MATH 3310

Grading Rubric for Homework Assignment #3: A non-intersection graph, an intersection graph, tournaments.

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1. $G = (\{\text{2-sets of } [5]\}, E)$, where, for $x, y \in V$, $xy \in E$ if and only if $x \cap y = \emptyset$.

(a) $[5] \setminus \{a, b\}$ leaves 3 elements and $\binom{3}{2} = 3$	0	1	$\boxed{2}$
(b) Author uses part (a) and the FTGT* to conclude $\lvert E\rvert = 15$	0	1	$\boxed{2}$
(c) Author argues G has a $K_3 \implies \lvert[5]\rvert \geq 6$	0	1	$\boxed{2}$
(d) Author uses Euler's formula and deduces $35 \leq 30^{\dagger}$, showing G ain't planar	0	1	$\boxed{2}$
(d) Author mentions (1 point) or argues (2 points) deg(any face) ≥ 5	0	1	$\boxed{2}$
(e) Author shows G contains a subgraph homeomorphic to $K_{3,3}$	$\boxed{0}$	1	2
• Readability: Seem to've found a K_{3,3} subgraph, but there isn't one in this graph.	0	1	$\boxed{2}$
• Fluency:	0	1	$\boxed{2}$

2. $\Gamma = (\{\text{3-sets of } [5]\}, E)$, where, for $x, y \in V$, $xy \in E$ if and only if $\lvert x \cap y\rvert = 2$.

(a) Author claims $\deg(v) = 6$	0	1	$\boxed{2}$
(a) ... arguing from Γ's definition and set relationships	0	1	$\boxed{2}$
(b) Author claims $\lvert E\rvert = 30$ ...	0	1	$\boxed{2}$
(b) .... using (a) and the FTGT	0	1	$\boxed{2}$
(c) Author correctly argues Γ has a K_4 (probably displaying a correctly constructed K_4 subgraph)	0	1	$\boxed{2}$
• Readability:	0	1	$\boxed{2}$
• Fluency:	0	1	$\boxed{2}$

3. There is no 4-vertex tournament in which every vertex is a queen.

• proof attempt is general: no examples, and all 4-tournaments are accounted for	0	1	$\boxed{2}$
• Author rules out vertices with score 0 and with score 3	0	1	$\boxed{2}$
• ... and forces the construction of the 4-tournament	0	1	$\boxed{2}$
• ... and finds a vertex that is not a queen	0	1	$\boxed{2}$
• Readability:	0	1	$\boxed{2}$
• Fluency:	0	1	$\boxed{2}$

4. *If a vertex in a tournament is beaten, it is beaten by a queen.* A common error is to consider I_v, for some v and $I_v \neq \emptyset$, and use the theorem which guarantees a queen in any flock to find a queen of I_v, but not show that vertex is a queen of F.

• Author assumes v is beaten, $I(v) \neq \emptyset$.	0	1	$\boxed{2}$
• Author finds q a queen of $I(v)$ ...	0	1	$\boxed{2}$
• ... and proves q is a queen of T	0	1	$\boxed{2}$
• Readability:	0	1	$\boxed{2}$
• Fluency:	0	1	$\boxed{2}$

Subtotal X = 50 /52; Scale: $\lceil X \times \frac{30}{52} \rceil =$ 29 ./30

*Fundamental Theorem of Graph Theory: *For any graph* G, $\sum_{x \in V(G)} \deg(x) = 2\lvert E(G)\rvert$.

†Or some other contradiction.