

BB 101: Biology. End-semester test

19 Nov 2023

Duration: 2 Hours.

Instructions:

- You can get maximum 33 Marks from this. Total marks (quiz+end-semester)=50.
- You may refer to one **hand-written A4 sheet**. You are NOT allowed to refer to other notes or other students. All questions are self-explanatory; context is based on discussions in the class.
- Do not share the calculator. If you do not have a calculator, write approximate numbers.
- Take Boltzmann constant $k_B = 1.4 \times 10^{-23}$ J/K. Viscosity of water = 10^{-3} Pa.s. Unless specified otherwise, use room temperature $T = 300\text{K}$.

1. The under-lined part in the sentences below has a mistake. Correct it. ($7 \times 1 = 7$ Marks)

(A) Inside cells, in the world of molecular biology, force is proportional to acceleration.

(a) Inside cells, in the world of molecular biology, force is proportional to velocity

(B) The Hecht-Shaler-Pirenne experiment suggests that rod cells in our eye require at least a thousand photons to produce detectable signal.

(a) The Hecht-Shaler-Pirenne experiment suggests that rod cells in our eye require at least six photons to produce detectable signal (a few or one are also given marks)

(C) Electrostatic repulsion energy is what leads to the diffusion of proteins.

(a) Thermal energy is what leads to the diffusion of proteins.

(D) *In vitro*, vesicles can be assembled in a beaker by adding lipids, water, etc. This self-assembled state represents the state of the system with minimum potential energy.

(a) ... the state of the system with minimum free energy.

(E) The primary function of F_0 - F_1 motor is to produce proton current using GDP

(a) The primary function of F_0 - F_1 motor is to produce ATP using proton current

(F) A funnel-shaped free energy implies that a protein will take a large number of configurations.

(a) ...implies that a protein will take one (or a few or native or unique) configuration. Or will quickly reach the native/unique/one configuration

(G) Typical force generated by a single molecular motor is of the order of mN (milli Newton), which is approximately 10^{-3} N.

(a)approximately pico Newton or 10^{-12} N

2. During the developmental process in *Drosophila* embryo, the diffusion coefficient (D) and degradation rate (K_d) of the Bicoid protein can set a scale (a ruler) for measuring lengths in tissues, leading to pattern formation. (3x2 = 6 Marks)

(A) Write the expression for a length-scale emerging from D and K_d ?

$$l_0 = \sqrt{D/k_d}$$

(B) Compute D for a typical globular protein of size $a=5$ nm at $T=300$ K in water.

$$D = \frac{k_B T}{6\pi\eta a} \text{ (1 Mark)}$$

$$D \approx \frac{4.2 \times 10^{-21}}{18.84 \times 10^{-3} \times 5 \times 10^{-9}}$$

$$\approx 42 \times 10^{-12} \frac{m^2}{s} \text{ (number of this order is given full mark)}$$

(C) If we assume that a similar process sets the size of the stripes on Tigers, what must be the value of K_d ? Assume that the size of the tiger stripe is 1cm.

$$\sqrt{\frac{D}{k_d}} = 1 \text{ cm (writing this relation gets 1 mark)}$$

Substitute above numbers and get

$$\Rightarrow k_d \approx 42 \times 10^{-8} s^{-1} \text{ (number of this order is given full mark)}$$

3. The figure below represents the current through an ion-channel. Answer the following question: (4 Marks)

Assume that the system can be found only in two discrete states: open state and closed state. Based on the figure, let us conclude that the probability of finding the ion channel in the closed state is 0.75. Assuming that the system obeys Boltzmann probability, calculate the energy of the closed state, if the energy of the open state is $0 k_B T$

$$P_c = 0.75 = \frac{e^{-E_c/k_B T}}{e^{-E_o/k_B T} + e^{-E_c/k_B T}}$$

Or equivalently

$$P_o = 0.25 = \frac{e^0}{e^0 + e^{-E_c/k_B T}}$$

Substitute and solve to get

$$E_c = -k_B T \ln 3 \text{ (anything equivalent is given mark)}$$

4. Consider a double stranded DNA of N basepairs (bp) at temperature T.

Assume that each base pairing gains energy E. The thermal fluctuations will break some of the base-pairing interactions to "melt" the double-stranded DNA into single stranded DNAs. Consider a situation where m base-pairs are intact and (N-m) base-pairs are broken (due to thermal fluctuations). Assuming $E = 1 k_B T$, compute and plot:

- (A) Entropy of the system, S(c), where $c=m/N$ is the fraction of the intact base-pairs. (2 Marks)

$$S = k_B \ln W = k_B \ln \left(\frac{N!}{m!(N-m)!} \right)$$

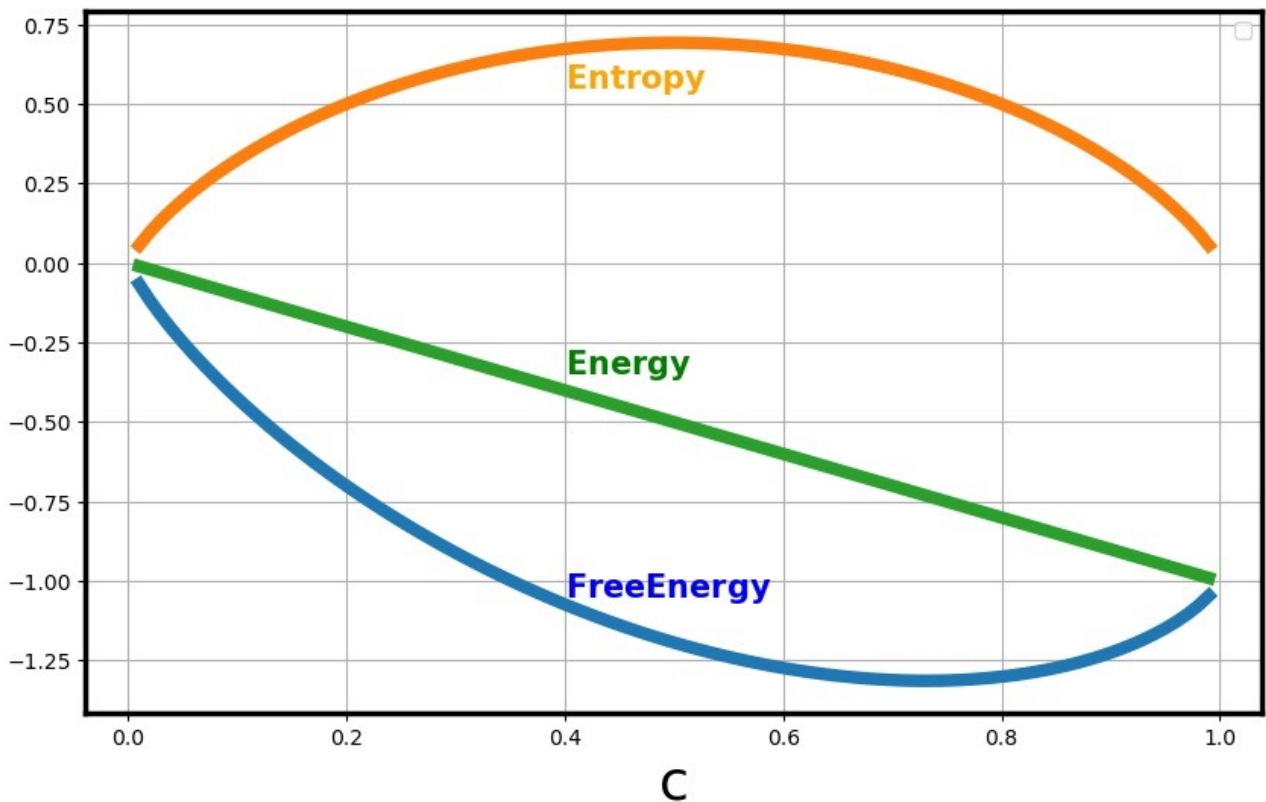
$$\text{Entropy, } S = -Nk_B [c \ln c + (1-c) \ln(1-c)]$$

- (B) Energy of the system, E(c). (2 Marks)

$$\text{Energy} = -m\epsilon = -Nc\epsilon$$

- (C) The Free energy of the system (2 Marks)

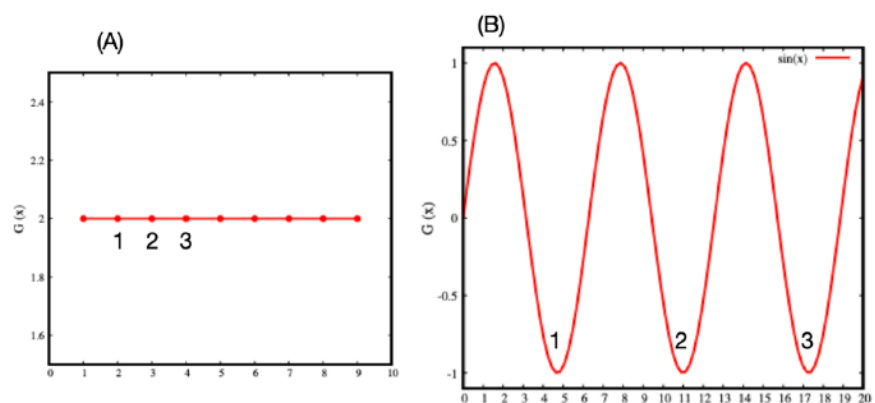
$$\text{Free Energy} = -Nc\epsilon + Nk_B T [c \ln c + (1-c) \ln(1-c)]$$



5. Consider the two free energy plots shown below. For each case, write an expression for the rate of the movement from state 2 to state 1 (r_{21}) and from state 2 to state 3 (r_{23}). Also, state what is the direction of net movement. (2x2 = 4 Marks)

(A) In the first figure, $G(x)$ = a constant. The three states are marked as 1,2,3 in the figure.

(B) In the second figure $G(x) = \sin(x)$. The three states are marked as 1,2,3 in the figure.



Answer (A)

$$r_{21} = r_0 e^0 = r_0$$

$$r_{12} = r_0 e^0 = r_0$$

(1.5 Marks if both the above equations are written correctly. Half mark for direction below)

No net movement (or movement in random direction or do not move or anything equivalent)

Answer (B)

$$r_{21} = r_0 \exp\left(\frac{-2 \text{ unit}}{k_B T}\right) \text{ (whatever unit of 2 you write, you will get mark)}$$

$$r_{21} = r_0 \exp\left(\frac{-2 \text{ unit}}{k_B T}\right)$$

No net movement (or movement in random direction or do not move or anything equivalent)

6. Many of the biopolymers, like actin and DNA, will appear like a bendable wire (cable) under a microscope. Assume that the picture below represents actin and DNA bent into an arc or circle, due to thermal fluctuations, at room temperature $T=300\text{K}$. The energy to bend such filaments of length L to an arc of a circle of radius R is $E = BL/R^2$ where B is the bending stiffness of the filament (it is the elastic constant of the filament related to the Young's modulus).

- (A) An actin filament of length $L_a = 10$ micrometer is bent to arc which is a part of a circle of radius $R_a = 10$ micrometer (see the left side picture). Calculate the approximate value of the bending stiffness. Use SI units. Assume actin is stable and not polymerising or depolymerising. (3 Marks)

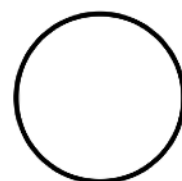
- (B) A DNA of length $L_d = 2$ micrometer is bent to a full circle of radius R_d . Calculate the approximate value of the bending stiffness. Use SI units. (3 Marks)

Actin under microscope seen as an arc of length L_a



Imagine that the arc is part of a circle (dashed line) of radius R_a

DNA of length L_d , under microscope, seen as bent to a circle



The radius of the circle is R_d

$$k_B T = \frac{BL}{R^2} \text{ (1 mark for equating this; mark is given even if you write } (3/2) k_B T)$$

Solve for B by substituting the values given

(A) $B = 10\mu m k_B T = 42 \times 10^{-27} Jm$

(B) $L = 2\pi R$

$$B \approx 2 \times 10^{-28} Jm$$