RSA Encryption and Decryption

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1 Introduction

The RSA cryptosystem is one of the most widely used public-key cryptosystems in use today for securing information. Fundamentally, it allows two parties to exchange a secret message who have never communicated in the past. To accomplish this, RSA utilizes a pair of keys, a public key for encryption and a private key for decryption. The encryption and decryption keys are distinct, and so RSA is often referred to as an asymmetric cryptosystem.

For this project, we studied the RSA cryptosystem to understand how and why it works. As one of the most mature cryptosystems, RSA has been studied extensively, and there are plenty of interesting resources on attacks and how to prevent them [1]. These attacks provide an excellent exposition for the dangers of improperly implementing RSA, which makes such a project well-suited for learning.

We focused on the number theory behind the algorithm, well-known attacks on the RSA cryptosysem, and secure coding practices associated with implementing cryptosystems more broadly. We implemented the RSA encryption and decryption algorithms according to cryptographic considerations for security and performance and according the well-established specifications. This provided a better understanding of the nuances of cryptographic coding in practice.

2 Implementation

We first detail how we handle multiple precision numbers, then we detail our implementation of RSA key generation and encryption and decryption functions.

2.1 Handling Multiple Precision Numbers

Even before starting the implementation of PKCS #1 [2] itself, the first major challenge we faced was deciding how to store the numbers that would be used for encryption. Typical RSA integers are on the order of 1000 bits in size, which far exceeds the capacity of standard C data types. Thus, some custom BigInteger data type was necessary to store integers of arbitrary precision. Though less of a security concern, this was nonetheless a fundamental part of implementing the encryption scheme.

To gain experience working with arbitrary precision integers, we initially attempted to create the BigInteger library ourselves. Three primary design

decisions guided the process. First of all, to make memory usage efficient, we used dynamically-sized integers. This allowed integers to occupy only the memory they required, and freed up any they didn't. It also had the additional benefit of placing no limit on the capacity of a BigInteger. Secondly, intending to replicate the behavior of primitive C data types, we did not use in-place operations on BigIntegers. That is, the output of any BigInteger operation was a newly allocated BigInteger, and the operands were unchanged. Finally, we decided not to represent negative integers. This is sufficient for RSA, and had the advantage of simplicity.

The dynamic sizing ultimately proved to be very cumbersome to work with. For most operations, it wasn't possible to predict the number of bytes of storage that would be needed until after the result was computed. This resulted in excessive memory management (for example, reallocating memory after the operation to fit the size of the result) and significant performance overhead. It would have been better to assign a maximum size for a multi-precision integer, allocate a fixed block of that size, and let it grow or shrink as needed. Although this is a less efficient use of memory, the lack of overhead for managing memory would have cleaned up the code and increase performance significantly.

Likewise, avoiding in-place operations proved to be an inconvenience. On several occasions, it would have been more convenient to write back the result of an operation to one of the operands (for example, to use immediately afterword). But our library did not support this, so we were forced to allocate a new integer whether or not it wasn't necessary. This again resulted in unnecessary overhead due to memory management, and made the library more difficult to use.

In the end, our custom solution was quite inefficient, and fixing all its issues would have likely required a complete redesign. Thus, we decided instead to incorporate a preexisting library to handle multiple-precision integers. For this purpose, we settled on GMP (the GNU Multi-Precision library) [3].

2.2 Key Pair Generation

We follow the Digital Signature Standard (DSS) [4] issued by the National Institute of Standards and Technology (NIST) to generate key pairs.

2.2.1 Pseudorandom Number Generator

In order to generate random primes, it is important that we use a cryptographically secure pseudorandom number generator. We decide to use the UNIX-based special file /dev/random, which generates high-quality pseudorandom numbers that are well-suited for key generation.

The semantics for /dev/random vary based on the operating system. In Linux, /dev/random is generated from entropy created by keystrokes, mouse movements, IDE timings, and other kernel processes. In macOS, /dev/random data is generated using the Yarrow-160 algorithm, which is a cryptographic pseudorandom number generator. Yarrow-160 outputs random bits using a combination of the SHA1 hash function and three-key triple-DES.

We believe <code>/dev/random</code>, as prescribed, is sufficient for our purposes, but the entropy pool can be further improved using specialized programs or hardware random number generators.

2.2.2 Primality Testing

We use the Miller-Rabin probabilistic primality test to validate the generation of prime numbers. There are two approaches for using Miller-Rabin primality testing: (1) using several iterations of Miller-Rabin alone; (2) using several iterations of Miller-Rabin followed by a Lucas primality test. For simplicity, we use the iterative Miller-Rabin implementation available in the GNU MP Library. Instead, we find it more interesting to learn how to use Miller-Rabin testing correctly in practice, as specified in the DSS.

For example, different modulus lengths for RSA require varying rounds of Miller-Rabin testing. We reproduce the number of rounds necessary for various auxiliary prime (see Section 2.2.3) lengths in Table 1, and we follow this in our implementation.

Auxiliary Prime Length	Rounds of M-R Testing
> 100 bits	28
> 140 bits	38
> 170 bits	41

Table 1: The table shows the number of Miller-Rabin rounds necessary as a function of the lengths of auxiliary primes p_1 , p_2 , q_1 , and q_2 .

2.2.3 Criteria for Key Pairs

The key pair for RSA consists of the public key (n, e) and the private key (n, d). The RSA modulus n is the product of two distinct prime numbers p and q. RSA's security rests on the primality and secrecy of p and q, as well as the secrecy of the private exponent d. The methodology for generating these parameters varies based on the desired number of bits of security and the desired quality of primes. However, several desideratum must hold true for all methods.

Public Exponent e. The following constraints must hold true for the public exponent e.

- 1. The public verification exponent e must be selected prior to generating the primes p and q, and the private signature exponent d.
- 2. The public verification exponent e must be an odd positive integer such that $2^{16} < e < 2^{256}$.

It is immaterial whether or not e is a fixed value or a random value, as long as it satisfies constraint 2 above. For simplicity, we fix $e = 2^{16} + 1 = 65537$.

Primes p and q. The following constraints must hold true for random primes p and q.

- 1. Both p and q shall be either provable primes or probable primes.
- 2. Both p and q shall be randomly generated prime numbers such that all of the following subconstraints hold:
 - (p+1) has a prime factor p_1

- (p-1) has a prime factor p_2
- (q+1) has a prime factor q_1
- (q-1) has a prime factor q_2

where p_1 , p_2 , q_1 , q_2 are auxiliary primes of p and q. Then, one of the following shall also apply:

- (i) p_1, p_2, q_1, q_2, p , and q are all provable primes
- (ii) p_1, p_2, q_1, q_2 are provable primes, and p and q are probable primes
- (iii) p_1, p_2, q_1, q_2, p , and q are all probable primes

For our implementation, we choose to generate probable primes p and q with conditions based on auxiliary probable primes p_1 , p_2 , q_1 , and q_2 . In other words, we choose the method (iii) listed above. While this method offers the lowest quality of primes, it offers the best performance. It would be interesting future work to benchmark key generation times and quality of primes among these three methods.

Method (iii) supports key sizes of length 1024, 2048, and 3072, which offers more utility over method (i), which offers only key sizes of length 2048 and 3072. For different key sizes, various lengths of auxiliary primes must be satisfied, which is reproduced in Table 2. Table 2 can be joined with Table 1 for a comprehensive view of parameters as a function of the key size nlen.

Key Size (nlen)	Minimum Length of Auxiliary Primes
1024 bits	> 100 bits
2048 bits	> 140 bits
3072 bits	> 170 bits

Table 2: The table shows the minimum length of auxiliary primes p_1 , p_2 , q_1 , and q_2 as a function of the key size nlen.

Regarding our actual implementation of method (iii), we closely follow the constraints above and how probable primes are generated from probable auxiliary primes as specified in the DSS [4]. There are further constraints to the above, which are specific to method (iii), that we satisfy but do not fully detail here. However, one important aspect of method (iii) is that it leverages the Chinese Remainder Theorem to improve performance for key generation.

Private exponent d. The following constraints must hold true for the private exponent d.

1. The private exponent d must be a positive integer between

$$2^{nlen/2} < d < LCM(p-1, q-1). \tag{1}$$

2.
$$1 \equiv (ed) \pmod{LCM(p-1, q-1)}$$
.

Implementing constraints for the private exponent d is relatively straightforward. However, we do consider that in the rare case when $d \leq 2^{nlen/2}$, new primes must be generated.

2.3 Encryption and Decryption

The PKCS #1 standard outlines two difference schemes for RSA encryption RSAES-OAEP and RSAES-PKCS1-v1_5. The former is required for new applications, and the latter is an older scheme kept around for backwards compatibility. For our project, we chose to implement the OAEP scheme. As indicated by its name, this scheme incorporates OAEP (Optimal Asymmetric Encryption Padding) which turns the otherwise deterministic RSA encryption into probabilistic encryption. This makes the scheme CPA-secure.

2.4 Data Primitives

As specified in the standard, there are two primary data types used for RSA encryption octet strings and multiple-precision integers. Octet strings are used to represent messages (i.e. plaintext and ciphertext) and the multiple-precision integers are used to perform the basic mathematical operations of the RSA scheme (i.e. exponentiation). To convert between the two, the standard specifies two data conversion primitives I2OSP (Integer to Octet String) and OS2IP (Octet String to Integer). To represent the multiple-precision integers, we already decided on the GMP library. However, there were a couple options for how to represent the octet strings.

- 1. Represent each octet as a single character: this is the most efficient way of representing octets, since each character can be any one of its possible 256 values. However, with this representation, string manipulation of octet strings became a challenge. Since '\0' is a valid octet, a NULL character cannot be used to represent the end of an octet string. This would require storing the length of the octet string separately.
- 2. Represent each octet as two hex characters: this method is less efficient, as it requires two characters for each octet. But it allows for NULL-terminated octet strings, which is the standard way of representing strings in C. Furthermore, the GMP library has little support for base-256 octet strings, so this is the option we chose.

2.5 Cryptographic Primitives

The two cryptographic primitives are RSAEP, which is the encryption primitive, and RSADP, which is the decryption primitive. We implement these as prescribed in the specification, adapting the GMP Library. These two cryptographic primitives perform the modular exponentiation portion of RSA.

2.6 RSAES-OAEP

RSAES-OAEP combines both of the cryptographic primitives aforementioned, and uses an encoding method based on Bellare and Rogaway's Optimal Assymetric Encryption Scheme [5]. RSAES-OAEP is parameterized by a hash function and mask generation function that we describe further in Section 2.7. Both the RSAES-OAEP-Encryption and RSAES-OAEP-Decryption operations are implemented as prescribed in the PKCS specification.

To accomplish the OAEP padding, the RSA-OAEP scheme makes use of a Mask Generation Function, which in turn is based on a secure hash. The PKCS #1 standard recommends the use of a hashing algorithm from the SHA-2 hash family, but does not require any hash algorithm in particular. For our project, we decided to use the SHA-256 hash algorithm. Since the algorithm is not given in the standard, we used the implementation from the OpenSSL library.

The representation of octets as pairs of hex characters worked fairly well, but there were some issues it caused. In particular, the OpenSSL SHA256 hash function uses outputs a base-256 octet string, which was not compatible with our octet string representation. Consequently, we had to convert back and forth between these formats, decreasing the performance of our implementation. It would have been better to have a single common format for octet strings. The simplest way to do this would probably have been to store the length of an octet string along with the data and represent all octet strings in base-256.

2.7 Hash and Mask Generation

While there are numerous acceptable hash functions, SHA-512 and SHA-256 are recommended. Thus, erring for performance, we choose to use the OpenSSL SHA-256 implementation. The Mask Generation Function (MGF) is crucial for the security of the RSA encryption scheme as specified. The MGF takes an octet string of varying length, and then outputs a pseudorandom octet string of a desired length. This means that the output cannot be predicted, and the provable security of RSAES-OAEP relies on the MGF's randomness.

3 Crypto Learning

Here, we overview a number of strengths and weaknesses of our RSA implementation. In particular, we discuss attacks that we do protect against, and attacks that would cause our implementation to fail.

3.1 Attacks via Insecure PRNGs

We generate pseudorandom numbers using the /dev/random file, as specified in Section 2.2.1. This is considered a cryptographically secure method for generating pseudorandom numbers and is widely used in practice. Even so, there exist several theoretical attacks on Linux's implementation of this PRNG.

Gutterman et al. perform an analysis of Linux's pseudorandom number generator (LRNG) and expose a number of security vulnerabilities [6]. More specifically, they reverse engineer LRNG and show that given the current state of the generator, it is possible to reconstruct previous states, thereby compromising the security of past usage. Further, they show that it is possible to measure and analyze the entropy created by the kernel. Bernstein presents a related attack in which monitoring one source of entropy could compromise the randomness of other sources of entropy [7].

While the latter attacks are theoretical, and to our knowledge have not been successful in practice, Gutterman also presents a denial of service attack that our implementation is susceptible to [6]. Since Linux's implementation of /dev/random may block the output of bits when the entropy is low, one simple

attack would be to simply read all the bits from /dev/random, thereby blocking other users' access to new bits for a long period of time. More interestingly, an attack can also be performed remotely by triggering system requests for get_random_bytes, which will block both /dev/random and the non-blocking /dev/urandom pool.

One possible solution is to limit the per user consumption of random bits. Alternatively, we could avoid using /dev/random altogether and instead generate pseudorandom numbers via hardware random number generators.

3.2 Common Modulus Attack

While the common modulus attack is simple, it is a case in point for the dangers of misusing RSA [1].

In order to prevent having to generate a different modulus n for different users, a developer might choose to fix n for a number of users or for all users. This is insecure, since a user could use his/her own exponents e and d to factor the fixed n, thereby recovering the private key d from some other user. Thus, the common modulus attack shows that the RSA modulus should not be fixed. Our implementation precludes this attack by generating a random modulus every time. This is done through calls to the gen_primes function.

3.3 Low Private Exponent Attack

In order to reduce the decryption time, a developer might choose a smaller value for the private exponent d rather than a random value. Choosing a small d can improve decryption performance (modular exponentation) by a factor of at least 10 for a 1024-bit modulus. However, Weiner shows that such a simplification is completely insecure [8]. Boneh and Durfee further improve the bounds of Weiner's attack, showing that $d < n^{0.292}$ is susceptible to attack [9]. There are two techniques to prevent this attack; both of which our implementation supports.

The first technique is to use a large public exponent e. Weiner shows that as long as $e>n^{1.5}$, this attack cannot be performed. In our implementation, we fix e=65537. Thus, for nlen=1024, our implementation supports this technique. However, this technique does not hold true for nlen=2048 or nlen=3072. This can be easily fixed by increasing e to satisfy nlen=3072, however, the downside is that it will increase encryption time. Nonetheless, the second technique, using the Chinese Remainder Theorem to speed up decryption, is fully supported by our implementation.

3.4 Low Public Exponent Attack

Similar to the latter attack, in order to reduce the encryption time, a developer might choose a smaller value for the public exponent e. This engenders a number of attacks on low public exponents, most of which are based on Coppersmith's theorem [10]. While the smallest e possible is 3, $e \ge 2^{16} + 1$ is recommended to prevent certain attacks. This is the value of e that we use in our implementation. It is simple to increase e for security, but this will result in a performance decline.

3.5 Partial Key Exposure Attack

Suppose that for a given private key (n, d), some portion of the private exponent d is exposed. Boneh $et\ al.$ show that recovering the rest of the private exponent d is possible when the corresponding private exponent e is small. Specifically, they show that it is possible to reconstruct all of d as long as $e < \sqrt{n}$. In our implementation, e = 65537 and all nlen are secure from such an attack. However, partial key exposure attacks do illustrate the importance of keeping the entire private key secret. This is one consideration that our implementation is lacking, and it will be interesting to explore this in the future.

3.6 Side-Channel Attacks

Kocher's seminal cryptanalysis of RSA via a timing attack shows that a clever attacker could measure the amount of time it takes for RSA decryption, thereby recovering the private exponent d [11]. Our implementation does not protect against such timing attacks, but there are two solutions that can be considered.

The first is to introduce a delay so that decryption (modular exponentiation, in particular) takes a fixed amount of time. However, this would cause a decline in performance. The second solution is based on blinding, by which a randomization is introduced such that decryption is performed on a random message unknown to the attacker. Thus, such timing attacks cannot be performed.

Kocher also discovered another side-channel attack by measuring the amount of power consumed during decryption. Since multiprecision multiplication causes greater power consumption, it is simple to detect the number of multiplications, thereby revealing information about the private exponent d.

Another security concern we encountered when implementing the BigInteger library was a potential timing attack when performing modular exponentiation. The larger the encryption exponent, the long a naïve implementation will take, and this leaks information about the length of the encryption exponent if an attacker can observe the time it takes to perform the exponentiation. For exponentiation using Montgomery multiplication, there is a modification called Montgomerys Ladder which is used to make exponentiation run in constant time. We did not implement this modification, but it would be interesting to do so in the future.

4 Secure Coding

We next overview secure coding practices that we considered for our implementation, as well as practices that could have further improved our code. These are mostly based on the SEI CERT C Coding Standard [12].

4.1 Integers and Floats

Handling multiple precision integers and multiple precision floats and understanding conversions between these data types is crucial in implementing RSA.

In regards to integers, we use different types of integers (i.e. int, unsigned long int, and mpz_t (multiple precision integers) for different purposes. For general purpose counters, we can safely use the int data type. For representing the size of an object, we can safely use the size_t data type, since this generally

covers the entire address space. For any integers that may be used in multiple precision arithmetic, we err on the side of caution and use the unsigned long int data type. Then finally, for any integers that require multiple precision, we use the mpz_t data type from the GMP Library.

In regards to floating point numbers, we simply use the mpf_t data type from the GMP Library, since their use is limited and the multiple precision float data type offers enough utility for the required use cases.

Further, we also perform adequate range checking, integer overflow checking, and truncation checking. For the generation of key parameters, it is crucial that we perform range checking thoroughly, since a single misstep could lead to an incorrect encryption or decryption. Additionally, we err on the side of caution and instantiate integers as either long int or mpz_t to prevent integer overflows. Finally, we pay attention to any truncation that may occur as a result of conversions between integers and floats. For example, it is important to consider that while a multiple precision integer square root function is available, the result is truncated to an integer. Thus, we must handle such operations more precisely using the mpf_t (multiple precision float) object.

4.2 Memory Management

Since memory owned by our process can be accessed and reused by another process in the absence of proper memory management, this could potentially reveal information about secret keys to other processes. Even further, systems with multiple users make it possible for one user to sniff keys from another users' process. Thus, proper memory management is crucial for the secrecy of private keys.

In this regard, we free dynamically allocated memory whenever it is no longer needed. This occurs throughout our implementation in two fashions. First, consider when a new block of memory is allocated using malloc. Once the allocated block of memory is no longer in use, memory is freed using the function call free. Second, when using the GMP Library to instantiate multiple precision numbers, these numbers are also dynamically allocated. Thus, this memory must either be freed using the function call mpz_clear (for integers) or "zeroized" to ensure that no information about the secret keys are revealed.

4.3 Characters and Strings

One secure coding practice that we should have considered is to cast characters to unsigned char before converting them to larger integer sizes. One instance of this is when generating pseudorandom numbers from /dev/random, since we sample random characters from this file and then convert it to a pseudorandom multiple precision integer. More broadly, any arguments to character-handling functions should be represented as an unsigned char. However, this is only applicable to platforms in which char data types have the same representation as signed char data types.

4.4 Error Handling

Another secure coding practice that we should have considered is to handle errors throughout the entire program. Although there are instances in which we do handle errors, our program would be much more robust if it detected and handled all standard library errors and GMP Library errors. Having a consistent and comprehensive error-handling policy would improve our implementation's resilience in the face of erroneous or malicious inputs, hardware or software faults, and unexpected environment changes. This would be advantageous both to the developers as well as the end-users of our implementation.

4.5 Test Suite

It would have been beneficial to set up a comprehensive test suite, which could rigorously test the modules within our implementation. Alternatively, we could have used a fuzzer to exercise the logic of our implementation. In the future, we can leverage static analysis techniques and a binary fuzzer, such as American Fuzzy Lop (AFL), to discover any bugs or vulnerabilities in our code.

5 Summary

Taken as a whole, this project illuminated many of the intricacies involved in a real-world implementation of the RSA cryptosystem, and cryptosystems more broadly. Truly, what we learn as "textbook" RSA is a tremendous oversimplification to what RSA is in practice. As expected, the learning outcomes from this project were innumerable as we were confronted with both number theoretic attacks, as well as implementation attacks. In regards to secure coding practices, perhaps the most important learning outcome was realizing that the vast space of considerations makes cryptographic coding especially difficult, and mistakes devastating for the security of the cryptosystem. This project skimmed the surface of an RSA implementation, and it will be interesting future work to improve upon cryptographic coding practices, general coding practices, and performance.

References

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A Code

Listing 1: Code for rsa.h.

```
1
2
     * Data Types
3
4
    struct RSAPublicKey {
5
            mpz_t modulus;
            mpz_t publicExponent;
7
   };
8
9
    struct RSAPrivateKey {
10
            mpz_t modulus;
11
            mpz_t privateExponent;
12
   };
13
14
15
      Methods for Generating Key Pairs
16
17
18
    void
            gen_e
                                  (mpz_t e);
19
    void
            gen_d
                                  (mpz_t d, mpz_t p_minus_1, mpz_t
        q_minus_1, mpz_t e, int n);
20
    void
            gen_probable_prime
                                 (mpz_t p, mpz_t p1, mpz_t p2, mpz_t e,
        int n);
21
                                  (mpz_t p, mpz_t e, int n);
            gen_primes
22
   int
            coprime
                                  (mpz_t a, mpz_t b);
23
            PRNG
                                               (mpz_t rand, int n);
    void
24
25
26
     * Methods for Encryption and Decryption
27
28
            I20SP
   char*
                                               (mpz_t x, int xLen);
29
            OS2IP
   void
                                               (char *X, mpz_t x);
30
                     RSAEP
                                                       (struct
    int
        RSAPublicKey *K, mpz_t m, mpz_t c);
31
                    RSADP
                                                       (struct
   int.
        RSAPrivateKey *K, mpz_t c, mpz_t m);
                                               (char *mgfSeed, unsigned
32
           MGF1
    char*
        long long maskLen);
33
    char*
           RSAES_OAEP_ENCRYPT
                                      (struct RSAPublicKey *K, char *M,
        char *L);
34
          RSAES_OAEP_DECRYPT
                                      (struct RSAPrivateKey *K, char *C,
        char *L);
```

Listing 2: Code for rsa.c.

```
1 #include <stdio.h>
 2 #include <stdlib.h>
 3 #include <stdarg.h>
    #include <string.h>
 5 #include <time.h>
 6 #include <gmp.h>
 7 #include <openssl/sha.h>
 8 #include "rsa.h"
 9
    #include <sys/types.h>
10 #include <sys/stat.h>
11 #include <fcntl.h>
12 #include <math.h>
13 #include <assert.h>
14
15
   // Convert nonnegative integer x to a zero-padded octet string of
         length xLen.
16
    char* I2OSP(mpz_t x, int xLen) {
17
         size_t osLen = mpz_sizeinbase(x, 16);
18
         xLen *= 2;
19
         if (xLen < osLen) {
20
             printf("integer utoo large \n");
21
             return NULL;
22
23
         char *os = malloc((xLen + 1) * sizeof(char));
24
         memset(os, '0', xLen - osLen);
25
         mpz_get_str(os + xLen - osLen, 16, x);
26
         os[xLen] = '\0';
27
         return os;
28 }
29
30
   // Convert octet string to a nonnegative integer
31
   void OS2IP(char *X, mpz_t x) {
32
             mpz_set_str(x, X, 16);
33
   }
34
35
   // RSA Encryption Primative
36
   int RSAEP(struct RSAPublicKey *K, mpz_t m, mpz_t c) {
37
             if (mpz_cmp(m, K->modulus) >= 0) {
38
                        \begin{array}{c} -1 \\ \text{printf("message} \sqcup \text{representative} \sqcup \text{out} \sqcup \text{of} \sqcup \text{range} \backslash \text{n")}; \end{array} 
39
                       return 0;
40
41
             mpz_powm_sec(c, m, K->publicExponent, K->modulus);
42
              return 1;
   }
43
    // RSA Decryption Primative
45
    int RSADP(struct RSAPrivateKey *K, mpz_t c, mpz_t m) {
46
             if (mpz_cmp(c, K->modulus) >= 0) {
47
48
                        \bar{\text{printf}} \, (\, "\, \text{ciphertext} \, \bot \, \text{representative} \, \bot \, \text{out} \, \bot \, \text{of} \, \bot \, \text{range} \, \backslash \, n \, " \, ) \, ; \\
49
                       return 0;
50
             }
51
             mpz_powm_sec(m, c, K->privateExponent, K->modulus);
52
             return 1;
53
54
55
    // Mask generation function specified in PKCS #1 Appendix B.
56
    char* MGF1(char *mgfSeed, unsigned long long maskLen) {
57
58
         // Step 1: Verify maskLen <= (hLen * 2^32)
         unsigned long long hLen = SHA256_DIGEST_LENGTH;
59
60
         if (maskLen > (hLen << 32)) {
```

```
printf("mask_{\perp}too_{\perp}long_{n}");
61
62
             return NULL;
63
64
         maskLen *= 2;
65
         hLen *= 2;
66
67
         // Step 2: Init T to empty octet string. T consists of TLen
             SHA256 hashes.
         int TLen = (maskLen + hLen - 1) / hLen;
68
69
         char *T = malloc((TLen * hLen) * sizeof(char));
70
71
         char *TPtr = T;
72
         char *hashOp;
73
         size_t mgfSeedLen = strlen(mgfSeed);
74
         hashOp = malloc((mgfSeedLen + 4 * 2) * sizeof(char));
75
         memcpy(hashOp, mgfSeed, mgfSeedLen);
76
77
         // Step 3: Generate mask
78
         int i, j;
79
         char *C;
         unsigned char *hash;
80
81
         unsigned char hChar;
82
         hash = malloc(SHA256_DIGEST_LENGTH * sizeof(char));
83
         mpz_t counter;
         mpz_init(counter);
84
85
         for (i = 0; i < TLen; ++i) {
86
             mpz_set_ui(counter, i);
87
             C = I2OSP(counter, 4);
88
             memcpy(hashOp + mgfSeedLen, C, 4 * 2);
             SHA256(hashOp, mgfSeedLen + 4 * 2, hash);
89
             for (j = 0; j < hLen; j += 2)
    sprintf(TPtr + j, "%02x", hash[j/2]);</pre>
90
91
92
             TPtr += hLen;
93
             free(C);
94
95
96
         // Step 4: Output mask
97
         char *mask = malloc(maskLen + 1);
98
         memcpy(mask, T, maskLen);
         mask[maskLen] = '\0';
99
100
         free(hash); free(hashOp); free(T);
101
         return mask;
102
103
104
     // RSA Encryption with OAEP. Section 7.1.1 in PKCS #1
     char* RSAES_OAEP_ENCRYPT(struct RSAPublicKey *K, char* M, char *L)
105
106
107
             // Step 1: Length checking (*_o stores size in octets; *_h
                  in hex chars)
108
             size_t k_o = (mpz_sizeinbase(K->modulus, 16) + 1) / 2;
             size_t hLen_o = SHA256_DIGEST_LENGTH;
109
110
             size_t mLen_o = strlen(M) / 2;
111
             size_t maxmLen_o = k_o - 2 * hLen_o - 2;
112
             if (mLen_o > maxmLen_o) {
                      printf("message_{\sqcup}too_{\sqcup}long \n");
113
114
                      return NULL;
115
116
             size_t k_h = k_o * 2;
117
             size_t hLen_h = hLen_o * 2;
             size_t mLen_h = mLen_o * 2;
                                                // If M is valid, then
118
                  mLen_h = strlen(M)
```

```
119
120
             // Step 2: EME-OAEP encoding
             if (L == NULL) L = "";
121
            char *lHash = SHA256(L, strlen(L), NULL);
122
123
124
             125
             size_t PSLen_h = (maxmLen_o - mLen_o) * 2;
126
             char *PS = malloc(PSLen_h * sizeof(char));
127
             memset(PS, '0', PSLen_h);
128
129
             // c. Generate data block (DB)
130
             size_t DBLen_o = k_o - hLen_o - 1;
             size_t DBLen_h = DBLen_o * 2;
131
132
             char *DB = malloc((DBLen_h + 1) * sizeof(char));
133
            int i;
            134
135
                        ]);
136
             memcpy(DB + hLen_h, PS, PSLen_h);
            memcpy(DB + hLen_h + PSLen_h, "01", 2);
memcpy(DB + DBLen_h - mLen_h, M, mLen_h);
137
138
139
            DB[DBLen_h] = '\0';
140
141
             // d. Generate random seed
142
            mpz_t seed;
143
             mpz_init(seed);
             PRNG(seed, hLen_o * 8);
144
             char *seedStr = I2OSP(seed, hLen_o);
145
146
147
             // ef. Generate dbMask and compute DB XOR dbMask
148
             char *dbMask = MGF1(seedStr, DBLen_o);
149
            mpz_t op1, op2, rop;
150
            mpz_init_set_str(op1, DB, 16);
151
             mpz_init_set_str(op2, dbMask, 16);
152
             mpz_init(rop);
153
             mpz_xor(rop, op1, op2);
154
             char *maskedDB = I2OSP(rop, DBLen_o);
155
156
             // gh. Generate seedMask and compute seed XOR seedMask
157
             char *seedMask = MGF1(maskedDB, hLen_o);
             mpz_set_str(op1, seedStr, 16);
158
159
            mpz_set_str(op2, seedMask, 16);
160
             mpz_xor(rop, op1, op2);
161
             char *maskedSeed = I2OSP(rop, hLen_o);
162
163
             // i. Generate encoded message (EM)
             size_t EMLen_h = hLen_h + DBLen_h + 2;
164
165
             char *EM = malloc((EMLen_h + 1) * sizeof(char));
166
             memset(EM, '0', 2);
            memcpy(EM + 2, maskedSeed, hLen_h);
memcpy(EM + hLen_h + 2, maskedDB, DBLen_h);
167
168
169
             EM[EMLen_h] = '\0';
170
171
             // Step 3-4: RSA encryption
172
            mpz_t m, c;
173
            mpz_init(m);
174
            mpz_init(c);
175
             OS2IP(EM, m);
176
            RSAEP(K, m, c);
177
             char *C = I2OSP(c, k_o);
178
179
             // Free memory
```

```
180
             free(PS); free(DB); free(dbMask); free(maskedDB);
181
             free(seedMask); free(maskedSeed); free(EM);
182
             mpz_clear(op1); mpz_clear(op2); mpz_clear(rop);
183
             mpz_clear(m); mpz_clear(c);
184
185
             return C;
186
187
188
    // RSA Decryption with OAEP. Section 7.1.2 in PKCS #1
189
    char *RSAES_OAEP_DECRYPT(struct RSAPrivateKey *K, char* C, char *L)
190
191
             // Step 1: Length checking (*_o stores sizes in octets; *_h
                  in hex chars)
             size_t k_o = (mpz_sizeinbase(K->modulus, 16) + 1) / 2;
192
193
             size_t CLen_o = strlen(C) / 2;
194
             if (k_o != CLen_o) {
195
                     printf("decryption uerror \n");
196
                     return NULL;
197
             }
198
             size_t hLen_o = SHA256_DIGEST_LENGTH;
             if (k_o < (2 * hLen_o + 2)) {
199
200
                     \verb|printf("decryption|| error \n");\\
201
                     return NULL;
202
             }
203
204
             // Step 2: RSA Decryption
205
             mpz_t c, m;
206
             mpz_init(c);
207
             mpz_init(m);
208
             OS2IP(C, c);
209
             if (!RSADP(K, c, m)) {
                     printf("decryption_error\n");
210
211
                     return NULL;
212
             }
213
             char *EM = I2OSP(m, k_o);
214
215
             // Step 3: EME-OAEP decoding
             if (L == NULL) L = "";
216
217
             size_t hLen_h = hLen_o * 2;
218
             char *lHash_o = malloc(hLen_o * sizeof(char));
219
             char *lHash_h = malloc(hLen_h * sizeof(char));
220
             SHA256(L, strlen(L), lHash_o);
221
             int i;
222
             for (i = 0; i < hLen_o; ++i)
223
                     sprintf(lHash_h + 2 * i, "%02x", (unsigned char)
                         lHash_o[i]);
224
225
             // b. Separate encoded message (EM) into its component
                 parts
             size_t DBLen_o = k_o - hLen_o - 1;
226
227
             size_t DBLen_h = DBLen_o * 2;
228
             char *maskedSeed = malloc((hLen_h + 1) * sizeof(char));
229
             char *maskedDB = malloc((DBLen_h + 1) * sizeof(char));
230
             memcpy(maskedSeed, EM + 2, hLen_h);
231
             memcpy(maskedDB, EM + 2 + hLen_h, DBLen_h);
             maskedSeed[hLen_h] = '\0';
232
233
             maskedDB[DBLen_h] = '\0';
234
235
             // cd. Generate seedMask and compute maskedSeed XOR
                 seedMask
236
             char *seedMask = MGF1(maskedDB, hLen_o);
```

```
237
                          mpz_t op1, op2, rop;
238
                          mpz_init_set_str(op1, maskedSeed, 16);
239
                           mpz_init_set_str(op2, seedMask, 16);
240
                          mpz_init(rop);
241
                           mpz_xor(rop, op1, op2);
242
                           char *seed = I2OSP(rop, hLen_o);
243
244
                           // ef. Generate dbMask and compute maskedDB XOR dbMask
                          char *dbMask = MGF1(seed, DBLen_o);
245
246
                           mpz_set_str(op1, maskedDB, 16);
247
                          mpz_set_str(op2, dbMask, 16);
248
                          mpz_xor(rop, op1, op2);
249
                          char *DB = I2OSP(rop, DBLen_o);
250
251
                           // g. Separate data block (DB) into component parts to
                                   recover message
252
                           size_t PSLen_h = strstr(DB + hLen_h, "01") - DB - hLen_h;
                           int mLen_h = DBLen_h - PSLen_h - hLen_h - 1;
253
254
                           int errCount = 0;
255
                           errCount += (mLen_h < 0);</pre>
                           errCount += !(EM[0] == '0' && EM[1] == '0');
256
                           errCount += (strncmp(DB, lHash_h, hLen_h) != 0);
257
258
                          if (errCount > 0) {
259
                                           printf("decryption uerror \n");
260
                                            return NULL;
261
262
                          char *M = malloc((mLen_h + 1) * sizeof(char));
263
                          memcpy(M, DB + DBLen_h - mLen_h + 1, mLen_h);
264
                          M[mLen_h] = '\0';
265
266
                           // Free memory
267
                           free(EM); free(lHash_o); free(lHash_h); free(maskedSeed);
                                   free(maskedDB);
268
                           free(seedMask); free(seed); free(dbMask); free(DB);
269
                           mpz_clear(op1); mpz_clear(op2); mpz_clear(rop);
270
                           mpz_clear(m); mpz_clear(c);
271
272
                          return M;
273 }
274
275
         // Generates pseudorandom n bits from /dev/random file
276
         void PRNG(mpz_t rand, int n) {
277
                  int devrandom = open("/dev/random", O_RDONLY);
278
                  char randbits[n/8];
                  size_t randlen = 0;
279
280
                  while (randlen < sizeof randbits) {
281
282
                           ssize_t result = read(devrandom, randbits + randlen, (
                                   sizeof randbits) - randlen);
283
                           if (result < 0)
284
                                   printf("%s\n", "Could_\underlinead_\underlinead_\underlinedfrom_\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\underlinedfrom\
285
                          randlen += result;
286
287
                  close(devrandom);
288
                  mpz_import(rand, sizeof(randbits), 1, sizeof(randbits[0]), 0,
289
                         0, randbits);
290
                  // Make sure rand is odd
                  if (mpz_odd_p(rand) == 0) {
291
292
                          unsigned long int one = 1;
293
                          mpz_add_ui(rand, rand, one);
294
```

```
295 }
296
297
     // Generate (constant) public exponent e
298
    void gen_e(mpz_t e) {
299
         // Set e to 2^16 + 1
300
         unsigned long int e_int = pow(2,16)+1;
301
         mpz_set_ui(e, e_int);
302
    }
303
304
    // Generate private exponent d
305
    void gen_d(mpz_t d, mpz_t p_minus_1, mpz_t q_minus_1, mpz_t e, int
         n) {
306
307
         unsigned long int one = 1;
308
         mpz_t lower_bound, upper_bound, base;
309
         mpz_init(lower_bound); mpz_init(upper_bound); mpz_init_set_str(
             base, "2", 10);
310
         mpz_pow_ui(lower_bound, base, n/2);
311
         mpz_lcm(upper_bound, p_minus_1, q_minus_1);
312
313
         mpz_invert(d, e, upper_bound);
if (mpz_cmp(d, lower_bound) < 0 || mpz_cmp(d, upper_bound) > 0)
314
315
             fprintf(stderr, "Private_uexponent_ud_too_usmall,_utry_again\\"
                 ):
             exit(-1);
316
317
318
319
         mpz_t ed, check_d;
320
         mpz_init(ed); mpz_init(check_d);
321
322
         mpz_mul(ed, e, d);
323
         mpz_mod(check_d, ed, upper_bound);
324
325
         assert(mpz_cmp_ui(check_d, one) == 0);
326
327
    }
328
329
    // Generate probable prime from auxiliary primes
330
     void gen_probable_prime(mpz_t p, mpz_t p1, mpz_t p2, mpz_t e, int n
         ) {
331
332
         // Step 1: Check if p1 and p2 are coprime
333
         mpz_t gcd, twop1;
334
         mpz_init(gcd); mpz_init(twop1);
335
         unsigned long int one = 1;
336
         unsigned long int two = 2;
337
         mpz_mul_ui(twop1, p1, two);
338
         mpz_gcd(gcd, twop1, p2);
         if (mpz_cmp_ui(gcd, one) != 0) {
    fprintf(stderr, "Auxiliaries_p1_and_p2_not_coprime\n");
339
340
341
             exit(-1);
342
343
         // Step 2: Chinese remainder theorem
344
         mpz_t R; mpz_t R1; mpz_t R2;
345
346
         mpz_init(R); mpz_init(R1); mpz_init(R2);
347
348
         mpz_invert(R1, p2, twop1);
349
         mpz_mul(R1, R1, p2);
350
351
         mpz_invert(R2, twop1, p2);
```

```
352
         mpz_mul(R2, R2, twop1);
353
354
         mpz_sub(R, R1, R2);
355
356
         // Check for CRT
357
         mpz_t check1; mpz_t check2; mpz_t mpz_one;
358
         mpz_init(check1); mpz_init(check2); mpz_init(mpz_one);
         mpz_set_str(mpz_one, "1", 10);
359
360
         mpz_mod(check1, R, twop1);
361
         mpz_mod(check2, R, p2);
362
         mpz_sub(check2, p2, check2);
363
         assert(mpz_cmp(check1, mpz_one) == 0);
364
         assert(mpz_cmp(check2, mpz_one) == 0);
365
366
367
         // Step 3: Generate random X between lower_bound and
             upper\_bound
368
         mpz_t lower_bound; mpz_t upper_bound; mpz_t base; mpz_t X;
             mpz_t temp; mpz_t Y;
369
         mpz_init(lower_bound); mpz_init(upper_bound); mpz_init(base);
             mpz_init(X); mpz_init(temp); mpz_init(Y);
370
371
         mpz_set_str(base, "2", 10);
372
         mpz_pow_ui(upper_bound, base, n/2);
373
         mpz_sub_ui(upper_bound, upper_bound, one);
374
375
376
         mpf_t f_lb, f_sqrt, f_base;
377
378
         mpf_init(f_lb); mpf_init(f_sqrt); mpf_init_set_str(f_base, "2",
379
380
         mpf_sqrt(f_sqrt, f_base);
381
         mpf_pow_ui(f_lb, f_base, n/2-1);
382
         mpf_mul(f_lb, f_lb, f_sqrt);
383
         mpz_set_f(lower_bound, f_lb);
384
385
386
         mpz_t cond;
387
         mpz_init(cond);
         mpz_pow_ui(cond, base, n/2);
388
389
390
         mpz_t Y_minus_1;
391
         mpz_init(Y_minus_1);
392
         mpz_sub_ui(Y_minus_1, Y, one);
393
394
395
         int i = 0;
396
         do {
397
398
             PRNG(X, n/2);
399
             while (mpz_cmp(X, lower_bound) < 0 || mpz_cmp(X,
                  upper_bound) > 0) {
400
                  PRNG(X, n/2);
401
402
403
             // Step 4: Calculate Y
404
             mpz_mul(temp, twop1, p2);
             mpz_sub(Y, R, X);
mpz_mod(Y, Y, temp);
mpz_add(Y, Y, X);
405
406
407
408
```

```
409
             // Step 5: i = 0
410
             i = 0;
411
412
             mpz_gcd(gcd, Y_minus_1, e);
413
414
             // Step 11: Go to Step 6
415
             while (mpz_cmp(Y, cond) < 0) {
416
                  i += 1;
417
                  if (mpz_cmp_ui(gcd, one) != 0) {
418
                      if (i >= 5*(n/2)) {
                          printf("%s\n", "FAILURE");
419
420
                          exit(-1);
421
422
                      mpz_add(Y, Y, temp);
423
                      mpz_gcd(gcd, Y_minus_1, e);
424
                  // Step 7: If GCD(Y-1, e) = 1
425
426
                  else {
427
                      if (mpz_probab_prime_p(Y, 28) >= 1) {
428
                          mpz_set(p, Y);
429
                          return;
430
431
432
                      //Step 8: Check if failure
433
                      if (i >= 5*(n/2)) {
434
                          printf("%s\n", "FAILURE");
435
                          exit(-1);
436
437
438
                      //Step 10: Update Y
                      mpz_add(Y, Y, temp);
mpz_gcd(gcd, Y_minus_1, e);
439
440
441
                 }
442
             }
443
         // Step 6: Check condition for Y > cond
444
         } while (mpz_cmp(Y, cond) >= 0);
445
446
         mpz_clear(gcd); mpz_clear(twop1); mpz_clear(R); mpz_clear(R1);
             mpz_clear(R2);
447
         mpz_clear(check1); mpz_clear(check2); mpz_clear(mpz_one);
448
         mpz_clear(lower_bound); mpz_clear(upper_bound); mpz_clear(base)
              ; mpz_clear(X); mpz_clear(temp); mpz_clear(Y);
449
         mpz_clear(cond); mpz_clear(Y_minus_1);
450
451
         mpf_clear(f_lb); mpf_clear(f_sqrt); mpf_clear(f_base);
452
    }
453
454
455
     // Generate auxiliary primes
     void gen_auxiliary_primes(mpz_t p, mpz_t e, int n) {
456
         if (n != 1024 && n != 2048 && n != 3072) {
457
             fprintf(stderr, "Invalid\_bit\_length\_for\_RSA\_modulus._{\sqcup}
458
                 Exiting...\n");
             exit(-1);
459
460
461
         mpz_t xp, xp1, xp2, p1, p2;
462
         mpz_init(xp); mpz_init(xp1); mpz_init(xp2); mpz_init(p1);
             mpz_init(p2);
463
         unsigned long int two = 2;
464
465
         int len_aux = 0;
466
         int mr_rounds = 0;
```

```
467
         if (n == 1024) {
468
             len_aux = 104;
469
             mr_rounds = 28;
470
471
         else if (n == 2048) {
472
             len_aux = 144;
473
             mr_rounds = 38;
474
         else if (n == 3072) {
475
476
             len_aux = 176;
477
             mr_rounds = 41;
478
479
480
         PRNG(xp1, len_aux);
481
         PRNG(xp2, len_aux);
482
483
         while (mpz_probab_prime_p(xp1, mr_rounds) != 1) {
484
             mpz_add_ui(xp1, xp1, two);
485
486
         while (mpz_probab_prime_p(xp2, mr_rounds) != 1) {
487
             mpz_add_ui(xp2, xp2, two);
488
489
         //gmp\_printf("\%s\n\%Zd\n\%Zd\n", "Auxiliary primes for p: ", xp1,
              xp2);
490
         mpz_set(p1, xp1);
491
         mpz_set(p2, xp2);
492
493
         gen_probable_prime(p, p1, p2, e, n);
494
         mpz_clear(xp); mpz_clear(xp1); mpz_clear(xp2); mpz_clear(p1);
             mpz_clear(p2);
495
496
497
     // Check if gcd(a,b) = 1 (coprime)
498
     int coprime(mpz_t a, mpz_t b) {
         int coprime = 1;
499
500
         mpz_t gcd; mpz_init(gcd);
501
         mpz_t one; mpz_init_set_str(one, "1", 10);
502
503
         mpz_gcd(gcd, a, b);
504
         if (mpz\_cmp(gcd, one) != 0) {
505
             coprime = 0;
506
507
         mpz_clear(gcd); mpz_clear(one);
508
         return coprime;
509
    }
510
    /*
    int main() {
511
512
             struct RSAPublicKey pubK;
513
             struct RSAPrivateKey privK;
514
         \textit{mpz\_init(pubK.modulus); mpz\_init(pubK.publicExponent);}
515
         mpz_init(privK.modulus); mpz_init(privK.privateExponent);
516
             mpz_t \mod, e, d, p, q;
517
         mpz_init(mod); mpz_init(e); mpz_init(d); mpz_init(p); mpz_init(
             q);
518
519
520
          * Key generation
521
522
523
         // Generate public exponent e
524
         gen_e(e);
         gmp\_printf("%s%Zd\n\n", "Public exponent e: ", e);
525
```

```
526
527
         // Generate primes p and q for modulus n
528
         gen_auxiliary_primes(p, e, 1024);
         gen_auxiliary_primes(q, e, 1024);
529
530
531
         // Check if (p-1) and (q-1) are coprime with e
532
         unsigned long int one = 1;
533
         mpz_t p_minus_1, q_minus_1;
534
         mpz_init(p_minus_1); mpz_init(q_minus_1);
535
         mpz\_sub\_ui(p\_minus\_1, p, one);
536
         mpz_sub_ui(q_minus_1, q, one);
537
538
         assert(coprime(p_minus_1, e) == 1);
539
         assert(coprime(q_minus_1, e) == 1);
540
          gmp\_printf("\%s\%Zd \n\n", "Prime p: ", p); \\ gmp\_printf("\%s\%Zd \n\n", "Prime q: ", q); \\
541
542
543
544
         mpz_mul(mod, p, q);
545
546
         gmp\_printf("%s%Zd \setminus n \setminus n", "Modulus n: ", mod);
547
548
         // Generate private exponent d
549
         gen_d(d, p_minus_1, q_minus_1, e, 1024);
550
551
         gmp\_printf("%s%Zd\n\n", "Private exponent d: ", d);
552
553
         mpz_set(pubK.modulus, mod);
554
         mpz_set(pubK.publicExponent, e);
555
556
         mpz_set(privK.modulus, mod);
557
         mpz_set(privK.privateExponent, d);
558
559
         gmp\_printf("%Zd\n", pubK.modulus);
560
              gmp\_printf("%Zd \ n", pubK.publicExponent);
561
              return 0;
562 }*/
                             Listing 3: Code for test.c.
 1 #include <stdio.h>
 2 #include <stdlib.h>
 3 #include <string.h>
    #include <gmp.h>
#include "rsa.h"
 4
 5
 6
    int Test1(struct RSAPublicKey*, struct RSAPrivateKey*);
 8
 9
    int main() {
              struct RSAPublicKey pubK;
 10
              struct RSAPrivateKey privK;
 11
 12
              mpz_init(pubK.modulus);
 13
              mpz_init(pubK.publicExponent);
 14
              mpz_init(privK.modulus);
              mpz_init(privK.privateExponent);
 15
 16
 17
              if (Test1(&pubK, &privK))
 18
                       printf("Test1: □ Passed!\n");
 19
              else
                       printf("Test2: □ Failed!\n");
20
 21
 22
              return 0;
 23 }
```

```
24
25
                                     int Test1(struct RSAPublicKey *pubK, struct RSAPrivateKey *privK) {
26
                                                                                                                         char *message = "6628194
                                                                                                                                                                 e12073db03ba94cda9ef9532397d50dba79b987004afefe34";
27
                                                                                                                         \mathtt{char} \ \ast \mathtt{mStr} \ = \ "\mathtt{a8} \sqcup \mathtt{b3} \sqcup \mathtt{b2} \sqcup \mathtt{84} \sqcup \mathtt{af} \sqcup \mathtt{8e} \sqcup \mathtt{b5} \sqcup \mathtt{0b} \sqcup \mathtt{38} \sqcup \mathtt{70} \sqcup \mathtt{34} \sqcup \mathtt{a8} \sqcup \mathtt{60} \sqcup \mathtt{f1} \sqcup \mathtt{46} \sqcup \mathtt{56} \sqcup \mathtt{106} \sqcup
                                                                                                                                                                 \mathtt{c4} \sqcup 91 \sqcup 9\mathtt{f} \sqcup 31 \sqcup 87 \sqcup 63 \sqcup \mathtt{cd} \sqcup 6\mathtt{c} \sqcup 55 \sqcup 98 \sqcup \mathtt{c8} \sqcup \mathtt{ae} \sqcup 48 \sqcup 11 \sqcup \mathtt{a1} \sqcup \mathtt{e0} \sqcup \mathtt{ab} \sqcup \mathtt{c4} \sqcup
                                                                                                                                                                 c7 \cup e0 \cup b0 \cup 82 \cup d6 \cup 93 \cup a5 \cup e7 \cup fc \cup ed \cup 67 \cup 5c \cup f4 \cup 66 \cup 85 \cup 12 \cup 77 \cup 2c \cup 0
                                                                                                                                                                 \verb|c_0bc_064_0a7_042_0c6_0c6_030_0f5_033_0c8_0cc_072_0f6_02a_0e8_033_0c4_00b|
                                                                                                                                                                 _{\sqcup}\mathtt{f2}_{\sqcup}58_{\sqcup}42_{\sqcup}\mathtt{e9}_{\sqcup}84_{\sqcup}\mathtt{bb}_{\sqcup}78_{\sqcup}\mathtt{bd}_{\sqcup}\mathtt{bf}_{\sqcup}97_{\sqcup}\mathtt{c0}_{\sqcup}10_{\sqcup}7\mathtt{d}_{\sqcup}55_{\sqcup}\mathtt{bd}_{\sqcup}\mathtt{b6}_{\sqcup}62_{\sqcup}\mathtt{f5}_{\sqcup}
                                                                                                                                                                 \mathtt{c4} \sqcup \mathtt{e0} \sqcup \mathtt{fa} \sqcup \mathtt{b9} \sqcup \mathtt{84} \sqcup \mathtt{5} \mathtt{c} \sqcup \mathtt{b5} \sqcup \mathtt{14} \sqcup \mathtt{8} \mathtt{e} \sqcup \mathtt{f7} \sqcup \mathtt{39} \sqcup \mathtt{2d} \sqcup \mathtt{d3} \sqcup \mathtt{aa} \sqcup \mathtt{ff} \sqcup \mathtt{93} \sqcup \mathtt{ae} \sqcup \mathtt{1e} \sqcup \mathtt{6}
                                                                                                                                                                 \texttt{b} \llcorner \texttt{166} \llcorner \texttt{7b} \llcorner \texttt{b3} \llcorner \texttt{d4} \llcorner \texttt{24} \llcorner \texttt{76} \llcorner \texttt{16} \llcorner \texttt{d4} \llcorner \texttt{f5} \llcorner \texttt{ba} \llcorner \texttt{10} \llcorner \texttt{d4} \llcorner \texttt{cf} \llcorner \texttt{d2} \llcorner \texttt{26} \llcorner \texttt{de} \llcorner \texttt{88} \llcorner \texttt{d3}
                                                                                                                                                                 \square 9f \square 16 \square fb";
28
                                                                                                                         char *eStr = "01_{\sqcup}00_{\sqcup}01";
                                                                                                                        char *dStr = "53_{||}33_{||}9c_{||}fd_{||}b7_{||}9f_{||}c8_{||}46_{||}6a_{||}65_{||}5c_{||}73_{||}16_{||}ac_{||}a8_{||}
                                                                                                                                                                 5c_{\sqcup}55_{\sqcup}fd_{\sqcup}8f_{\sqcup}6d_{\sqcup}d8_{\sqcup}98_{\sqcup}fd_{\sqcup}af_{\sqcup}11_{\sqcup}95_{\sqcup}17_{\sqcup}ef_{\sqcup}4f_{\sqcup}52_{\sqcup}e8_{\sqcup}fd_{\sqcup}8e_{\sqcup}
                                                                                                                                                                 25 \, {}_{\sqcup} 8 \, d_{\sqcup} f \, 9_{\sqcup} 3 \, f_{\sqcup} e \, e_{\sqcup} 18 \, {}_{\sqcup} 0 \, f_{\sqcup} a \, 0_{\sqcup} e \, 4_{\sqcup} a \, b_{\sqcup} 29 \, {}_{\sqcup} 69 \, {}_{\sqcup} 3 \, c_{\sqcup} d \, 8_{\sqcup} 3 \, b_{\sqcup} 15 \, {}_{\sqcup} 2 \, a_{\sqcup} 55 \, {}_{\sqcup} 3
                                                                                                                                                                  d_{\sqcup} 4 a_{\sqcup} c 4_{\sqcup} d1_{\sqcup} 81_{\sqcup} 2 b_{\sqcup} 8 b_{\sqcup} 9 f_{\sqcup} a 5_{\sqcup} a f_{\sqcup} 0 e_{\sqcup} 7 f_{\sqcup} 5 5_{\sqcup} f e_{\sqcup} 7 3_{\sqcup} 0 4_{\sqcup} d f_{\sqcup} 4 1_{\sqcup} 5 7_{\sqcup} 6 e_{\sqcup} 6 e_{\sqcup}
                                                                                                                                                                 {\scriptstyle \sqcup} 09 {\scriptstyle \sqcup} 26 {\scriptstyle \sqcup} f3 {\scriptstyle \sqcup} 31 {\scriptstyle \sqcup} 1f {\scriptstyle \sqcup} 15 {\scriptstyle \sqcup} c4 {\scriptstyle \sqcup} d6 {\scriptstyle \sqcup} 5a {\scriptstyle \sqcup} 73 {\scriptstyle \sqcup} 2c {\scriptstyle \sqcup} 48 {\scriptstyle \sqcup} 31 {\scriptstyle \sqcup} 16 {\scriptstyle \sqcup} ee {\scriptstyle \sqcup} 3d {\scriptstyle \sqcup} 3d {\scriptstyle \sqcup} 2d {\scriptstyle \sqcup}
                                                                                                                                                                 0 \verb|a|| f 3 | 5 4 | 9 \verb|a|| d 9 | b f | 7 c | b f | b 7 | 8 a | d 8 | 8 4 | f 8 | 4 d | 5 b | e b | 0 4 | 7 2 | 4 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 6 d | 
                                                                                                                                                                 \mathtt{d}_{\sqcup}\mathtt{c7}_{\sqcup}36_{\sqcup}9\mathtt{b}_{\sqcup}31_{\sqcup}\mathtt{de}_{\sqcup}\mathtt{f3}_{\sqcup}7\mathtt{d}_{\sqcup}0\mathtt{c}_{\sqcup}\mathtt{f5}_{\sqcup}39_{\sqcup}\mathtt{e9}_{\sqcup}\mathtt{cf}_{\sqcup}\mathtt{cd}_{\sqcup}\mathtt{d3}_{\sqcup}\mathtt{de}_{\sqcup}65_{\sqcup}37_{\sqcup}29
                                                                                                                                                                 _{\sqcup}ea_{\sqcup}d5_{\sqcup}d1";
30
                                                                                                                        mpz_set_str(pubK->modulus, mStr, 16);
31
                                                                                                                        mpz_set(privK->modulus, pubK->modulus);
                                                                                                                        mpz_set_str(pubK->publicExponent, eStr, 16);
32
33
                                                                                                                        mpz_set_str(privK->privateExponent, dStr, 16);
34
                                                                                                                        char *C = RSAES_OAEP_ENCRYPT(pubK, message, NULL);
35
36
                                                                                                                        char *M = RSAES_OAEP_DECRYPT(privK, C, NULL);
37
                                                                                                                        if (!M) return -1;
38
39
                                                                                                                        return (strcmp(M, message) == 0);
40
                                  }
```

B Crypto Coding Practices

- 1. We learned how to use a cryptographically secure pseudorandom number generator and that even this has inherent disadvantages.
- 2. We learned how to prevent the elementary common modulus attack.
- 3. We learned how to prevent low private exponent attacks and that the security of our implementation can be improved further by choosing a larger public exponent.
- 4. We learned how to prevent low public exponent attacks and that the security of our implementation can be improved further by choosing a larger public exponent.
- 5. We learned how to prevent partial key exposure attacks and that our implementation is lacking in privacy provisions for the private exponent d.
- 6. We learned that our implementation is not immune to timing attacks and power consumption attacks and that precluding these attacks is difficult.

C Secure Coding Practices

- 1. We learned the importance of freeing dynamically allocated memory, especially in a cryptographic setting where unfreed memory can contain sensitive data.
- 2. We learned to correctly size memory allocation for an object; using GMP Library for most instances greatly reduces developer errors.
- 3. We learned the importance of converting char data types to unsigned char data types whenever it is being passed to a character-handling function.
- 4. We learned retrospectively that consistent and comphrehensive error handling would have made the development effort much easier.
- 5. We would have liked to create a comprehensive test suite that could ensure correctness through future iterations of our implementation. This would have been tremendously helpful during the development process.
- 6. It would be interesting to leverage static analysis techniques and a binary fuzzer, such as AFL, to discover any unintended behavior in our implementation.