

# 载流子的输运

## 载流子的运动类型

### Three primary types of carrier behavior inside a semiconductor:

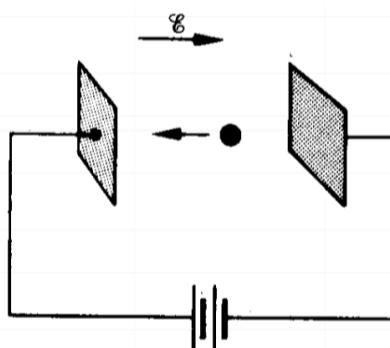
- **Drift (漂移)**: charged particle motion under the influence of an electric field. 电场
- **Diffusion (扩散)**: particle motion due to concentration gradient or temperature gradient. 浓度梯度或者温度梯度
- Electron-Hole **Recombination (复合)** & **Generation (产生)**

### Other Carrier Transport Phenomena:

- Thermionic emission (热离子发射)
- Quantum Tunneling (量子隧穿)
- Impact ionization (碰撞离化)

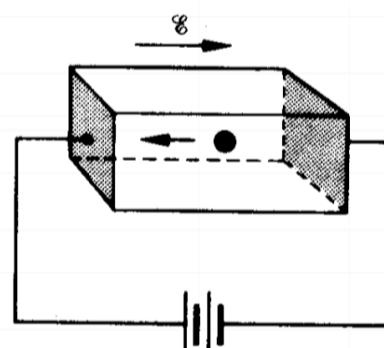
## 半导体中的载流子运动方程

### Electron in vacuum



$$F = (-q)E = m_0 a$$

### Electron in semiconductor



$$F = (-q)E = m_n^* a$$

Why  $m_n^*$ , not  $m_0$ ?

Charge carriers moving in a crystal are affected by the periodic potential of the lattice.

**conductivity effective mass**

在晶体中移动的电荷载流子受到晶格周期性电势的影响。

	Si	Ge	GaAs
$m_n^*/m_0$	0.26	0.12	0.068
$m_p^*/m_0$	0.39	0.30	0.50

**Electron and hole conductivity effective masses**

# Carrier Drift

## 载流子漂移

### 无外电场时的运动载流子

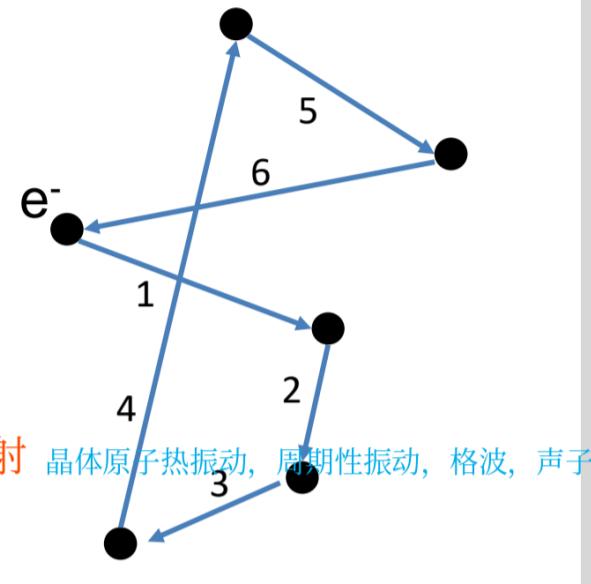
晶格中的移动载流子总是以平均动能进行无规则热运动:

Mobile carriers in the lattice are always in random thermal motion with the **average kinetic energy**:  $\frac{3}{2} kT = \frac{1}{2} m_n^* V_{th}^2$  平均动能

$$V_{th} = 2.3 \times 10^7 \text{ cm/s for silicon}$$

Mobile carriers are scattered intrinsically and extrinsically: 散射

- Lattice (phonon) scattering 晶格散射/声子散射
- Ionized impurity scattering 电离杂质散射
- Carrier-carrier scattering 载流子散射
- Defect scattering
- Substrate scattering

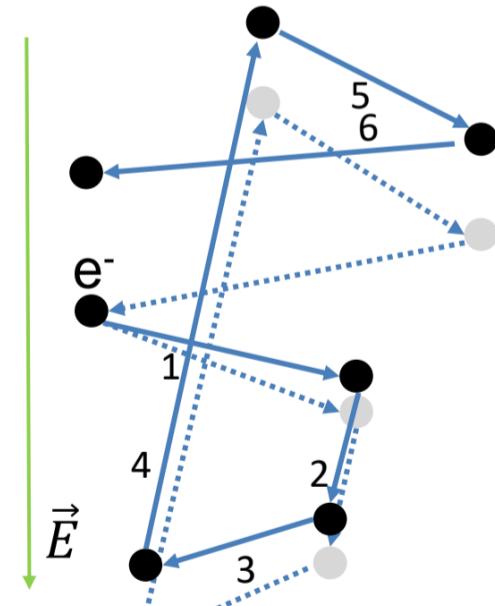


There is no net current at zero field.

### 有外电场时的运动载流子

- Charge carriers are accelerated by the electrostatic force due to the electric field.
- Electrons/Holes drift in the direction opposite/parallel to the field, generating a net current flow.  
漂移电流产生
- Charge carriers are not constantly accelerated due to scattering. The **drift velocity  $v_d$**  is used to describe the average motion of the carriers.

载流子运动的平均速度:  $v_d$



Carriers are drifted under the electric field.

# $v_d$ 与E的关系

不考虑其他因素时，载流子的动量守恒方程：

$$\vec{p}(t+dt) = \vec{p}(t) + (-q\vec{E}) dt$$

考虑散射（可能来自多种原因）后，电子的最终动量可能沿各个方向

因此，考虑引入散射时间： $\tau$

其物理意义是：相邻两次散射的时间间隔

因此，载流子运动时，发生散射的概率： $dt/\tau$

发生散射的载流子的最终动量，沿各个方向，因此在统计意义上为零。

因此，在上述动量守恒方程中，初动量仅考虑未发生散射的载流子即可。

未发生散射的概率： $1-dt/\tau$

因此，考虑散射后的载流子动量守恒方程

(统计意义上)：

$$\vec{p}(t+dt) = (1 - \frac{dt}{\tau}) \vec{p}(t) - q\vec{E} dt$$

求解：

$$\vec{p}(t) + d\vec{p}(t) = (1 - \frac{dt}{\tau}) \vec{p}(t) - q\vec{E} dt$$

$$d\vec{p}(t) = -\frac{dt}{\tau} \vec{p}(t) - q\vec{E} dt$$

$$d\vec{p}(t) = -\frac{dt}{\tau} (\vec{p}(t) + q\vec{E}\tau)$$

$$\frac{d\vec{p}(t)}{\vec{p}(t) + q\vec{E}\tau} = -\frac{dt}{\tau}$$

$$\ln(\vec{p}(t) + q\vec{E}\tau) = -\frac{dt}{\tau} + C$$

$$\vec{p}(t) = C e^{-\frac{dt}{\tau}} - q\vec{E}\tau$$

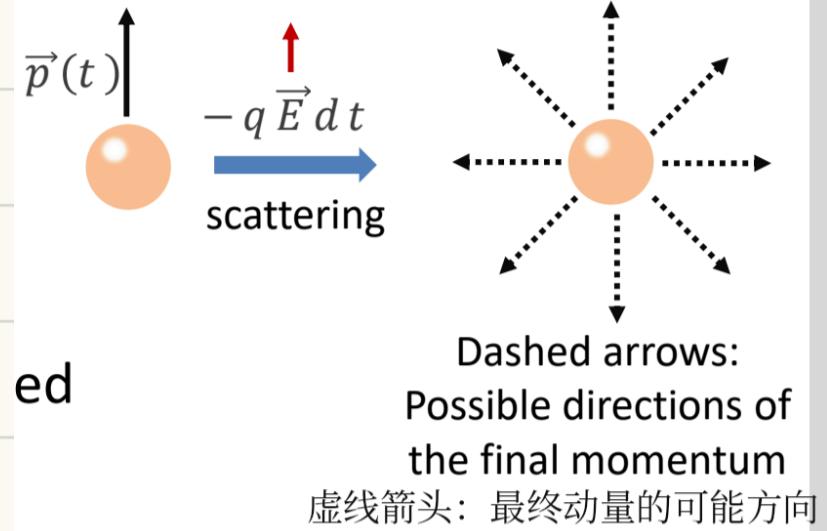
$$\text{稳定时 } t \rightarrow +\infty \quad \vec{p}(t) = -q\vec{E}\tau \quad \rightarrow \quad \vec{v}_d = -\frac{q\vec{E}\tau}{m^*} = -\frac{q\tau}{m^*} \vec{E}$$

$$\text{载流子迁移率} \quad \text{Carrier Mobility } \mu = \frac{q\tau}{m^*}$$

物理意义：带电粒子响应于拉动它们的电场而在介质中移动的能力。

载流子迁移率 ≠ 电导率 = 导电能力

$$\mu_n = \frac{e\tau_{mn}}{m_n^*} \quad \mu_p = \frac{e\tau_{mp}}{m_p^*}$$



Define  $\mu \equiv q\tau / m^*$  as the carrier mobility, the ability of charged particles to move through a medium in response to an electric field that is pulling them.

**Electron mobility:**  $\mu_n \equiv [q\tau_{mn} / m_n^*]$

**Hole mobility:**  $\mu_p \equiv [q\tau_{mp} / m_p^*]$

**Electron/hole mobility for intrinsic semiconductors/semimetals @ 300K**  
电子迁移率普遍大于空穴迁移率

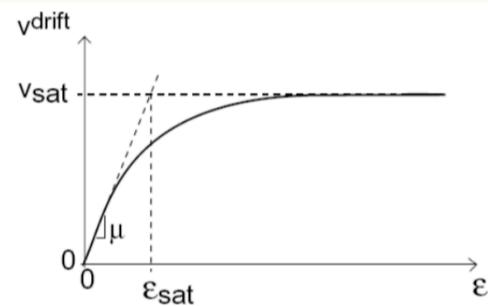
	Si	Ge	GaAs	InAs	Graphene
$\mu_n$ (cm <sup>2</sup> /V·s)	1400	3900	8500	30,000	140,000
$\mu_p$ (cm <sup>2</sup> /V·s)	470	1900	400	500	140,000

当电场强度极大时，载流子漂移速度 $v_d$ 有最大值而不会无限增加，此时 $v_d = \mu E$ 不再适用，因为 $\tau$ 还跟电场强度有关，此时不能视为常数：

$$\vec{v}_d = \begin{cases} \mu \vec{E} & \text{Low field: } \vec{E} \rightarrow 0 \\ \vec{v}_{sat} & \text{High field: } \vec{E} \rightarrow \infty \end{cases}$$

$\rightarrow \tau \text{ is not a constant for large } \mathcal{E}.$

$$m^* v_{max}^2 / 2 = E_{phonon(opt.)}$$



## 案例

- What is the hole scattering time on intrinsic Si at room temperature?
- The mean free path (平均自由程)  $l$  is defined as the average distance travelled by a carrier between successive collisions. Calculate  $l$  for the thermal motion of the intrinsic Si at room temperature.

Solution: a).  $\mu_p = \frac{q\tau_{mp}}{m_p^*} \Rightarrow \tau_{mp} = \frac{m_p^* \mu_p}{q}$

$$= \frac{0.39 \times 9.1 \times 10^{-31} kg \cdot 470 cm^2 / V \cdot s}{1.6 \times 10^{-19} C} \approx 0.1 ps$$

b).  $V_{th} = \sqrt{\frac{3kT}{m_p^*}} = \sqrt{\frac{3 \times 0.026 eV \times (1.6 \times 10^{-19} J/eV)}{0.39 \times 9.1 \times 10^{-31} kg}}$

$$= 1.9 \times 10^5 m/s = 1.9 \times 10^7 cm/s$$

$$l = V_{th} \tau_{mp} = 1.9 \times 10^7 cm / s \cdot 0.1 ps = 19 nm$$

# 散射机制

半导体中载流子的散射有多种类型

## Low electric field:

- Ionized impurity (dopant) scattering 摊杂离子杂质
- Lattice (acoustic phonon) scattering 晶格（声学声子）

## High electric field:

- Lattice (optical phonon) scattering 晶格（光学声子）

## High doping:

- Carrier-carrier scattering 载流子-载流子散射

## Extrinsic scattering:

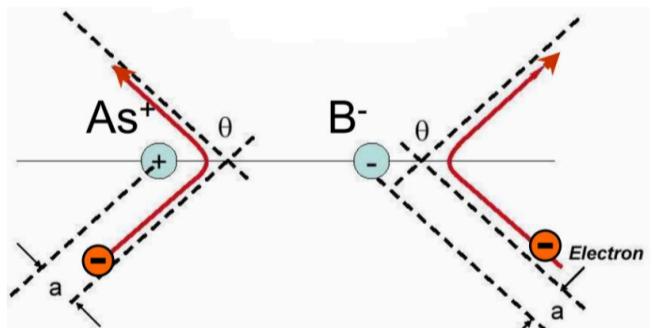
- Defect scattering 缺陷散射
- Substrate scattering 衬底散射

## Ionized Impurities Scattering

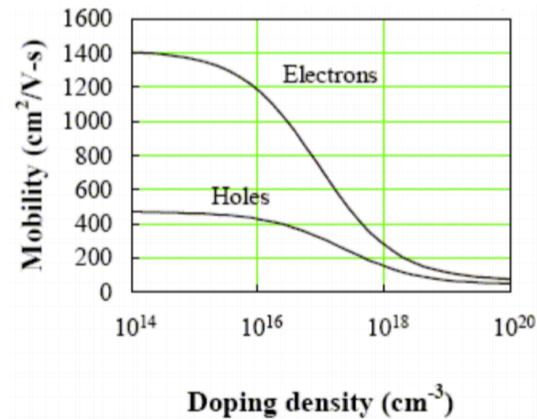
## 掺杂离子散射

单独考虑掺杂离子散射时

- There is less change in the electron's direction if the electron travels by the ion at a higher speed.
- More scattering events happen with increasing impurity density.



$$\mu_{impurity} \propto \frac{V_{th}^3}{N_A + N_D} \propto \frac{T^{3/2}}{N_A + N_D}$$
$$\frac{3}{2} kT = \frac{1}{2} m_n^* V_{th}^2$$



离子散射限制迁移率随T的增加而增加  
**Ion scattering limited mobility increases with increasing T:**

温度越高，载流子迁移率越高(速度V<sub>th</sub>增加，收到离子作用时速度方向的改变较小)  
掺杂离子浓度增加，载流子迁移率降低

单独考虑声子散射时

➤ Phonon scattering limited mobility decreases with increasing  $T$ :

$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{\text{phonon density} \times v_{th}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

温度越高，晶振增强，载流子迁移率越低

考虑多种散射机制时——马西森法则

➤ Matthiessen's Rule: 马西森法则

Probability that a carrier will be scattered by any mechanism within a time period  $dt$  is  $\sum_i dt/\tau_i$  在时间段  $dt$  内，载体被任何机制散射的概率为  $\sum_{i=1}^n \frac{dt}{\tau_i}$

$\tau_i \equiv$  mean time between scattering events due to mechanism  $i$

$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}} \Rightarrow \frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

Mobility vs Temperature

$$\mu_{impurity} \sim \frac{T^{\frac{3}{2}}}{N_T}$$

迁移率与温度?

$$\mu_{lattice} \sim T^{-\frac{3}{2}}$$

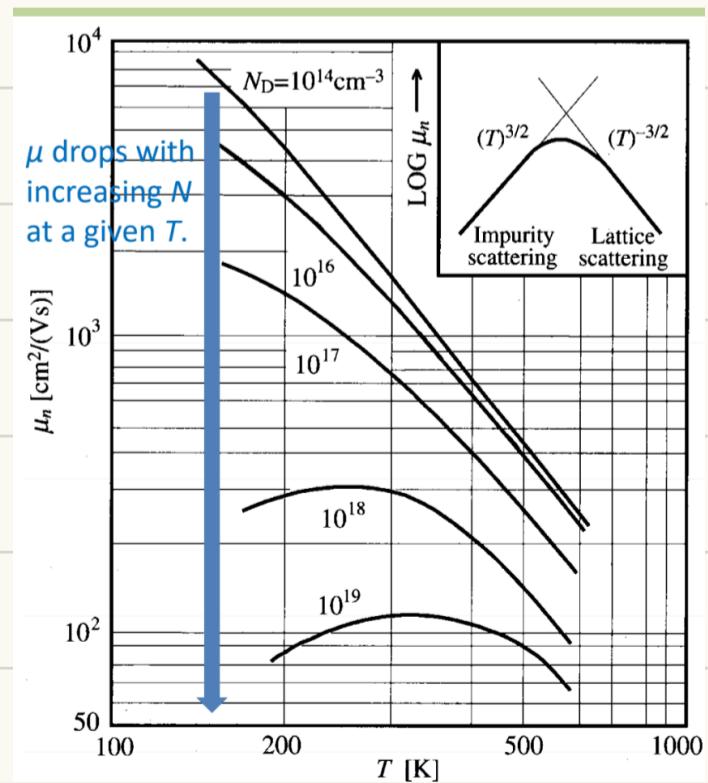
对于轻掺杂半导体

声子散射为主要因素

$$T \uparrow \quad \mu \downarrow$$

对于重掺杂半导体

低温时掺杂离子为主要因素，  
高温时声子散射为主要因素



## Mobility vs Impurity Concentration

迁移率与掺杂离子浓度?

➤ **Lightly doped semiconductor**

Lattice scattering dominant

➤ **Heavily doped semiconductor**

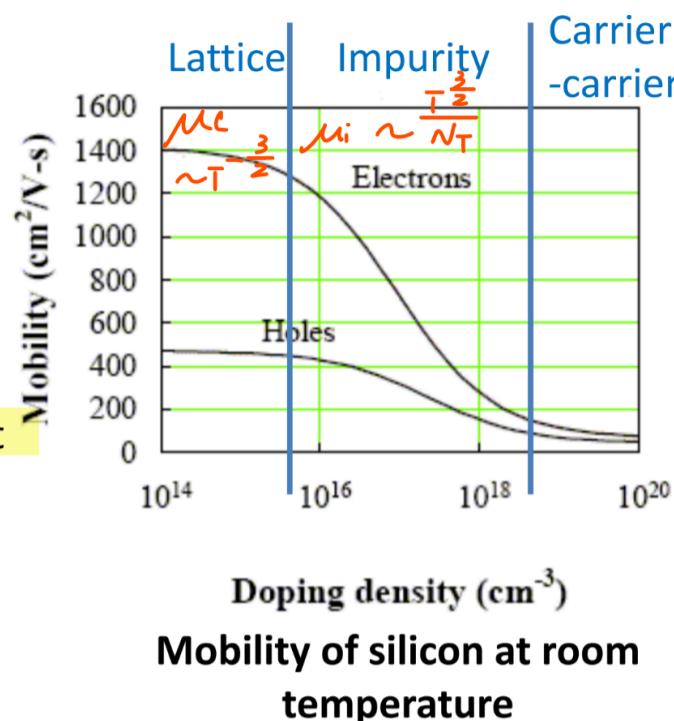
Impurity scattering dominant

➤ **Super Heavily doping**

Carrier-carrier scattering dominant

载流子散射

- The distinction between the electron and hole mobility originates from the different effective masses.



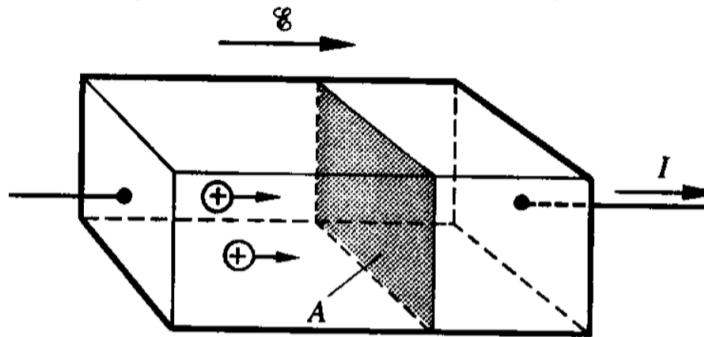
可以发现，在掺杂浓度比较低时，载流子迁移率几乎不变

# 漂移电流密度

# Drift Current Density

$$\text{漂移电流密度 } J = q p v_{dp}$$

Consider the hole transportation as an example:



$v_{dp} \Delta t A$  = volume from which all holes cross plane in time  $\Delta t$

$p v_{dp} \Delta t A$  = number of holes crossing plane in time  $\Delta t$

$q p v_{dp} \Delta t A$  = hole charge crossing plane in time  $\Delta t$

$q p v_{dp} A$  = hole charge crossing plane per unit time = hole current

$$\rightarrow \text{Hole drift current per unit area } J_{p,drift} = q p v_{dp}$$

## 电导率和电阻率，欧姆定律

In a semiconductor, both electrons and holes conduct current:

$$J_{p,drift} = qp\mu_p E \quad J_{n,drift} = -qn(-\mu_n E)$$

电子所受电场力方向相反

$$J_{drift} = J_{p,drift} + J_{n,drift} = qp\mu_p E + qn\mu_n E$$

$$J_{drift} = (qp\mu_p + qn\mu_n)E \equiv \sigma E$$

- The **conductivity** of a semiconductor is  $\sigma \equiv qp\mu_p + qn\mu_n$

$\nabla T \uparrow \mu \downarrow n(p) \uparrow$

欧姆定律的微分形式:  $\vec{J} = \sigma \vec{E}$

迁移率高, 电导率不一定高

$$\Leftrightarrow \rho \vec{J} = \vec{E}$$

$$\Leftrightarrow \rho J_s = E_s$$

$$\Leftrightarrow \rho I = \frac{U}{L} s$$

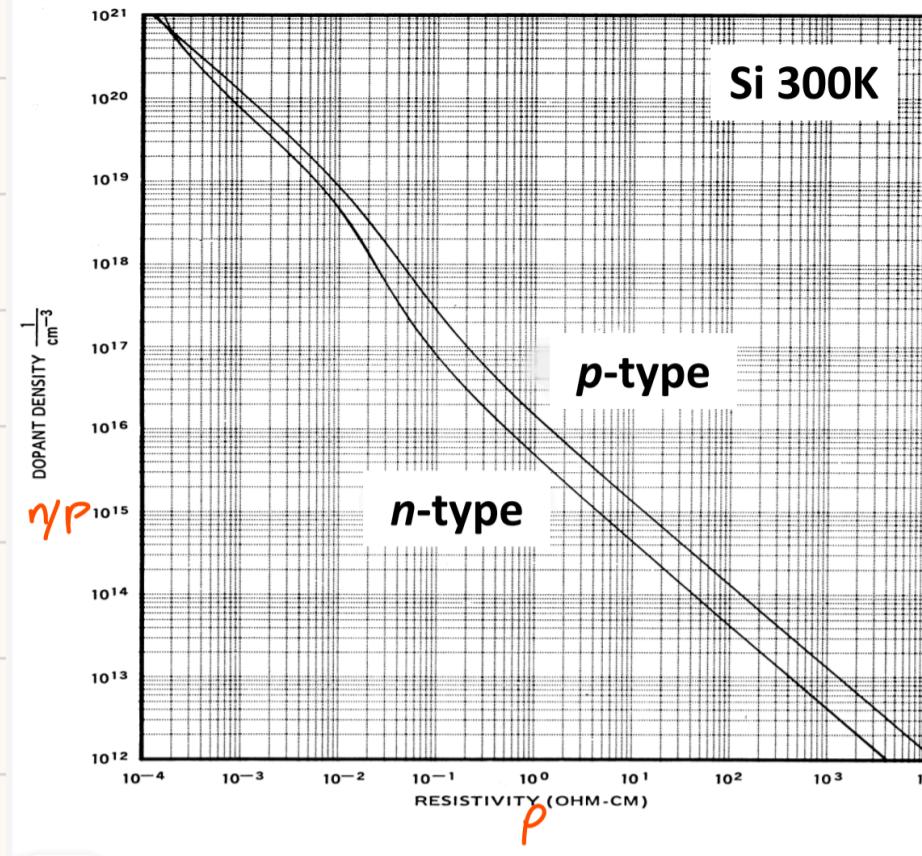
$$\Leftrightarrow U = \rho \frac{L}{s} I \quad (R = \rho \frac{L}{s})$$

- The **resistivity** of a semiconductor is  $\rho \equiv \frac{1}{\sigma} \Rightarrow R \equiv \frac{V}{I} = \rho \frac{L}{Wt}$

宏观形式

For a resistor with regular shape and uniform doping

## 电阻率与掺杂浓度



For n-type material:

$$\rho \approx \frac{1}{qn\mu_n}$$

$\sigma \approx qn\mu_n$

$$\rho \approx \frac{1}{qp\mu_p}$$

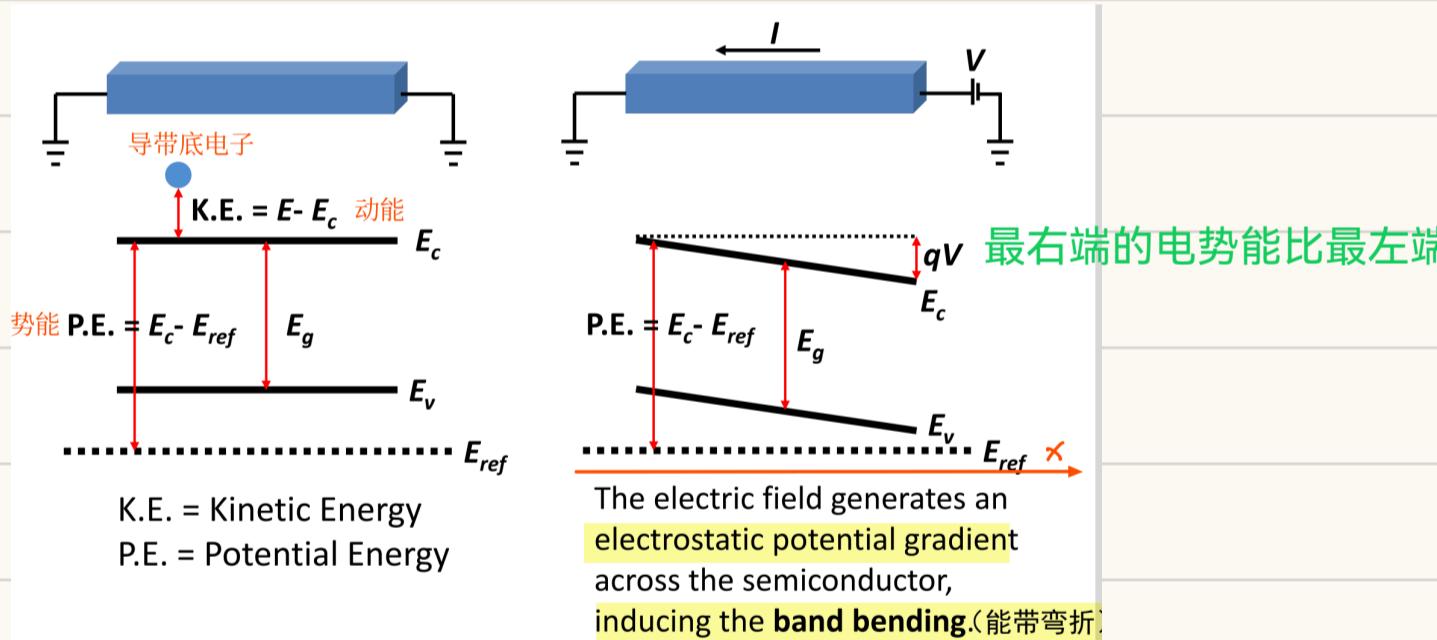
$$\sigma \approx qp\mu_p$$

The resistivity of p-Si is larger than that of n-Si at a fixed doping level.

室温下硅的空穴迁移率小于电子迁移率  $\mu_p < \mu_n$

## 用能带模型解释载流子漂移

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设x处电势为V(x)，则电子势能可以表示为： P.E. = -qV(x)

$$\rightarrow V(x) = -\frac{P.E.}{q}$$

$$\rightarrow V(x) = -\frac{E_c - E_{ref}}{q}$$

$$\rightarrow E = -\nabla V(x) = -\frac{dV(x)}{dx} = \frac{1}{q} \frac{dE_c}{dx}$$

场强，一维情况

电场与带梯度(斜率)成正比。

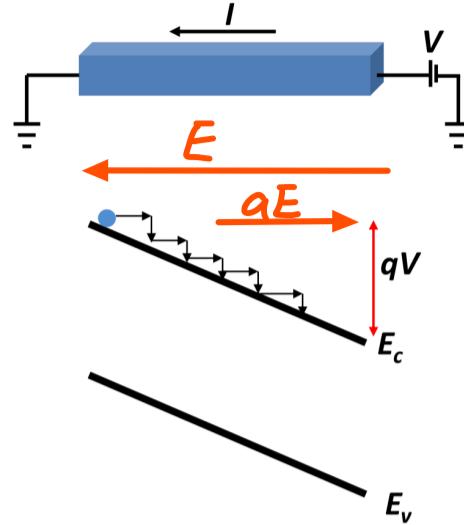
Q: 为什么Ec是一条直线？

A: 导体形状均匀，掺杂均匀，在偏置电压V的作用下，内部产生的漂移电场为匀场电场，故  $dE_c/dx$  为常数

# 弯曲能带解释漂移电流

For a n-type semiconductor:

- The electron is accelerated by the electric field  $\rightarrow$  K.E. increases;
- K.E. get reduced due to the scattering;
- The electron is accelerated again by the electric field again .....
- The electron gradually moves to the anode, resulting in a current flow.



Similar analysis can be made for the hole drift in a p-type semiconductor.

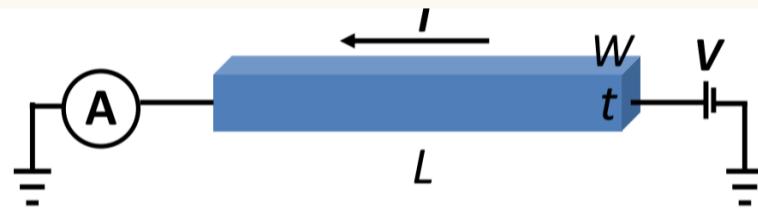
## 几个应用

电阻测定  $\longrightarrow \rho \text{ } \sigma$  的测定

### 二端法

Two probe measurement:

$$\rho = \frac{V}{I} \frac{Wt}{L}$$



缺点：接触电阻很大，干扰测量

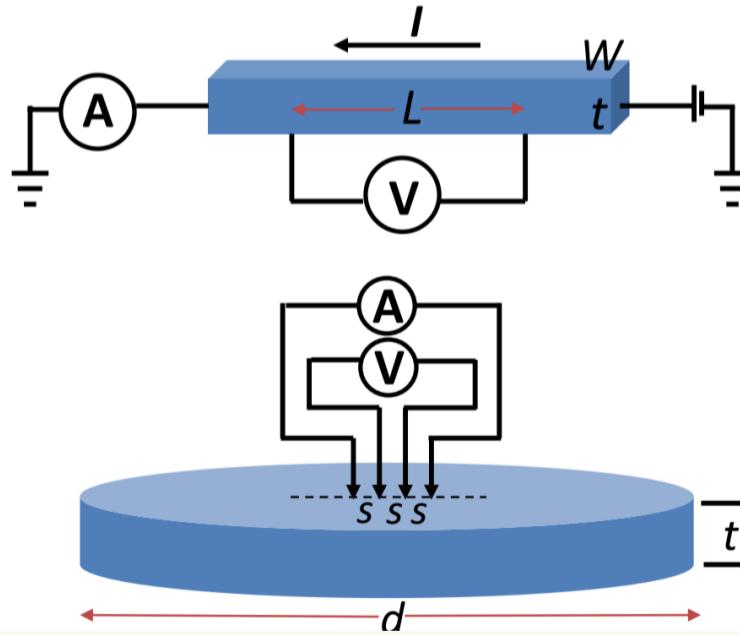
四端法

Four probe measurement:

$$\rho = \frac{V}{I} \frac{Wt}{L}$$

$$\rho = \frac{V}{I} \cdot 2\pi s \cdot f(d, t)$$

$f(d, t)$ : correction factor



# 霍尔效应

→  $\mu$  的测定

## ➤ Calculation steps

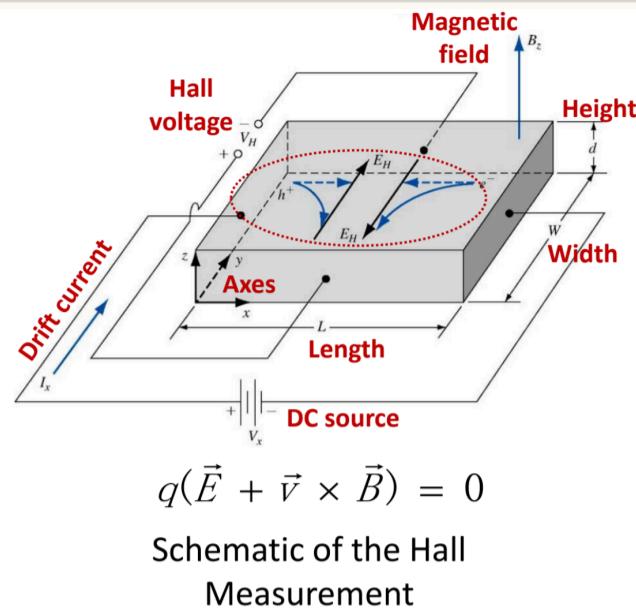
$$E_H = v_x B_z$$

$$v_x = \frac{J_p}{qp} = \frac{I_x}{qpWd}$$

$$V_H = E_H W = \frac{I_x B_z}{qp d}$$

$$\Rightarrow \boxed{p = \frac{I_x B_z}{q V_H d}}$$

$$n = -\frac{I_x B_z}{q V_H d}$$



$$① \text{ 霍尔电压 } V_H = \frac{I_x B_z}{q p d}$$

d: 磁场方向长度 ( $R_H = \frac{1}{q p}$ )

$V_H$  is known

$$② \text{ 载流子浓度}$$

$$p = \frac{I_x B_z}{q V_H d}$$

$$n = -\frac{I_x B_z}{q V_H d}$$

符号来源于霍尔电压的方向

$$\text{欧姆定律积分形式: } \sigma = \frac{I_x}{V_x} \frac{L}{Wd} \quad \left. \right\} q p \mu_p = \frac{I_x}{V_x} \cdot \frac{L}{Wd}$$

$$\text{欧姆定律微分形式: } \sigma = q p \mu_p$$

$p, n, \sigma$  are known

$$③ \text{ 载流子迁移率}$$

$$\mu_p = \frac{\sigma}{q p} = \frac{\frac{I_x}{V_x} \frac{L}{Wd}}{q \cdot \frac{I_x B_z}{q V_H d}} = \frac{V_H L}{V_x W B_z}$$

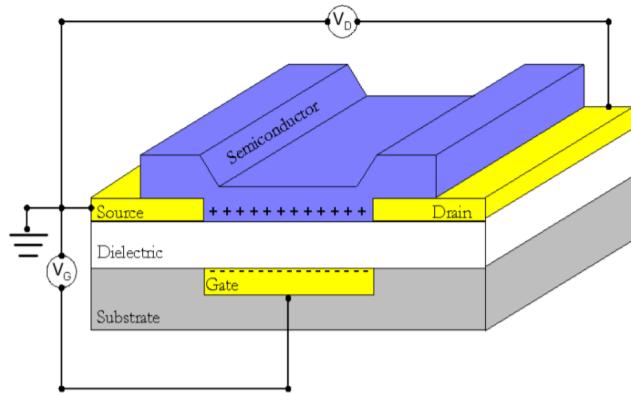
$$\mu_n = -\frac{V_H L}{V_x W B_z}$$

符号来源于霍尔电压的方向

L为电流方向长度

W为霍尔电压方向长度

# Field Effect Measurement

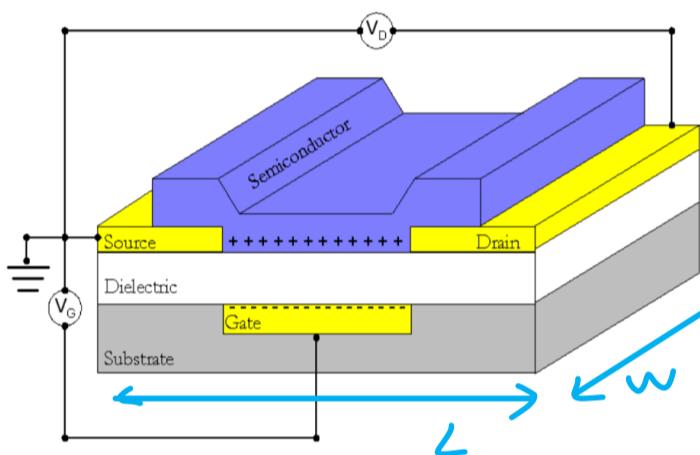


Schematic of the Field Effect Measurement

➤ The **field-effect transistor (FET)** is an electronic device which uses an electric field to control the flow of current.

➤ This is achieved by the application of a voltage to the **gate** terminal, which in turn alters the conductivity between the **drain** and **source** terminals.

➤ To determine the carrier mobility.



Schematic of the Field Effect Measurement

$C_i$ : gate dielectric capacitance per unit area  
 $L, W, t$ : length, width and thickness of the semiconductor

半导体、电介质和栅极接触形成平行板电容器

➤ The semiconductor, the dielectric and the gate contact form a parallel plate capacitor:

$$\Delta Q = C_i \Delta V_G = \Delta p q t$$

➤ For the semiconductor:

$$\Delta \sigma = \frac{\Delta I_{DS}}{V_{DS}} \frac{L}{Wt}$$

➤ Field effect mobility:

$$\mu = \frac{\Delta \sigma}{\Delta p q} = \frac{\Delta I_{DS}}{V_{DS}} \frac{L}{Wt} / \frac{C_i \Delta V_G}{t}$$

$$\Rightarrow \mu = \frac{\Delta I_{DS}}{\Delta V_G} \frac{L}{W} \frac{1}{V_{DS}} \frac{1}{C_i}$$

场效应管参数  
可测

场效应迁移率

固定源漏电压不动，测定场效应迁移率

$$① C_i \Delta V_G = \Delta p \cdot q \cdot \frac{t}{s} = \Delta p \cdot q \cdot t$$

$$② \sigma = \frac{I_{DS}}{V_{DS}} \cdot \frac{L}{Wt} \longrightarrow \Delta \sigma = \frac{\Delta I_{DS}}{V_{DS}} \cdot \frac{L}{Wt}$$

$$③ \mu = \frac{\sigma}{qP} = \frac{\Delta \sigma}{\Delta p \cdot q}$$

联立①②③：

$$\mu_P = \frac{\frac{\Delta I_{DS}}{V_{DS}} \cdot \frac{L}{Wt}}{\frac{C_i \Delta V_G}{t}} = \frac{\Delta I_{DS}}{\Delta V_G} \cdot \frac{1}{V_{DS}} \cdot \frac{1}{C_i} \cdot \frac{L}{W}$$

# Diffusion

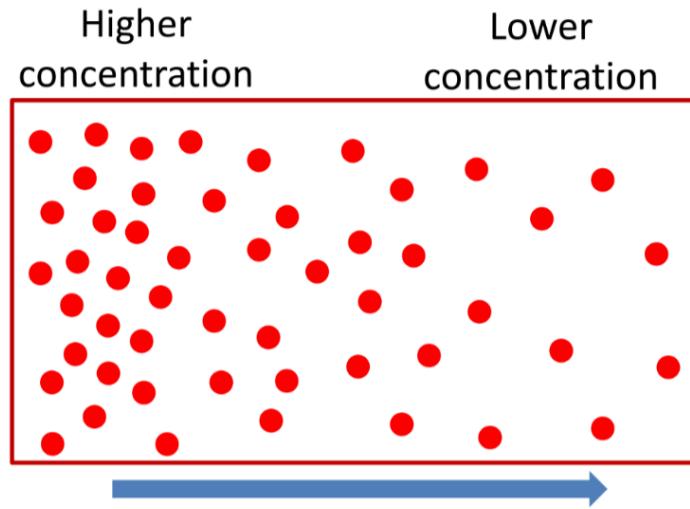
## 扩散

### 扩散

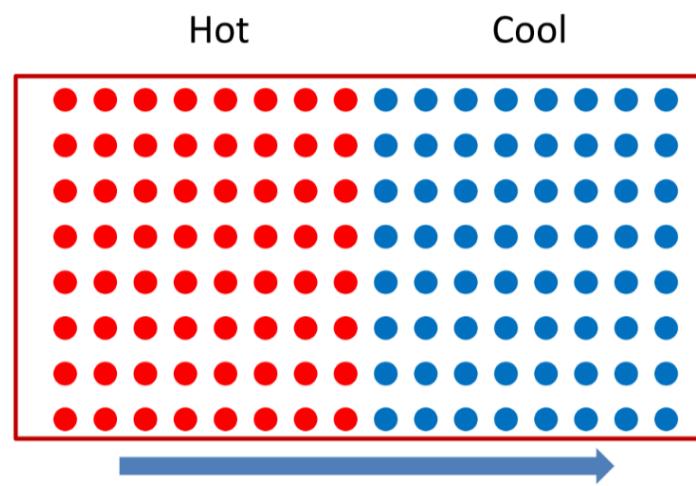
两种类型的扩散——基于浓度与基于温度

物理性质：随机热运动——布朗运动

Physical nature: Random thermal motion — Brownian motion



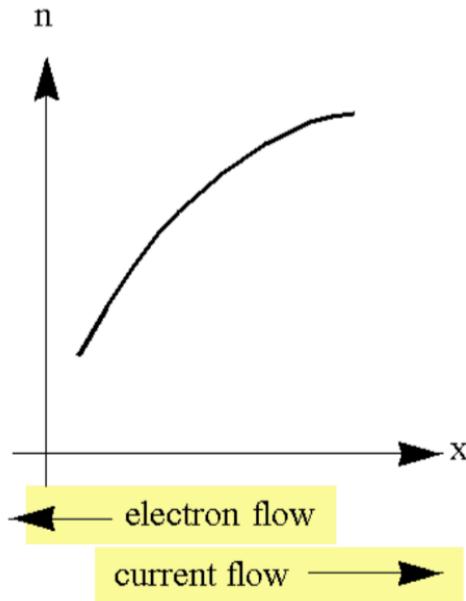
Particles diffuse from regions of higher concentration to regions of lower concentration.



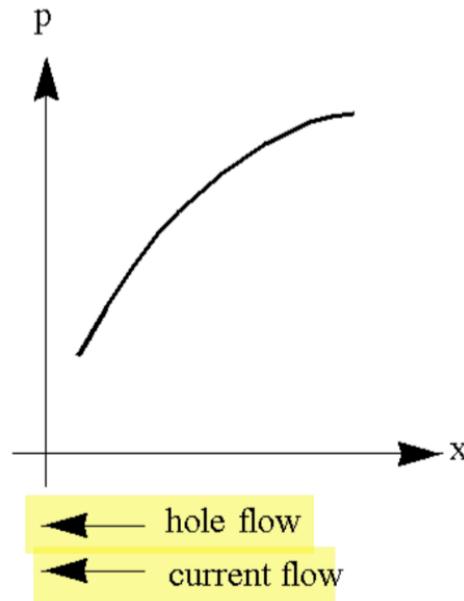
Particles diffuse from regions with higher temperature to regions with lower temperature.

## 扩散电流——菲克第一定律(Fick's First Law)

$$J_{n,diff} = qD_n \frac{dn}{dx}$$



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$



$D$  is the **diffusion constant**, or **diffusivity**.  
(扩散系数) (扩散率)

# 总电流——基于漂移和扩散

$$J = J_n + J_p$$

$$J_n = J_{n,drift} + J_{n,diff} = q_n v_{dn} + q D_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diff} = q_p v_{dp} - q D_p \frac{dp}{dx}$$

什么样的半导体存在载流子浓度梯度？

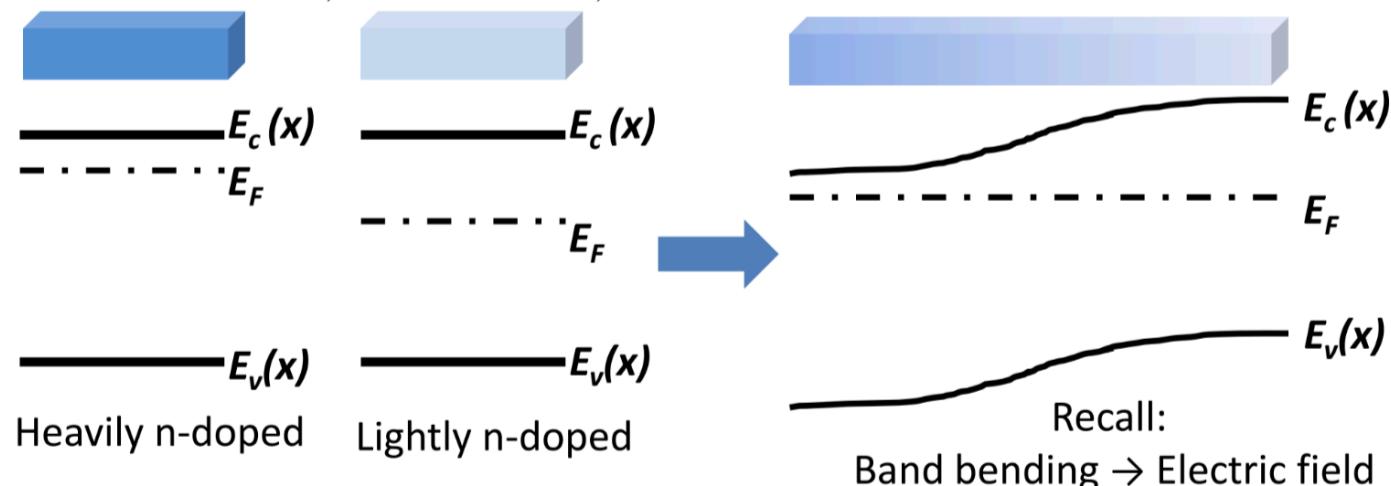
## 非均匀掺杂半导体

### 非均匀掺杂半导体的能带模型

## Non-Uniformly-Doped Semiconductor

### 非均匀掺杂半导体

- The position of  $E_F$  relative to the band edges is determined by the carrier concentrations, which is determined by the net dopant concentration. EF 相对于带边的位置由载流子浓度决定，载流子浓度由净掺杂剂浓度决定
- In equilibrium  $E_F$  is constant; therefore, the band-edge energies vary with position in a non-uniformly doped semiconductor: 在平衡状态下，EF是常数；因此，带边能量在非均匀掺杂半导体中随位置而变化：



$$E_F = E_i + kT \ln \frac{n}{n_i}$$

$$E_c = E_F + kT \ln \frac{N_c}{n}$$

为什么EF是水平的？

EF的含义是绝对零度下，电子从最低能级开始向上填充能填充到的最高能级。非均匀掺杂时，如果不是水平的，那么高能级电子会向低能级填充。

为什么能带弯曲？

能带与EF之间的距离由载流子浓度决定：  $n = N_c e^{-(E_c - E_F)/kT}$

本征费米能级？

形状与导带和价带相同，因为本征费米能级几乎为导带和价带之和的一半

# 非均匀掺杂半导体与内建电场

## 内建电场

用能带模型解释载流子漂移中提到，外电场会使能带发生弯折，而此处在不均匀分布的载流子浓度作用下，能带自己发生了偏折，可以料想到，必然半导体内部必然存在着一个电场，这个电场就是内建电场。

$$\text{Recall: } n = n_i e^{(E_F - E_i) / kT}$$

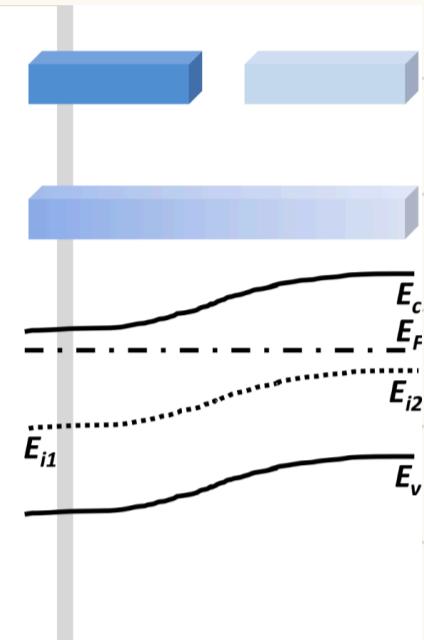
$$\Rightarrow E_F - E_{i1} = kT \ln\left(\frac{n_1}{n_i}\right) \Rightarrow E_{i1} = E_F - kT \ln\left(\frac{n_1}{n_i}\right)$$

$$\text{Similarly, } E_{i2} = E_F - kT \ln\left(\frac{n_2}{n_i}\right)$$

$$\text{Therefore } E_{i1} - E_{i2} = kT \left[ \ln\left(\frac{n_2}{n_i}\right) - \ln\left(\frac{n_1}{n_i}\right) \right] = kT \ln\left(\frac{n_2}{n_1}\right)$$

$$\text{电势差 } V_2 - V_1 = \frac{1}{q} (E_{i1} - E_{i2}) = \frac{kT}{q} \ln\left(\frac{n_2}{n_1}\right)$$

A built-in electric field is generated due to the carrier concentration gradient. (内建电场)



$$\text{半导体两端电势差: } V_2 - V_1 = -\frac{1}{q} (E_{i2} - E_{i1}) = \frac{kT}{q} \ln \frac{n_2}{n_1}$$

## 载流子浓度与内建电场

用能带模型解释载流子漂移中提到电场与导带的关系：

$$E = \frac{1}{q} \frac{dE_c}{dx}$$

$$n = N_c e^{-c(E_c - E_F)/kT}$$

$$\rightarrow \frac{dn}{dx} = -\frac{N_c}{kT} e^{-c(E_c - E_F)/kT} \cdot \frac{dE_c}{dx}$$

$$\rightarrow \frac{dn}{dx} = -\frac{n}{kT} \frac{dE_c}{dx}$$

$$\rightarrow \frac{dn}{dx} = -\frac{n}{kT} q \epsilon$$

$$\rightarrow \epsilon = -\frac{1}{q} \cdot \frac{kT}{n} \cdot \frac{dn}{dx}$$

## 三种方式理解内建电场以及电场方向

我们考虑左侧掺杂浓度高，右侧掺杂浓度低的N型半导体

- ① 左侧掺杂浓度高，初始载流子浓度高，向右侧扩散后形成阳离子区，同理，右侧形成阴离子区。热平衡状态就是在浓度梯度与电场的平衡下完成的。因此电场方向向右

$$② \quad \varepsilon = \frac{1}{q} \cdot \frac{dE_c}{dx}$$

对于左侧掺杂浓度高的半导体，导带从左至右升高，斜率为正，因此电场方向向右

$$③ \quad \varepsilon = -\frac{1}{q} \cdot \frac{kT}{n} \frac{dn}{dx}$$

对于左侧掺杂浓度高的半导体， $dn/dx$ 为负，因此电场方向向右

## 爱因斯坦关系式

### 重要结论

热平衡状态下，载流子电流密度为零，即：

$$J_n = J_p = 0$$

理解：由内建电场产生的漂移电流与由浓度梯度产生的扩散电流相互抵消

### 爱因斯坦关系式（常用）

$$\begin{cases} J_n = qn\mu_n \varepsilon + qD_n \frac{dn}{dx} = 0 \\ J_p = qp\mu_p \varepsilon - qD_p \frac{dp}{dx} = 0 \end{cases} \quad \begin{array}{l} \varepsilon = -\frac{kT}{q} \frac{1}{n} \frac{dn}{dx} \\ \varepsilon = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \end{array} \rightarrow$$

$$\begin{cases} \frac{kT}{q} = \frac{D_n}{\mu_n} \\ \frac{kT}{q} = \frac{D_p}{\mu_p} \end{cases}$$

$$J_n = qn\mu_n \varepsilon + qD_n \frac{dn}{dx} \quad \varepsilon = -\frac{kT}{q} \frac{1}{n} \cdot \frac{dn}{dx}$$

↓  
漂移电流：  
微元法+动  
量守恒推导

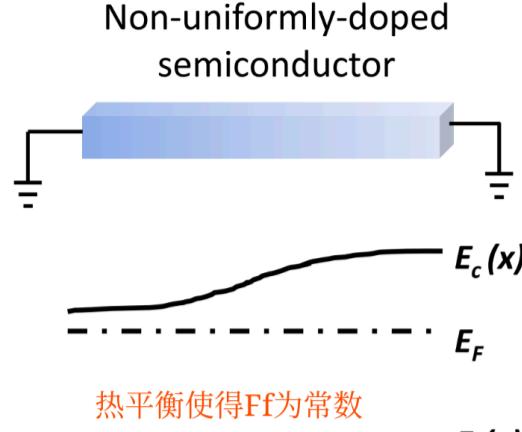
↓  
菲克第  
一定律

↓  
内建电场与载流  
子浓度的关系

对于非热平衡状态，爱因斯坦关系式也成立

# 对比：漂移和扩散中的能带弯曲

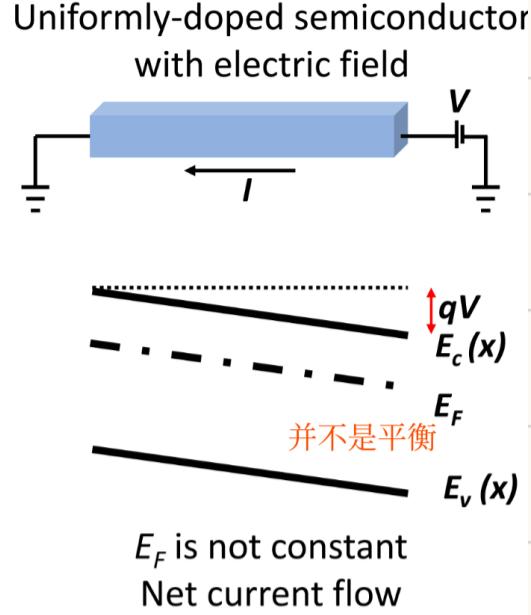
非均匀掺杂半导体



热平衡使得  $E_F$  为常数

$E_F$  is constant  
No net current flow

均匀掺杂半导体



扩散

热平衡使得  $E_F$  为常数，但  $E_i$  不是

漂移

并非热平衡状态，但均匀掺杂，导带与价带与费米能级平行

# 什么是热平衡状态?

## 定义

“热平衡状态”是指半导体系统在没有外界扰动（如外加电压、光照、温度梯度等）的情况下，内部载流子（电子和空穴）的统计分布达到动态平衡的状态。

## 核心特征与标志

- (1) 载流子浓度稳定:  $np=ni^2$  (对于非简并掺杂半导体)
- (2) 无静电流流动:  $Jn=Jp=0$  (对于非简并掺杂和简并掺杂都是)
- (3) 费米能级 ( $E_F$ ) 在整个材料中保持恒定。

即：整个半导体内部的费米能级  $E_F$  在空间上呈现一条水平直线，无空间变化( $dE_F/dx=0$ )

## 能带什么时候弯折？

能带弯折的本质原因是空间电荷的存在，能带弯折由电势梯度(即电场)决定：

$$E_{c(x)} = E_{c0} - q\Phi(x) \quad E_{v(x)} = E_{v0} - q\Phi(x)$$

$E_{c0}$ 与 $E_{v0}$ 是材料本征的导带和价带位置，保持水平

是否弯折可由斜率直接体现：

$$\frac{dE_{c(x)}}{dx} = qE \quad \frac{dE_{v(x)}}{dx} = qE$$

即若存在电场，则能带必弯折；若电场为零，则能带平直

## 本征费米能级什么时候弯折？

在绝大多数情况下，本征费米能级、导带、价带的弯折同步且平行。

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \frac{N_v}{N_c}$$

即本征费米能级的弯折也由电势梯度决定：

$$E_{i(x)} = E_{i0} - q\Phi(x) \longrightarrow \frac{dE_{i(x)}}{dx} = qE$$

## 费米能级什么时候弯折？

### 平衡态

热平衡状态下，费米能级一定保持水平

### 常见误区与回答

**误区1：**“非均匀掺杂导致平衡态下费米能级弯折”。

**纠正：**非均匀掺杂会导致能带 $E_c$ 和 $E_v$ 弯折，但费米能级仍水平。弯折的能带通过内建电场平衡载流子分布，维持费米能级的统一。

**误区2：**“只要载流子浓度梯度存在，费米能级就会弯折”。

**纠正：**平衡态下，载流子浓度梯度被内建电场抵消，费米能级保持水平；非平衡态下，浓度梯度导致准费米能级弯折，但此时费米能级已不适用。

## 非平衡态

单一的费米能级不再适用，需引入准费米能级

$$n(x) = N_c e^{[E_F(x) - E_c(x)] / kT} \quad , \quad p(x) = N_v e^{[E_v(x) - E_F(x)] / kT}$$

准费米能级什么时候弯折？

准费米能级需满足：

$$\textcircled{1} \quad n(x) = N_c e^{[E_F(x) - E_c(x)] / kT} \quad , \quad p(x) = N_v e^{[E_v(x) - E_F(x)] / kT}$$
$$\textcircled{2} \quad F_N(x) = E_i(x) + kT \ln \frac{n(x)}{n_i} \quad , \quad F_P(x) = E_i(x) - kT \ln \frac{p(x)}{p_i}$$

①和②实际上是等效的。

即在非平衡态下，准费米能级的弯折由掺杂浓度梯度以及电势分布的平衡条件决定，即由载流子浓度和能带弯折情况共同决定。

从方程形式上看，准费米能级的空间导数为：

$$\frac{dF_N(x)}{dx} = \frac{dE_c(x)}{dx} + \frac{kT}{n(x)} \frac{dn(x)}{dx} \quad , \quad \frac{dF_P(x)}{dx} = \frac{dE_v(x)}{dx} - \frac{kT}{p(x)} \frac{dp(x)}{dx}$$

# Recombination and Generation

## 电子空穴对的复合与产生

复合 Recombination : A process by which electrons and holes are annihilated **in pairs**

产生 Generation : A process by which electrons and holes are created **in pairs.**

Q: 本节我们在讨论半导体中的电流，无论是漂移还是扩散都直接提供了产生电流的途径。  
那么这里为什么我们要考虑电子空穴对的复合与产生这个看似与电流无关的过程呢？

A: Generation and recombination processes act to change the **carrier concentrations**,  
and thereby indirectly affect current flow.

热平衡:  $n_p = n_i^2$

非热平衡: 过剩载流子

**excess carriers** are introduced to a semiconductor by light or electrical bias voltage, which is called **carrier injection**  
**载流子注入**

The system tends to restore the equilibrium when getting disturbed.

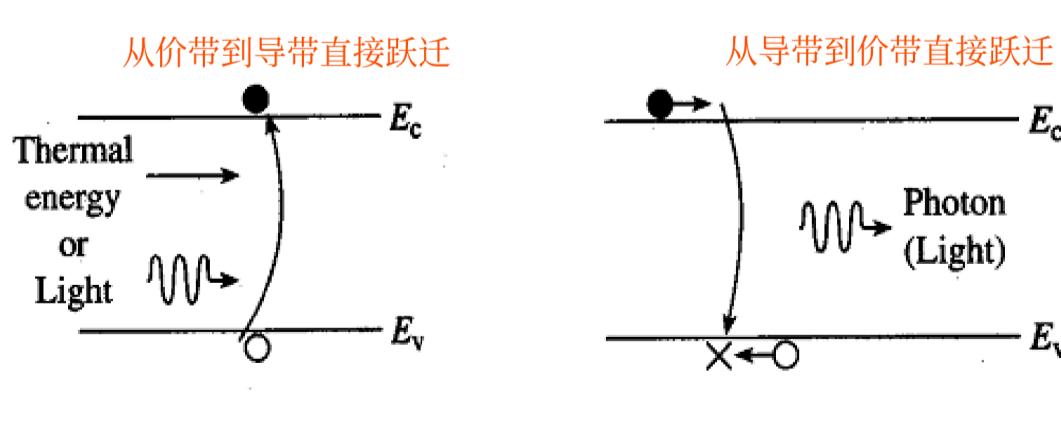
系统在受到干扰时倾向于恢复平衡。

The released energy from the recombination process can be used to emit a photon (radiative recombination) or dissipate to the lattice as heat (nonradioactive recombination)

复合过程释放的能量可用于发射光子（辐射复合）或作为热量消散到晶格（非放射性复合）。

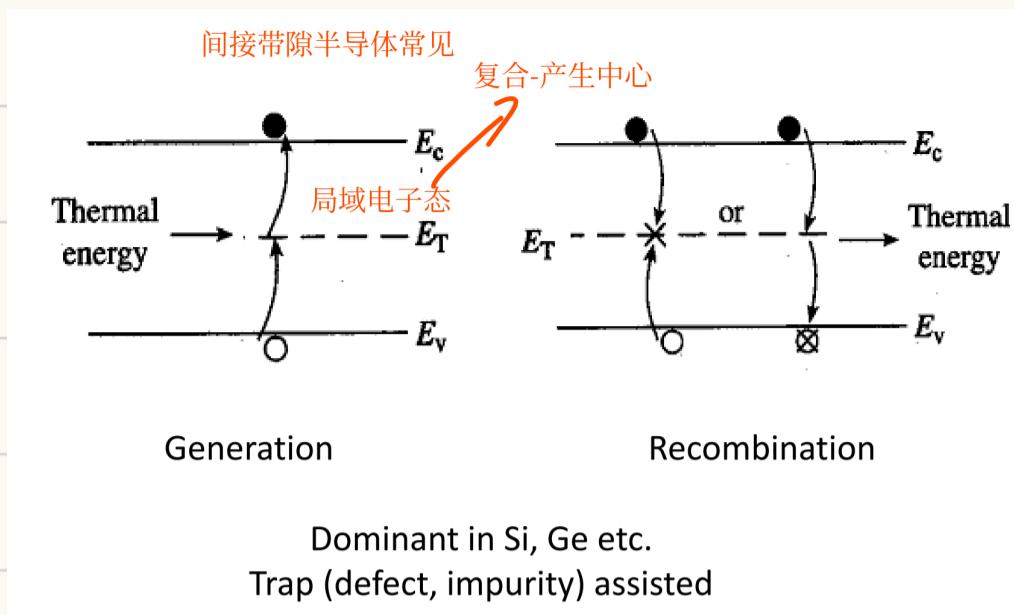
## 电子空穴对复合的方式

### Band-to-Band Processes (带-带过程, 直接跃迁)



Occurs in GaAs, GaN etc.

## R-G Center Processes (复合-产生中心过程，间接跃迁)



① 在物理上，复合中心是晶格缺陷或特殊的杂质原子

② R-G复合中心最重要的性质是在带隙中心处引入了允许电子能级

### 复合过程

如图 3.15(b)所示，在 R-G 中心的复合是一个两步过程。首先，由一种类型的载流子（如电子）漂移进入 R-G 中心附近，被与 R-G 中心相关的势阱所捕获，失去能量，且被俘获于中心里面。随后，出现空穴，被俘获的电子吸引，失去能量，并在中心内与电子一同湮灭。换句话说，在图 3.15(b)中，电子失去能量后，在价带中与空穴一起湮灭。R-G 中心复合也称为间接复合，其特点是复合过程中释放热能（热量），或者相当于产生晶格振动。

Q: 直接跃迁只会发生在直接带隙半导体，间接跃迁只会发生在间接带隙半导体吗？

A: 不是

### Band- to-Band Process:

直接带隙半导体(如GaAs、InP): 电子从导带直接跃迁到价带，与空穴复合，同时发射光子（辐射复合），该过程无需声子参与。

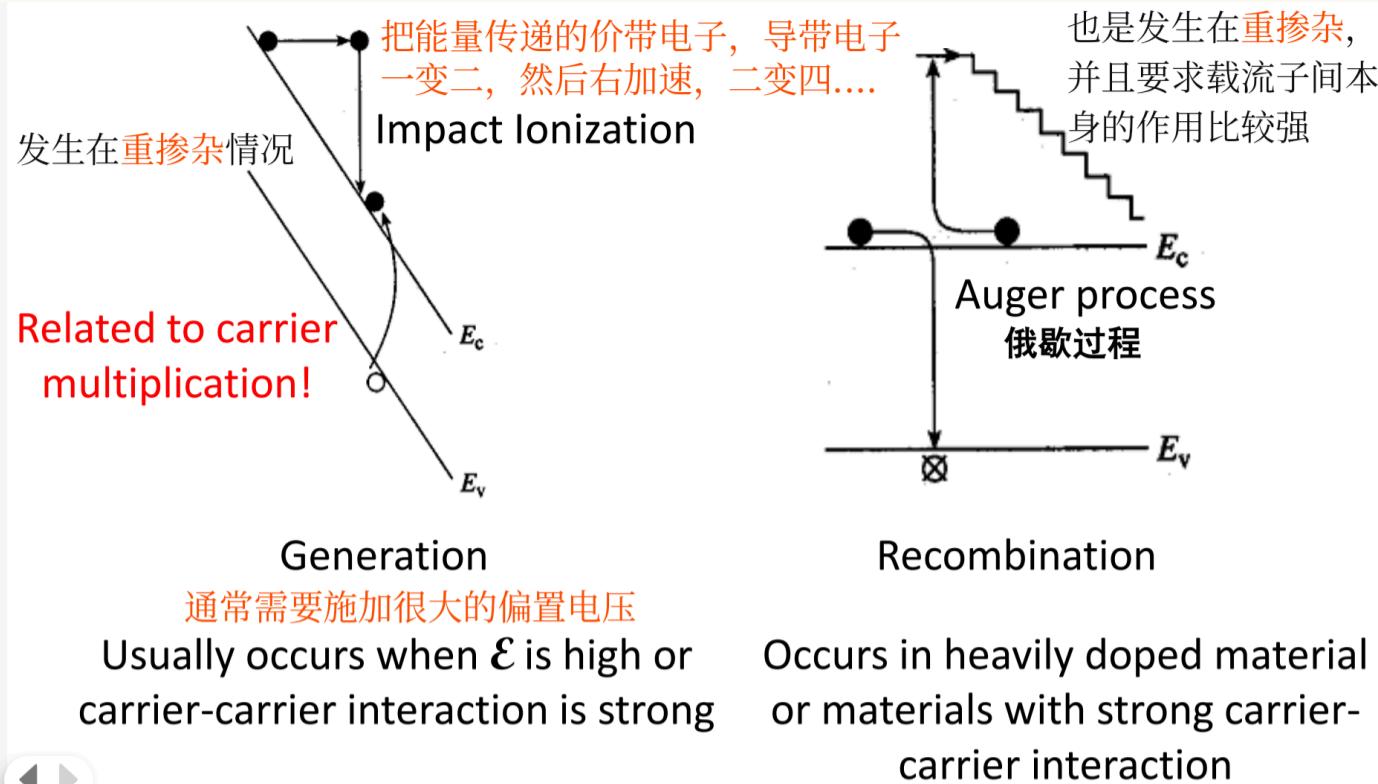
间接带隙半导体(如Si、Ge): 导带底和价带顶动量不匹配 ( $\Delta k \neq 0$ )，直接带间复合需要声子辅助，导致辐射复合概率极低，通常不是主要的复合机制。

### R-G Center Process:

间接带隙半导体(如Si): 复合中心(如杂质、缺陷)作为中介，电子和空穴分步通过复合中心完成复合，同时发射声子（非辐射复合）。

直接带隙半导体: 复合中心机制可发生，但带间辐射复合效率更高，因此复合中心的影响相对较弱（除非材料缺陷浓度极高）。

## Carrier-Carrier Processes (载流子-载流子过程)



### 俄歇复合

两个同种类型的载流子发生碰撞，从而发生直接复合。复合所释放的能量传递给经过碰撞保存下来的载流子。然后这个高能载流子与晶格碰撞产生热量，从而失去能量。

## 电子空穴对产生的方式

任何一个前面所描述的复合过程都存在逆过程来产生载流子。如图3.15(d)所示，当电子被激发后，直接从价带进入导带，这就是直接产生过程。要指明的是，热能或光能所提供的能量必须满足能带间的跃迁。若热能被吸收，这个过程就称之为直接热产生；若外部输入的光被吸收，这个过程就称为光产生。如图3.15(e)所示，热能如同中间媒介，它可以帮助在R-G中心内产生载流子。与俄歇复合相反的碰撞电离如图3.15(f)所示。这个过程是由于高能载流子与晶格发生碰撞，能量释放的结果产生一对电子和空穴。通过碰撞电离产生的载流子，通常是在器件的高电场区。相关的详细说明将在pn结的击穿中阐述。

# 过剩载流子与R-G数学描述

## 过剩载流子浓度( $\Delta n$ $\Delta p$ )

①  $\Delta n = n - n_0$   $\Delta p = p - p_0$   $n_0, p_0$ : 热平衡时的载流子浓度

② 电中性条件:  $\Delta n = \Delta p$

### Low-Level Injection 小注入

Often the disturbance from equilibrium is small, such that the majority-carrier concentration is not affected significantly.

对于n型半导体:  $|\Delta n| = |\Delta p| \ll n_0 \rightarrow n \approx n_0$

对于p型半导体:  $|\Delta n| = |\Delta p| \ll p_0 \rightarrow p \approx p_0$

过剩载流子浓度必须远小于多子浓度, 但可以远大于少子浓度

### High level injection:

The excess carriers induces the majority increasing. 大注入

### Indirect Recombination Rate (间接跃迁复合率)

为什么我们这里要单独研究R-G过程?

绝大多数的器件都是使用间接半导体材料Si制造的, 在间接半导体中:

- ① 直接跃迁需要借助声子, 相比R-G可完全忽略;
- ② 俄歇复合需要高掺杂, 可忽略;
- ③ 碰撞电离需要高电场, 可忽略;

R-G过程对在室温下非简并掺杂间接带隙半导体的低电场区域起主导作用

Suppose excess carriers are introduced into an n-type Si sample (e.g. by temporarily shining light onto it) at time  $t = 0$ . How does  $p$  vary with time  $t > 0$ ?

既有符合, 也有产生, 只是可能复合快

1. Consider the rate of hole recombination via traps:

$$\text{复合率} \quad R: \text{Recombination} \quad \frac{\partial p}{\partial t} \Big|_R = -C_p N_T p$$

捕获截面  $\text{Capture cross-section}$       复合中心浓度  $\text{No. of R-G center in unit volume}$       空穴浓度  $\text{Hole density}$

2. Under low-level injection conditions, the hole generation rate is not significantly affected:

$$\text{产生率} \quad \frac{\partial p}{\partial t} \Big|_G \cong \frac{\partial p}{\partial t} \Big|_{G-equilibrium} = -\frac{\partial p}{\partial t} \Big|_{R-equilibrium} = C_p N_T p_0$$

不太依赖空穴浓度      因此产生率等于热平衡时的复合率

空穴浓度变化率：

$$\frac{\partial p}{\partial t} \Big|_{R-G} = \frac{\partial p}{\partial t} \Big|_R + \frac{\partial p}{\partial t} \Big|_q = -C_p N_T P + C_p N_T P_0 = -C_p N_T \Delta P$$

① 这实际上是一个微分方程：  $\frac{\partial p}{\partial t} \Big|_{R-G} = \frac{d \Delta P}{dt} \longrightarrow \frac{d \Delta P}{\Delta P} = -C_p N_T t \rightarrow \Delta P = \Delta P_0 e^{-C_p N_T t}$

②  $\tau_p = \frac{1}{C_p N_T}$  过剩空穴寿命  $\Delta P = \Delta P_0 e^{-\frac{t}{\tau_p}}$

③ 类似的，有： $\Delta n = \Delta n_0 e^{-\frac{t}{\tau_n}}$   $\tau_n = \frac{1}{C_n N_T}$   $\longrightarrow$  统计意义上的少子平均寿命

考虑一个没有电流流动的半导体，其中热平衡被突然产生的多余空穴和电子所扰乱。系统将通过这种R-G机制放松回到平衡状态。

### τ：少数载流子寿命

the average time an excess minority carrier “survives” in a sea of majority carriers

过剩的少子在多子的海洋中“生存”的平均时间

τ在Si中的范围为1ns至1ms并且取决于诸如Au和Pt的金属杂质（污染物）的密度以及晶体缺陷的密度。这些杂质/缺陷在带隙深处产生局域能态。这种深陷阱捕获电子或空穴以促进复合，被称为复合生成中心。

### 少子浓度变化率

#### ➤ Low level injection and minority carrier

$$\frac{\partial p}{\partial t} \Big|_{R-G} = -\frac{\Delta p}{\tau_p} \quad \frac{\partial n}{\partial t} \Big|_{R-G} = -\frac{\Delta n}{\tau_n}$$

#### ➤ General case

$$\frac{\partial p}{\partial t} \Big|_{R-G} = \frac{\partial n}{\partial t} \Big|_{R-G} = -\frac{n_i^2 - np}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

$$n_1 = n_i \exp[(E_T - E_i)/kT]$$

$$p_1 = n_i \exp[(E_i - E_T)/kT]$$

## Example: Photoconductor(光电导)

### P掺杂

Consider a sample of Si doped with  $10^{16} \text{ cm}^{-3}$  boron, with recombination lifetime  $1 \mu\text{s}$ . It is exposed continuously to light, such that electron-hole pairs are generated throughout the sample at the rate of  $10^{20} \text{ per cm}^3$  per second, i.e. the **generation rate  $G_L = 10^{20}/\text{cm}^3/\text{s}$**

**What are  $p_0$  and  $n_0$ ?**

$$p_0 = 10^{16} \text{ cm}^{-3} \quad n_0 = n_i^2 / p_0 = 10^4 \text{ cm}^{-3}$$

处于一个稳态，宏观物理量不随时间变化  
但处于稳态不一定处于热平衡态（热平衡要求无光无电无力无热），即有过剩载流子，不满足  $np = n_i^2$

**What are  $\Delta n$  and  $\Delta p$ ?**

Exposed continuously to light -> Steady-state-> $G_L = R_L$

$$\Delta n = \Delta p = G_L \cdot \tau_n = 10^{14} \text{ cm}^{-3}$$

np is very different from  $n_i^2$ .

**What are  $n$  and  $p$  and  $np$ ?**

$$n = n_0 + \Delta n \approx 10^{14} \text{ cm}^{-3} \quad p = p_0 + \Delta p \approx 10^{16} \text{ cm}^{-3} \quad np = 10^{30} \text{ cm}^{-6}$$

理解:  $\frac{\partial p}{\partial t} = -c_p N_T p$

此时对  $\partial p / \partial t|_R$  影响最大的因子和空穴复合率是多少? 以空穴为例, 要消去一个空穴, 这个空穴必须从价带跃迁到充满电子的 R-G 中心。理论上, 充满电子的 R-G 中心数量越多, 空穴湮灭跃迁的几率越大, 复合率也越大。在平衡条件下, 平衡费米能级一定在  $E_T$  位置之上, 所以全部的 R-G 中心基本上都被电子填满。由于  $\Delta p \ll n_0$ , 电子在数量上总是远远大于空穴并且迅速地填充失去电子的 R-G 能级。在保持充满电子的 R-G 中心数量不变的过程中, 弛豫过程近似等于  $N_T$ 。于是,  $\partial p / \partial t|_R$  与  $N_T$  近似地成比例。正如理论描述的那样, 空穴湮灭跃迁数随空穴数是线性增加的。湮灭所用的空穴越多, 单位时间内从价带上移的空穴数也越多。因此,  $\partial p / \partial t|_R$  与  $p$  也近似地成比例。考虑到增加因子没有依赖性, 所以引入正的比例常数  $c_p$ , 并且因为  $p$  是减小的, 实现的  $\partial p / \partial t|_R$  是负的, 最终可得

$$\left. \frac{\partial p}{\partial t} \right|_R = -c_p N_T p \quad (3.29)$$

理解: 少子寿命  $\tau$

在 (3.34) 式中没有介绍时间常数  $\tau_n$  和  $\tau_p$ , 这里可以把它们看成是与复合 - 产生相关的“作用常数”。为了寻求与  $\tau$  的命名和标准解释有关的信息, 重新考虑图 3.21 所示的情况。考察空穴浓度随时间的变化关系, 可以认为过剩的空穴不会在同一时刻全部消失。不过, 在  $t=0$  时空穴过剩, 被系统地湮灭, 并且只在短周期内存在一些空穴而其他一些则在相对长的周期内“存活”。若热复合 - 产生是作用在半导体内惟一的衰减过程中, 则过剩空穴的平均寿命  $\langle t \rangle$  可以通过相关的直接方式来计算。这里不再赘述, 平均寿命可由  $\langle t \rangle = \tau_n$  (或  $\tau_p$ ) 计算。在物理上,  $\tau_n$  和  $\tau_p$  被认为是多数载流子 (多子) 内过剩少子存活的平均时间。为了便于识别,  $\tau_n$  和  $\tau_p$  称为少子寿命。

和迁移率  $\mu$  和扩散系数  $D$  一样,  $\tau$  也是器件建模过程中必需考虑的重要材料参数。但与  $\mu$  和  $D$  不同的是, 已知器件中  $\tau$  的数值可用图像来显示。事实上, 辅助实验设备 (参考下一节) 可以用来决定已知半导体样品中的少子寿命。对变化急剧的参数  $\tau_n$  和  $\tau_p$ , 可由示波器来监测这些信息。参考关系式 (3.33) 式, 可知载流子寿命常常依赖于难以控制的 R-G 中心的浓度 ( $N_T$ ), 而不依赖于实际可控的掺杂参数 ( $N_A$  和  $N_D$ )。并且已知样品内, 占有支配地位 R-G 中心浓度的物理性质是在器件制造过程中改变的。器件制造工艺过程称为掺杂法, 这种方法可使 R-G 中心浓度减少至很低的水平, 并可导致硅中的  $\tau_n$  ( $\tau_p$ ) 大约在  $1 \text{ ms}$ 。另一方面, 将金元素掺入硅中可控制 R-G 中心浓度并可使得  $\tau_n$  ( $\tau_p$ ) 大约在  $1 \text{ ns}$  左右。所有硅器件的少数载流子寿命大约趋向于上述两个极限值的中间值。

# 状态方程

## 连续性方程

所有类型载流子的输运，不管它是漂移 扩散 间接或直接热复合，间接或直接产生，还是其他类型载流子的输运，它们都会使载流子浓度随时间而变化。因而，可以把所有类型载流子输运的总效果看成是相同的，即单位时间内载流子浓度的总变化 ( $\partial n/\partial t$  或  $\partial p/\partial t$ ) 等于与电子的  $\partial n/\partial t$  相关的各个过程或与空穴的  $\partial p/\partial t$  相关的各个过程的总和，即

$$\frac{\partial n}{\partial t} = \left. \frac{\partial n}{\partial t} \right|_{drift} + \left. \frac{\partial n}{\partial t} \right|_{diff} + \left. \frac{\partial n}{\partial t} \right|_{R-G}^{thermal} + \left. \frac{\partial n}{\partial t} \right|_{other}^{(light,etc)}$$

$$\frac{\partial p}{\partial t} = \left. \frac{\partial p}{\partial t} \right|_{drift} + \left. \frac{\partial p}{\partial t} \right|_{diff} + \left. \frac{\partial p}{\partial t} \right|_{R-G}^{thermal} + \left. \frac{\partial p}{\partial t} \right|_{other}^{(light,etc)}$$

## 连续性方程的简化与推导

在上述四个浓度变化中，扩散和漂移带来的浓度变化可以由电流密度改写：

纯数学推导：

$$\begin{aligned}\left. \frac{\partial n}{\partial t} \right|_{drift} + \left. \frac{\partial n}{\partial t} \right|_{diff} &= \frac{1}{q} \left[ \left. \frac{\partial Q}{\partial t} \right|_{drift} + \left. \frac{\partial Q}{\partial t} \right|_{diff} \right] \text{ 这里的Q是电荷体密度} \\ &= \frac{1}{qV} \cdot \left[ \left. \frac{\partial Q}{\partial t} \right|_{drift} + \left. \frac{\partial Q}{\partial t} \right|_{diff} \right] \text{ 这里的Q是电荷量} \\ &= \frac{1}{qV} \cdot \vec{J}_n \cdot \vec{s} \quad \text{这里的电流密度} \vec{J}_n \text{指向外, 对应电流向内} \\ &= \frac{1}{q} \nabla \cdot \vec{J}_n\end{aligned}$$

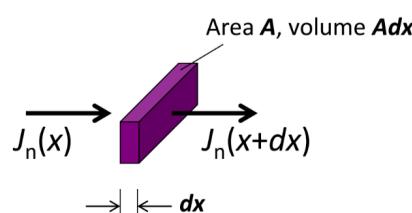
故：  $\left. \frac{\partial n}{\partial t} \right|_{drift} + \left. \frac{\partial n}{\partial t} \right|_{diff} = \frac{1}{q} \cdot \nabla \cdot \vec{J}_n$  内部电子浓度增加对应电流流向外部

同理有：  $\left. \frac{\partial p}{\partial t} \right|_{drift} + \left. \frac{\partial p}{\partial t} \right|_{diff} = -\frac{1}{q} \cdot \nabla \cdot \vec{J}_p$  内部空穴浓度增加对应电流流向内部

一维形式：  
 $\left. \frac{\partial n}{\partial t} \right|_{drift} + \left. \frac{\partial n}{\partial t} \right|_{diff} = \frac{1}{q} \cdot \frac{\partial J_n(x)}{\partial x}$   
 $\left. \frac{\partial p}{\partial t} \right|_{drift} + \left. \frac{\partial p}{\partial t} \right|_{diff} = -\frac{1}{q} \cdot \frac{\partial J_p(x)}{\partial x}$

## 物理推导：

考虑进入/离开无穷小体积的载流子通量：  
Consider the carrier-flux into/out-of an infinitesimal volume:



内部总的电子数量变化为：

$$A dx \frac{\partial n}{\partial x} = -\frac{1}{q} J_n(x) A + \frac{1}{q} J_n(x+dx) A - \frac{\Delta n}{\tau_n} A dx + G_L A dx$$

体内湮灭的  
电子数量      体内产生的  
电子数量

$$J_n(x+dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x} dx \longrightarrow \text{微分的定义}$$

$$\rightarrow A dx \frac{\partial n}{\partial x} = \frac{1}{q} \cdot \frac{\partial J_n(x)}{\partial x} dx A - \frac{\Delta n}{\tau_n} A dx + G_L A dx$$

$$\longrightarrow \frac{\partial n}{\partial x} = \frac{1}{q} \frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L$$

故连续性方程简化形式：

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p(x)}{\partial x} - \frac{\Delta p}{\tau_n} + G_L$$

注：连续性方程适用于所有载流子，实际应用侧重于少子，因多子行为稳定，少子变化显著影响器件性能。

## 少子的扩散方程

在输运理论中起主要作用的少子扩散方程由连续性方程基于以下假设推出：

- (1) 分析所使用的系统是一维的，即所有的变量一定只是一维坐标的函数（ $x$ 坐标）。
- (2) 分析仅限于少子。
- (3) 在半导体或半导体的某一区域内，分析是在  $E \approx 0$  的条件下进行。
- (4) 平衡少子浓度不是位置的函数，即  $n_0 \neq n_0(x)$ ,  $p_0 \neq p_0(x)$ 。均匀掺杂
- (5) 小注入条件成立。
- (6) 间接热复合 - 产生是主要的热 R-G 机制。
- (7) 在系统内没有“其他过程”发生，这可能排除了光产生。

$$\text{假设(1): } \frac{1}{q} \nabla \cdot J_N(x) \longrightarrow \frac{1}{q} \cdot \frac{\partial J_N(x)}{\partial x}$$

$$\text{假设(3): } J_N = q n_m e + q D_n \frac{dn}{dx} \approx q D_n \frac{dn}{dx}$$

这里还需要用假设(2)，否则即便电场很小，第一项值也很大

$$\longrightarrow \frac{1}{q} \cdot \frac{\partial J_N(x)}{\partial x} = D_n \frac{\partial^2 n}{\partial x^2}$$

$$\text{假设(4): } n = n_0 + \Delta n \longrightarrow \frac{\partial n}{\partial x} = \frac{\partial \Delta n}{\partial x} \longrightarrow \frac{\partial^2 n}{\partial x^2} = \frac{\partial^2 \Delta n}{\partial x^2}$$

$$\longrightarrow D_n \frac{\partial^2 n}{\partial x^2} = D_n \frac{\partial^2 \Delta n}{\partial x^2} \quad (\text{同理: } \frac{1}{q} \cdot \frac{\partial J_p(x)}{\partial x} = -D_p \frac{\partial^2 \Delta p}{\partial x^2})$$

$$\text{假设(5)(6): } \left. \frac{\partial n}{\partial t} \right|_{\substack{\text{thermal} \\ \text{R-G}}} = -\frac{\Delta n}{\tau_n}$$

$$\text{假设(7): } \left. \frac{\partial n}{\partial t} \right|_{\substack{\text{other} \\ \text{process}}} = G_L$$

综上，我们得到了少子扩散方程：

$$\frac{\partial \Delta n_p}{\partial t} = D_p \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_p} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_n \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_n} + G_L$$

注：少子扩散方程只能应用于半导体中的少子，即N型半导体中的空穴，P型半导体中的电子

### 特殊情况——进一步简化少子扩散方程

稳态 ➤ Steady state:	$\frac{\partial \Delta n_p}{\partial t} = 0$	$\frac{\partial \Delta p_n}{\partial t} = 0$
无扩散电流 ➤ No diffusion current:	$D_n \frac{\partial^2 \Delta n_p}{\partial x^2} = 0$	$D_p \frac{\partial^2 \Delta p_n}{\partial x^2} = 0$
无热R-G ➤ No R-G:	$\frac{\Delta n_p}{\tau_n} = 0$	$\frac{\Delta p_n}{\tau_p} = 0$
无光 ➤ No light:		$G_L = 0$

### 案例

- Consider an n-type Si sample at room temperature with  $N_D = 10^{15} \text{ cm}^{-3}$  and  $\tau_p = 10^{-6} \text{ s}$ 
  - at  $t = 0$ , the sample is uniformly illuminated. The photo-generated electrons and holes per unit volume and per unit time is  $10^{17} \text{ cm}^{-3} \cdot \text{s}^{-1}$ . Calculate  $\Delta p_n(t)$  at  $t > 0$ .

少子(空穴)扩散方程:  $\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L$

$$\tau_p = 10^{-6} \text{ s} \quad G_L = 10^{17} \text{ cm}^{-3} \cdot \text{s}^{-1}$$

热平衡时:  $n_0 = N_D = 10^{15} \text{ cm}^{-3}$   $p_0 = \frac{n_i^2}{n_0} = 10^{-5} \text{ cm}^{-3}$

均匀掺杂，均匀光照，则载流子浓度与空间位置无关，即:  $D_p \frac{\partial^2 \Delta p}{\partial x^2} = 0$

少子扩散方程化为:  $\frac{\partial \Delta p_n}{\partial t} = - \frac{\Delta p_n}{\tau_p} + G_L \longrightarrow \frac{d \Delta p_n}{dt} + \frac{1}{\tau_p} \Delta p_n = G_L$

解得:  $\Delta p_n = G_L \tau_p + A e^{-\frac{t}{\tau_p}}$

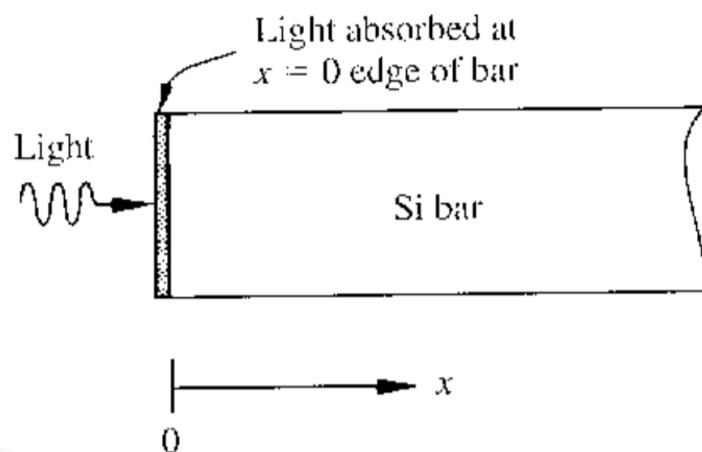
边界条件:  $\Delta p_n = 0$

解:  $\Delta p_n = G_L \tau_p (1 - e^{-\frac{t}{\tau_p}})$

➤ Consider an n-type Si sample illuminated at one end:

- constant minority-carrier injection at  $x = 0$   $\Delta p_n(0) = \Delta p_{n0}$
- no light absorption for  $x > 0$

Calculate the steady state  $\Delta p_n(x)$



少子(空穴)扩散方程:  $\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L$

稳态时, 浓度不再随时间变化:  $\frac{\partial \Delta p_n}{\partial t} = 0$

$x > 0$ , 无光:  $G_L = 0$

少子扩散方程简化为:  $D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{1}{\tau_p} \Delta p = 0$

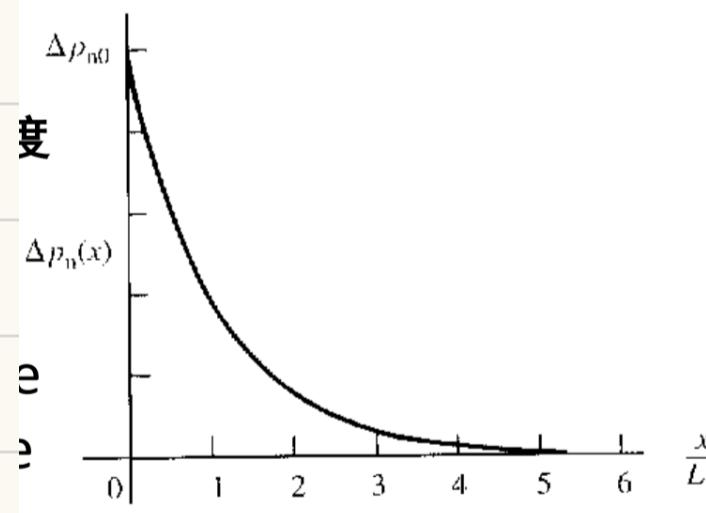
解:  $\Delta p = A e^{-x/L_p} + B e^{x/L_p}$   $L_p = \sqrt{D_p \tau_p}$  扩散长度

边界条件:  $\Delta p_n(0) = \Delta p_{n0}$   $\Delta p_n(\infty) = 0$

解得:  $\Delta p_n = \Delta p_{n0} e^{-\frac{x}{L_p}}$

Physically,  $L_p$  and  $L_n$  represent the average distance that minority carriers can diffuse into a sea of majority carriers before being annihilated.

从物理上讲,  $L_p$  和  $L_n$  表示少数载流子在被湮灭之前可以扩散到多数载流子海洋中的平均距离



## 准费米能级

- Whenever  $\Delta n = \Delta p \neq 0$ ,  $np \neq n_i^2$ . However, we would like to preserve and use the relations:

$$n = n_i e^{(E_F - E_i)/kT} \quad p = n_i e^{(E_i - E_F)/kT}$$

- These equations imply  $np = n_i^2$ , however. The solution is to introduce two **quasi-Fermi levels**  $F_N$  and  $F_P$  such that

$$n = n_i e^{(F_N - E_i)/kT} \quad p = n_i e^{(E_i - F_P)/kT}$$

FN, FP: 准费米能级

## 案例

Consider a Si sample with  $N_D = 10^{17} \text{ cm}^{-3}$  and  $\Delta n = \Delta p = 10^{14} \text{ cm}^{-3}$ .

**What are  $p$  and  $n$  and the  $np$  product?**

$$n_0 = N_D = 10^{17} \text{ cm}^{-3} \quad p_0 = n_i^2 / n_0 = 10^3 \text{ cm}^{-3}$$

$$n = n_0 + \Delta n \approx 10^{17} \text{ cm}^{-3} \quad p = p_0 + \Delta p \approx 10^{14} \text{ cm}^{-3}$$

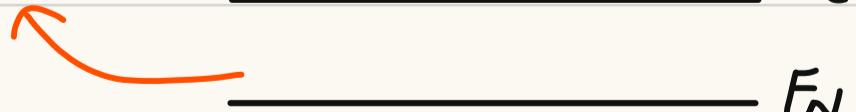
$$np = 10^{17} \text{ cm}^{-3} \cdot 10^{14} \text{ cm}^{-3} = 10^{31} \text{ cm}^{-6}$$

**Find  $F_N$  and  $F_P$ :**

$$F_N \equiv E_i + kT \ln\left(\frac{n}{n_i}\right) = E_i + 0.42 \text{ eV}$$

$$F_P \equiv E_i - kT \ln\left(\frac{p}{n_i}\right) = E_i - 0.24 \text{ eV}$$

对于多数载流子，准费米能级非常接近EF!



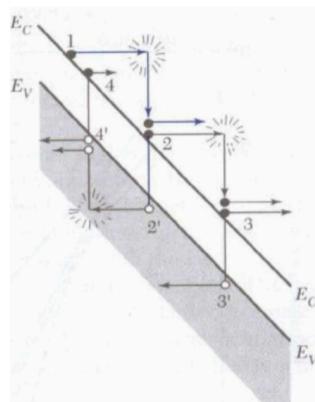
对于少数载流子，FP的相对Ei的位置几乎由Δp决定， $\Delta p > n_i$ 时，FP甚至可以向下偏离Ei、FF很多



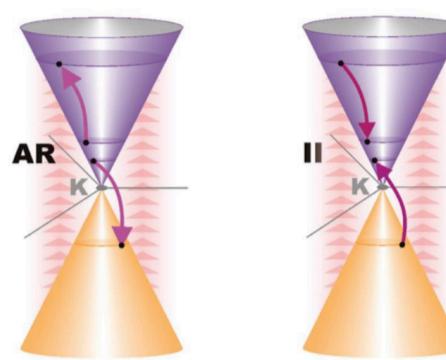
# 其他运输机制

## 碰撞离化与俄歇过程

### Impact Ionization (碰撞离化) and Auger process (俄歇过程)



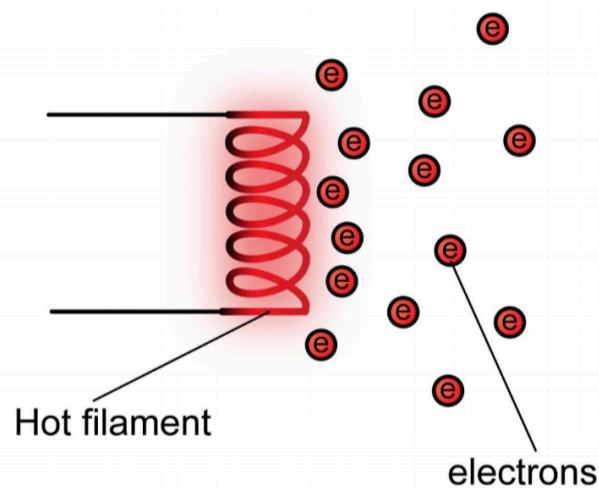
Impact ionization with high electric field and the avalanche effect



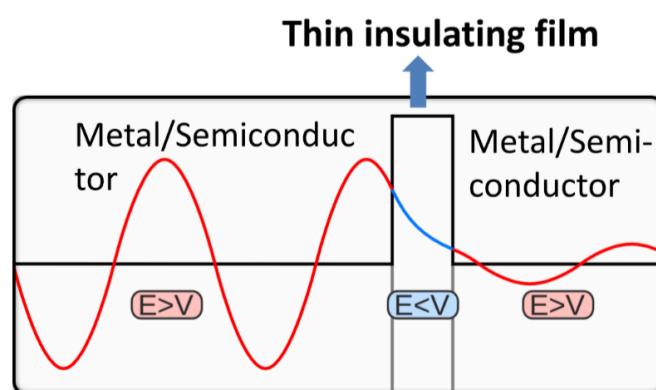
Auger process and Impact Ionization in material with strong carrier-carrier interaction

## 热电子发射

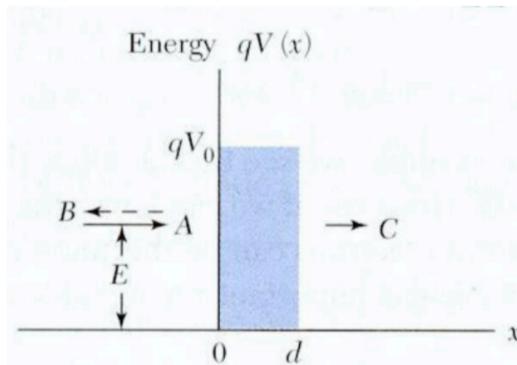
**Thermionic emission** is the thermally induced flow of charge carriers from a surface or over a potential-energy barrier. This occurs because the thermal energy given to the carrier overcomes the work function of the material.



## 量子隧穿



Quantum tunneling is the quantum mechanical phenomenon where a particle tunnels through a barrier that it classically cannot surmount.

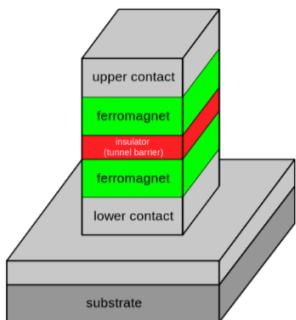


$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x) + V(x)\Psi(x) = E\Psi(x)$$

$$\left(\frac{C}{A}\right)^2 = \left[1 + \frac{(qV_0 \sinh \beta d)^2}{4E(qV_0 - E)}\right]^{-1}$$

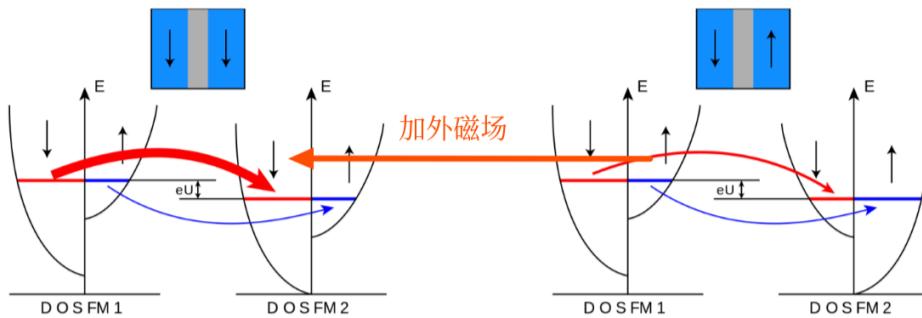
$$\text{where } \beta = [2m(qV_0 - E)]^{1/2}/\hbar$$

## 磁隧道结



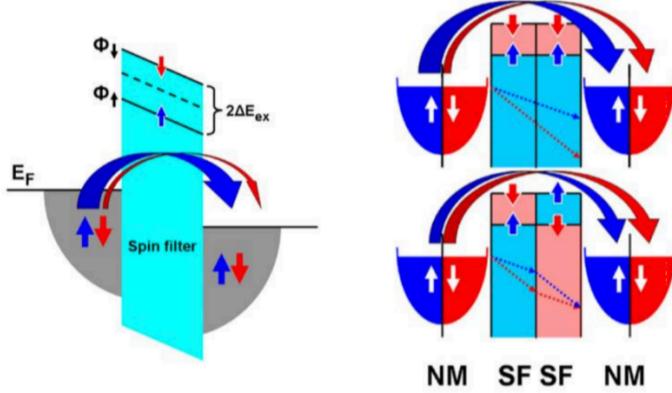
➤ **Magnetic tunnel junction (MTJ)**: a component consisting of two ferromagnets separated by a thin insulator (typically a few nm). Electrons can tunnel from one ferromagnet into the other.

➤ MTJ is the basis of **MRAM** – Magnetoresistive random-access memory.

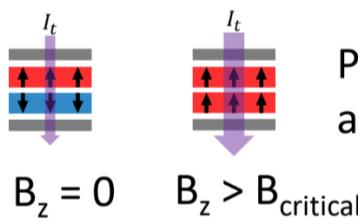


Parallel aligned magnetization assists the electron tunneling.

## 自旋过滤磁隧道结



➤ **Spin-Filter Magnetic tunnel junction (sf-MTJ)**: a component consisting of two non-magnetic metal contacts separated by multiple thin ferromagnetic insulating films (typically a few nm).



Parallel aligned magnetization assists the electron tunneling.

## 弹道输运

在介观物理学中，**弹道传导（弹道输运）**是指电荷载流子在介质中的运输，散射引起的电阻率可以忽略不计。

➤ In mesoscopic physics, **ballistic conduction (ballistic transport)** is the transport of charge carriers in a medium, having negligible electrical resistivity caused by scattering.

当电子的平均自由程（远）长于电子穿过的介质的尺寸时，就会观察到弹道输运。

➤ Ballistic transport is observed when the mean free path of the electron is (much) longer than the dimension of the medium through which the electron travels.

弹道传输通常在准1D结构中观察到，如碳纳米管或硅纳米线，因为这些材料中存在极端的尺寸量化效应

➤ Ballistic transport is typically observed in quasi-1D structures, such as carbon nanotubes or silicon nanowires, because of the extreme size quantization effects in these materials.

➤ Ballistic transport is different from the superconductivity.

超导