Algorithm Design and Analysis-CS403

Programming Assignment

Second Programming Assignment

Total: 25 Marks.

1. **House Robbing:** Alice is a skilled thief who plans to rob houses down a lane. Each house has a certain amount of money stashed, and the only thing keeping Alice from robbing each of them is that neighbouring houses have security systems attached, and if two adjacent houses are broken into on the same night, the security systems will immediately notify the police. Write a program with dynamic approach which return the maximum amount of money you can steal tonight without alerting the cops, given an integer array `wealth` representing the amount of money lying in each home.
   Example : If there as 5 houses in a street, and their `wealth` array is given as: $[2, 7, 9, 3, 1]$.
   Output : 12
   Explanation: Rob house 1 (give `wealth` $= 2$), rob house 3 (give `wealth` $= 9$) and rob house 5 (give `wealth` $= 1$). Total profit Alice $= 2 + 9 + 1 = 12$.
   2 *Marks.*

2. **Award problem:** Rahul teaches an algebraic courses. He wants to honour the students in his class with some awards. All the students stand in a line and each of them has a grade according to his or her participation in the class. Rahul wants to give each student at least one award. If two students are standing adjacent to each other, then the one with the higher rating must get more awards than the other. Rahul wants to buy the fewest number of awards possible. Your task is to determine the minimum number of awards Rahul must purchase in order to distribute them among his students according to the criteria listed above, given an array GRADES of size N containing each student's grades.
   Example: Assume his students' ratings are $[5, 8, 1, 5, 9, 4]$. He gives the students awards in the following minimal amounts: $[1, 2, 1, 2, 3, 1]$. He must buy a minimum of 10 awards. *2 Marks.*

3. **Minimum steps to one problem:** You can perform any one of the three steps mentioned below on a positive integer.
   (a) Take away one. $(n = n - 1)$

(b) Divide by 2, if it's divisible by two. (if $n\%2 == 0$, then $n = n/2$),

(c) If the number is divisible by three, divide by three.(if $n\%3 == 0$. then $n = n/3$),

Now, given a positive integer n, calculate the minimum number of steps that takes from n to 1. Solve only by using dynamic approach and comment in your code where you have applied it.

3 *Marks.*

4. **Cutting a gold bar:** Bob robbed a gold bar from a wealthy businessman, and in order to avoid being arrested, he decided to shell the gold bar, but not in one piece, so he cut the bar into small pieces and plan to sell them to various jewellery stores to maximise his profit. He has a gold bar of length n inches and an array of prices containing prices of all pieces of size smaller than n. Write a program using dynamic programming approach that determine the maximum value obtainable by cutting up the gold bar and selling the pieces.

   Example : If length of the rod is 10 inches and the values of different pieces are given as following,

length	1	2	3	4	5	6	7	8
price	10	15	20	25	30	50	60	70

   then the maximum obtainable value is 100 (by cutting in ten pieces of length 1). *3 Marks.*

5. **Vegetable cost Problem:** Rahul wants to inspect the quality of the vegetables in each store of a vegetable market. Suppose there are N different vendors. Every seller has a single kilogram of onion, carrot, and tomato, but of different prices. Rahul wants to buy only one vegetable from a single shop, avoiding buying the same vegetable from adjacent shops. Given the cost of each vegetable in each shop in a $N \times 3$ matrix, calculate the minimum amount of money that Rahul must spend in the inspection.
   Example: If there are three shops, and the corresponding cost matrix is given as follows:

x	onion	carrot	tomato
*shop*1	50	50	1
*shop*2	50	50	50
*shop*3	1	50	50

   Then minimum cost will be 52 (onion from *shop*3, carrot from any *shop*2, and tomato from *shop*1). Write a program with dynamic approach for N shops and any given $N \times 3$ cost matrix. *5 Marks.*

6. **Investment Problem:** Your knowledge of algorithms helps you obtain an exciting job with the Software Company, along with a $\$20,000$ signing bonus. You decide to invest this money with the goal of maximizing your return at the end of 10 years. You decide to use the ABC Investment Company to manage your investments. ABC Investments requires you to observe the following rules. It offers n different investments, numbered 1 through n. In each year j , investment i provides a return rate of r_{ij} . In other words, if you invest d dollars in investment i in year j, then at the end of year j , you have dr_{ij} dollars. The return rates are guaranteed, that is, you are given all the return rates for the next 10 years for each investment. You make investment decisions only once per year. At the end of each year, you can leave the money made in the previous year in the same investments, or you can shift money to other investments, by either shifting money between existing investments or moving money to a new investment. If you do not move your money between two consecutive years, you pay a fee of f_1 dollars, whereas if you switch your money, you pay a fee of f_2 dollars, where $f_2 > f_1$.
   Design a program that plans your optimal investment strategy, for any given amount and years. *5 Marks.*

7. Write an algorithm by modify the `KMP algorithm` (not DP algorithm!) to find the longest common substring in string S and string T. The algorithm should return the longest common substring if present otherwise return *"Not Found"*. What is the time complexity of this modified KMP algorithm?

   For example: if the string S is `report`, and the string T is `airport`, then algorithm should output the longest common substring: `port` as output. *3 Marks.*

8. Given a string S, you are allowed to convert it to a palindrome by adding 0 or more characters in front of it. Find the length of the shortest palindrome that you can create from S by applying the above transformation. *2 Marks.*