Exemple 1
Au pays d'Oz

1. Pour l'exemple « Le temps d'Oz », exprimer les probabilités d'être dans les différents états pour $t = 1$ en fonction des mêmes probabilités pour $t = 0$.
2. Traduire ces relations sous forme matricielle.

$\{P, S, N\}$

R1 : $P(A|B) = \frac{P(A\cap B)}{P(B)} \Leftrightarrow P(A\cap B) = P(B)P(A|B)$, $P(A|B) = P_B(A)$

1°) * $P(X_1 = P) = ?$

Les évènements $[X_0 = P]$, $[X_0 = S]$, $[X_0 = N]$ forment un s.c.e

Donc $[X_1 = P] = [X_1 = P]\cap[X_0 = P] \uplus [X_1 = P]\cap[X_0 = S] \uplus [X_1 = P]\cap[X_0 = N]$

D'où

$$\begin{aligned}
P(X_1 = P) &= P(X_1 = P \cap X_0 = P) + P(X_1 = P \cap X_0 = S) + P(X_1 = P \cap X_0 = N)\\
&= P(X_0 = P)\times P(X_1 = P \mid X_0 = P) \longrightarrow 0{,}6 \times P(X_0 = P)\\
&+ P(X_0 = S)\times P(X_1 = P \mid X_0 = S) \longrightarrow 0{,}2 \times P(X_0 = S)\\
&+ P(X_0 = N)\times P(X_1 = P \mid X_0 = N) \longrightarrow 0{,}4 \times P(X_0 = N)
\end{aligned}$$

$$\begin{aligned}
* \ P(X_1 = S) &= P(X_0 = P)\times P(X_1 = S \mid X_0 = P) \longrightarrow 0{,}1 \ P(X_0 = P)\\
&+ P(X_0 = S)\times P(X_1 = S \mid X_0 = S) \longrightarrow 0{,}6 \ P(X_0 = S)\\
&+ P(X_0 = N)\times P(X_1 = S \mid X_0 = N) \longrightarrow 0 \cdot P(X_0 = N)
\end{aligned}$$

$$\begin{aligned}
* \ P(X_1 = N) &= P(X_0 = P)\times P(X_1 = N \mid X_0 = P) \longrightarrow 0{,}3 \ P(X_0 = P)\\
&+ P(X_0 = S)\times P(X_1 = N \mid X_0 = S) \longrightarrow 0{,}2 \ P(X_0 = S)\\
&+ P(X_0 = N)\times P(X_1 = N \mid X_0 = N) \longrightarrow 0{,}6 \ P(X_0 = N)
\end{aligned}$$

2°)

$$\underbrace{\begin{pmatrix} P(X_1 = P) \\ P(X_1 = S) \\ P(X_1 = N) \end{pmatrix}}_{\mu_1} = \underbrace{\begin{array}{ccc} P & S & N \\ \hline 0{,}6 & 0{,}2 & 0{,}4 \\ 0{,}1 & 0{,}6 & 0 \\ 0{,}3 & 0{,}2 & 0{,}6 \end{array}}_{= T} \underbrace{\begin{pmatrix} P(X_0 = P) \\ P(X_0 = S) \\ P(X_0 = N) \end{pmatrix}}_{\mu_0}$$

μ_0 distribution initiale