Metody numeryczne

Komentarz 3. zadania laboratoryjnego

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1. Wybór pierwiastka równania kwadratowego x_{n+3}

2. Wybór wzoru obliczeniowego wartości x_{n+3}

3. Obsługa liczb zespolonych

Nie zaimplementowano.

4. Dowody równoważności

W poniższych przekształceniach pamiętamy z treści zadania, że zachodzi:

$$C = f[x_{n+2}, x_{n+1}] + f[x_{n+2}, x_n] - f[x_{n+1}, x_n]$$
$$\Delta = C^2 - 4f[x_{n+2}]f[x_{n+2}, x_{n+1}, x_n]$$

- (1) ⇔ (2)

  Za pomocą odpowiednich przekształceń pokażemy, że (1) = (2). Zapiszmy tę równość, od razu redukując składnik $f[x_{n+2}]$:

$$f[x_{n+2}, x_{n+1}] \cdot (x - x_{n+2}) + f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2})(x - x_{n+1}) =$$
$$= C(x - x_{n+2}) + f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2})^2$$

$\Leftrightarrow$

$$f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2})(x - x_{n+1}) - f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2})^2 =$$
$$= C(x - x_{n+2}) - f[x_{n+2}, x_{n+1}] \cdot (x - x_{n+2})$$

$\Leftrightarrow$

$$f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2}) \cdot (x - x_{n+1} - x + x_{n+2}) =$$
$$= (x - x_{n+2}) \cdot (C - f[x_{n+2}, x_{n+1}])$$

$\Leftrightarrow$

$$f[x_{n+2}, x_{n+1}, x_n] \cdot (x - x_{n+2}) \cdot (x_{n+2} - x_{n+1}) =$$
$$= (x - x_{n+2}) \cdot (f[x_{n+2}, x_{n+1}] + f[x_{n+2}, x_n] - f[x_{n+1}, x_n] - f[x_{n+2}, x_{n+1}])$$

$\Leftrightarrow$

$$(x - x_{n+2}) \cdot \frac{f[x_{n+1}, x_n] - f[x_{n+2}, x_{n+1}]}{x_n - x_{n+2}} \cdot (x_{n+2} - x_{n+1}) =$$
$$= (x - x_{n+2}) \cdot (f[x_{n+2}, x_n] - f[x_{n+1}, x_n])$$

$\Leftrightarrow$

$$(x - x_{n+2}) \cdot \frac{\frac{f[x_n] - f[x_{n+1}]}{x_n - x_{n+1}} - \frac{f[x_{n+1}] - f[x_{n+2}]}{x_{n+1} - x_{n+2}}}{x_n - x_{n+2}} \cdot (x_{n+2} - x_{n+1}) =$$
$$= (x - x_{n+2}) \cdot \left(\frac{f[x_n] - f[x_{n+2}]}{x_n - x_{n+2}} - \frac{f[x_n] - f[x_{n+1}]}{x_n - x_{n+1}} \right)$$

$\Leftrightarrow$

$$(x - x_{n+2}) \cdot \frac{\frac{(f[x_n]-f[x_{n+1}])\cdot(x_{n+1}-x_{n+2})-(f[x_{n+1}]-f[x_{n+2}])\cdot(x_n-x_{n+1})}{(x_n-x_{n+1})\cdot(x_{n+1}-x_{n+2})}}{x_n - x_{n+2}} \cdot (x_{n+2} - x_{n+1}) =$$

$$= (x - x_{n+2}) \cdot \frac{(f[x_n] - f[x_{n+2}]) \cdot (x_n - x_{n+1}) - (f[x_n] - f[x_{n+1}]) \cdot (x_n - x_{n+2})}{(x_n - x_{n+2}) \cdot (x_n - x_{n+1})}$$

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{(f[x_n]-f[x_{n+1}])\cdot(x_{n+1}-x_{n+2})-(f[x_{n+1}]-f[x_{n+2}])\cdot(x_n-x_{n+1})}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})\cdot(x_{n+1}-x_{n+2})}\cdot(x_{n+2}-x_{n+1}) =$$

$$= (x-x_{n+2})\cdot\frac{f[x_n]\cdot x_n-f[x_n]\cdot x_{n+1}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}-f[x_n]\cdot x_n+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

redukujemy w pierwszym równaniu $(x_{n+1} - x_{n+2})$ z $(x_{n+2} - x_{n+1})$, pamiętając o pomnożeniu licznika przez (-1) :

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{(f[x_{n+1}]-f[x_{n+2}])\cdot(x_n-x_{n+1})-(f[x_n]-f[x_{n+1}])\cdot(x_{n+1}-x_{n+2})}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})} =$$

$$= (x-x_{n+2})\cdot\frac{f[x_n]\cdot x_n-f[x_n]\cdot x_n-f[x_n]\cdot x_{n+1}+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+1}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}-f[x_n]\cdot x_{n+1}+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_{n+1}-f[x_{n+1}]\cdot x_{n+2}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})} =$$

$$= (x-x_{n+2})\cdot\frac{-f[x_n]\cdot x_{n+1}+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{-f[x_n]\cdot x_{n+1}+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+1}+f[x_{n+1}]\cdot x_{n+1}-f[x_{n+1}]\cdot x_{n+2}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})} =$$

$$= (x-x_{n+2})\cdot\frac{f[x_n]\cdot x_{n+2}-f[x_n]\cdot x_{n+1}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}+f[x_{n+2}]\cdot x_{n+1}-f[x_{n+2}]\cdot x_n}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{-f[x_n]\cdot x_{n+1}+f[x_n]\cdot x_{n+2}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}-f[x_{n+2}]\cdot x_n+f[x_{n+2}]\cdot x_{n+1}}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})} =$$

$$= (x-x_{n+2})\cdot\frac{f[x_n]\cdot(x_{n+2}-x_{n+1})+f[x_{n+1}]\cdot(x_n-x_{n+2})+f[x_{n+2}]\cdot(x_{n+1}-x_n)}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

$\Leftrightarrow$

$$(x-x_{n+2})\cdot\frac{f[x_n]\cdot x_{n+2}-f[x_n]\cdot x_{n+1}+f[x_{n+1}]\cdot x_n-f[x_{n+1}]\cdot x_{n+2}+f[x_{n+2}]\cdot x_{n+1}-f[x_{n+2}]\cdot x_n}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})} =$$

$$= (x-x_{n+2})\cdot\frac{f[x_n]\cdot(x_{n+2}-x_{n+1})+f[x_{n+1}]\cdot(x_n-x_{n+2})+f[x_{n+2}]\cdot(x_{n+1}-x_n)}{(x_n-x_{n+2})\cdot(x_n-x_{n+1})}$$

$\Leftrightarrow$

$$(x - x_{n+2}) \cdot \frac{f[x_n] \cdot (x_{n+2} - x_{n+1}) + f[x_{n+1}] \cdot (x_n - x_{n+2}) + f[x_{n+2}] \cdot (x_{n+1} - x_n)}{(x_n - x_{n+2}) \cdot (x_n - x_{n+1})} =$$

$$= (x - x_{n+2}) \cdot \frac{f[x_n] \cdot (x_{n+2} - x_{n+1}) + f[x_{n+1}] \cdot (x_n - x_{n+2}) + f[x_{n+2}] \cdot (x_{n+1} - x_n)}{(x_n - x_{n+2}) \cdot (x_n - x_{n+1})}$$

Tym samym pokazaliśmy, że (1) $\Leftrightarrow$ (2). $\square$

- (3) $\Leftrightarrow$ (4)

  Wymnażamy licznik i mianownik ułamka w równości (3) przez $(C \mp \sqrt{\Delta})$ (uwaga - zmiana znaków) i korzystamy z wzoru skróconego mnożenia na różnicę kwadratów:

$$\begin{aligned}
x_{n+3} &= x_{n+2} - \frac{C \pm \sqrt{\Delta}}{2f[x_{n+2}, x_{n+1}, x_n]} = \\
&= x_{n+2} - \frac{C \pm \sqrt{\Delta}}{2f[x_{n+2}, x_{n+1}, x_n]} \cdot \frac{C \mp \sqrt{\Delta}}{C \mp \sqrt{\Delta}} = \\
&= x_{n+2} - \frac{C^2 - \Delta}{2f[x_{n+2}, x_{n+1}, x_n] \cdot (C \mp \sqrt{\Delta})} = \\
&= x_{n+2} - \frac{C^2 - (C^2 - 4f[x_{n+2}]f[x_{n+2}, x_{n+1}, x_n])}{2f[x_{n+2}, x_{n+1}, x_n] \cdot (C \mp \sqrt{\Delta})} = \\
&= x_{n+2} - \frac{4f[x_{n+2}]f[x_{n+2}, x_{n+1}, x_n]}{2f[x_{n+2}, x_{n+1}, x_n] \cdot (C \mp \sqrt{\Delta})} = \\
&= x_{n+2} - \frac{2f[x_{n+2}]}{C \mp \sqrt{\Delta}} = \\
&= x_{n+3}
\end{aligned}$$

  Tym samym pokazaliśmy, że (3) $\Leftrightarrow$ (4). $\square$