First Year Maths and Further Maths combined Test B9
Lower sixth differentiation including exponentials/logarithms
32 minutes

Throughout the entire test all working must be shown and solutions based entirely on graphical or numerical methods may not be acceptable.

1.

The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

2.

Fig. 9 shows the curve $y = f(x)$, where $f(x) = e^{2x} + k\,e^{-2x}$ and k is a constant greater than 1.

The curve crosses the y-axis at P and has a turning point Q.

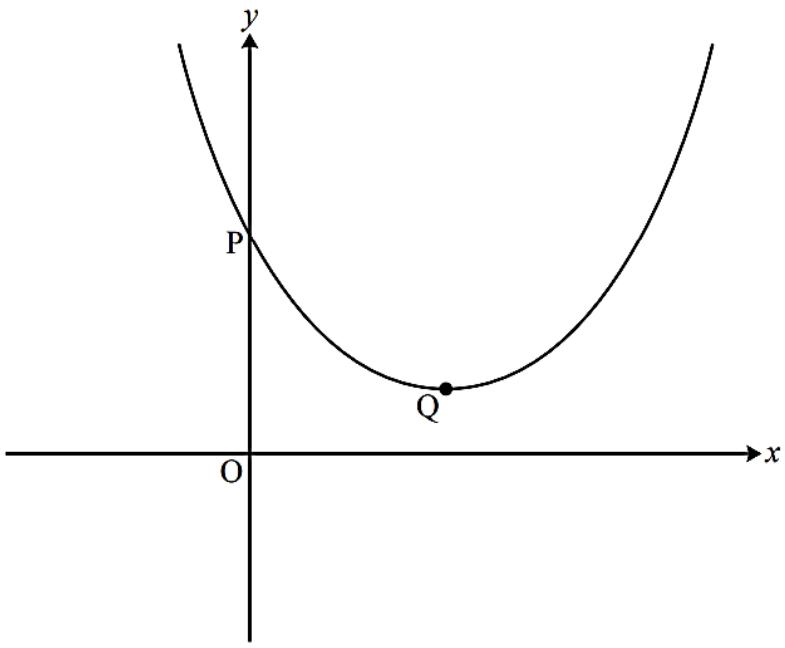

Fig. 9

(i) Find the y-coordinate of P in terms of k. **[1]**

(ii) Show that the x-coordinate of Q is $\frac{1}{4}\ln k$, and find the y-coordinate in its simplest form. **[5]**

3.

A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\,\pi$ cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

