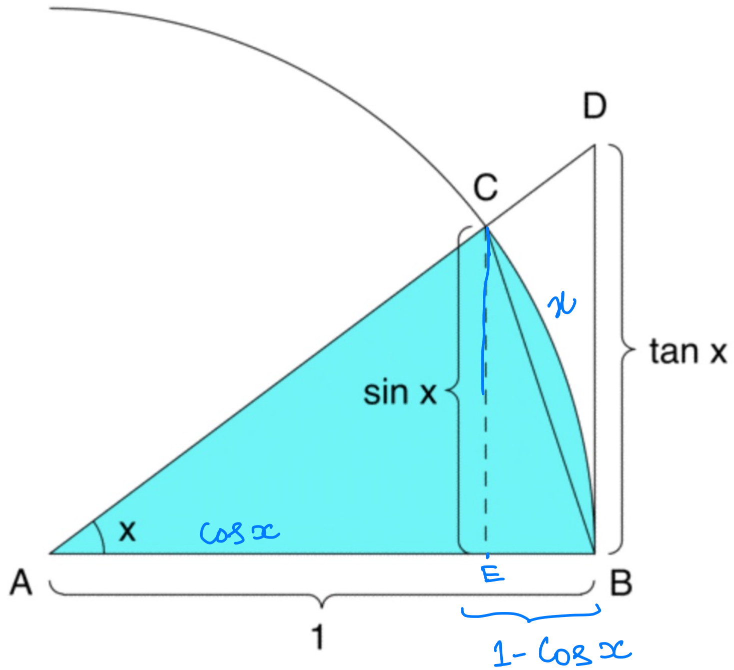

Area ACE $\le$ Area of sector ACB $\le$ Area ABD

$$\Rightarrow \quad \frac{1}{2}\sin x\cos x \le \frac{1}{2}x \le \frac{1}{2}\tan x$$

$$\Rightarrow \quad \cos x \le \frac{x}{\sin x} \le \frac{1}{\cos x}$$

$$\Rightarrow \quad \boxed{\cos x \le \frac{\sin x}{x} \le \frac{1}{\cos x}}$$

You also get $|\sin x| \le |x|$ & $|1-\cos x| \le |x|$ for $x \in [-\pi/2, \pi/2]$

Define $e^x = \lim_{n\to\infty} (1+x/n)^n$ and

Show that $\lim_{x\to 0} e^x = 1$ and $\lim_{x\to 0} \frac{e^x-1}{x} = 1.$

Bernoulli's Inequality $\Rightarrow$

For $n \in \mathbb{N}$, $\left(1+\frac{x}{n}\right)^n \geq 1+x$, $x > -1$

(prove it using induction)

$\Downarrow$

$$e^x \geq 1+x, \quad |x|<1 \quad \text{—— (1)}$$

By putting $-x$ in (1), we get

$$e^{-x} \geq 1-x \Rightarrow e^x \leq \frac{1}{1-x}, \quad \text{—— (2)}$$

for $|x|<1$.

(1) & (2) $\Rightarrow$

$$\boxed{1+x \leq e^x \leq \frac{1}{1-x}, \text{ for } |x|<1}$$

$\Downarrow$

$$\boxed{x \leq e^x-1 \leq \frac{x}{1-x}, \text{ for } |x|<1}$$

$\Downarrow$

$$\boxed{1 \leq \frac{e^x-1}{x} \leq \frac{1}{1-x}, \text{ for } 0<x<1}$$

& $$\boxed{\frac{1}{1-x} \leq \frac{e^x-1}{x} \leq 1, \text{ for } -1<x<0}$$

Now, you can use the Sandwich theorem to compute the limits of functions given in slides.