A Project report on

CFD Analysis of Converging Diverging Nozzle

For the course of Gas Dynamics MECH6111 of Master of Engineering in Mechanical Engineering

Prepared for

Dr Pierre Q Gauthier

Prepared by

Punitbhai Manjibhai Limbani Student ID: 40204393

Smit Mehul Lakdawala Student ID: 40205041

DEPARTMENT OF MECHANICAL, INDUSTRIAL & AEROSPACE ENGINEERING (MIAE),
GINA CODY SCHOOL OF ENGINEERING AND COMPUTER SCIENCE,

CONCORDIA UNIVERSITY, MONTREAL

Summer – 2022

Table of Contents

Table of Figures

List of Tables

Abstract

In this project we have analysed the flow in C-D Nozzle in Ansys and compare the CFD result with the theoretical calculation in the alter stage. For the analysis of the nozzle first we have fixed the inlet, throat and exit area. According to these values from the isentropic flow table we got the pressure ratios and Mach number after that we tried to match both theoretical and analytical values for the various back pressure ranges to achieve different conditions like subsonic choked flow, supersonic flow at exit, normal shock inside diverging part of the nozzle, oblique shock outside the nozzle and expansion wave outside the nozzle.

However, the theoretical calculations are done using 1-Dimensional data but the analytical results are based on 2-Dimensioins.

Chapter 1: Introduction

As the name suggest it is made up of two sections one is convergent section and another is divergent section and both sections are connected with the throat of the nozzle. Also, it is a variable are passage which is used to accelerate gases to higher supersonic speed.

1.1 Operating stages in the Nozzle

When the back pressure is same as the reservoir pressure there will be no flow in the nozzle. Hence by reducing back pressure we can get different scenario for various back pressure ranges.

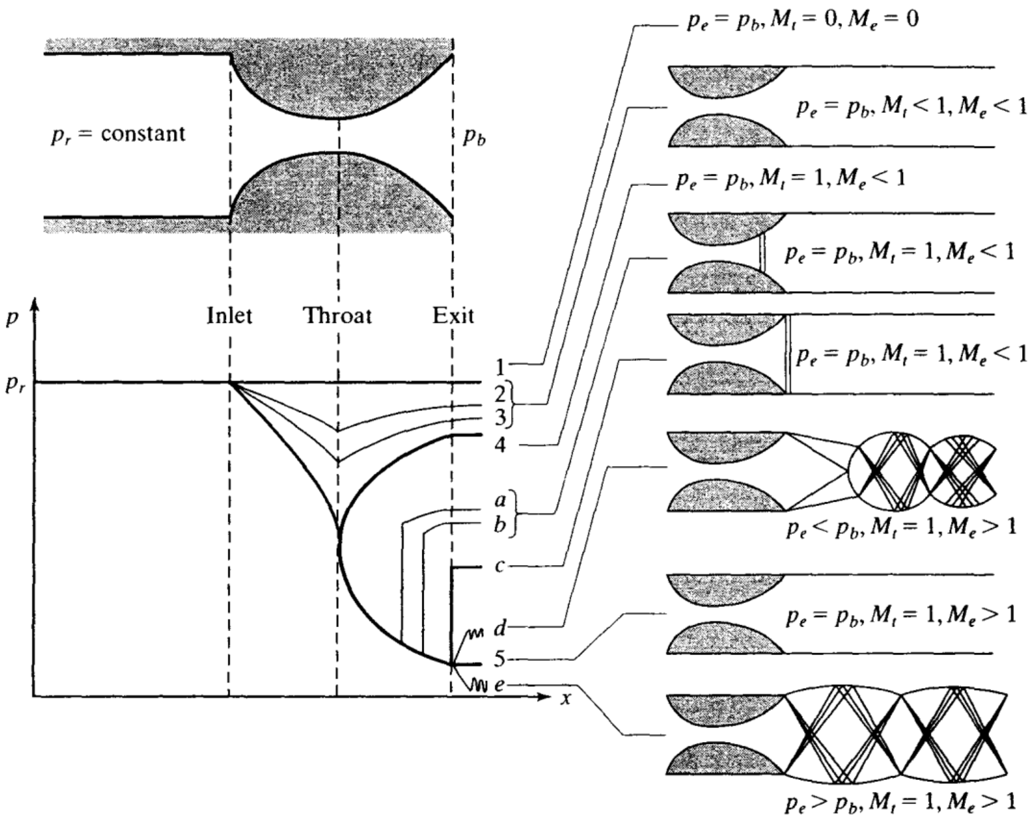

Figure 1. Pressure variation across C-D Nozzle [1]

As, shown in the figure when the exit pressure is same as the back pressure (Curve 1) there will be no flow in the nozzle. After that decrement in the back pressure will lead to flow in the nozzle and the state of the flow is subsonic at the inlet and outlet of the nozzle (Curve 2 & 3). Further reduce in the back pressure, nozzle will be chocked and will give maximum mass flow rate and subsonic Mach number at the outlet (Curve 4). Once curve 4 achieve after that decrement in the back pressure will lead to the normal shock in the diverging part of at the exit of the nozzle (curve a & b). Once shock occurs in the nozzle there will be subsonic flow in the remaining nozzle and entropy will increase. Still decrement in the back pressure will result in the oblique shock outside the nozzle (curve d). Further reducing the back pressure will result the supersonic flow in the diverging part of the nozzle and that Mach number and exit pressure is called as the design parameters of the nozzle (curve 5). After curve 5 all back pressure will result in the expansion wave outside the nozzle (curve e).

Chapter 2: Theoretical 1-D Calculations

2.1 Assumptions

- Theoretical calculations are done according to 1-dimensional flow but in analysis it is assumed as 2-dimensional flow.
- The flow is assumed as the isentropic so the stagnation pressure is going to be same until the normal shock occurs in the nozzle or oblique shock occurs outside the nozzle.
- Inviscid and steady flow is considered.
- Flow is adiabatic, i.e. No heat adding or removal
- Isentropic expansion wave is assumed.

2.2 Supersonic Flow at exit

Total pressure at inlet P_{01}= 1 MPa.

Total Temperature at inlet T_{01}= 300 K.

We have assumed design A_e/A^*=1.22 for exit radius of 17.26 mm and throat radius of 14.33 mm.

So, from the IFT we found exit Mach number M_e= 1.567461.

P_e/P_{0e} = 0.246877 and T_e/T_{0e} = 0.670199.

So, from above data we can calculate P_e = 246876.8 Pa and T_e = 201.0597 K.

As, we have inlet radius of 22.65 mm and throat radius of 17.26 mm.

Which leads to inlet Mach number of M_1 = 0.40.

P_1/P_{01} = 0.89415 and T_1/T_{01} = 0.968546

So, from above data we can calculate P_1 = 894150.4 Pa and T_1 = 290.5638 K

2.3 Subsonic choked Flow at exit

We have designed Ae/A*=1.22, from IFT Me = 0.57

Also, Pe/P0e = 0.801363 and Te/T0e = 0.938721

So, Pe = 801362.6 Pa and Te = 281.6163 K

2.4 Normal Shock inside diverging part of the Nozzle

From the supersonic exit Mach number M_{ex}=M_e = 1.56 which leads to M_{ey} = 0.67 from normal shock table.

Also, Py/Px = 2.69 so, Py = 666528.9379 Pa

Ty/Tx = 1.365 so, Ty = 274.58 K

P0y/P0x = 0.907 so, P0y = 907051.345 Pa

From above calculation we can say that the pressure range for the normal shock which is 666528.9379 < Pb < 801362.6.

From the above pressure range we chose 700000 Pa back pressure which will create the shock inside the diverging part of the nozzle.

Conservation of mass,

$\rho * v * A = \rho* * v* * A*$

$P*/(T*)^{1/2} * A* * M* = P_e/(T_e)^{1/2} * A_e * M_e$

From Isentropic relation, $T_{0e}/Te = 1 + ((\gamma-1)/2) * M_e^2$

$(P*/P_e) * (A*/A_e) * (T_{0e}/T*)^{1/2} = [1 + ((\gamma-1)/2) * M_e^2]^{1/2} * M_e$

$[(P*/P_e) * (A*/A_e)]^2 * (T_{0e}/T*) = M_e^2 + ((\gamma-1)/2) * M_e^4$

$[(528300/700000) * (1/1.2247)] * (1.2) = M_e^2 + (1.2) * M_e^4$

$M_e = 0.6483$

From IFT at $M_e = 0.6483$, $P_2/P_{02} = 0.7539$

So, P_{02} = 926774.2431 Pa

Hence, we get P_{02}/P_{01} = 926774.2431 / 1000000 = 0.926774243 and from NST we got M_x = 1.50931 and M_y = 0.697841

P_y/P_x = 2.489721 So, P_y = 669061.7 Pa

T_x/T_{01} = 0.6870 So, T_x = 206.11 K

T_y/T_x = 1.326438 So, T_y = 273.39 k

2.5 Oblique Shock outside the Nozzle

We have back pressure P_b = 275000 Pa and supersonic design pressure Pe = 246876.8 Pa So, P_b/P_e = 1.11391

At this pressure ratio from NST M_{en} = 1.04763 and M_{An} = 0.95527 also P_{0A}/P_{0e} = 0.9998

M_{en} = Me *sin (θ)

1.04763 = 1.567461 *sin (θ)

$\theta = 41.9^0$ and corresponding $\delta = 2^0$ also P_{0A} = 999800 Pa

At, M_{An} = 0.95527 from IFT T_A/T_{0A} = 0.8457 So, T_A = 253.71 K

Also, $M_A = M_{An} / \sin(\theta - \delta)$ = 1.53

2.6 Expansion Wave outside the Nozzle

For the expansion wave outside the nozzle the pressure must be less than supersonic design pressure which is 246876.8 Pa. So, we have assumed the back pressure as 100000 Pa.

M_e = 1.567461 (supersonic design) from PMT $\nu_1 = 13.8965^0$

Now we have P_b/P_{01} = 100000/1000000 = 0.1 from PMT $\nu_A = 30.6167^0$

Hence M_A = 2.158

$\Delta\nu = \nu_A - \nu_1 = 16.7202^0$

At pressure ratio of 0.1 as mentioned we found $T_A/T_{01} = 0.5179$

So, $T_A = 155.37$ K and $P_A = P_b = 100000$ Pa

2.7 Pressure ranges for various conditions

From the above calculation we got the various back pressure ranges to operate the nozzle.

(All pressure values are in Pa)

$P_b = 1000000$ - No flow will be there

801362.6 < P_b < 1000000 – Subsonic choked flow

666528.9 < P_b < 801362.6 – Normal shock inside the nozzle

246876.8 < P_b < 666528.9379 – Oblique shock outside the nozzle

0 < P_b < 246876.8 – Expansion wave outside the nozzle

Chapter 3: CFD Analysis

3.1 Nozzle Geometry

As shown in the below snap, Converging-diverging nozzle was modelled in Ansys Design Modeler. This geometry without downstream Tank was used to simulate the flow with Supersonic Design, Subsonic choked design and Normal Shock inside the nozzle.

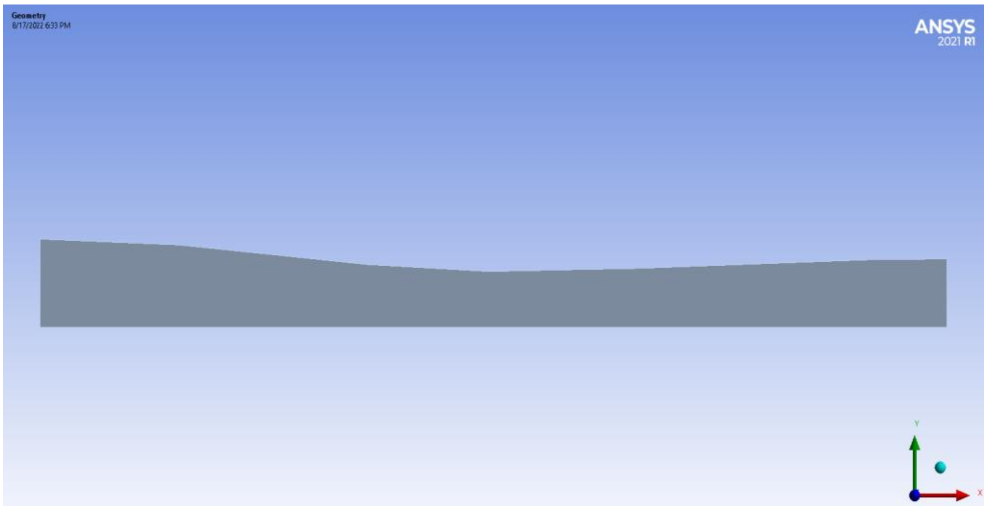

Figure 2. CD Nozzle Geometry

To capture Oblique shock and Expansion waves, a tank was attached downstream of the Nozzle as shown below.

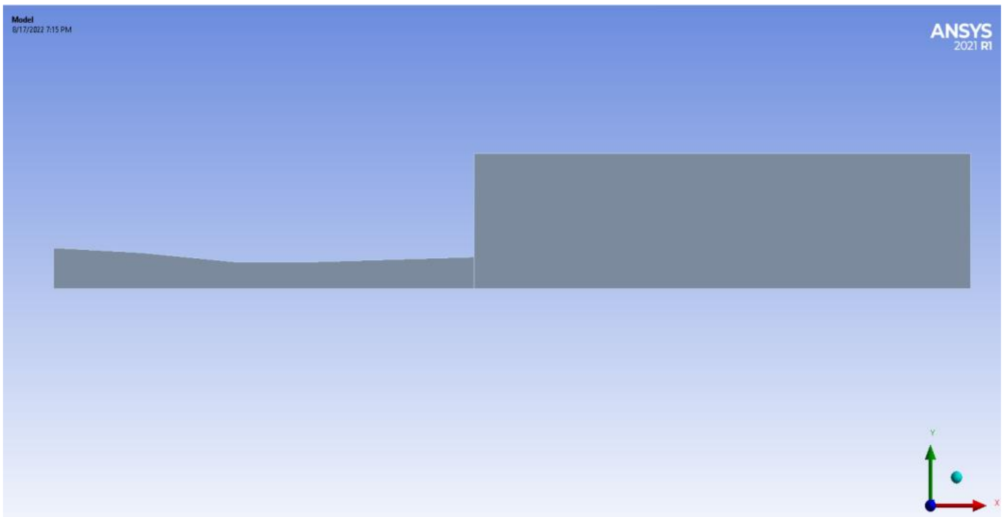

Figure 3. CD Nozzle Geometry with Tank

Nozzle dimensions used are tabulated below.

Wall Point	X location mm	Y Location mm
1	0	22.66
2	12.7	22.49
3	38.1	21.21
4	63.5	18.96
5	88.9	16.40
6	114.3	14.59
7	139.7	14.42
8	165.1	15.05
9	190.5	16.02
10	215.9	16.94
11	241.3	17.48
12	254	17.55

Table 3-1 C-D Nozzle Dimensions

3.2 Meshing

For CFD analysis, meshing is very important aspect which can affect the final results.

Following mesh features were used to create structured mesh.

i. Automatic Quadrilateral Dominant Meshing Method

ii. Edge Sizing with Hard Behaviour

iii. Face Meshing to generate structured mesh

- Bias was suitably used in meshing to capture enough details at throat, exit and in the tank.
- Named selections were utilised such that it'll be easy to apply boundary conditions in the setup. Symmetry, Wall, Inlet, outlet and fluid etc were defined.

Shown below is the meshed geometry for the both cases.

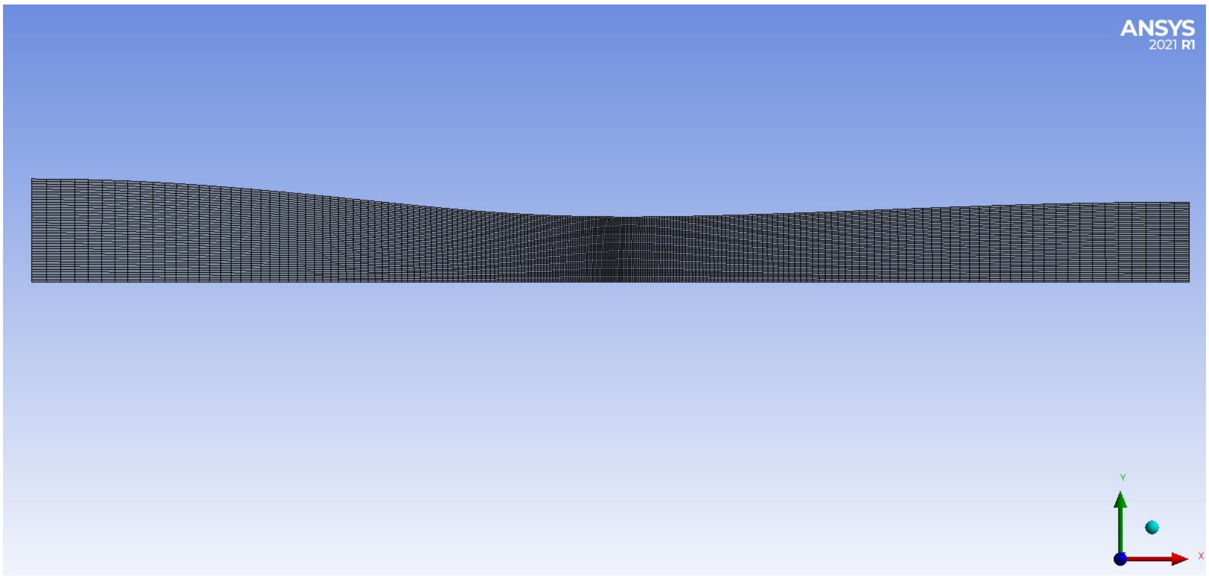

Figure 4. Meshing for flow with Supersonic, Subsonic and Normal Shock

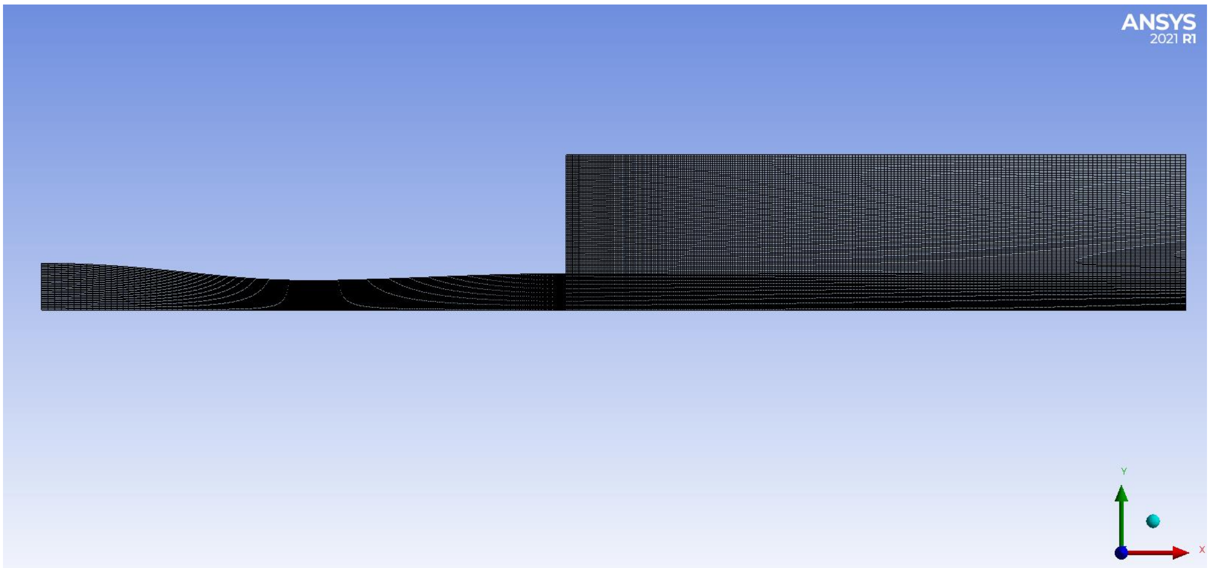

Figure 5. Meshing for flow with Oblique and Expansion Wave

3.3 Set-up

Following setup is necessary to carry out satisfactory CFD analysis for the given case.

- Density based solver (for Compressible flow)
- Steady Analysis
- Energy Model On
- Inviscid flow
- Air as Ideal Gas fluid
- Gravity Off
- Hybrid Initialisation
- Set Space discretization as First-order upwind.

Also, one of the important parts is setting proper boundary conditions.

Pressure inlet and outlet were chosen.

Inlet Total Pressure: 1 MPa

Outlet pressure varies based on the Flow to be simulated

3.4 Results

A series of results are shown in this section and comparison is made in form of table at the end of each flow case.

3.4.1 Result of Subsonic Choked Flow

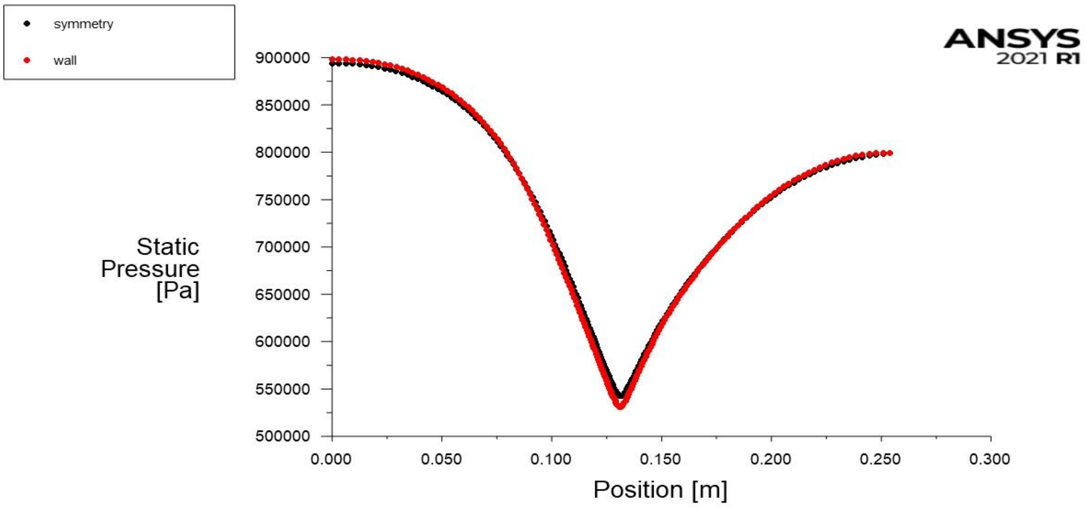

Figure 6. Static Pressure, XY Plot, Subsonic choked flow

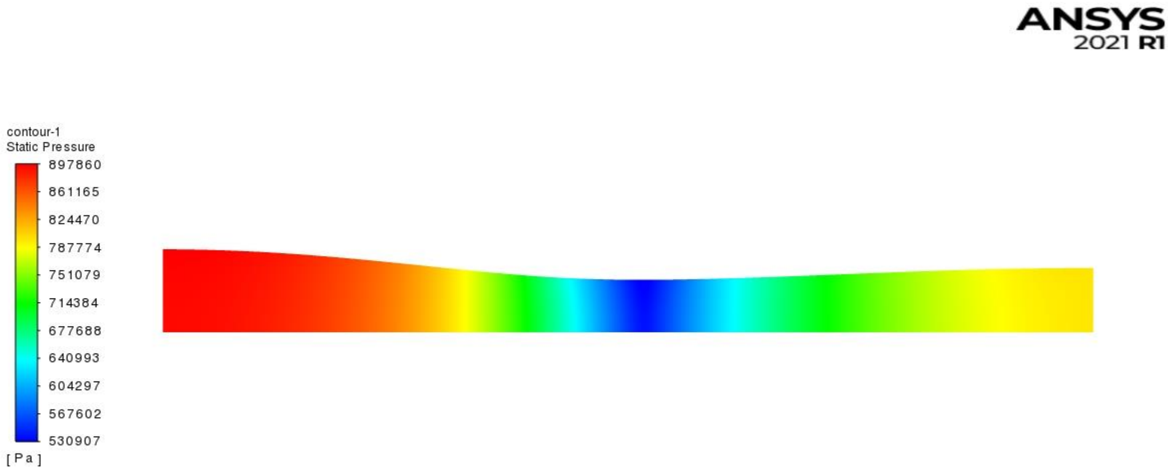

Figure 7. Static Pressure, Contour, Subsonic choked flow

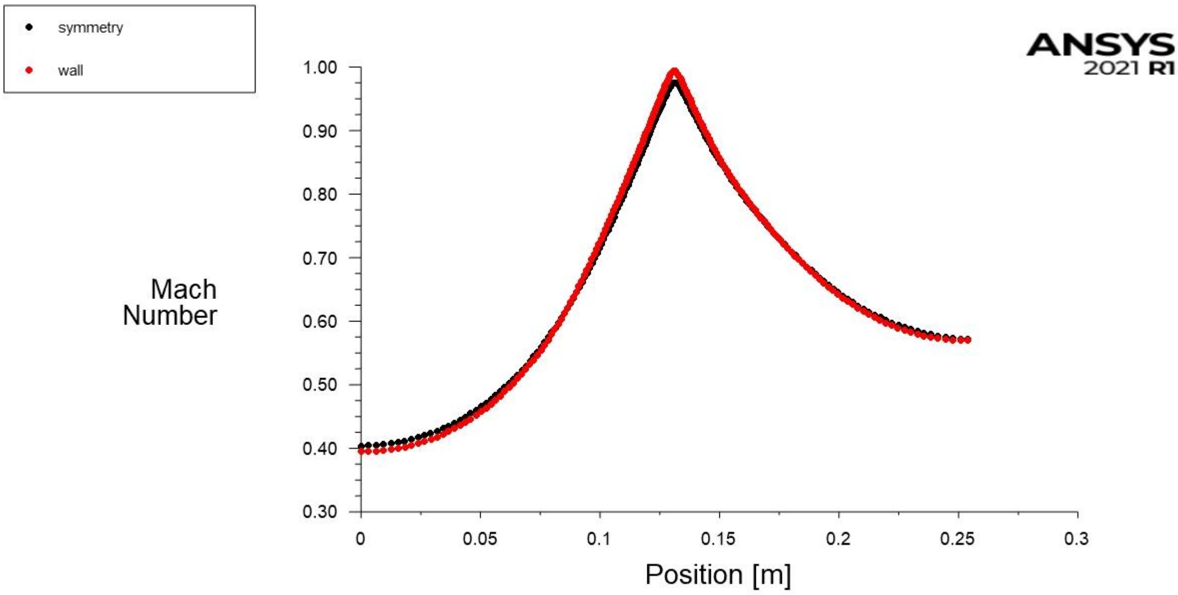

Figure 8. Mach Number, XY Plot, Subsonic choked flow

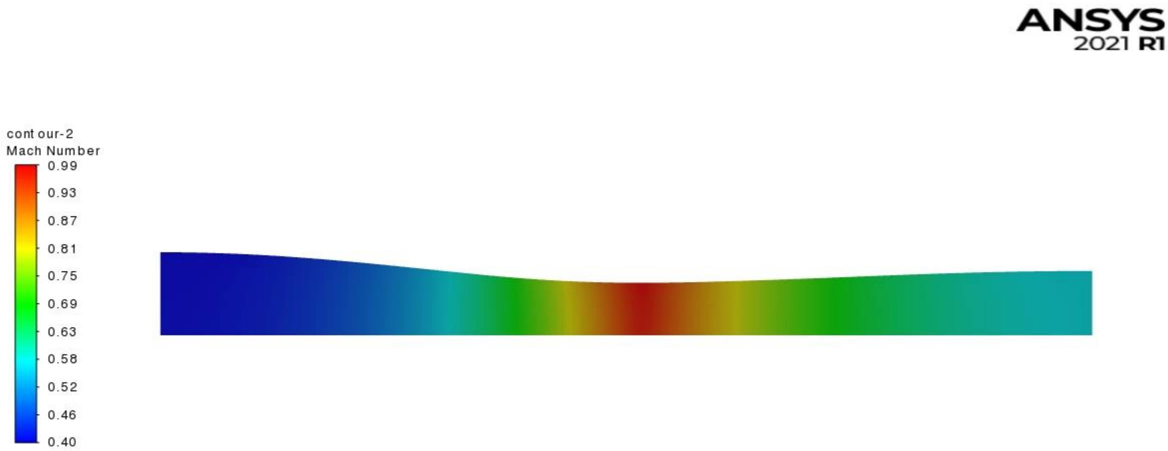

Figure 9. MACH Number, Contour, Subsonic choked flow

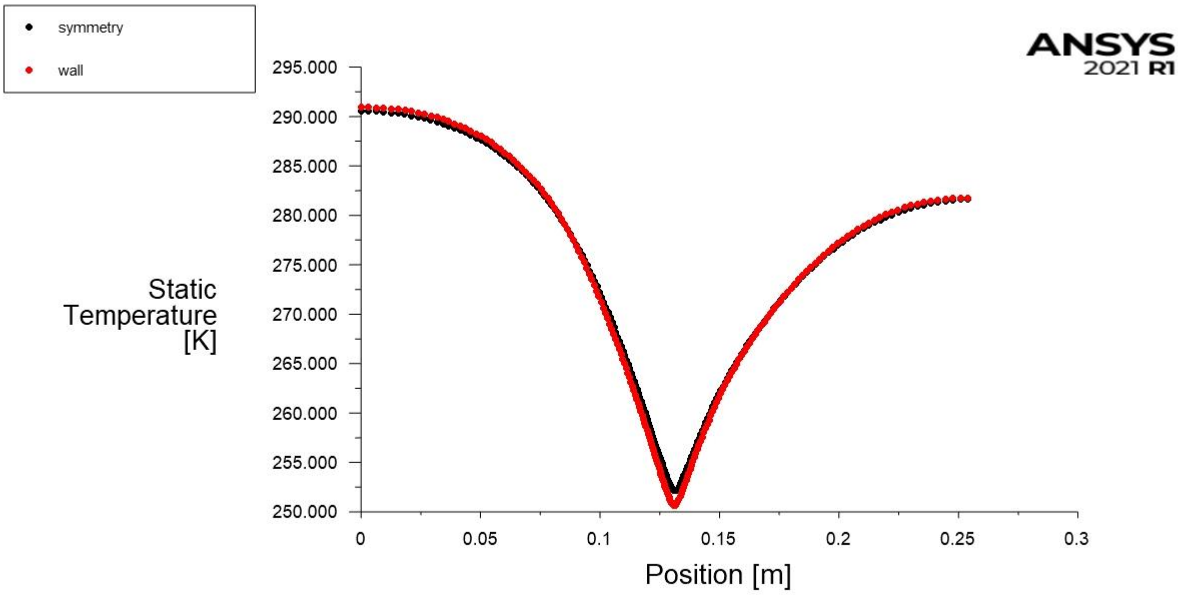

Figure 10. Temperature, XY Plot, Subsonic choked flow

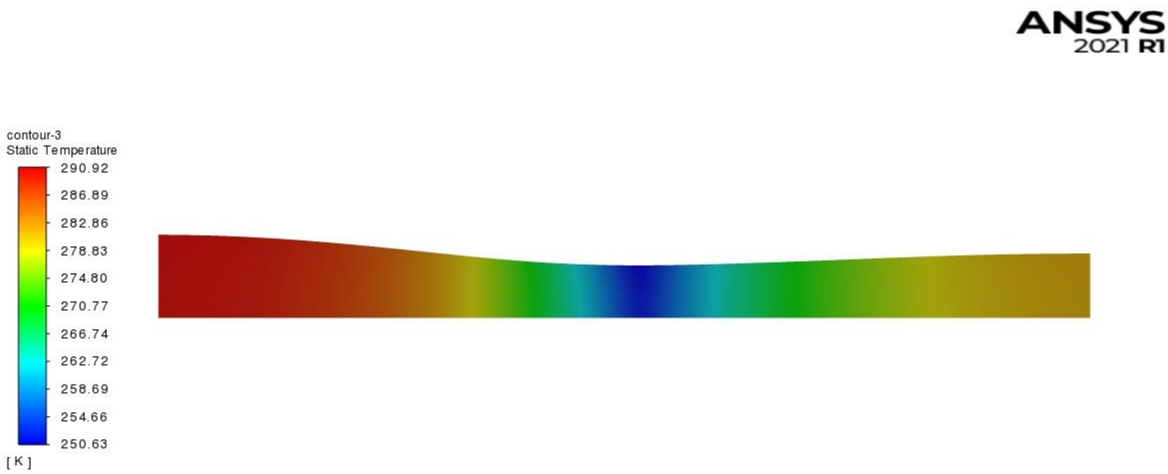

Figure 11. Temperature, Contour, Subsonic choked flow

Exit Plane	Theoretical Value	Simulation Value	Error
Static Pressure (KPa)	801.36	800	-0.17%
Mach Number	0.57	0.55	-3.51%
Static Temperature(K)	281.6	283	0.50%

Table 3-2 Comparison of Theoretical 1D Calculations and CFD Simulation Results, Subsonic flow

3.4.2 Result of Supersonic Design Flow

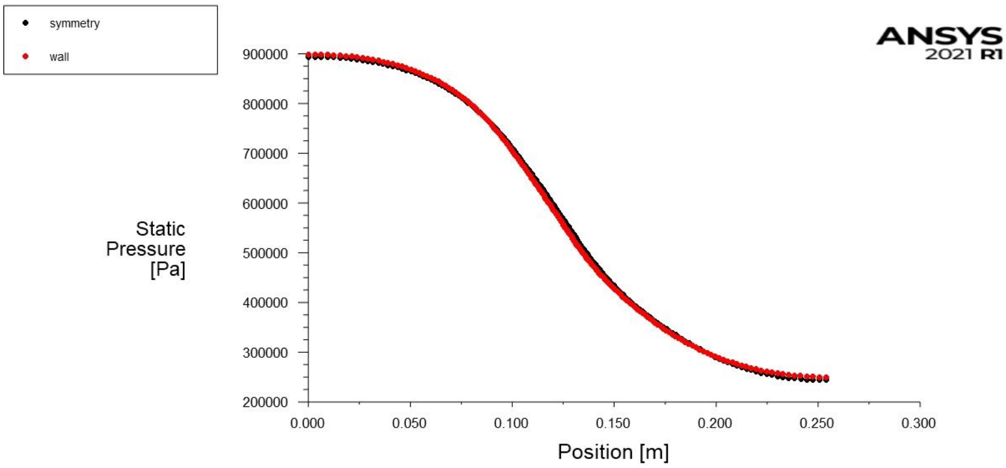

Figure 12. Static Pressure, XY Plot, Supersonic design flow

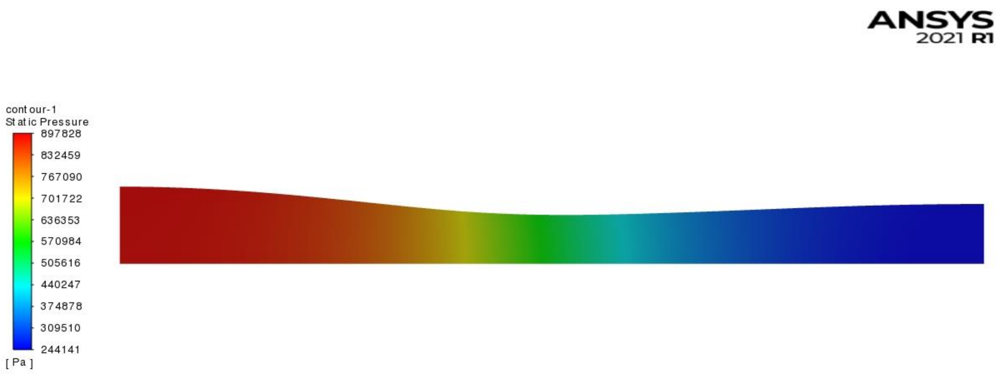

Figure 13. Static Pressure, Contour, Supersonic design flow

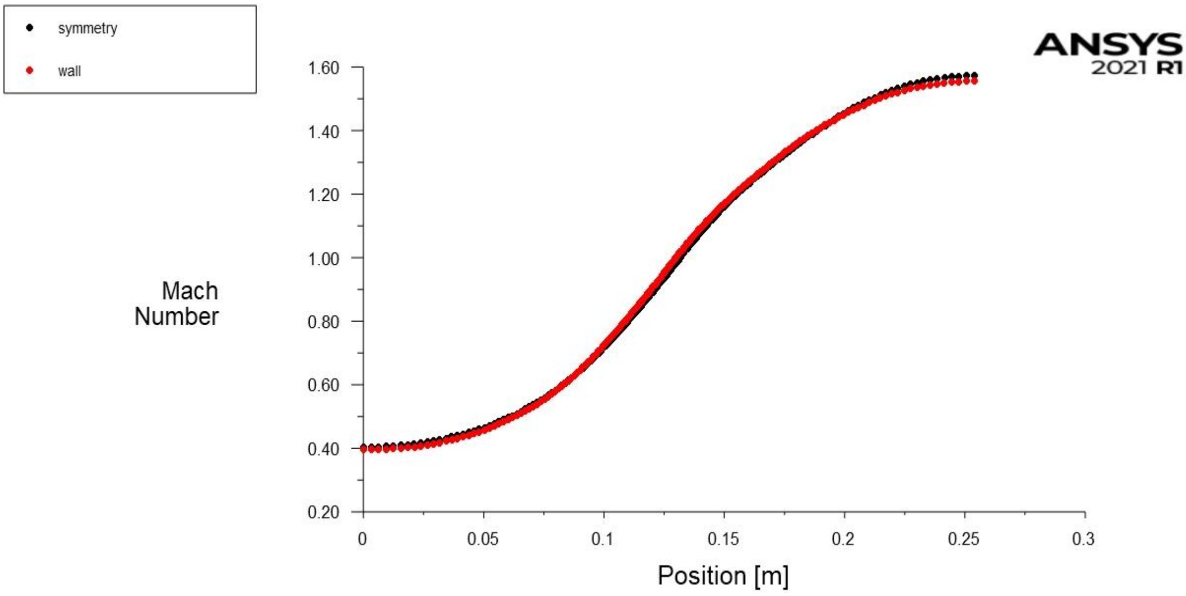

Figure 14. Mach Number, XY Plot, Supersonic design flow

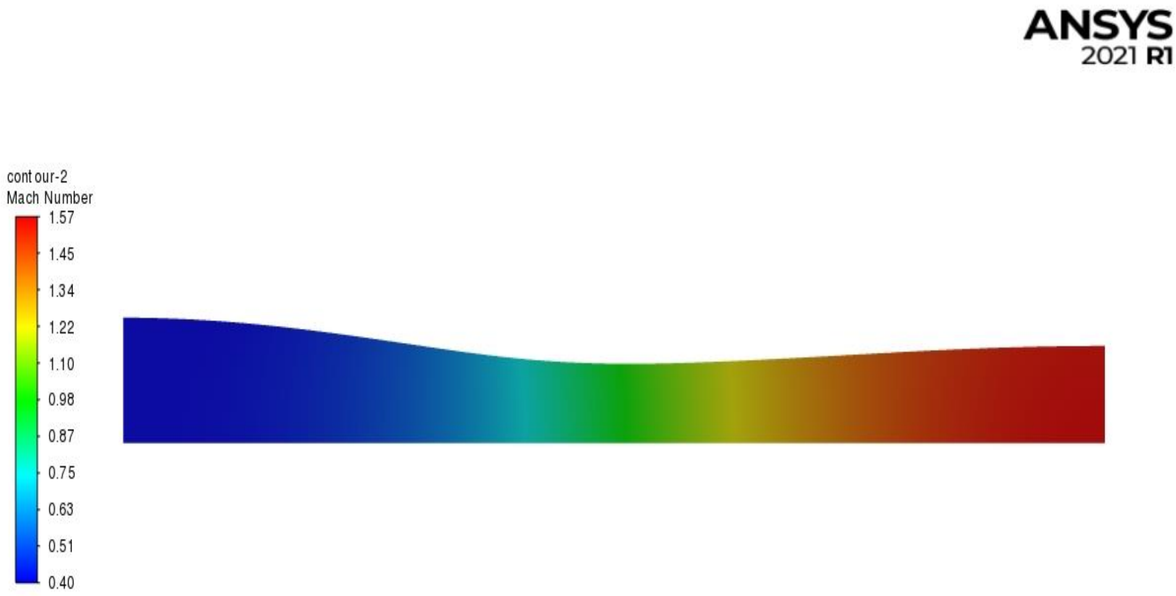

Figure 15. Mach Number, Contour, Supersonic design flow

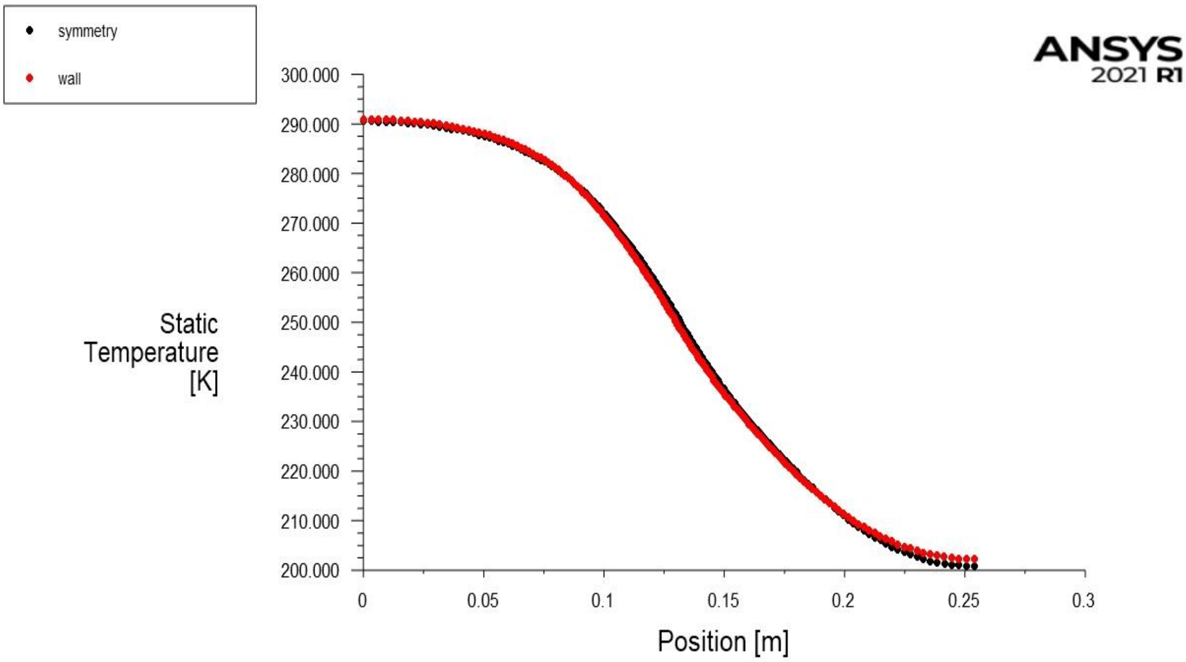

Figure 16. Static Temperature, XY Plot, Supersonic design flow

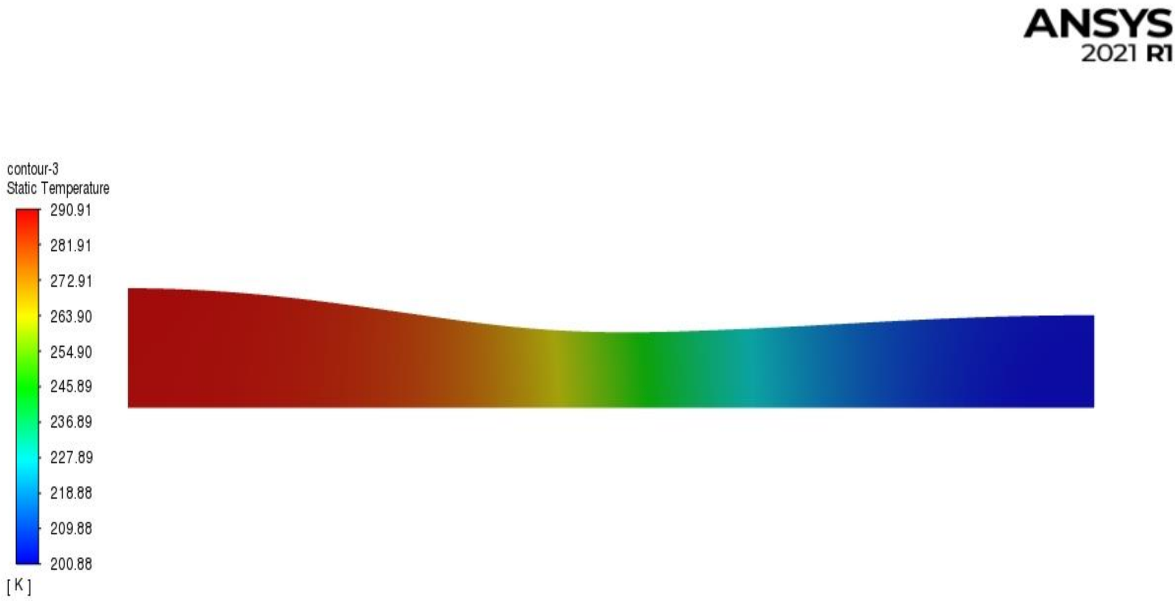

Figure 17. Static Temperature XY Plot, Supersonic design flow

Exit Plane	Theoretical Value	Simulation Value	Error
Static Pressure (KPa)	246.88	244.14	-1.11%
Mach Number	1.57	1.56	-0.45%
Static Temperature(K)	201.06	200.88	-0.09%

Table 3-3 Comparison of Theoretical 1D Calculations and CFD Simulation Results, Supersonic flow

3.4.3 Result of Normal Shock inside Nozzle

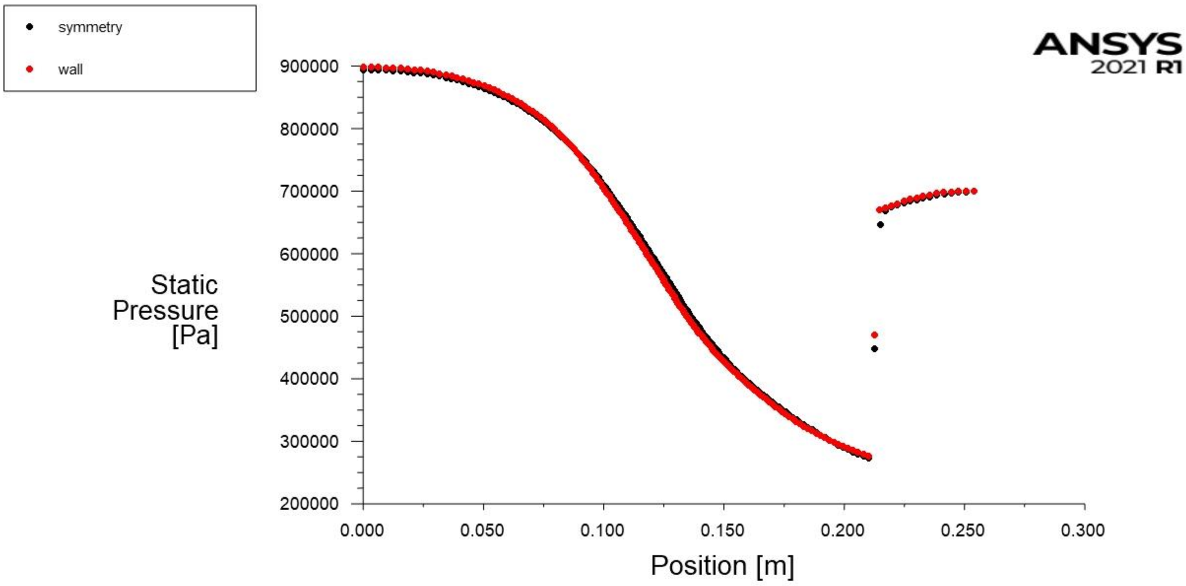

Figure 18. Static Pressure, XY Plot, Normal Shock Inside Nozzle

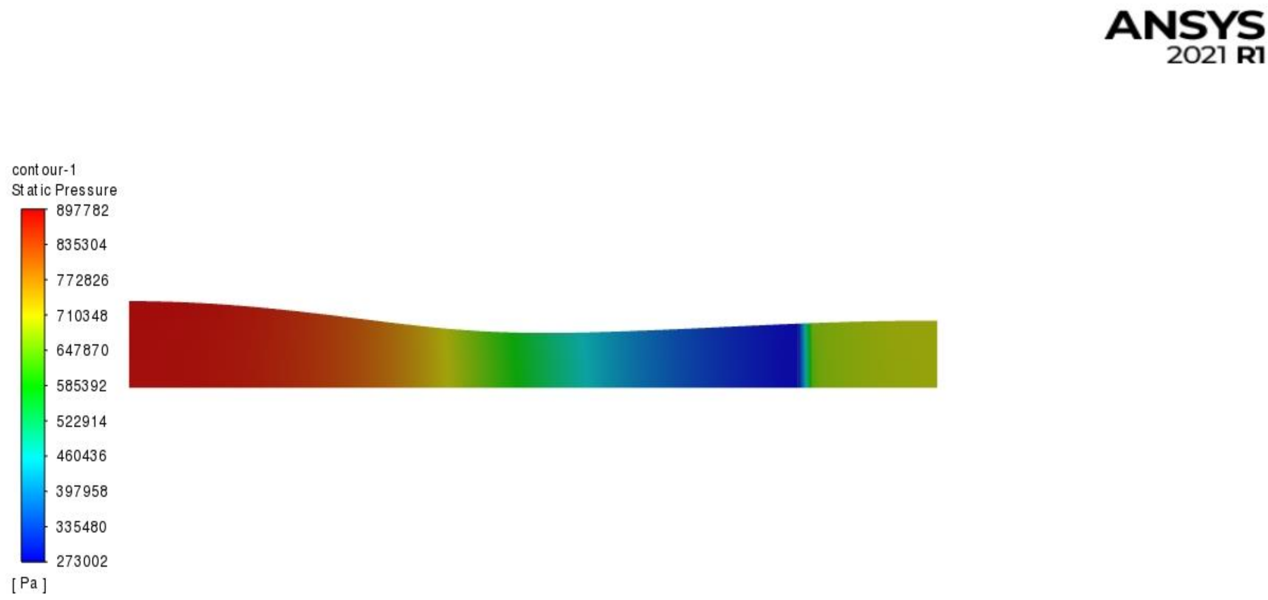

Figure 19. Static Pressure, Contour, Normal Shock Inside Nozzle

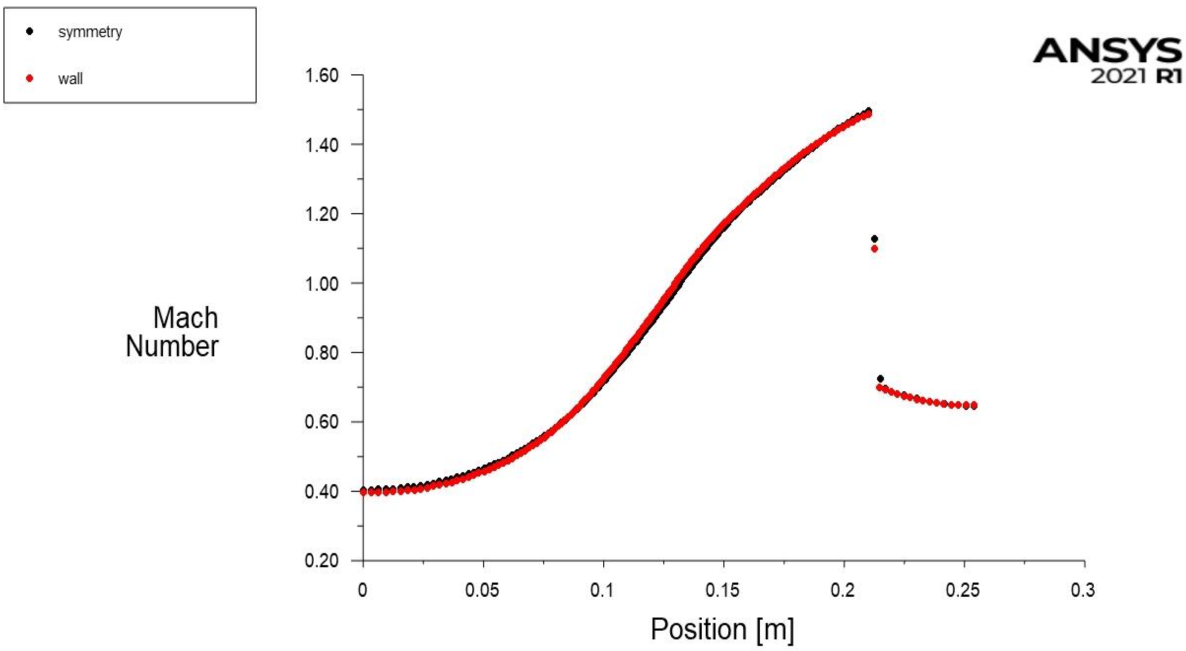

Figure 20. Mach Number, XY Plot, Normal Shock Inside Nozzle

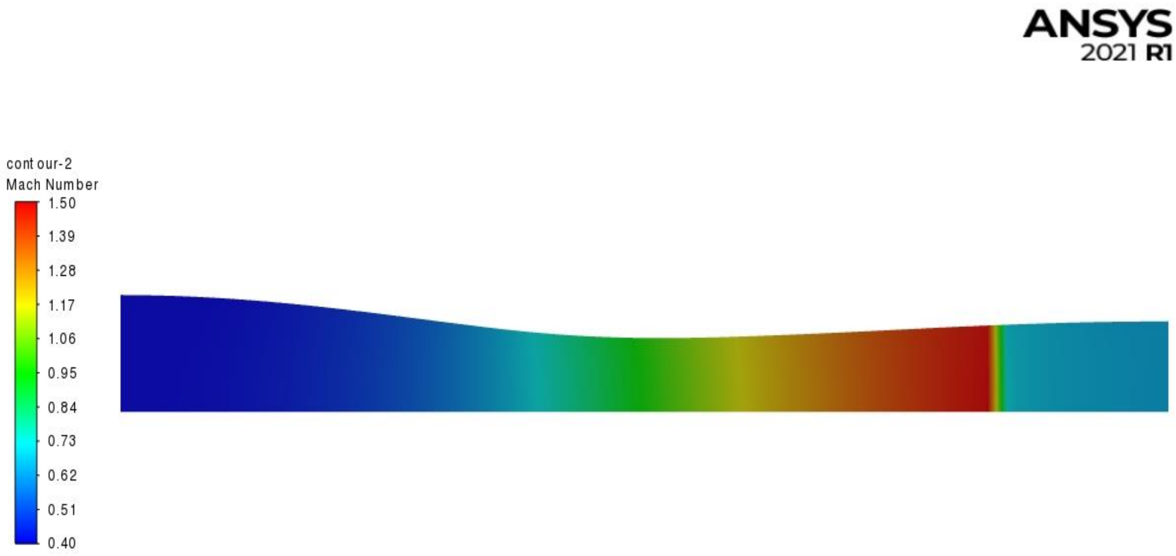

Figure 21. Mach Number, Contour, Normal Shock Inside Nozzle

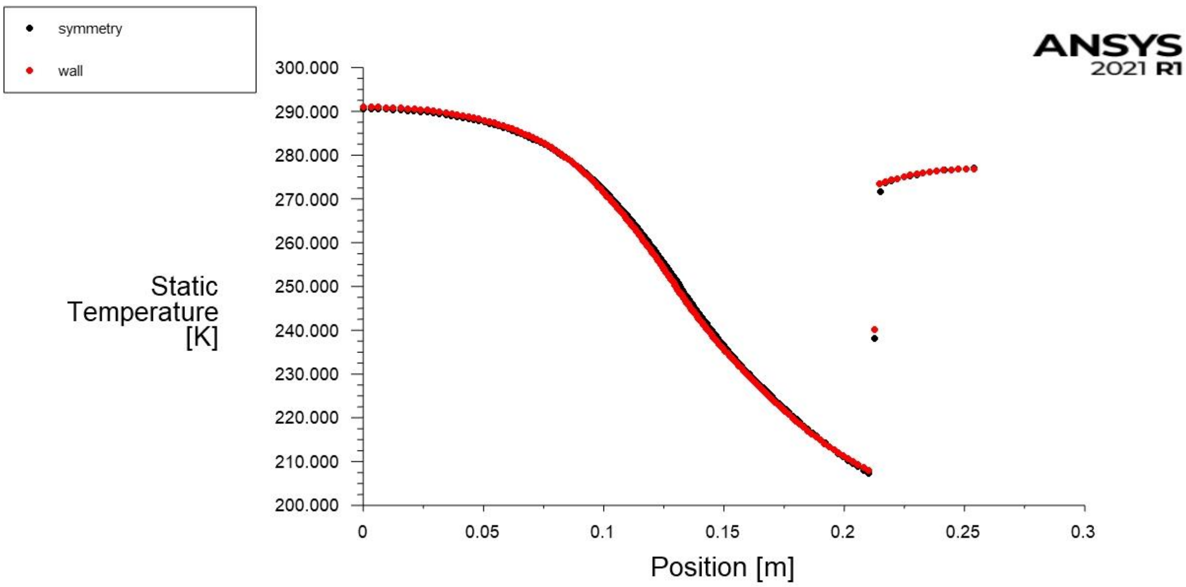

Figure 22. Static Temperature, XY Plot, Normal Shock Inside Nozzle

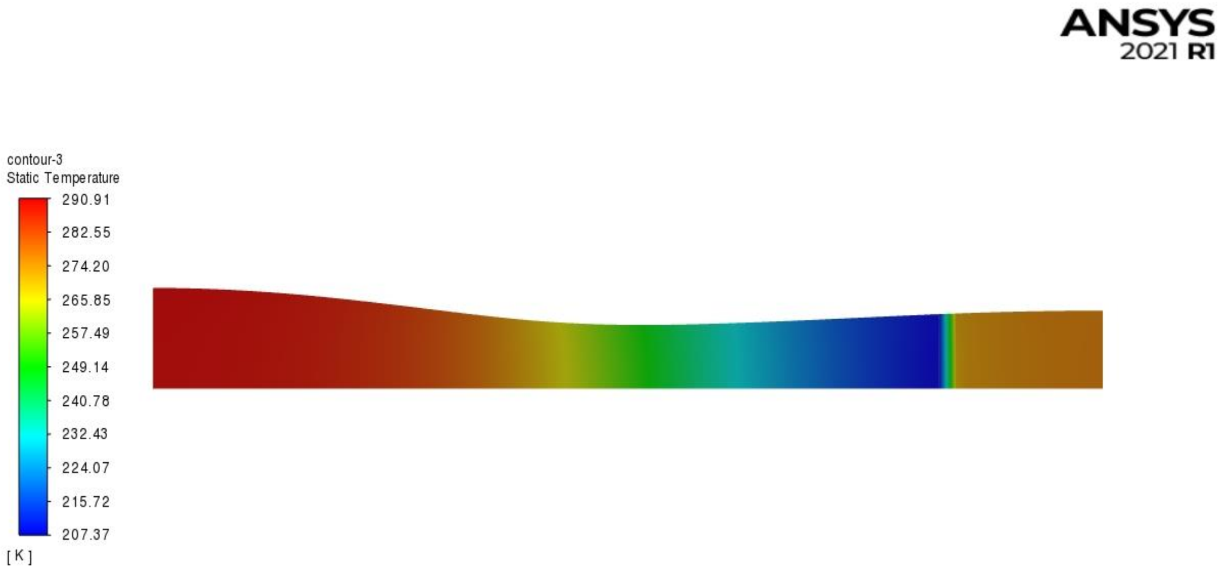

Figure 23. Static Temperature, Contour, Normal Shock Inside Nozzle

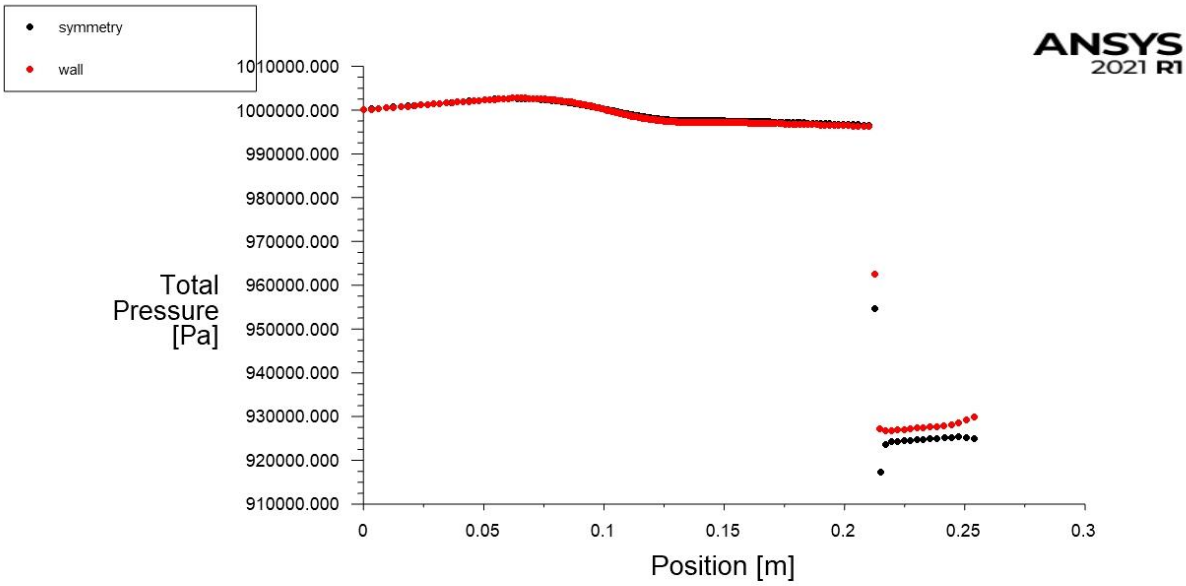

Figure 24. Total Pressure, XY Plot, Normal Shock Inside Nozzle

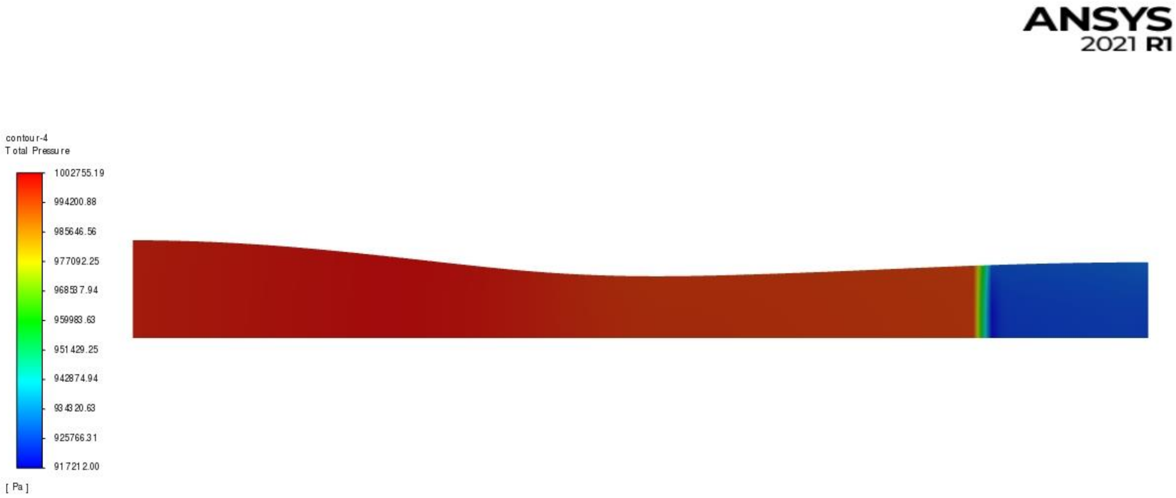

Figure 25.   Total Pressure, Contour, Normal Shock Inside Nozzle

Flow Property	Theoretical Value		Simulation Value		Error	
	Before Shock	**After Shock**	**Before Shock**	**After Shock**	**Before Shock**	**After Shock**
Static Pressure (KPa)	268.73	669.06	275	675	-2%	1%
Mach Number	1.51	0.70	1.50	0.68	1%	-3%
Static Temperature(K)	206.11	273.40	207.50	275	-1%	1%
Total Pressure drop (KPa)	1000.00	926.77	1000.00	928.00	0%	0%

Table 3-4 Comparison of Theoretical 1D Calculations and CFD Simulation Results, Normal Shock

3.4.4 Result of Oblique Shock

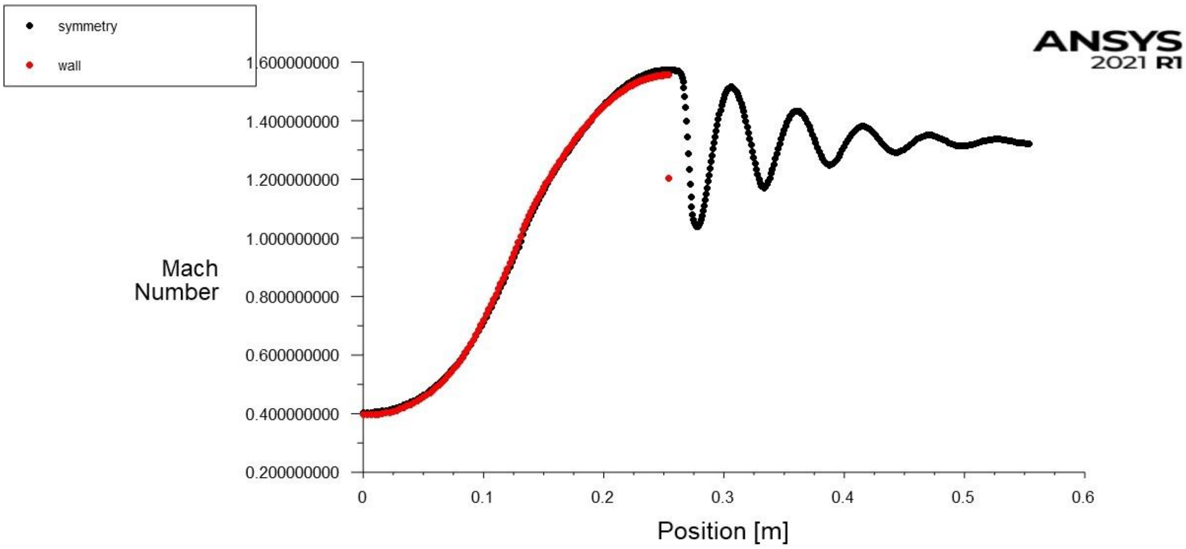

Figure 26. Mach Number, XY Plot, Oblique Shock outside Nozzle

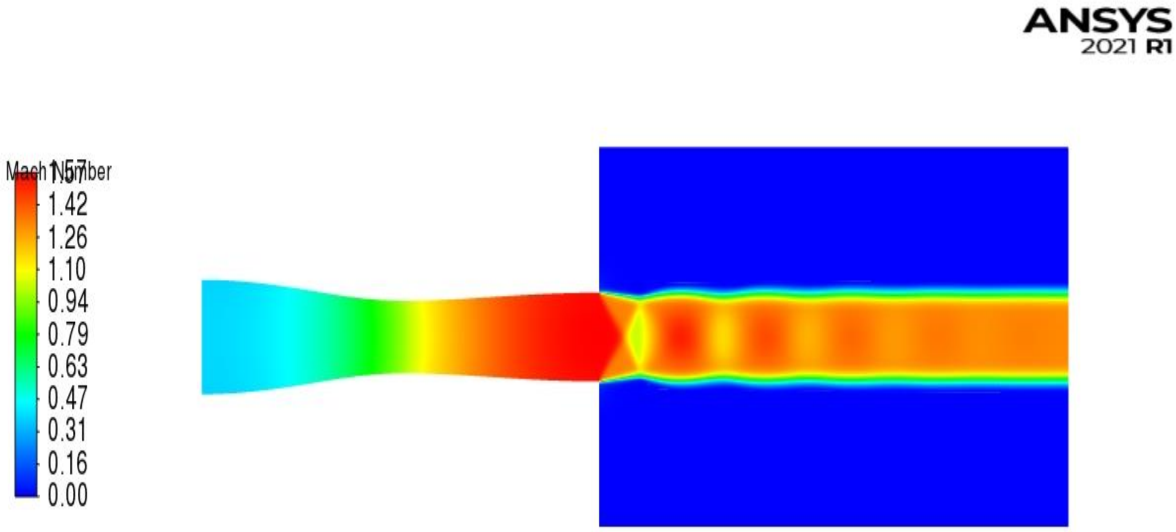

Figure 27. Mach Number, Contour, Oblique Shock outside Nozzle

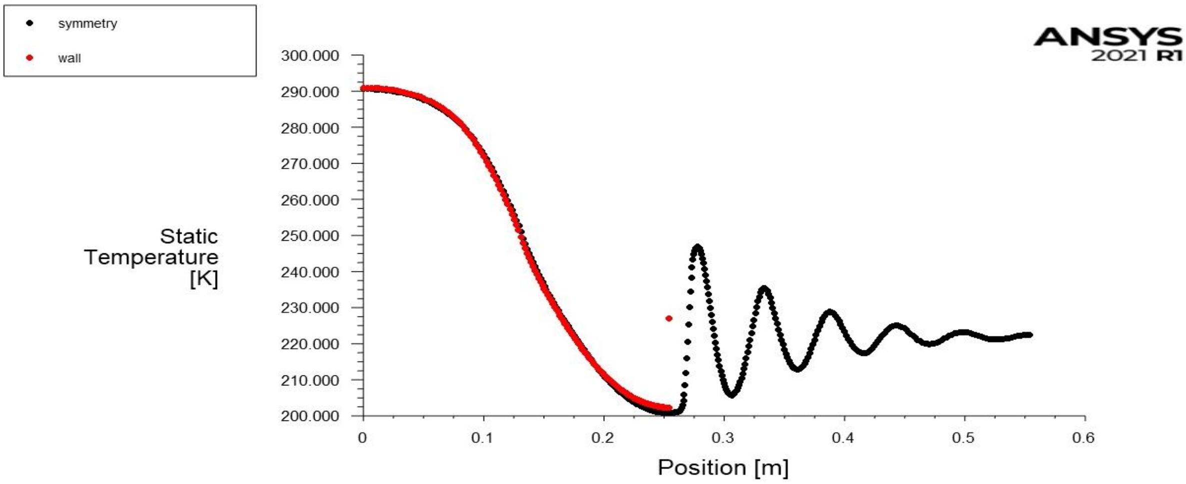

Figure 28. Static Temperature, XY Plot, Oblique Shock outside Nozzle

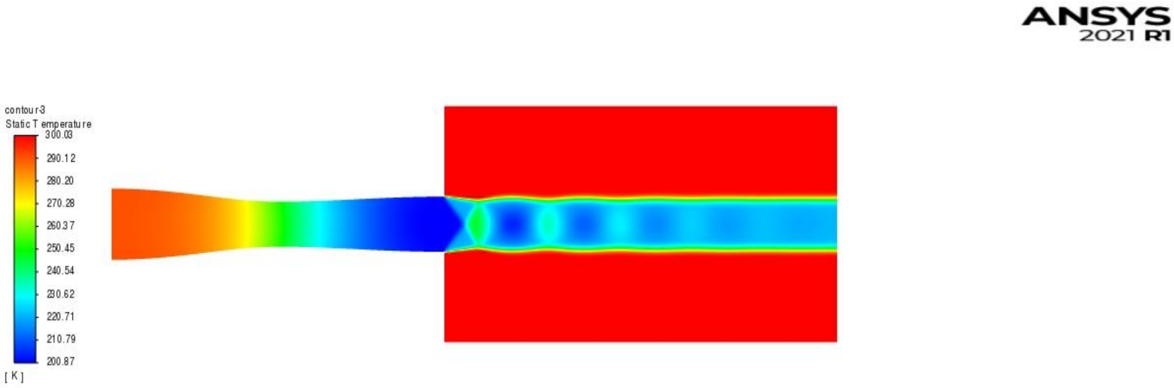

Figure 29. Static Temperature, Contour, Oblique Shock outside Nozzle

After Oblique Shock	Theoretical Value	Simulation Value	Error
Mach Number	1.53	1.50	-1.96%
Static Temperature(K)	201.60	205	1.69%

Table 3-5 Comparison of Theoretical 1D Calculations and CFD Simulation Results, Oblique Shock

3.4.5 Result of Expansion Wave

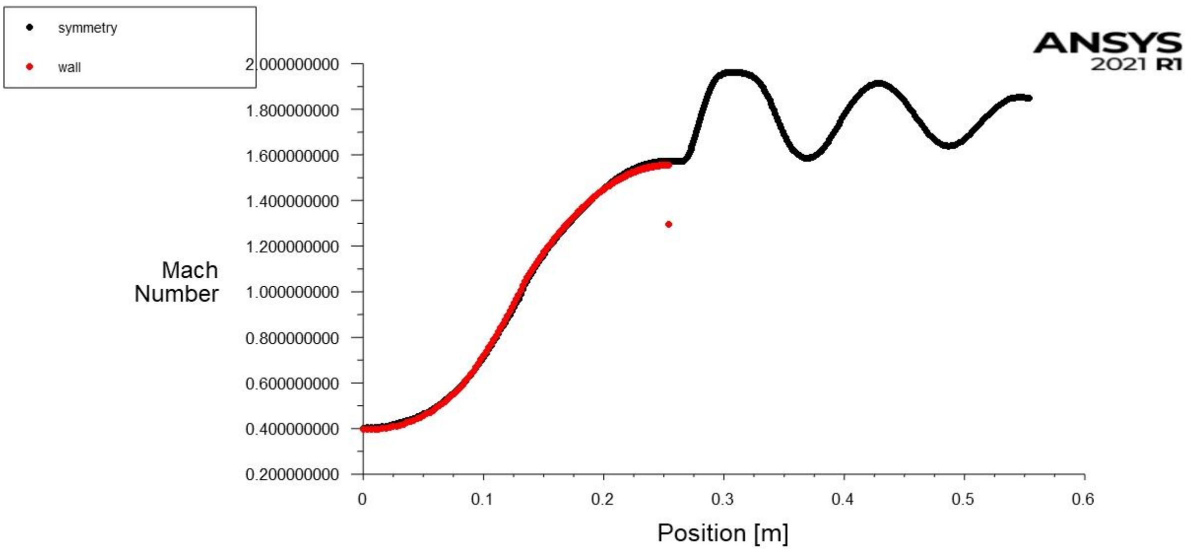

Figure 30. Mach Number, XY Plot, Expansion Wave

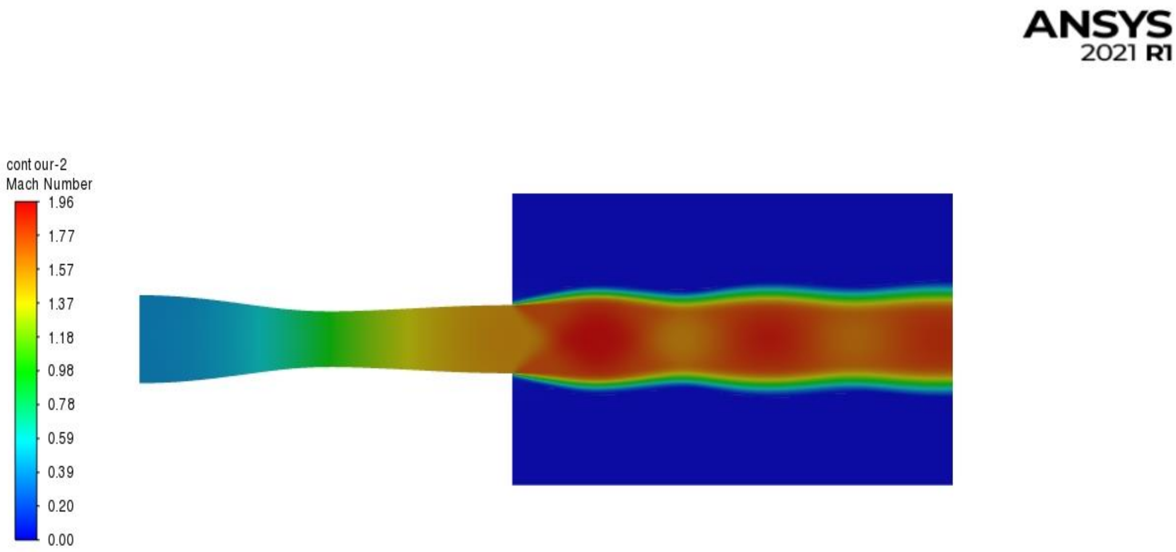

Figure 31. Mach Number, Contour, Expansion Wave

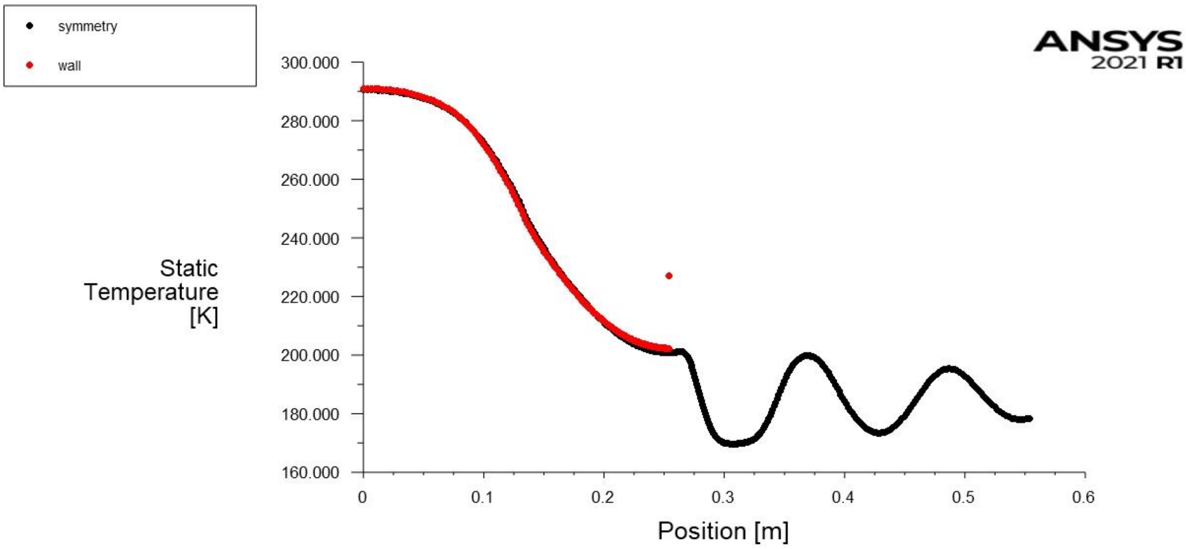

Figure 32. Static Temperature, XY Plot, Expansion Wave

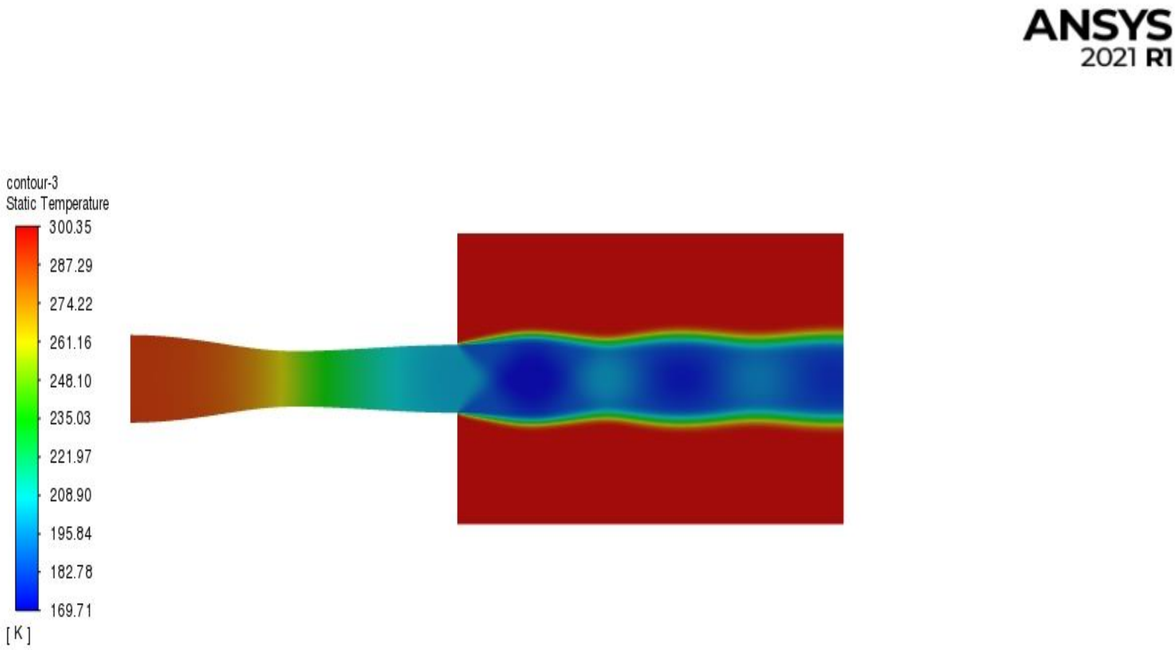

Figure 33. Static Temperature, Contour, Expansion Wave

After first Expansion Fan	Theoretical Value	Simulation Value	Error
Mach Number	2.16	1.96	-9.18%
Static Temperature(K)	155.37	167	7.49%

Table 3-6 Comparison of Theoretical 1D Calculations and CFD Simulation Results, Expansion Wave

Chapter 4: Conclusion

In this report, theoretical and CFD analysis results are reported for various back pressure ranges such that different flows can be simulated in Converging Diverging Nozzle. It is observed that both the approaches (i.e. Theoretical and CFD) are in good agreement. Values are compared in a table and error is less than 10%.

The error observed between the two is because of the software limitation where only 2D flows can be modelled, whereas theoretical calculation done are on 1-D basis.

References

[1] John E. John, Theo G. Keith, Gas dynamics. 1984;