MA 105 Part II (IIT Bombay) Tutorial Sheet 2 : Multiple integrals

1. For the following, write an equivalent iterated integral with the order of integration reversed:
   (a) $\displaystyle\int_0^1 \left[\int_1^{e^x} dy\right] dx$
   (b) $\displaystyle\int_0^1 \left[\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dx\right] dy$

2. Evaluate the following integrals
   (a) $\displaystyle\int_0^{\pi} \left[\int_x^{\pi} \frac{\sin y}{y} dy\right] dx$
   (b) $\displaystyle\int_0^1 \left[\int_y^1 x^2 e^{xy} dx\right] dy$
   (c) $\displaystyle\int_0^2 (\tan^{-1}\pi x - \tan^{-1} x)dx.$

3. Find $\displaystyle\iint_D f(x,y)d(x,y)$, where $f(x,y) = e^{x^2}$ and D is the region bounded by the lines $y = 0,\ x = 1$ and $y = 2x$.

4. (a) Compute the volume of the solid enclosed by the ellipsoid:
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$
   where a, b, c are given real numbers.
   (b) Find the volume of the region under the graph of $f(x,y) = e^{x+y}$ over the region
   $$D := \{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \le 1\}.$$

5. Find
   $$\lim_{r\to\infty} \iint_{D(r)} e^{-(x^2+y^2)} d(x,y),$$
   where $D(r)$ equals:
   (a) $\{(x,y) : x^2 + y^2 \le r^2\}$.
   (b) $\{(x,y) : x^2 + y^2 \le r^2,\ x \ge 0,\ y \ge 0\}$.
   (c) $\{(x,y) : |x| \le r,\ |y| \le r\}$.
   (d) $\{(x,y) : 0 \le x \le r,\ 0 \le y \le r\}$.

6. Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ using double integral over a region in the plane. (Hint: Consider the part in the first octant.)

7. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the region $x^2 + y^2 = 2x$ in $x - y$ plane.

8. Express the solid $D = \{(x, y, z) | \sqrt{x^2 + y^2} \leq z \leq 1\}$ as

$$\{(x, y, z) \mid a \leq x \leq b, \quad \phi_1(x) \leq y \leq \phi_2(x), \quad \xi_1(x, y) \leq z \leq \xi_2(x, y)\}.$$

9. Evaluate

$$I = \int_0^{\sqrt{2}} \left(\int_0^{\sqrt{2-x^2}} \left(\int_{x^2+y^2}^{2} x dz \right) dy \right) dx.$$

Sketch the region of integration and evaluate the integral by expressing the order of integration as $dxdydz$.

10. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

11. Describe the solid whose volume is given by the integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho^2 \sin \phi d\rho d\phi d\theta,$$

and evaluate the integral.