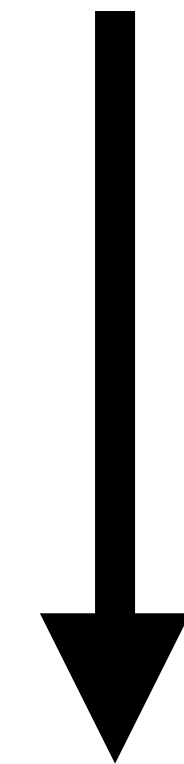
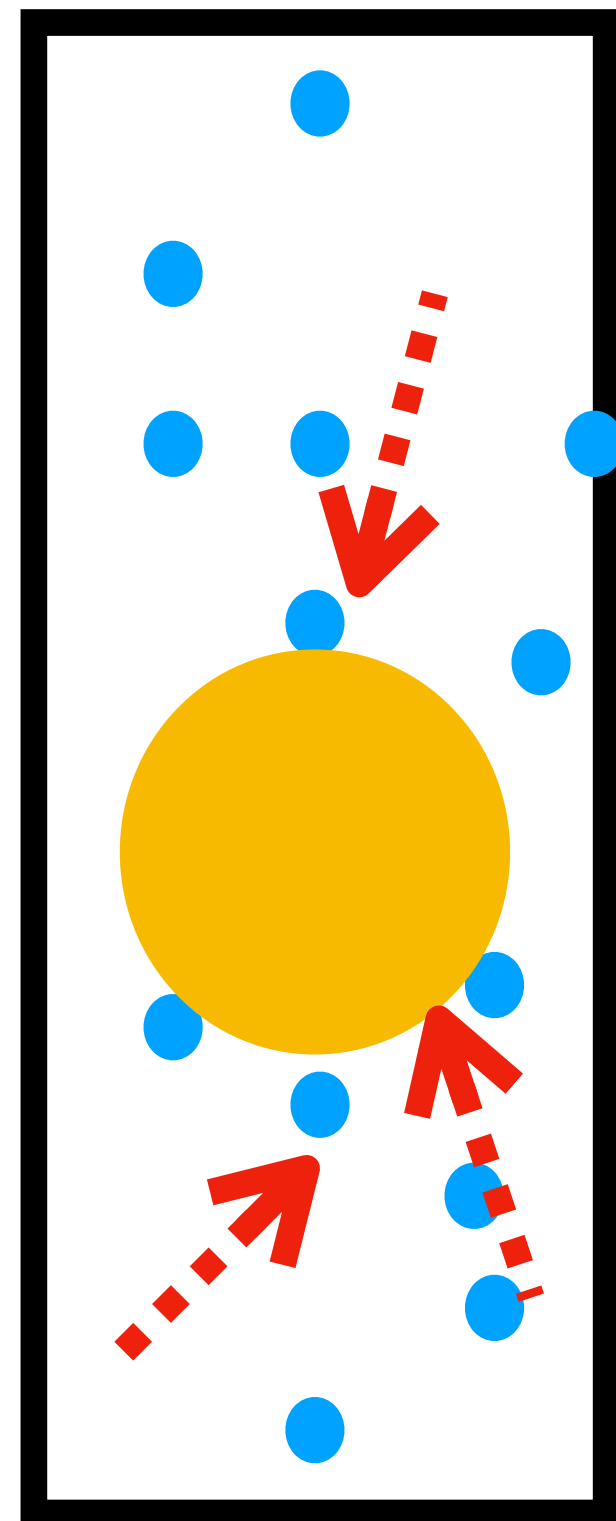


**“Life” (or Molecular biology) exists
at low Reynolds number, in salty
water, and in a thermal bath!**

What are the laws of physics and chemistry there?

**What is the consequence of
the thermal energy?**

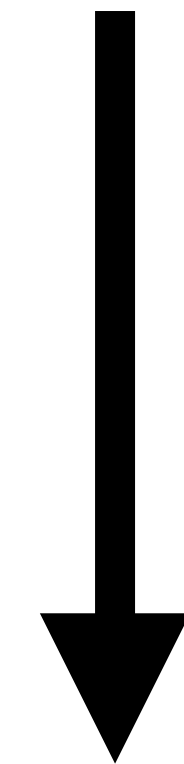
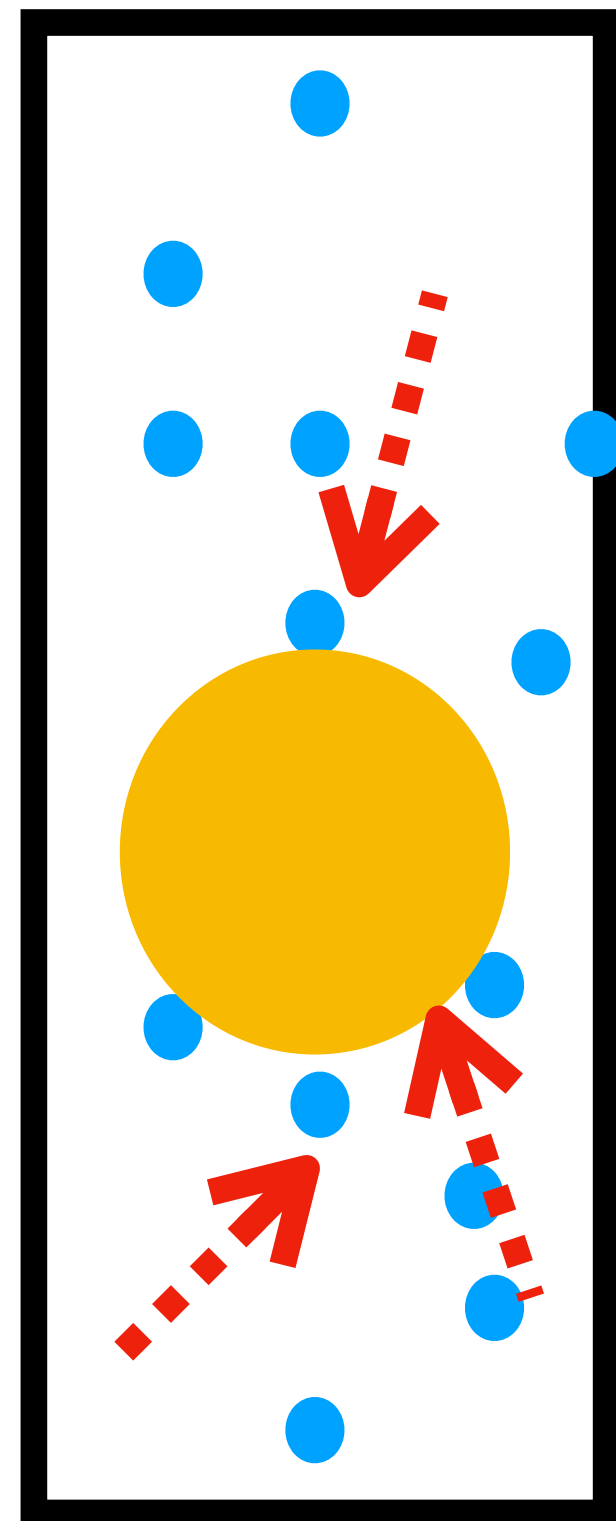
A cell or a biomolecule falling in a test tube under an external force (gravity or electric force)



Force, f

Where will you find the cell?

A cell or a biomolecule falling in a test tube under an external force (gravity or electric force)



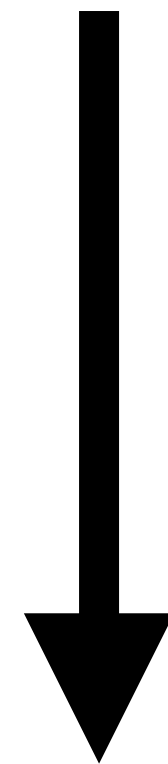
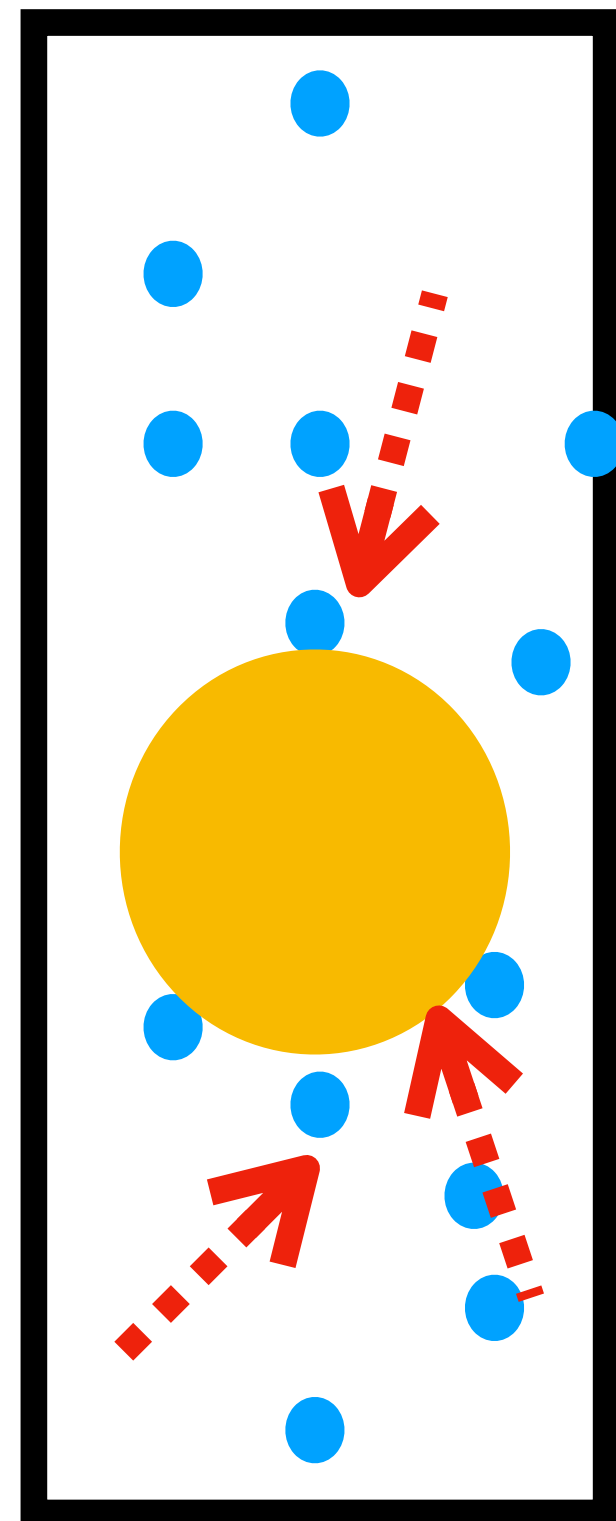
Force, f

Where will you find the cell?

At the bottom

(old understanding; no thermal effect)

A cell or a biomolecule falling in a test tube under an external force (gravity or electric force)

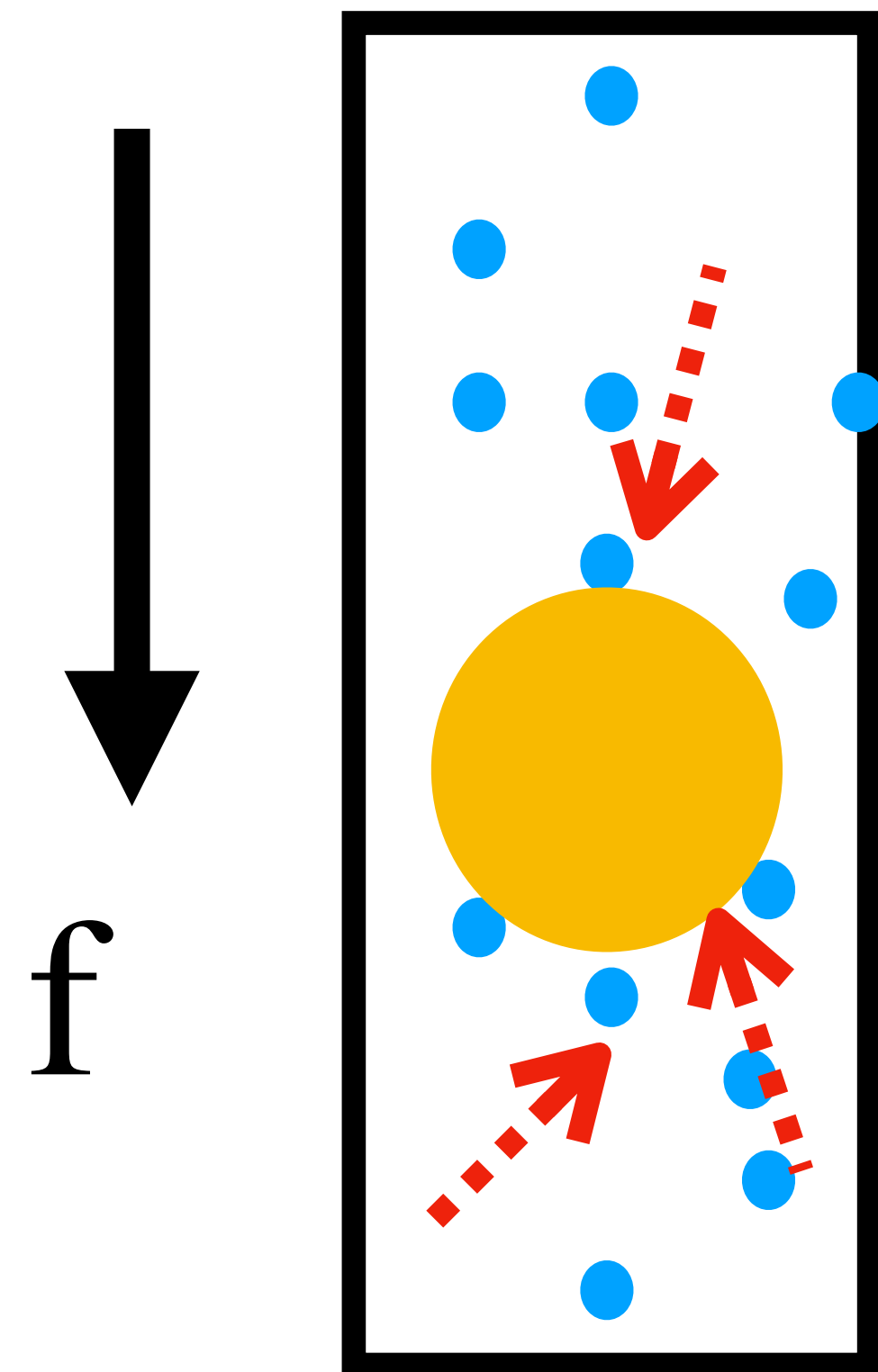


Force, f

Where will you find the cell?

With thermal kicks, it can be anywhere!

A cell or a biomolecule falling in a test tube under an external force (gravity or electric force)



Where will you find the cell?

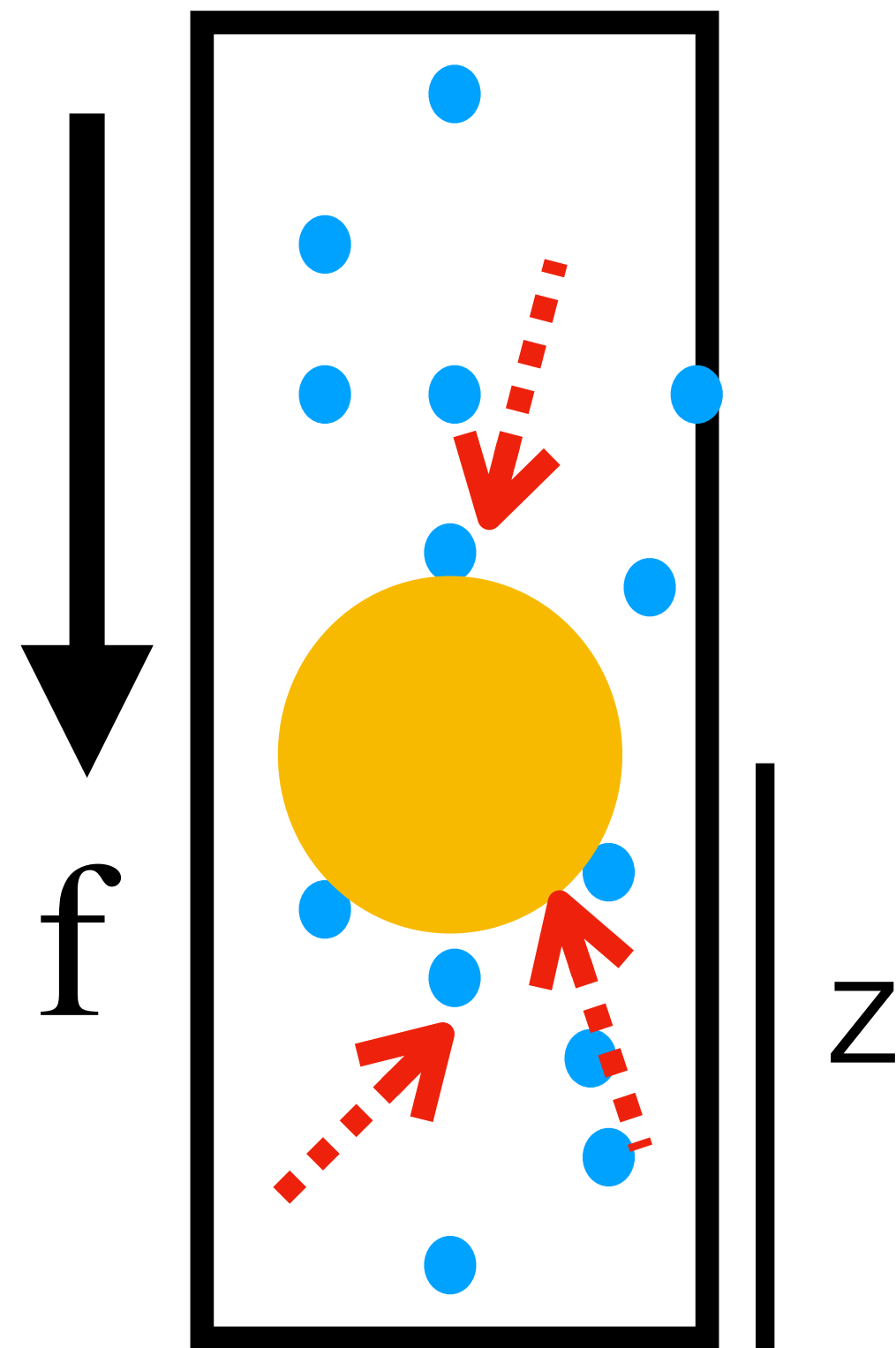
With thermal kicks, it can be anywhere!

We cannot be certain! Hence we have to use “probability” ideas!

Biology => probabilistic; stochastic

With thermal kicks, it can be anywhere!

Probability of finding the a height, z



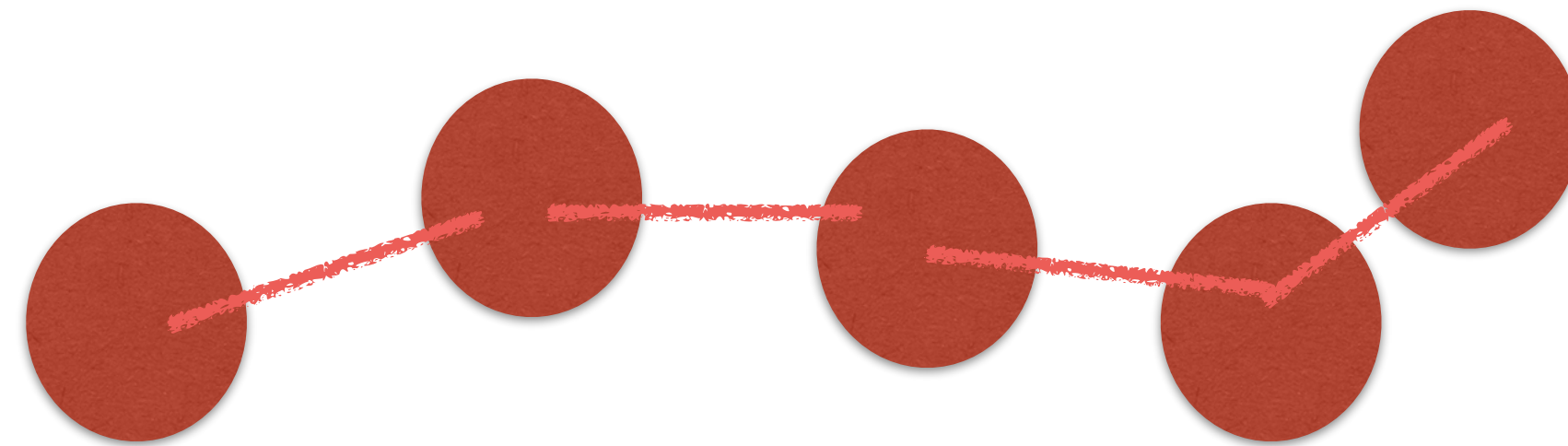
$$P(z) = A \exp \left(\frac{-E(z)}{k_B T} \right)$$

A = normalisation constant

Boltzmann probability

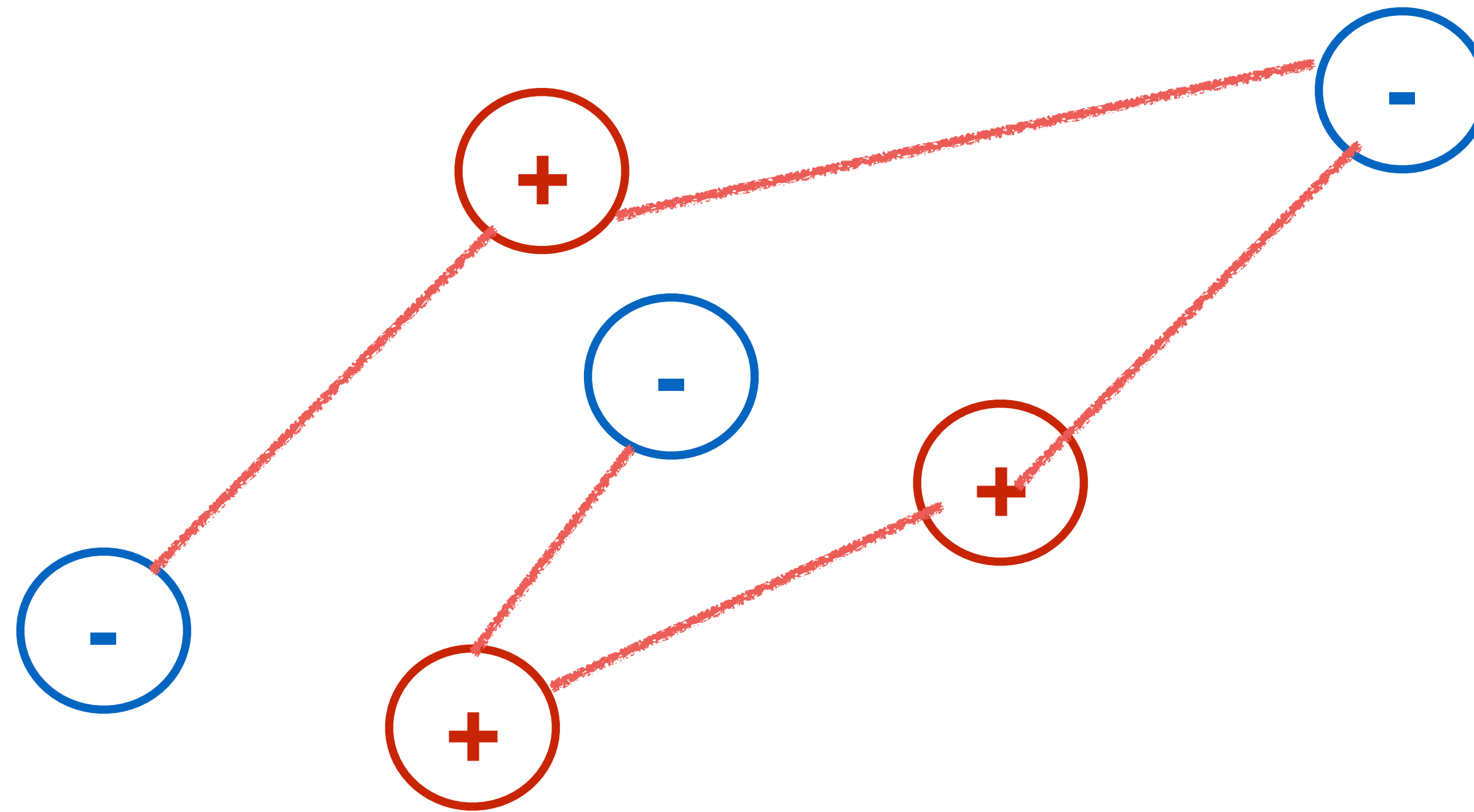
$E(z)$ = Potential energy at height z

A protein is a small chain of molecules.



Proteins do their job
by folding up to a nearly unique shape in 3D

What will be the preferred conformation of such proteins ?



Probability of finding protein in a conformation
(say, conformation “n”)

$$P_n = \frac{\exp\left(\frac{-E_n}{k_B T}\right)}{Z}$$

E_n is the total potential energy of the protein, in that conformation

Boltzmann probability

$$Z = \sum_n \exp \left(\frac{-E_n}{k_B T} \right)$$

Z=partition function

**Unlike what you learned so far, the
system does not find itself in its
minimum “energy” state!**

Unlike what you learned so far, the system does not find itself in its minimum “energy” state!

System (biomolecule, cells) can be in many different “states”.

Each has a certain probability

State = arrangements of atoms; also known as microstate

Dynamics: What is the equation of motion that you have studied?

In the world we see around us,

force \propto acceleration

JEE question: If I take a dead bacterium and leave it with an initial velocity “ u ”, in water, how far will it move?

(neglect thermal fluctuation; neglect gravity)

Assume bacterium to be a sphere of radius, $a = 1$ micrometer, filled with water.

$$m \frac{dv}{dt} = -6\pi\eta a v$$

$$v = u \exp \left(\frac{-6\pi\eta a}{m} t \right)$$

$$v = u \exp \left(\frac{-6\pi\eta a}{m} t \right)$$

$$v = u \exp \left(\frac{-t}{\tau} \right)$$

$$\tau = \frac{m}{6\pi\eta a}$$

$$\text{mass, } m = \text{density} \times \text{volume} = 1000 \times \frac{4}{3}\pi(1\mu m)^3 \text{ SI units}$$

$$\approx 4 \times 10^{-15} kg$$

$$\tau = \frac{m}{6\pi\eta a} \approx \frac{4 \times 10^{-15}}{20 \times 10^{-3} \times 10^{-6}} = 2 \times 10^{-7} \text{ seconds}$$

$$v = u \exp \left(\frac{-t}{\tau} \right)$$

$$\tau = \frac{m}{6\pi\eta a}$$

$$\text{Total distance moved by bacterium} = \int_0^{\infty} v dt = u\tau$$

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$$u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

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$$\tau = \frac{m}{6\pi\eta a}$$

$$\text{Total distance moved by bacterium} = \int_0^{\infty} v dt = u\tau$$

$$u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

Total distance moved by the bacterium is smaller than the size of an atom!!

$$m \frac{dv}{dt} = -6\pi\eta a v$$

$$u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

A bacterium without any external force, cannot even move the size of an atom!

$$m \frac{dv}{dt} = -6\pi\eta a v$$

$$u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

A bacterium without any external force, cannot even move the size of an atom!

If you want to move (if you want velocity), apply force!

$$m \frac{dv}{dt} = -6\pi\eta a v \qquad u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

A bacterium without any external force, cannot even move the size of an atom!

If you want to move (if you want velocity), apply force!

$$m \frac{dv}{dt} + 6\pi\eta a v = f$$

$$m \frac{dv}{dt} = -6\pi\eta a v$$

$$u\tau \approx 10^{-6}(m/s) \times 10^{-7}s = 10^{-13}m$$

A bacterium without any external force, cannot even move the size of an atom!

If you want to move (if you want velocity), apply force!

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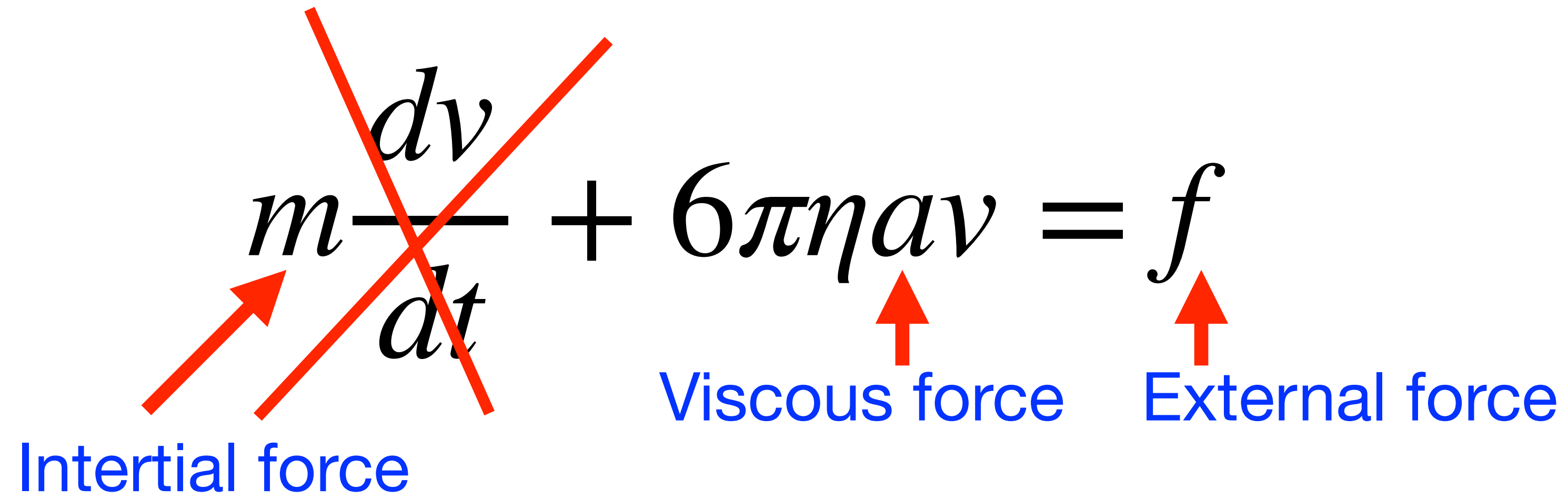
Inertial force Viscous force External force

For the parameters relevant to molecular biology, inertial force contribution is negligible compared to viscous force

$$\cancel{m \frac{dv}{dt}} + 6\pi\eta a v = f$$

Inertial force Viscous force External force

For the parameters relevant to molecular biology, inertial force contribution is negligible



The diagram shows the equation $m \frac{dv}{dt} + 6\pi\eta a v = f$. A large red 'X' is drawn over the term $m \frac{dv}{dt}$. A red arrow points from the text 'Inertial force' below to the $m \frac{dv}{dt}$ term. Another red arrow points from the text 'Viscous force' below to the $6\pi\eta a v$ term. A third red arrow points from the text 'External force' below to the f term.

$$\cancel{m \frac{dv}{dt}} + 6\pi\eta a v = f$$

Inertial force Viscous force External force

\Rightarrow force \propto velocity

For the parameters relevant to molecular biology, inertial force contribution is negligible

The diagram shows the equation $m \frac{dv}{dt} + 6\pi\eta a v = f$. A large red 'X' is drawn over the first term, $m \frac{dv}{dt}$. Three red arrows point upwards to the terms in the equation: one to m , one to v in the second term, and one to f . Below each arrow is a blue label: 'Inertial force' under the arrow to m , 'Viscous force' under the arrow to v , and 'External force' under the arrow to f .

$$\cancel{m \frac{dv}{dt}} + 6\pi\eta a v = f$$

Inertial force Viscous force External force

\Rightarrow force \propto velocity

(Not acceleration!)

For the parameters relevant to molecular biology, inertial force contribution is negligible

$$\Rightarrow \text{force} \propto \text{velocity}$$

Experiments probing motion in biology will typically measure the relation between force and velocity

Force-Velocity relation

At low Reynolds number

\Rightarrow force \propto velocity

$$\text{Reynolds number, } R \approx \frac{u\tau}{a} \approx \frac{\rho u a}{\eta}$$

$$R = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

At low Reynolds numbers

\Rightarrow force \propto velocity

$$\text{Reynolds number, } R \approx \frac{\rho u a}{\eta} = \frac{10^3 u a}{10^{-3}} = 10^6 u a$$

	Typical speed, u	Typical size, a	Approximate R	R compared to 1
Bacterium in water	1 micron/s	Micron	10^{-6}	Low R
Molecular motor in water	1 micron/s	10 nm	10^{-8}	Low R
Fish in water	10cm/s	10cm	10^4	High R
Human in water	0.1 m/s	1m	10^5	High R
Human in tar (viscosity 100 billion times of water)	cm/s	1m	10^{-8}	Low R

Advanced reading recommendation

Life at low Reynolds number

E. M. Purcell

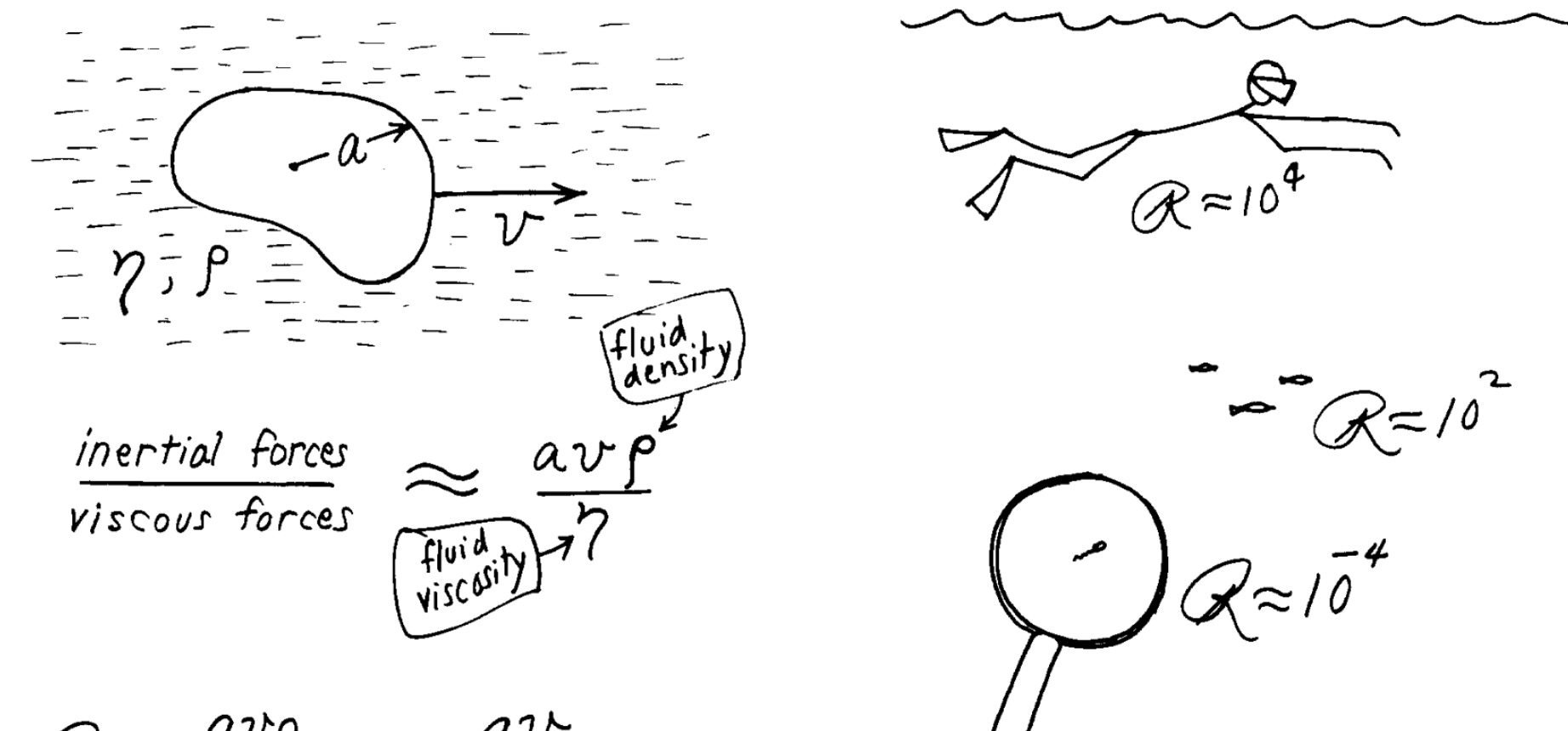
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Summary

- Not minimum energy!
- Thermal force would make all configurations probable
- Some configurations are more probable than the others
- Boltzmann probability
- In the microscopic biology world, force is needed to maintain velocity
- Viscous forces are much larger than inertial forces.
- In the low Reynold's number world, F is proportional to v .