Homework 8

Due date: None.

Reading assignment. Chapter 4 and 5 of the textbook.

Finals. Sections 1.1–1.6, 2.1–2.4, 3.1–3.4, 4.1–4.3, 5.1–5.3. Make sure that you look at the problems in these sections and in the review sections of these chapters. No notes or cheat sheets are allowed in the finals. Only simple calculators are allowed. In particular calculators that can do matrix algebra are not allowed.

Problem 1. Find the eigenvalues and eigenvectors of

$$\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} a & b \\ b & a \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

Problem 2. Find the eigenvalues of the following matrices (and their rank):

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}.$$

Find the eigenvectors of non-zero eigenvalues.

Problem 3. Consider an LTI system whose input signal $x[n]$ and output signal $y[n]$ are related by the second-order difference equation

$$y[n] = x[n] + \frac{y[n-1] + y[n-2]}{2}.$$

a) Write the equation as a system of first-order difference equations of the form

$$\begin{pmatrix} y[n] \\ y[n-1] \end{pmatrix} = A\begin{pmatrix} y[n-1] \\ y[n-2] \end{pmatrix} + \begin{pmatrix} x[n] \\ 0 \end{pmatrix}.$$

b) Find the limit as $n \to \infty$ of A^n.

c) Show that you can pick a causal impulse response of the form $H[n] = A^n u[n]$, where $u[n]$ is the unit step function:

$$u[n] = \begin{cases} 1, & n \geqslant 0, \\ 0, & n < 0. \end{cases}$$

d) Find the limit as $n \to \infty$ of $y[n]$, if $x[n] = 0$, $y[0] = 1$ and $y[-1] = 0$.