CH 111 Tutorial 1

Solve these problems BEFORE the tutorial session

1. Consider the eigenvalue equation $C^2\Psi = \Psi$ where C is a quantum mechanical operator, and Ψ is an eigenfunction. What are the eigenvalues of the operator C?

2. The eigenvalue equation is given as $\hat{A}\Psi = a\Psi$. Suggest eigenfunctions for the following operators

   (i) $-i\hbar\dfrac{\partial}{\partial q}$ (ii) $\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$

3. Plot the following functions and hence, explain which of these CANNOT be a valid wavefunction. (x is real)

   (i) $x\sin x$ (ii) $\dfrac{1}{x}\sin x$ If time permits, try (iii) e^{-x^2} (iv) $1-e^{-x}$

4. Suppose that the wavefunction for a system can be written as

$$\psi(x) = \frac{\sqrt{11}}{4}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{2+\sqrt{2}\,i}{2}\phi_3(x)$$

   where, $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$ are orthogonal to each other and are normalized eigenfunctions of an arbitrary Hermitian operator $\hat{A}$, with eigenvalues a_1, a_2 and a_3 respectively.

   a) Is $\psi(x)$ normalized?
   b) Suppose we have 36,000 identically prepared systems in the normalised $\psi(x)$ state. If we perform measurements corresponding to the operator $\hat{A}$ on these systems, what are the possible values of the observable that you could obtain and how many times do they occur (approximately)?
   c) What is the (i) average value and (ii) most probable value of the observable that will be obtained for infinite number of measurements?

5. Estimate the value: $\frac{d}{dt}\int_{-\infty}^{\infty}\Psi_i^*(x,t)\Psi_j^*(x,t)dx$, where Ψ_i and Ψ_j are the stationary state solutions of the Schrodinger equation with energy eigenvalue E_i and E_j respectively.

6. (a) An electron is moving towards positive x direction with a speed of 200m/s to an accuracy of 1%. What is the minimum uncertainty with which its position is known? (mass of electron $m_e = 9.1 \times 10^{-31}$kg, Planck's constant h=6.624$\times$ 10^{-34}J. s).

(b) For a cricket ball of mass m=0.05kg with similar speed with measurement accuracy of 1%, what would be the approximate uncertainty in position? Rationalize your answer.