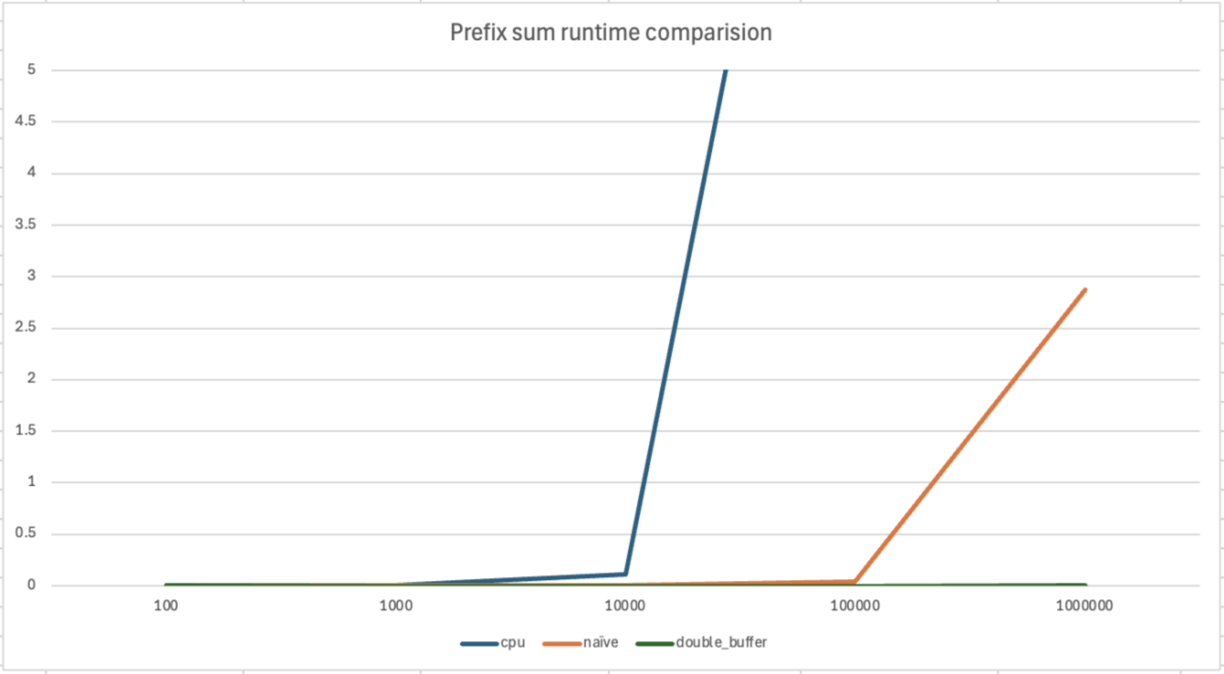
The naive scan algorithm on the GPU has a time complexity of O(n). This is demonstrated in the data where the runtime scales linearly with the input size. For example, when the input size increases from 100000 to 1000000 (a 10x increase), the runtime grows from 0.038163s to 2.869606s, which is roughly a factor of 10 increase. This behavior matches the O(n) time complexity, where the algorithm needs to process each element in the array sequentially, leading to linear scaling with the input size.

Time complexity for double buffer approach: O(log n)

The double buffer approach exhibits a much more efficient time complexity of O(log n). This is clearly reflected in the data, where the runtime increases very slowly as the input size grows. Interestingly, the runtime decreases from 0.00012s for size 100 to 0.000012s for size 1000, likely due to better GPU utilization at slightly larger sizes. As the input size continues to grow, the runtime increases much more gradually compared to the other two approaches. For example, the runtime for size 100000 is 0.000069s and for size 1000000 is 0.000412s. This is consistent with the logarithmic time complexity, where the work done by the algorithm grows much slower thanlinearly as the input size increases.

Graph is on following page.

| | 100 | 1000 | 10000 | 100000 | 1000000 |
|---|---|---|---|---|---|
| cpu | 0.000013 | 0.001184 | 0.113064 | 11.331399 | 1123.04855 |
| naïve | 0.000158 | 0.000159 | 0.001535 | 0.038163 | 2.869606 |
| double_buffer | 0.00012 | 0.000012 | 0.000014 | 0.000069 | 0.000412 |


Prefix sum runtime comparision
cpu
naïve
double_buffer