Following problems are from *chapters 3,4* Sheldon Ross, Introduction to Probability and Statistics for Engineers and Scientists, 4th Edition, Academic Press. The problems are on concepts of sample space, events, conditional probability, Bayes rule, law of total probability, indepdence, distribution function, density function etc.
Note: (i) In Ross book, the $\cap$ symbol (intersection symbol) is not explicitly used. Thus, interpret set AB as $A \cap B$.
(ii) Random variable is denoted as X (upper case letter) in the book by Ross. So, we use the same notation in the problems below (and in subsequent slides). Also, instead of ξ the book uses x. Thus, the probability distribution function is denoted as $F_X(x)$ which we know is defined as $P(\{\omega : X(\omega) \leq x\})$. Sometimes the subscript X on F is also ignored when we are dealing with only one random variable. We also follow this convention in this tutorial.

1. Problem 6, chapter 3: Let E, F, G be three events. Find expressions (in terms of unions, intersections, complements, etc.) for the events that of E, F, G
   (a) only E occurs.
   (b) both E and G but not F occur.
   (c) at least one of the events occurs.
   (d) at least two of the events occur.
   (e) all three occur.
   (f) none of the events occur.
   (g) at most one of them occurs.
   (h) at most two of them occur.
   (i) exactly two of them occur.
   (j) at most three of them occur.

2. Problem 12, chapter 3:
   If $P(E) = 0.9$ and $P(F) = 0.9$, show that $P(E \cap F) \geq 0.8$. In general, prove Bonferroni's inequality, namely that $P(E \cap F) \geq P(E) + P(F) - 1$.

3. Problem 21, chapter 3:
   There is a 60 percent chance that the event A will occur. If A does not occur, then there is a 10 percent chance that B will occur.
   (a) What is the probability that at least one of the events A or B occurs?
   (b) If A is the event that the democratic candidate wins the presidential election in US in 2012 and B is the event that there is a 6.2 or higher earthquake in Los Angeles sometime in 2013, what would you take as the probability that both A and B occur? What assumption are you making?

4. Problem 23, chapter 3:
   Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

5. Problem 24, chapter 3:
   A couple has 2 children. What is the probability that both are girls if the eldest is a girl?

6. Problem 47, chapter 3:
   Let A, B, C be events such that $P(A) = 0.2, P(B) = 0.3, P(C) = 0.4$.
   Find the probability that at least one of the events A and B occurs if

(a) A and B are mutually exclusive.

(b) A and B are independent.

Find the probability that all of the events A, B, C occur if:

(a) A, B, C are independent.

(b) A, B, C are mutually exclusive.

7. Problem 4, chapter 4:
The distribution function of the random variable X is given

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \leq x < 1 \\ \frac{2}{3}, & 1 \leq x < 2 \\ \frac{11}{12}, & 2 \leq x < 3 \\ 1, & 3 \leq x \end{cases}$$

   (a) Plot this distribution function.
   (b) What is $P(X > 12)$?
   (c) What is $P(2 < X \leq 4)$?
   (d) What is $P(X < 3)$?
   (e) What is $P(X = 1)$?

8. Problem 6, chapter 4:
The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{-x/100}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

What is the probability that a computer will function between 50 and 150 hours before breaking down? What is the probability that it will function less than 100 hours? Note: You need to obtain value of λ first.

9. Problem 7, chapter 4:
The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} 0, & x \leq 100 \\ \frac{100}{x^2}, & x > 100 \end{cases}$$

What is the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within the first 150 hours of operation? Assume that the events $E_i,\ i = 1, 2, 3, 4, 5$, that the i^{th} such tube will have to be replaced within this time are independent.

Following are some other problems:

10. Suppose the random variable X has distribution function

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \exp(-x^2), & x > 0 \end{cases}$$

What is the probability that X exceeds 1?

11. Suppose that X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} C(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of C?

(b) Find $P(X > 1)$?

12. Show that if $X(\omega) \leq Y(\omega)$ for all $\omega \in \Omega$, then $F_X(a) \geq F_Y(a)$ for all $a \in \mathbb{R}$. Here $X(\omega)$ and $Y(\omega)$ are two different random variables, each mapping the sample space Ω to $\mathbb{R}$.

13. Show that if $a \leq X(\omega) \leq b$ for all $\omega \in \Omega$, then $F_X(x) = 1$ for $x > b$ and $F_X(x) = 0$ for $x < a$.