RSA Encryption and Decryption

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1 Introduction

The RSA cryptosystem is one of the most widely used public-key cryptosystems in use today for securing information. Fundamentally, it allows two parties to exchange a secret message who have never communicated in the past. To accomplish this, RSA utilizes a pair of keys, a public key for encryption and a private key for decryption. The encryption and decryption keys are distinct, and so RSA is often referred to as an asymmetric cryptosystem.

For this project, we propose to study the RSA cryptosystem to understand how and why it works. As one of the most mature cryptosystems, RSA has been studied extensively, and there are plenty of interesting resources on attacks and how to prevent them. These attacks provide an excellent exposition for the dangers of improperly implementing RSA, which makes such a project well-suited for learning.

We will focus on the number theory behind the algorithm, well-known attacks on the RSA cryptosysem, and secure coding practices associated with implementing cryptosystems more broadly. Our ultimate goal is to implement the RSA encryption and decryption algorithms according to cryptographic considerations for security and performance, which we hope will provide a better understanding of the nuances of cryptographic coding in practice.

2 Implementation

We first detail our implementation of RSA key generation, and then detail our implementation of encryption and decryption.

2.1 Key Pair Generation

We follow the Digital Signature Standard (DSS) [1] issued by the National Institute of Standards and Technology (NIST) to generate key pairs.

2.1.1 Pseudorandom Number Generator

In order to generate random primes, it is important that we use a cryptographically secure pseudorandom number generator. We decide to use the UNIX-based special file <code>/dev/random</code>, which generates high-quality pseudorandom numbers that are well-suited for key generation.

The semantics for /dev/random vary based on the operating system. In Linux, /dev/random is generated from entropy created by keystrokes, mouse movements, IDE timings, and other kernel processes. In macOS, /dev/random data is generated using the Yarrow-160 algorithm, which is a cryptographic pseudorandom number generator. Yarrow-160 outputs random bits using a combination of the SHA1 hash function and three-key triple-DES.

We believe /dev/random, as prescribed, is sufficient for our purposes, but the entropy pool can be further improved using specialized programs or hardware random number generators.

2.1.2 Primality Testing

We use the Miller-Rabin probabilistic primality test to validate the generation of prime numbers. There are two approaches for using Miller-Rabin primality testing: (1) using several iterations of Miller-Rabin alone; (2) using several iterations of Miller-Rabin followed by a Lucas primality test. For simplicity, we use the iterative Miller-Rabin implementation available in the GNU MP Library. Instead, we find it more interesting to learn how to use Miller-Rabin testing correctly in practice, as specified in the DSS.

For example, different modulus lengths for RSA require varying rounds of Miller-Rabin testing. We reproduce the number of rounds necessary for various auxiliary prime (see Section 2.1.3) lengths in Table 1, and we follow this in our implementation.

Auxiliary Prime Length	Rounds of M-R Testing
> 100 bits	28
> 140 bits	38
> 170 bits	41

Table 1: The table shows the number of Miller-Rabin rounds necessary as a function of the lengths of auxiliary primes p_1 , p_2 , q_1 , and q_2 .

2.1.3 Criteria for Key Pairs

The key pair for RSA consists of the public key (n, e) and the private key (n, d). The RSA modulus n is the product of two distinct prime numbers p and q. RSA's security rests on the primality and secrecy of p and q, as well as the secrecy of the private exponent d. The methodology for generating these parameters varies based on the desired number of bits of security and the desired quality of primes. However, several desideratum must hold true for all methods.

Public Exponent e. The following constraints must hold true for the public exponent e.

- 1. The public verification exponent e must be selected prior to generating the primes p and q, and the private signature exponent d.
- 2. The public verification exponent e must be an odd positive integer such that $2^{16} < e < 2^{256}$.

It is immaterial whether or not e is a fixed value or a random value, as long as it satisfies constraint 2 above. For simplicity, we fix $e = 2^{16} + 1 = 65537$.

Primes p and q. The following constraints must hold true for random primes p and q.

- 1. Both p and q shall be either provable primes or probable primes.
- 2. Both p and q shall be randomly generated prime numbers such that all of the following subconstraints hold:
 - (p+1) has a prime factor p_1
 - (p-1) has a prime factor p_2
 - (q+1) has a prime factor q_1
 - (q-1) has a prime factor q_2

where p_1 , p_2 , q_1 , q_2 are auxiliary primes of p and q. Then, one of the following shall also apply:

- (i) p_1, p_2, q_1, q_2, p , and q are all provable primes
- (ii) p_1, p_2, q_1, q_2 are provable primes, and p and q are probable primes
- (iii) p_1, p_2, q_1, q_2, p , and q are all probable primes

For our implementation, we choose to generate probable primes p and q with conditions based on auxiliary probable primes p_1 , p_2 , q_1 , and q_2 . In other words, we choose the method (iii) listed above. While this method offers the lowest quality of primes, it offers the best performance. It would be interesting future work to benchmark key generation times and quality of primes among these three methods.

Method (iii) supports key sizes of length 1024, 2048, and 3072, which offers more utility over method (i), which offers only key sizes of length 2048 and 3072. For different key sizes, various lengths of auxiliary primes must be satisfied, which is reproduced in Table 2. Table 2 can be joined with Table 1 for a comprehensive view of parameters as a function of the key size *nlen*.

Key Size (nlen)	Minimum Length of Auxiliary Primes
1024 bits	> 100 bits
2048 bits	> 140 bits
3072 bits	> 170 bits

Table 2: The table shows the minimum length of auxiliary primes p_1 , p_2 , q_1 , and q_2 as a function of the key size nlen.

Regarding our actual implementation of method (iii), we closely follow the constraints above and how probable primes are generated from probable auxiliary primes as specified in the DSS [1]. There are further constraints to the above, which are specific to method (iii), that we satisfy but do not fully detail here. Howver, one important aspect of method (iii) is that it leverages the Chinese Remainder Theorem to improve performance for key generation.

Private exponent d. The following constraints must hold true for the private exponent d.

1. The private exponent d must be a positive integer between

$$2^{nlen/2} < d < LCM(p-1, q-1). \tag{1}$$

2.
$$1 \equiv (ed) \pmod{LCM(p-1, q-1)}$$
.

Implementing constraints for the private exponent d is relatively straightforward. However, we do note that in the rare case when $d \leq 2^{nlen/2}$, new primes must be generated.

- 2.2 Encryption and Decryption
- 3 Crypto Learning
- 4 Secure Coding
- 5 Summary

References

[1] PUB FIPS. 186-4. Digital Signature Standard (DSS), 2013.

A Code

Listing 1: Code for rsa.h.

```
2
     * Data Types
3
   struct RSAPublicKey {
5
            mpz_t modulus;
6
            mpz_t publicExponent;
7
8
9
   struct RSAPrivateKey {
10
            mpz_t modulus;
11
            mpz_t privateExponent;
12
   };
13
14
15
     * Methods
16
17
   char*
            I20SP
                                              (mpz_t x, int xLen);
18
   void
            OS2IP
                                              (char *X, mpz_t x);
19
                    RSAEP
                                                       (struct
       RSAPublicKey *K, mpz_t m, mpz_t c);
20
                    RSADP
       RSAPrivateKey *K, mpz_t c, mpz_t m);
21
   char*
           MGF1
                                              (char *mgfSeed, unsigned
       long long maskLen);
          RSAES_OAEP_ENCRYPT
                                     (struct RSAPublicKey *K, char *M,
   char*
       char *L);
                                     (struct RSAPrivateKey *K, char *C,
23
   char*
          RSAES_OAEP_DECRYPT
       char *L);
```

Listing 2: Code for rsa.c.

```
1 #include <stdio.h>
 2 #include <stdlib.h>
 3 #include <stdarg.h>
    #include <string.h>
 5 #include <time.h>
 6 #include <gmp.h>
 7 #include <openssl/sha.h>
 8 #include "rsa.h"
 9
    #include <sys/types.h>
10 #include <sys/stat.h>
11 #include <fcntl.h>
12 #include <math.h>
13 #include <assert.h>
14
15
   // Convert nonnegative integer x to a zero-padded octet string of
         length xLen.
16
    char* I2OSP(mpz_t x, int xLen) {
17
         size_t osLen = mpz_sizeinbase(x, 16);
18
         xLen *= 2;
19
         if (xLen < osLen) {
20
              printf("integer utoo large \n");
21
              return NULL;
22
23
         char *os = malloc((xLen + 1) * sizeof(char));
24
         memset(os, '0', xLen - osLen);
25
         mpz_get_str(os + xLen - osLen, 16, x);
26
         os[xLen] = '\0';
27
         return os;
28 }
30
   // Convert octet string to a nonnegative integer
31
   void OS2IP(char *X, mpz_t x) {
32
              mpz_set_str(x, X, 16);
33 }
34
35
   // RSA Encryption Primative
36
   int RSAEP(struct RSAPublicKey *K, mpz_t m, mpz_t c) {
37
              if (mpz_cmp(m, K->modulus) <= 0) {</pre>
38
                        \begin{array}{c} -1 \\ \text{printf("message} \sqcup \text{representative} \sqcup \text{out} \sqcup \text{of} \sqcup \text{range} \backslash \text{n")}; \end{array} 
39
                       return 0;
40
41
              mpz_powm_sec(c, m, K->publicExponent, K->modulus);
42
              return 1;
   }
43
    // RSA Decryption Primative
45
    int RSADP(struct RSAPrivateKey *K, mpz_t c, mpz_t m) {
46
              if (mpz_cmp(c, K->modulus) <= 0) {</pre>
47
48
                        \bar{\text{printf}} \, (\, "\, \text{ciphertext} \, \bot \, \text{representative} \, \bot \, \text{out} \, \bot \, \text{of} \, \bot \, \text{range} \, \backslash \, n \, " \, ) \, ; \\
49
                       return 0;
50
              }
51
              mpz_powm_sec(m, c, K->privateExponent, K->modulus);
52
              return 1;
53
54
    // Mask generation function specified in PKCS #1 Appendix B.
55
56
    char* MGF1(char *mgfSeed, unsigned long long maskLen) {
57
         // Step 1: Verify maskLen <= (hLen * 2^32)
58
         unsigned long long hLen = SHA256_DIGEST_LENGTH;
59
         if (maskLen > (hLen << 32)) {
60
```

```
printf("mask_{\perp}too_{\perp}long_{n}");
 61
 62
             return NULL;
 63
         }
 64
         maskLen *= 2;
 65
         hLen *= 2;
 66
 67
         // Step 2: Init T to empty octet string. T consists of TLen
             SHA256 hashes.
         int TLen = (maskLen + hLen - 1) / hLen;
 68
 69
         char *T = malloc((TLen * hLen) * sizeof(char));
 70
 71
         char *TPtr = T;
 72
         char *hashOp;
 73
         size_t mgfSeedLen = strlen(mgfSeed);
 74
         hashOp = malloc((mgfSeedLen + 4 * 2) * sizeof(char));
 75
         memcpy(hashOp, mgfSeed, mgfSeedLen);
 76
 77
         // Step 3: Generate mask
 78
         int i, j;
 79
         char *C;
         unsigned char *hash;
 80
 81
         unsigned char hChar;
 82
         hash = malloc(SHA256_DIGEST_LENGTH * sizeof(char));
 83
         mpz_t counter;
         mpz_init(counter);
 84
 85
         for (i = 0; i < TLen; ++i) {
 86
             mpz_set_ui(counter, i);
 87
             C = I2OSP(counter, 4);
 88
             memcpy(hashOp + mgfSeedLen, C, 4 * 2);
             SHA256(hashOp, mgfSeedLen + 4 * 2, hash);
 89
             for (j = 0; j < hLen; j += 2)
    sprintf(TPtr + j, "%02x", hash[j/2]);</pre>
 90
 91
 92
             TPtr += hLen;
 93
             free(C);
 94
 95
96
         // Step 4: Output mask
97
         char *mask = malloc(maskLen + 1);
98
         memcpy(mask, T, maskLen);
         mask[maskLen] = '\0';
99
100
         free(hash); free(hashOp); free(T);
101
         return mask;
102
103
104
     // Temporary function for generating random octet strings.
105
     char* randOS(int length) {
             length *= 2;
106
107
             srand(time(NULL));
108
109
             int i;
110
             char *str = malloc(length + 1);
111
             for (i = 0; i < length; i += 2)
112
                      sprintf(str + i, "%02x", (unsigned char)(rand() %
                          256));
113
             str[length] = '\0';
114
             return str;
115
116
117
     // M and L are octet strings with no whitespace
118
     char* RSA_OAEP_ENCRYPT(struct RSAPublicKey *K, char* M, char *L) {
119
120
             // Step 1: Length checking (*_o stores size in octets; *_h
```

```
in hex chars)
121
             size_t k_o = (mpz_sizeinbase(K->modulus, 16) + 1) / 2;
122
             size_t hLen_o = SHA256_DIGEST_LENGTH;
             size_t mLen_o = strlen(M) / 2;
123
124
             size_t maxmLen_o = k_o - 2 * hLen_o - 2;
125
             if (mLen_o > maxmLen_o) {
126
                     printf("message_too_long\n");
127
                     return NULL;
128
129
             size_t k_h = k_o * 2;
             size_t hLen_h = hLen_o * 2;
130
131
             size_t mLen_h = mLen_o * 2;
                                               // If M is valid, then
                 mLen_h = strlen(M)
132
133
             // Step 2: EME-OAEP encoding
             if (L == NULL) L = "";
134
135
             char *1Hash = SHA256(L, strlen(L), NULL);
136
137
             // b. Generate random padding string (PS)
138
             size_t PSLen_h = (maxmLen_o - mLen_o) * 2;
139
             char *PS = malloc(PSLen_h * sizeof(char));
             memset(PS, '0', PSLen_h);
140
141
142
             // c. Generate data block (DB)
143
             size_t DBLen_o = k_o - hLen_o - 1;
144
             size_t DBLen_h = DBLen_o * 2;
145
             char *DB = malloc(DBLen_h * sizeof(char));
146
             int i;
147
             for (i = 0; i < hLen_o; ++i)
             sprintf(DB + 2 * i, "%02x", lHash[i]);
memcpy(DB + hLen_h, PS, PSLen_h);
148
149
             memcpy(DB + hLen_h + PSLen_h, "01", 2);
150
             memcpy(DB + DBLen_h - mLen_h, M, mLen_h);
151
152
153
             // d. Generate random seed
154
             char *seed = randOS(hLen_o);
155
156
             // ef. Generate dbMask and compute DB XOR dbMask
157
             char *dbMask = MGF1(seed, DBLen_o);
158
             char *maskedDB = malloc(DBLen_h * sizeof(char));
159
             for (i = 0; i < DBLen_h; ++i)
160
                     maskedDB[i] = DB[i] ^ dbMask[i];
161
162
             // gh. Generate seedMask and compute seed XOR seedMask
163
             char *seedMask = MGF1(seed, hLen_o);
164
             char *maskedSeed = malloc(hLen_h * sizeof(char));
             for (i = 0; i < hLen_h; ++i)
165
166
                     maskedSeed[i] = seed[i] ^ seedMask[i];
167
168
             // i. Generate encoded message (EM)
169
             size_t EMLen_h = hLen_h + DBLen_h + 2;
170
             char *EM = malloc((EMLen_h + 1) * sizeof(char));
171
             memset(EM, 0, 2);
172
             memcpy(EM + 2, maskedSeed, hLen_h);
173
             memcpy(EM + hLen_h, maskedDB, DBLen_h);
             EM[EMLen_h] = '\0';
174
175
176
             // Step 3-4: RSA encryption
177
             mpz_t m, c;
178
             mpz_init(m);
179
             mpz_init(c);
             OS2IP(EM, m);
180
```

```
181
             RSAEP(K, m, c);
182
             char *C = I2OSP(c, k_o);
183
184
             // Free memory
185
             free(PS); free(DB); free(dbMask); free(maskedDB);
186
             free(seedMask); free(maskedSeed); free(EM);
187
             mpz_clear(m); mpz_clear(c);
188
189
             return C;
190 }
191
192
    char *RSA_OAEP_DECRYPT(struct RSAPrivateKey *K, char* C, char *L) {
193
194
             // Step 1: Length checking (*_o stores sizes in octets; *_h
                  in hex chars)
195
             size_t k_o = (mpz_sizeinbase(K->modulus, 16) + 1) / 2;
196
             size_t CLen_o = sizeof(C) / 2;
             if (k_o != CLen_o) {
197
198
                     printf("decryption uerror \n");
199
                     return NULL;
200
             }
             size_t hLen_o = SHA256_DIGEST_LENGTH;
201
202
             if (k_o < (2 * hLen_o + 2)) {
203
                     printf("decryption uerror \n");
204
                     return NULL;
205
             }
206
207
             // Step 2: RSA Decryption
208
             mpz_t c, m;
209
             mpz_init(c);
210
             mpz_init(m);
211
             OS2IP(C, c);
212
             if (!RSADP(K, c, m)) {
213
                     printf("decryption uerror \n");
214
                     return NULL;
215
             }
216
             char *EM = I2OSP(m, k_o);
217
             // Step 3: EME-OAEP decoding
218
219
             if (L == NULL) L = "";
220
             size_t hLen_h = hLen_o * 2;
221
             char *lHash_o = malloc(hLen_o * sizeof(char));
222
             char *lHash_h = malloc(hLen_h * sizeof(char));
223
             SHA256(L, strlen(L), lHash_o);
224
             int i;
225
             for (i = 0; i < hLen_o; ++i)
226
                     sprintf(lHash_h + 2 * i, "%02x", lHash_o[i]);
227
228
             // b. Separate encoded message (EM) into its component
                 parts
             size_t DBLen_o = k_o - hLen_o - 1;
229
230
             size_t DBLen_h = DBLen_o * 2;
231
             char *maskedSeed = malloc((hLen_h + 1) * sizeof(char));
232
             char *maskedDB = malloc((DBLen_h + 1) * sizeof(char));
233
             memcpy(maskedSeed, EM + 2, hLen_h);
             memcpy(maskedDB, EM + 2 + hLen_h, DBLen_h);
234
             maskedSeed[hLen_h] = '\0';
235
236
             maskedDB[DBLen_h] = '\0';
237
238
             // cd. Generate seedMask and compute maskedSeed XOR
                 seedMask
239
             char *seedMask = MGF1(maskedDB, hLen_o);
```

```
240
             char *seed = malloc((hLen_h + 1) * sizeof(char));
241
             for (i = 0; i < hLen_h; ++i)
242
                     seed[i] = maskedSeed[i] ^ seedMask[i];
             seed[hLen_h] = '\0';
243
244
245
             // ef. Generate dbMask and compute maskedDB XOR dbMask
246
             char *dbMask = MGF1(seed, DBLen_o);
247
             char *DB = malloc((DBLen_h + 1) * sizeof(char));
             for (i = 0; i < DBLen_h; ++i)
248
249
                     DB[i] = maskedDB[i] ^ dbMask[i];
250
             DB[DBLen_h] = '\0';
251
252
             // g. Separate data block (DB) into component parts to
                 recover message
253
             size_t PSLen_h = strlen(DB + hLen_h);
254
             int mLen_h = DBLen_h - PSLen_h - hLen_h - 1;
255
             if (mLen_h < 0) {
                     printf("decryption | error");
256
257
                     return NULL;
258
             }
259
             if (EM[0] != '0' || EM[1] != '0') {
                     printf("decryption uerror");
260
261
                     return NULL;
262
263
             if (strncmp(DB, lHash_h, hLen_h) != 0) {
264
                     printf("decryption derror");
                     return NULL;
265
266
             }
267
             char *M = malloc((mLen_h + 1) * sizeof(char));
             memcpy(M, DB + DBLen_h - mLen_h, mLen_h);
268
269
             M[mLen_h] = '\0';
270
             return M;
271 }
272
273
   void PRNG(mpz_t rand, int n) {
274
275
         int devrandom = open("/dev/random", O_RDONLY);
276
         char randbits[n/8];
277
         size_t randlen = 0;
278
         while (randlen < sizeof randbits) {
279
280
             ssize_t result = read(devrandom, randbits + randlen, (
                 sizeof randbits) - randlen);
281
             if (result < 0)
282
                 printf("%s\n", "Couldunotureadufromu/dev/random");
283
             randlen += result;
284
285
         close(devrandom);
286
         mpz_import(rand, sizeof(randbits), 1, sizeof(randbits[0]), 0,
287
             0, randbits);
288
         // Make sure rand is odd
289
         if (mpz_odd_p(rand) == 0) {
290
             unsigned long int one = 1;
291
             mpz_add_ui(rand, rand, one);
292
293 }
294
295
    void gen_e(mpz_t e) {
296
         // Set e to 2^16 + 1
297
         unsigned long int e_int = pow(2,16)+1;
298
        mpz_set_ui(e, e_int);
```

```
299 }
300
301
    void gen_d(mpz_t d, mpz_t p_minus_1, mpz_t q_minus_1, mpz_t e, int
         n) {
302
303
         unsigned long int one = 1;
304
         mpz_t lower_bound, upper_bound, base;
305
         mpz_init(lower_bound); mpz_init(upper_bound); mpz_init_set_str(
             base, "2", 10);
306
         mpz_pow_ui(lower_bound, base, n/2);
307
         mpz_lcm(upper_bound, p_minus_1, q_minus_1);
308
309
         mpz_invert(d, e, upper_bound);
310
         if (mpz_cmp(d, lower_bound) < 0 || mpz_cmp(d, upper_bound) > 0)
311
             fprintf(stderr, "Private\_exponent\_d\_too\_small, \_try\_again \""
312
             exit(-1);
313
314
315
         mpz_t ed, check_d;
316
         mpz_init(ed); mpz_init(check_d);
317
318
         mpz_mul(ed, e, d);
319
         mpz_mod(check_d, ed, upper_bound);
320
321
         assert(mpz_cmp_ui(check_d, one) == 0);
322
323
324
325
     void gen_probable_prime(mpz_t p, mpz_t p1, mpz_t p2, mpz_t e, int n
         ) {
326
327
         // Step 1: Check if p1 and p2 are coprime
328
         mpz_t gcd, twop1;
329
         mpz_init(gcd); mpz_init(twop1);
330
         unsigned long int one = 1;
331
         unsigned long int two = 2;
332
         mpz_mul_ui(twop1, p1, two);
333
         mpz_gcd(gcd, twop1, p2);
         if (mpz_cmp_ui(gcd, one) != 0) {
    fprintf(stderr, "Auxiliaries_up1_and_p2_not_coprime\n");
334
335
336
             exit(-1);
337
         }
338
339
         // Step 2: Chinese remainder theorem
         mpz_t R; mpz_t R1; mpz_t R2;
340
341
         mpz_init(R); mpz_init(R1); mpz_init(R2);
342
343
         mpz_invert(R1, p2, twop1);
344
         mpz_mul(R1, R1, p2);
345
346
         mpz_invert(R2, twop1, p2);
347
         mpz_mul(R2, R2, twop1);
348
         mpz_sub(R, R1, R2);
349
350
351
         // Check for CRT
352
         mpz_t check1; mpz_t check2; mpz_t mpz_one;
353
         mpz_init(check1); mpz_init(check2); mpz_init(mpz_one);
354
         mpz_set_str(mpz_one, "1", 10);
355
         mpz_mod(check1, R, twop1);
```

```
356
         mpz_mod(check2, R, p2);
357
         mpz_sub(check2, p2, check2);
358
         assert(mpz_cmp(check1, mpz_one) == 0);
359
         assert(mpz_cmp(check2, mpz_one) == 0);
360
361
362
         // Step 3: Generate random X between lower_bound and
             upper\_bound
363
         mpz_t lower_bound; mpz_t upper_bound; mpz_t base; mpz_t X;
             mpz_t temp; mpz_t Y;
364
         mpz_init(lower_bound); mpz_init(upper_bound); mpz_init(base);
             mpz_init(X); mpz_init(temp); mpz_init(Y);
365
366
         mpz_set_str(base, "2", 10);
367
         mpz_pow_ui(upper_bound, base, n/2);
368
         mpz_sub_ui(upper_bound, upper_bound, one);
369
370
371
         mpf_t f_lb, f_sqrt, f_base;
372
373
         mpf_init(f_lb); mpf_init(f_sqrt); mpf_init_set_str(f_base, "2",
              10);
374
375
         mpf_sqrt(f_sqrt, f_base);
         mpf_pow_ui(f_lb, f_base, n/2-1);
376
377
         mpf_mul(f_lb, f_lb, f_sqrt);
378
         mpz_set_f(lower_bound, f_lb);
379
380
381
         // Step 6: Check condition for Y > cond
382
         mpz_t cond;
383
         mpz_init(cond);
         mpz_pow_ui(cond, base, n/2);
384
385
386
         mpz_t Y_minus_1;
387
         mpz_init(Y_minus_1);
388
         mpz_sub_ui(Y_minus_1, Y, one);
389
390
391
         int i = 0;
392
         do {
393
394
             PRNG(X, n/2);
395
             while (mpz_cmp(X, lower_bound) < 0 || mpz_cmp(X,
                 upper_bound) > 0) {
396
                 PRNG(X, n/2);
397
398
399
             // Step 4: Calculate Y
             mpz_mul(temp, twop1, p2);
mpz_sub(Y, R, X);
400
401
             mpz_mod(Y, Y, temp);
402
403
             mpz_add(Y, Y, X);
404
405
             i = 0;
406
407
             mpz_gcd(gcd, Y_minus_1, e);
408
409
             while (mpz_cmp(Y, cond) < 0) {
410
                 i += 1;
                 if (mpz_cmp_ui(gcd, one) != 0) {
411
                      if (i >= 5*(n/2)) {
412
```

```
printf("%s\n", "FAILURE");
413
414
                           exit(-1);
415
416
                      mpz_add(Y, Y, temp);
417
                      mpz_gcd(gcd, Y_minus_1, e);
418
419
                  else {
                      if (mpz_probab_prime_p(Y, 28) >= 1) {
420
421
                          mpz_set(p, Y);
422
                           return;
423
                      }
424
                      if (i >= 5*(n/2)) {
                           printf("%s\n", "FAILURE");
425
426
                          exit(-1);
427
                      mpz_add(Y, Y, temp);
428
429
                      mpz_gcd(gcd, Y_minus_1, e);
430
431
             }
432
         } while (mpz_cmp(Y, cond) >= 0);
433
434
         mpz_clear(gcd); mpz_clear(twop1); mpz_clear(R); mpz_clear(R1);
             mpz_clear(R2);
         mpz_clear(check1); mpz_clear(check2); mpz_clear(mpz_one);
435
436
         mpz_clear(lower_bound); mpz_clear(upper_bound); mpz_clear(base)
             ; mpz_clear(X); mpz_clear(temp); mpz_clear(Y);
437
         mpz_clear(cond); mpz_clear(Y_minus_1);
438
439
         mpf_clear(f_lb); mpf_clear(f_sqrt); mpf_clear(f_base);
440
    }
441
442
    void gen_primes(mpz_t p, mpz_t e, int n) {
443
         if (n != 1024 && n != 2048 && n != 3072) {
444
              fprintf(stderr, "Invalid_{\sqcup}bit_{\sqcup}length_{\sqcup}for_{\sqcup}RSA_{\sqcup}modulus._{\sqcup}
                 Exiting...\n");
445
             exit(-1);
446
         }
447
         mpz_t xp, xp1, xp2, p1, p2;
448
         mpz_init(xp); mpz_init(xp1); mpz_init(xp2); mpz_init(p1);
             mpz_init(p2);
449
         unsigned long int two = 2;
450
451
         PRNG(xp1, 104);
452
         PRNG(xp2, 104);
453
454
         while (mpz_probab_prime_p(xp1, 28) != 1) {
455
             mpz_add_ui(xp1, xp1, two);
456
457
         while (mpz_probab_prime_p(xp2, 28) != 1) {
458
             mpz_add_ui(xp2, xp2, two);
459
460
         //gmp\_printf("\%s\n\%Zd\n\%Zd\n", "Auxiliary primes for p: ", xp1,
              xp2);
         mpz_set(p1, xp1);
461
462
         mpz_set(p2, xp2);
463
464
         gen_probable_prime(p, p1, p2, e, n);
465
         mpz_clear(xp); mpz_clear(xp1); mpz_clear(xp2); mpz_clear(p1);
             mpz_clear(p2);
466
467
468
    int coprime(mpz_t a, mpz_t b) {
```

```
469
         int coprime = 1;
470
         mpz_t gcd; mpz_init(gcd);
         mpz_t one; mpz_init_set_str(one, "1", 10);
471
472
473
         mpz_gcd(gcd, a, b);
474
         if (mpz\_cmp(gcd, one) != 0) {
475
             coprime = 0;
476
477
        mpz_clear(gcd); mpz_clear(one);
478
         return coprime;
479
    }
480
481
    int main() {
482
            struct RSAPublicKey pubK;
483
             struct RSAPrivateKey privK;
484
         mpz_init(pubK.modulus); mpz_init(pubK.publicExponent);
485
         mpz_init(privK.modulus); mpz_init(privK.privateExponent);
486
            mpz_t \mod, e, d, m, c, p, q;
487
         mpz_init(mod); mpz_init(e); mpz_init(d); mpz_init(p); mpz_init(
488
            mpz_init(m); mpz_init(c);
489
490
491
          * Key generation
492
493
494
         // Generate public exponent e
495
         gen_e(e);
496
         gmp_printf("%s%Zd\n\n", "Public_exponent_e:_", e);
497
498
         // Generate primes p and q for modulus n
499
         gen_primes(p, e, 1024);
500
         gen_primes(q, e, 1024);
501
502
         // Check if (p-1) and (q-1) are coprime with e
503
         unsigned long int one = 1;
504
         mpz_t p_minus_1, q_minus_1;
505
         mpz_init(p_minus_1); mpz_init(q_minus_1);
506
         mpz_sub_ui(p_minus_1, p, one);
507
         mpz_sub_ui(q_minus_1, q, one);
508
509
         assert(coprime(p_minus_1, e) == 1);
510
         assert(coprime(q_minus_1, e) == 1);
511
        512
513
514
515
         mpz_mul(mod, p, q);
516
517
         gmp_printf("%s%Zd\n\n", "Modulus_n:_", mod);
518
519
         // Generate private exponent d
520
         gen_d(d, p_minus_1, q_minus_1, e, 1024);
521
522
         gmp_printf("%s%Zd\n\n", "Private_exponent_d:_", d);
523
524
         mpz_set(pubK.modulus, mod);
525
         mpz_set(pubK.publicExponent, e);
526
527
         mpz_set(privK.modulus, mod);
         mpz_set(privK.privateExponent, d);
528
529
```

- **B** Crypto Coding Practices
- C Secure Coding Practices