

What is a cube?

A cube is a square, which is then rotated then offset by a certain amount in all other directions to combine to make a cube

What do we need?

Therefore this is then rotated by the rotation matrix

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot R_x(\theta) \cdot R_y(\theta) \cdot R_z(\theta)$ is the equation required to describe how the cube rotates around all three axes.

This expands to:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Converting to Quaternions

A quaternion is of the shape (q_0, q_1, q_2, q_3)

$$\text{When given the rotation matrix, } R = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} \\ r_{2,1} & r_{2,2} & r_{2,3} \\ r_{3,1} & r_{3,2} & r_{3,3} \end{pmatrix}$$

We can find the equivalent quaternions by this method:

Step 1: Find the magnitude of each quaternion component.

$$|q_0| = \sqrt{\frac{1 + r_{1,1} + r_{2,2} + r_{3,3}}{4}}$$

$$|q_1| = \sqrt{\frac{1 + r_{1,1} - r_{2,2} - r_{3,3}}{4}}$$

$$|q_2| = \sqrt{\frac{1 - r_{1,1} + r_{2,2} - r_{3,3}}{4}}$$

$$|q_3| = \sqrt{\frac{1 - r_{1,1} - r_{2,2} + r_{3,3}}{4}}$$

Step 2: Resolve the signs

To resolve the signs, find the largest out of q_0, q_1, q_2, q_3 and assume the sign is positive.

Then follow this table

q_0 is largest	q_1 is largest	q_2 is largest	q_3 is largest
$q_1 = \frac{r_{3,2} - r_{2,3}}{4 \cdot q_0}$	$q_0 = \frac{r_{3,2} - r_{2,3}}{4 \cdot q_0}$	$q_0 = \frac{r_{1,3} - r_{3,1}}{4 \cdot q_2}$	$q_0 = \frac{r_{2,1} - r_{1,2}}{4 \cdot q_3}$

Quaternion for R_x :

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

Quaternion for $R_x(\theta) = (q_0, q_1, q_2, q_3)$

$$|q_0| = \sqrt{\frac{1 + 1 + \cos \theta + \cos \theta}{4}}$$

$$|q_1| = \sqrt{\frac{1 + 1 - \cos \theta - \cos \theta}{4}}$$

$$|q_2| = \sqrt{\frac{1 - 1 + \cos \theta - \cos \theta}{4}}$$

$$|q_3| = \sqrt{\frac{1 - 1 - \cos \theta + \cos \theta}{4}}$$

The maximum value is 1, and $|q_0| = |q_1|$

Bibliography