What is a cube?

A cube is a square, which is then rotated then offset by a certain amount in all other directions to combine to make a cube

What do we need?

Therefore this is then rotated by the rotation matrix

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot R_x(\theta) \cdot R_y(\theta) \cdot R_z(\theta)$ is the equation required to describe how the cube rotates around all three axes.

This expands to:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Converting to Quaternions

A quaternion is of the shape (q_0, q_1, q_2, q_3)

$$\text{When given the rotation matrix,} \quad R = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} \\ r_{2,1} & r_{2,2} & r_{2,3} \\ r_{3,1} & r_{3,2} & r_{3,3} \end{pmatrix}$$

$$\text{We can find the equivalent quaternions by this method:}$$

Step 1: Find the magnitude of each quaternion component.

$$|q_0| = \sqrt{\frac{1 + r_{1,1} + r_{2,2} + r_{3,3}}{4}}$$

$$|q_1| = \sqrt{\frac{1 + r_{1,1} - r_{2,2} - r_{3,3}}{4}}$$

$$|q_2| = \sqrt{\frac{1 - r_{1,1} + r_{2,2} - r_{3,3}}{4}}$$

$$|q_3| = \sqrt{\frac{1 - r_{1,1} - r_{2,2} + r_{3,3}}{4}}$$

Step 2: Resolve the signs

To resolve the signs, find the largest out of q_0, q_1, q_2, q_3 and assume the sign is positive.
Then follow this table

q_0 is largest	q_1 is largest	q_2 is largest	q_3 is largest
$q_1 = \frac{r_{3,2} - r_{2,3}}{4 \cdot q_0}$	$q_0 = \frac{r_{3,2} - r_{2,3}}{4 \cdot q_0}$	$q_0 = \frac{r_{1,3} - r_{3,1}}{4 \cdot q_2}$	$q_0 = \frac{r_{2,1} - r_{1,2}}{4 \cdot q_3}$

Quaternion for R_x:

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Quaternion for $R_x(\theta)$ = (q_0, q_1, q_2, q_3)

$$|q_0| = \sqrt{\frac{1 + 1 + \cos\theta + \cos\theta}{4}}$$

$$|q_1| = \sqrt{\frac{1 + 1 - \cos\theta - \cos\theta}{4}}$$

$$|q_2| = \sqrt{\frac{1 - 1 + \cos\theta - \cos\theta}{4}}$$

$$|q_3| = \sqrt{\frac{1 - 1 - \cos\theta + \cos\theta}{4}}$$

The maximum value is 1, and $|q_0| = |q_1|$

$$\begin{aligned} \boldsymbol{q} = &\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right) \\ &+\left(\sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} - \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)\hat{\imath} \\ &+\left(\cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)\hat{\jmath} \\ &+\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2} - \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2}\right)\hat{k} \end{aligned}$$

$$\boldsymbol{q^{-1}} = \left(w, -a\hat{\imath}, -b\hat{\jmath}, -c\hat{k}\right)$$

$$\begin{aligned} \therefore \boldsymbol{q^{-1}} = &\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right) \\ &-\left(\sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} - \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)\hat{\imath} \\ &-\left(\cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)\hat{\jmath} \\ &-\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2} - \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2}\right)\hat{k} \end{aligned}$$

$$p = (0, x, y, z)$$

$$
\begin{aligned}
q\cdot p = 0 - \Bigg(&\left(\sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} - \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)x \\
&+\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)y \\
&+\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2} - \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2}\right)z\Bigg) \\
&+\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix}
\end{aligned}
$$

$$
+\begin{pmatrix}
\left(\cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)z - \left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2} - \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2}\right)y \\
\left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2} - \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2}\right)x - \left(\cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)z \\
\left(\sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\cos\frac{\theta_z}{2} - \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)y - \left(\cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\cos\frac{\theta_z}{2} + \sin\frac{\theta_x}{2}\cos\frac{\theta_y}{2}\sin\frac{\theta_z}{2}\right)x
\end{pmatrix}
$$

$$
qpq^{-1} =
$$

Bibliography