Quantum computing, Assignment 3. Due 15 October

(Dated: September 15, 2020)

- **Problem 1: Quantum Fourier Transform [12 points]**
  - **(i)** [**4 points**] Suppose that there is a function $f(x)$ in the two-Qbit space spanned by $x = 0, 1, 2, 3$. Using QFT show that it is possible to determine if $f(x)$ has a period $f(x+2) = f(x)$.
  - **(ii)** [**8 points**] Write the code in Qiskit for QFT and check it for the case of $n = 3$ Qubits.

- **Problem 2: Quantum phase estimation [10 points]** Suppose that we have gate $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$ with unknown phase θ. Using the algorithm of quantum phase estimation, write the code in Qiskit which determines θ.

- **Problem 3: Quantum error correction [10 points]**

  Suppose the only types of errors are the bit-flip errors but we would like to correct not only single bit-flips $\mathbf{X}_i$ but also the double bit-flips $\mathbf{X}_i\mathbf{X}_j$ with $j \neq i$.

  - **(i)** [**4 points**] Find the size of the codewords n such that the dimension of the n-Qbit state space is just large enough to accommodate mutually orthogonal two-dimensional subspaces for the uncorrupted code words and all codewords produced by single- and double- bit flip corruptions.
  - **(ii)** [**6 points**] Show for the n found in **(i)** that there is indeed a perfect n-Qbit code that corrects all single and double bit-flip errors by writing down the states that encode $|\bar{0}\rangle$ and $|\bar{1}\rangle$, and writing down a set of commuting hermitian operators whose squares are unity, that preserve both codewords and have distinct patterns of commutations and anticommutations for each of the operators that produce all single and double bit-flip errors.