MATH3121 Midterm Review

Types of problems

(1) Determine a given subset of a group is a subgroup.

Example. Is $\{0,1,-1\}$ is a subgroup in $\mathbb{R}$.

Answer 1. No, as $1+1=2\notin\{0,1,-1\}$

(2) Computer the order of a given elements in a group, which include $\mathbb{C}^*,\mathbb{Z}_n,\mathrm{GL}(3,\mathbb{R})$

(3) Computation of S_n

- Decompose σ into a product of disjoint cycle
- Determine if σ is even or odd
- Compute the order of σ and σ^{2023}

(4) Determine if a map $\phi: G\to G'$ is a homomorphism (HW #4)

Include True or false

Proof questions exists. Probably 2.

Symbols used in exams

Additive group: $\mathbb{Z},\mathbb{Q},\mathbb{R},\mathbb{C}$

Multiplicative group: $\mathbb{Q}^*,\mathbb{R}^*,\mathbb{C}^*$

Finite group: $\mathbb{Z}_n, U_n, S_n, A_n$. Note that S_n, A_n are not Abelian.

Commonly used group: $\mathrm{GL}(n,\mathbb{R}), n\in\mathbb{Z}_{>0}$

Sample T/F question

1. If G is a finite group, $H\subset G$ is a subgroup, then $|H|$ is a divisor $|G|$. (T)
2. If G is a finite group, if n is a divisor of $|G|$, then there is a subgroup H such that $|H|=n$. (F)

(Converse of Lagrange theorem might not be true)

3. If G is a group, $a^{10}=e$, then order of $A=10$ (F)
4. If $a^{10}=e$, then the order of a is in $\{1,2,5,10\}$ (T)
5. If G is Abelian, then $\phi: G\to G$ given by $\phi(a)=a^2$ is a homomorphism.

Definition. *A map $\phi: G\to G'$ is homomorphism (of groups) if $\phi(a*b)=\phi(a)\star\phi(b)$*

Where $*$ is the operation on G, while $\star$ is the operation on G'.

Note that:

$$\phi(ab)=(ab)^2=(a)^2(b)^2=\phi(a)\phi(b)$$

By the definition of Abelian group, as a result the answer is T.

(6) If G is a group, H is a nonempty subset, if H is closed, then H is a subgroup. (F)

Let $\{2,4,\ldots,\}\in\mathbb{R}$. The identity element doesn't even exist. So must be false.

(6) If G is a group, H is a **FINITE** nonempty subset, if H is closed, then H is a subgroup. (T)

[END OF MATH3121 REVIEW]