Quantum computing, Assignment 1. Maximal score: 34 points. Due in 16 September 2020

(Dated: September 11, 2020)

Useful literature: 1

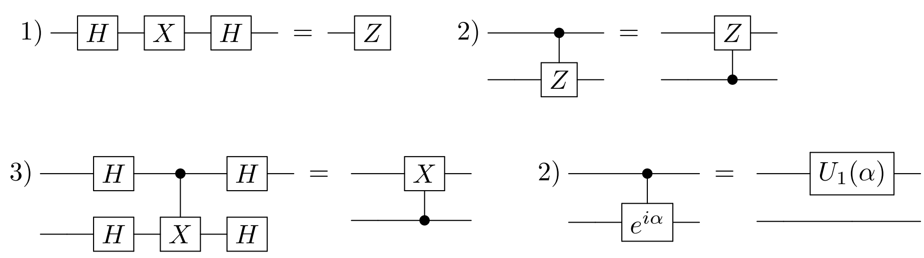
- **Problem 0 [2 points]:** Install Qiskit (see instructions in the Lecture 1 slides)
- **Problem 1 [4 points]: Extracting Qubit phase** Suppose that we have the Qubit in the state

$$|\psi\rangle = \sin\theta|0\rangle + e^{i\varphi}\cos\theta|1\rangle \tag{1}$$

Show how using the measurement in computational basis to measure the angle φ. Write the code using Qiskit which prepares the state (1) and performs this measurement. Visualise the circuit. Give link to jupyter notebook.

- **Problem 2 [8 points]: Single Qubit state preparation** Suggest a circuit which uses fixed number of standard gates: X, H, S and $R_z(\gamma)$ where $R_z(\gamma)$ is rotation around z axis by an arbitrary angle γ to prepare the state (1) modulo the total phase of the wave function. Compare the accuracy of this procedure with custom U_3 gate at IMB Quantum for the angles $\varphi = \theta = \pi/3$. Write the code using Qiskit, give link to jupyter notebook.
- **Problem 3 [4 points]: Useful identities for 1-Qubit and 2-Qubit gates**

Prove the following identities



- **Problem 4 [4 points]: Separable and entangled states**

Determine which of the following states are entangled. If the state is not entangled, show how to write that as a tensor product; if it is entangled, prove it.

  - (a) $\frac{2}{3}|00\rangle + \frac{1}{3}|01\rangle - \frac{2}{3}|11\rangle$
  - (b) $\frac{1}{2}\left(|00\rangle - i|01\rangle + i|10\rangle + |11\rangle\right)$
  - (c) $\frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)$

- **Problem 5 [8 points]: Preparation of the general 2-Qubit pure state**

Using Schmidt decomposition and its relation to the Singular value decomposition of matrices, suggest the algorithm to prepare general 2-Qubit pure state

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle \tag{2}$$

Apply this procedure to prepare the example 2-Qbit state

  - (a) $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$
  - (b) $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$
  - (c) $|\Phi\rangle = \frac{1}{\sqrt{12}}(3|00\rangle + |01\rangle + |10\rangle - |11\rangle)$

The states $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$ are called Bell states, they are used in quantum cryptography algorithms. Write code in Qiskit, give the link to the Jupyter notebook.

- **Problem 6 [4 points]: Measuring gate**

  Let us consider the general state of n Qbits $|\Psi\rangle_n$ which can be written in the form

  $$|\Psi\rangle_n = \sum_x \alpha_x |x\rangle_m |\Phi_x\rangle_{n-m}$$

  According to the Born rule if we measure $m < n$ Qbits then with probability $|\alpha_x|^2$ the result will be $|x\rangle$ and after this measurement the n Qbits will be in the sate

  $$|x\rangle_m |\Phi_x\rangle_{n-m}$$

  (a) Suppose that first m Qbits are measured and immediately after additional k Qbits are measured, so that $m + k \leq n$. Find the possible states after these two measurements and their associated probabilities by applying first m measurements and then k measurements.

  (b) Show that the result obtained in (a) coincides with that of the single measurement of $m + k$ Qbits.

  Hint: use the relation between joint $p(xy)$ and conditional $p(y|x)$ probabilities for two outcomes x and y:

  $$p(xy) = p(y|x)p(x)$$

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[1] Quantum Algorithm Implementations for Beginners, Abhijith J., Adetokunbo Adedoyin, arXiv:1804.03719