Reformulation: Summation of Some Polygamma Functions at Negative Integer Values

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ABSTRACT

In this paper we consider the summation of some Polygamma[n, f(x)] function from interval x equals to N_1 to interval x equals to N_2, where the value of f(x) along this interval includes some negative or asymptotically negative integer values. We also consider a normalized signal function that is continous and differentiable across all real domain unlike the sinc(x) function.

OUTLINE

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INTRODUCTION

Closed form partial summations of some elementary and non-elementary mathematical functions f(x,y) tend to resolve into non-polynomial functions comprising of the Polygamma, Factorial and other closely related functions g(y). These new functions when their x values are positive can be approximated by Maclaurin, Taylor and other series expansion methods at a specific value or at infinity. At negative values of x these functions can be expressed in terms of their positive values, and the co-tangent function or closely associated functions, which tend to positive or negative infinity at some or all integer values and that are discontinuous.

Let f(x,y) = 1/(x + y)

Sum[f(x,y), {x, 1, N}, {y, 1, N}] = Sum[Polygamma[1,y], {y, 1, N}] - Expression(1)

Where; g(y) = Polygamma[1,y]

Let f_a(x,y) = 1/(x - y)

Sum[f_a(x,y), {x, 1, N}, {y, 1, N}] = Sum[Polygamma[1, -y], {y, 1, N}]
- Expression(2)

Where; g_a(y) = Polygamma[1,-y]

Expression(1) & Expression(2) above are both very simple cases of functions f(x), which when partially summed resolve into new functions g(y) & g_a(y) which are wholly composed of Polygamma functions with positive variables and Polygamma functions with negative variables respectively.

GAMMA & POLYGAMMA FUNCTION

Let g(y) = Polygamma[1,y]

Taylor Series Expansion of g(y) at infinity =

The Gamma function and higher order Differential/Integral Polygamma functions are continous across all positive y real values

Let g_a(y) = Polygamma[1,-y]

g_a(y) = Polygamma[1,y] + 1/y + (Pi * y * Cot[Pi * y])

Because of the (Pi * y * Cot[Pi * y]) function in the g_a(y) function definition above, the Gamma function and higher order

Differential/Integral Polygamma functions are discontinuous at all negative integer values of y.

NP-COMPLETENESS

ELLIPTICAL CURVE CRYTOGRAPHY

Consider an elliptical curve (NIST curve secp256k1) [1];

$y^2 = x^3 + a * x + b$

where; ($a = 0$, $b = 7$).

$x^3 + 7 - y^2 \bmod p = 0$

$(x^3 + 7 - y^2)/p - 1 = 1$ - Expression(3)

where p is prime and $p > 3$

using A.S.E & A.S.M [2] we can find all integers; x & y that satisfy Expression(3) above from interval -P to interval P.

i.e: Sum[A.S.E(M), {x,-P,P}, {y,-P,P}]; where M = Expression 3 above gives us the total number of combimation of integer values of x and y that equals 1.

We can then use A.S.M to find the integer values.

CONTINOUS COSINC FUNCTION

The continous CoSinc function can be defined by;

2 * Cos[g(x)]/(g(x) + 1)

g(x) = (F * (Sin[Pi f(x)])^2)

we can reformulate the sums of Polygamma functions comprising of negative values that express a signal via the Alli-Smith Expression [2] into the continous CoSinc function above.

Sum[Sum[A.S.E(M), {x,1,N}] = Polygamma[J] , {y,1,N}]

Sum[Polygamma[J], {y,1,N}] = Sum[2 * Cos[F * (Sin[Pi J])^2]/(F * (Sin[Pi J])^2 + 1), {y,1,N}] - Expression(4), where J is a negative value from interval y equals 1 to N.

A continous signal function can be defined by;

2/(g(x) + 1)

Expression(4) above can be reformulated into Sum[Polygamma[J], {y,1,N}] = Sum[2 * 1/(F * (Sin[Pi J])^2 + 1), {y,1,N}], where J is a negative value from interval y equals 1 to N.

CONCLUSION

In conclusion we can sum Expression(5) above using the Euler Maclaurin Summation Formular;

F(x) = 2/(F * Sin^2(Pi * (d - x)/x) + 1)

R_f(x) = Integrate[BernoulliB[2, Frac(x)] * D[F(x), {x,2}] * 1/2, {}], where R_f (x) is the remainder part of E.M.S.F.

Local minimum g(x) of f(x) = 1/(1 + 4n)

We can utilise these points from g(x) above as intervals for R_f(x) integration; where d = 21 & F = 10^5.

Integrate[R_f(1/(1 + 4n)), {n, 1, 2}] + Integrate[R_f(1/(1 + 4n)), {n, 2, 3}] + ... + Integrate[R_f(1/(1 + 4n)), {n, k - 1, k}] (?)

Integrate[Taylor Series[R_f(1/(1 + 4n)), n=7], {n, 1, 2}] + ... +

Integrate[Taylor Series[R_f(1/(1 + 4n)), n=(1/(1 + 4(k - 1)) + 1/(1 + 4k))/2, {n, k-1, k}]

Sum[Integrate[Taylor Series[R_f(1/(1 + 4n)), {n=(1/(1 + 4(J-1)) + 1/(1 + 4J))/2}],{n, J-1, J}],{J, 1, k}]

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