

## Schrodinger's philosophy

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \Psi(x,t)}{\partial t^2} \quad \text{Classical Wave Equation}$$

$\Psi(x,t)$  = Amplitude

$\Psi(x,t) = Ce^{i\alpha}$  ; Where  $\alpha = 2\pi \left( \frac{x}{\lambda} - \nu t \right)$  is the phase

*Remember!*

$$E = h\nu = \hbar\omega$$

$$\lambda = \frac{h}{p} = \frac{2\pi}{k}$$

$$\alpha = 2\pi \left( \frac{x}{\lambda} - \nu t \right) = \frac{x \cdot p - E \cdot t}{\hbar}$$

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$$\frac{\partial \Psi(x,t)}{\partial t} = iCe^{i\alpha} \cdot \frac{\partial \alpha}{\partial t} = i \cdot \Psi(x,t) \cdot \frac{\partial \alpha}{\partial t} = i \cdot \Psi(x,t) \cdot \left( \frac{-E}{\hbar} \right)$$

$$\frac{-\hbar}{i} \frac{\partial \Psi(x,t)}{\partial t} = E \cdot \Psi(x,t)$$

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## Operators

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$$\frac{-\hbar}{i} \frac{\partial}{\partial t} = i\hbar \frac{\partial}{\partial t} = \hat{E}$$

$$\frac{\hbar}{i} \frac{\partial}{\partial x} = -i\hbar \frac{\partial}{\partial x} = \hat{p}_x \quad \text{Operators}$$

### **Operator**

A symbol that tells you to do something to whatever follows it

Operators can be real or complex,

Operators can also be represented as matrices

## Operators and Eigenvalues

Operator operating on a function results in re-generating the same function multiplied by a number

$$\hat{A} \cdot f(x) = a \cdot f(x) \quad \text{Eigen Value Equation}$$

The function  $f(x)$  is eigenfunction of operator  $\hat{A}$  and  $a$  its eigenvalue

$$f(x) = \sin(\alpha x)$$

$$\frac{d}{dx} f(x) = \alpha \cdot \cos(\alpha x)$$

$$\frac{d^2}{dx^2} f(x) = \frac{d}{dx} [\alpha \cdot \cos(\alpha x)] = -\alpha^2 \cdot \sin(\alpha x) = -\alpha^2 \cdot f(x)$$

$\sin(\alpha x)$  is an eigenfunction of operator  $\frac{d^2}{dx^2}$  and  $-\alpha^2$  is its eigenvalue

# Laws of Quantum Mechanics

The mathematical description of quantum mechanics is built upon the concept of an operator

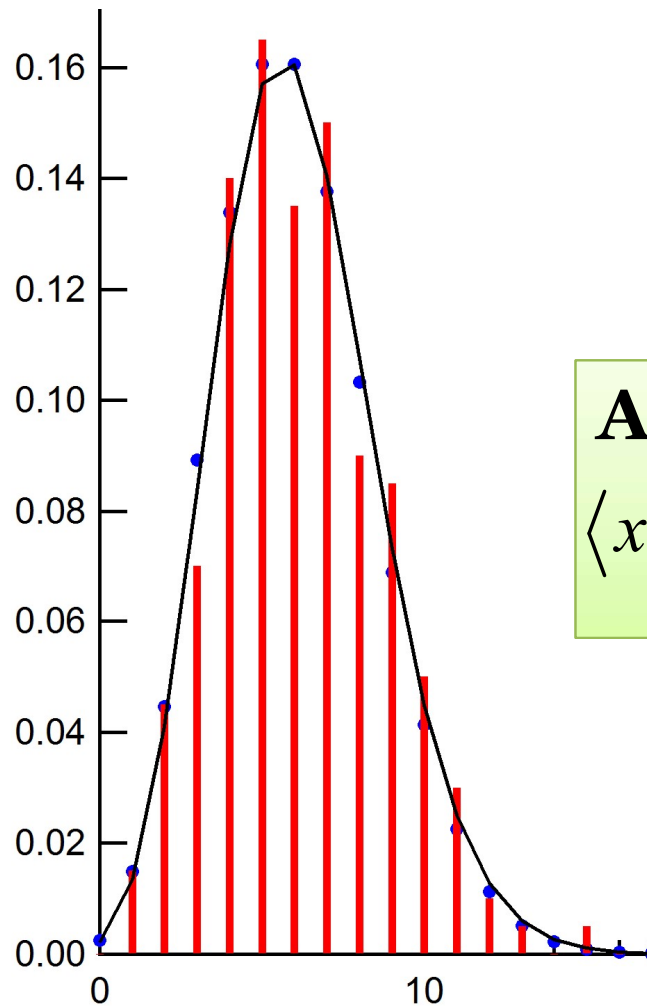
The values which come up as result of an experiment are the eigenvalues of the self-adjoint linear operator.

The average value of the observable corresponding to operator  $\hat{A}$  is

$$\langle a \rangle = \int \Psi^* \hat{A} \Psi d\nu$$

The state of a system is completely specified by the wavefunction  $\Psi(\mathbf{x}, \mathbf{y}, \mathbf{z}, t)$  which evolves according to time-dependent Schrodinger equation

# Probability Distribution



## Average Values

$$\langle x \rangle = \sum_{j=1}^n x_j P_j(x_j) \text{ and } \langle x^2 \rangle = \sum_{j=1}^n x_j^2 P_j(x_j)$$



# Probability Distribution

Let us consider Maxwell distribution of speeds

$$f(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

The mean speed is calculated by taking the product of each speed with the fraction of molecules with that particular speed and summing up all the products. However, when the distribution of speeds is continuous, summation is replaced with an integral

$$\bar{v} = \int_0^\infty v f(v) dv = \left( \frac{8RT}{\pi M} \right)^{1/2}$$

