First Year Maths and Further Maths combined Test B5
The factor theorem, rational functions and partial fractions
26 minutes

Throughout the entire test **all working must be shown** and solutions based entirely on graphical or numerical methods may not be acceptable

1.

You are given that $f(x) = (x+3)(x-2)(x-5)$.

(i) Sketch the curve $y = f(x)$. **[3]**

(ii) Show that $f(x)$ may be written as $x^3 - 4x^2 - 11x + 30$. **[2]**

2.

(i) Express $\frac{x+8}{x(x+2)}$ in partial fractions. **[3]**

(ii) express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P+\frac{Q}{x}+\frac{R}{x+2}$. **[3]**

3.

(a) Sketch the curve with equation $y = x^2(x-3)$.

[3 marks]

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^2(x-3) + 20$.

i.

Use the Factor Theorem to show that $x+2$ is a factor of $\mathrm{p}(x)$.

[2 marks]

ii.

Express $\mathrm{p}(x)$ in the form $(x+2)(x^2+bx+c)$, where b and c are integers.

[2 marks]

iii.

Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root and state its value.

[3 marks]

