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§1 Lagrangian Mechanics

Let us start from defining the **action**:

$$S\left[\boldsymbol{q}\left(t\right)\right]=\int_0^T L\left(\boldsymbol{q},\dot{\boldsymbol{q}},t\right)\,dt,$$

the **Eular-Lagrange equation** is derived from $\delta S=0$, with an additional restriction $\delta\boldsymbol{q}\left(0\right)=\delta\boldsymbol{q}\left(T\right)=0$:

$$\begin{aligned}\delta\int_0^T L\,dt&=\int_0^T\left(\nabla_{\boldsymbol{q}}L\cdot\delta\boldsymbol{q}+\nabla_{\dot{\boldsymbol{q}}}L\cdot\delta\dot{\boldsymbol{q}}\right)\,dt\\&=\int_0^T\left(\nabla_{\boldsymbol{q}}L-\frac{d}{dt}\nabla_{\dot{\boldsymbol{q}}}L\right)\cdot\delta\boldsymbol{q}\,dt=0.\end{aligned}$$

Since the choice of $\delta\boldsymbol{q}$ is arbitrary, we obtain the Eular-Lagrange equation (we place the highest order derivative at the beginning):

$$\frac{d}{dt}\boldsymbol{\nabla}_{\dot{\boldsymbol{q}}}L-\boldsymbol{\nabla}_{\boldsymbol{q}}L=\boldsymbol{0}.\tag{1}$$

But what is the **Lagrangian** function L?

†a Free Particle

First, there are some symmetry about the Lagrangian:

1) Space translation: $L\left(\boldsymbol{q},\dot{\boldsymbol{q}},t\right)=L\left(\dot{\boldsymbol{q}}\right)$.
2) Rotation: $L\left(\dot{\boldsymbol{q}}\right)=L\left(\left|\dot{\boldsymbol{q}}\right|^2\right)$.

In this sense:

$$2\frac{d}{dt}L'\left(\left|\dot{\boldsymbol{q}}\right|^2\right)\dot{\boldsymbol{q}}.$$

Compare to the Newtonian Mechanics:

$$m\ddot{\boldsymbol{q}}=0,$$

we make $L'\left(\left|\dot{\boldsymbol{q}}\right|^2\right)$ as a constant $m/2$:

$$L_{\text{free}}=T=\frac{1}{2}m\left|\dot{\boldsymbol{q}}\right|^2.\tag{2}$$

†b Conservative Force

In the case of conservative force:

$$m\ddot{\boldsymbol{r}}=\boldsymbol{F}=-\boldsymbol{\nabla}V\left(\boldsymbol{q}\right).$$

Then we have

$$\frac{d}{dt}\boldsymbol{\nabla}_{\dot{\boldsymbol{q}}}T-\boldsymbol{\nabla}_{\boldsymbol{q}}T+\boldsymbol{\nabla}V\left(\boldsymbol{q}\right)=\frac{d}{dt}\boldsymbol{\nabla}_{\dot{\boldsymbol{q}}}\left(T-V\right)-\boldsymbol{\nabla}_{\boldsymbol{q}}\left(T-V\right)=0.$$

So we define

$$L_{\text{conserve}}=T-V=\frac{1}{2}m\left|\dot{\boldsymbol{q}}\right|^2-V.\tag{3}$$

†c Constraint Force

Under the following conditions, we can use the **generalized coordinate**:

1) There is some holonomic constraint $f\left(\boldsymbol{q},t\right)$.
2) The constraint force satisfies $\boldsymbol{F}\cdot\delta\boldsymbol{q}=0$.

In Newtonian Mechanics:

$$m\ddot{\boldsymbol{q}}=\boldsymbol{F}_{\text{conserve}}+\boldsymbol{F}_{\text{constraint}}.$$

But in the variation of the action:

$$\begin{aligned}\delta S&=\int_0^T\left(m\ddot{\boldsymbol{q}}-\boldsymbol{F}_{\text{conserve}}-\boldsymbol{F}_{\text{constraint}}\right)\cdot\delta\boldsymbol{q}\,dt\\&=\int_0^T\left(m\ddot{\boldsymbol{q}}-\boldsymbol{F}_{\text{conserve}}\right)\cdot\delta\boldsymbol{q}\,dt=0\end{aligned}$$

Chapter 1

Introduction to Statistics

§1 Data Type

†a Object

Python is an **object-oriented** programing language. Everything is an **object** in Python:

$$\text{object} = \begin{cases} \text{identity}, \\ \text{type / class}, \\ \text{value / state}, \\ \text{methods / behaviors / operations}. \end{cases}$$

```
# print the identity, type, and the value for 4
print(id(4),type(4),4)
# type of any type is a type, the type itself is a type
print(type(type(4)))
print(type(type(type(4))))
```

```
140711759792664 <class 'int'> 4
<class 'type'>
<class 'type'>
```

- **Identity**: it guarantees that different objects have distinct identities at any given time.
- **Type**: objects of the same type support the same operations, and share the same properties.

†b Numeric

The following are numeric types:

$$\text{bool} \subset \text{int} \subset \text{float} \subset \text{complex}$$

```
# an example for the above data types
print(True,1,1.0,1+0j)
```

```
True 1 1.0 (1+0j)
```

Bibliography