PLEASE RIP THIS PAGE OFF THE TEST!

Appropriate formulae from the AQA formulae book

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{Q})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

First Year Maths and Further Maths combined Test B11
AS Mathematics Pure mock
2 hours

Materials

- You will need the rip off formula sheet on the front of the paper.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or a black ball-point pen. Pencil should only be used for drawing.
- Write your name at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), ask your teacher for extra paper. Write your name on every sheet and write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the rip off formula sheet.
- You do not necessarily need to use all the space provided.

LEAVE BLANK. FOR TEACHER USE					
Question	Marks	Question	Marks	Question	Marks
1		8		15	
2		9		16	
3		10		17	
4		11		18	
5		12		19	
6		13		20	
7		14		21	
				Total Marks	

Answer **all** questions in the spaces provided.

1 At a point P on a curve, the gradient of the tangent to the curve is 10

State the gradient of the normal to the curve at P

Circle your answer.

[1 mark]

-10 $\quad$ -0.1 $\quad$ 0.1 $\quad$ 10

2 Identify the expression below which is equivalent to $\left(\dfrac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

$\dfrac{8x^3}{125}$ $\quad$ $\dfrac{125x^3}{8}$ $\quad$ $\dfrac{125}{8x^3}$ $\quad$ $\dfrac{8}{125x^3}$

3 Simplify $\log_2 8^a$

Circle your answer.

[1 mark]

a^3 $2a$ $3a$ $8a$

4 It is given that $\sin\theta = \frac{4}{5}$ and $90° < \theta < 180°$

Find the value of $\cos\theta$

Circle your answer.

[1 mark]

$-\frac{3}{4}$ $-\frac{3}{5}$ $\frac{3}{5}$ $\frac{3}{4}$

5 The coefficient of x^2 in the binomial expansion of $(1 + ax)^6$ is $\frac{20}{3}$

Find the two possible values of a

[3 marks]

Turn over for the next question

6 (a) Find $\int \left(2x^3 + \frac{8}{x^2}\right) \mathrm{d}x$

[3 marks]

6 (b) A curve has gradient function $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 + \frac{8}{x^2}$

The x-intercept of the curve is at the point (2, 0)

Find the equation of the curve.

[2 marks]

Do not write outside the box

7 It is given that $5\cos^2\theta - 4\sin^2\theta = 0$

7 (a) Find the possible values of $\tan\theta$, giving your answers in exact form.

[3 marks]

7 (b) Hence, or otherwise, solve the equation

$$5\cos^2\theta - 4\sin^2\theta = 0$$

giving all solutions of θ to the nearest 0.1° in the interval $0° \leq \theta \leq 360°$

[2 marks]

Do not write outside the box

8 Find the exact solution of the equation $\ln(x+1) + \ln(x-1) = \ln 15 - 2\ln 7$

Fully justify your answer.

[5 marks]

9 (a) Given that $y = x\sqrt{x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

[2 marks]

9 (b) The line, *L*, has equation $6x - 2y + 5 = 0$

L is a tangent to the curve with equation $y = x\sqrt{x} + k$

Find the value of k

[5 marks]

10 It is given that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

Use these two expressions to show that $\tan 15° = 2 - \sqrt{3}$

Fully justify your answer.

[4 marks]

Turn over for the next question

11 (a) The curve C_1 has equation $y = 2x^2 - 20x + 42$

Express the equation of C_1 in the form

$$y = a(x - b)^2 + c$$

where a, b and c are integers.

[3 marks]

11 (b) Write down the coordinates of the minimum point of C_1

[1 mark]

11 (c) The curve C_1 is mapped onto the curve C_2 by a stretch in the y-direction.

The minimum point of C_2 is at $(5, -4)$

Find the equation of C_2

[2 marks]

Do not write outside the box

12 A curve has equation

$$y = 2x^2 + px + 1$$

A line has equation

$$y = 5x - 2$$

Find the set of values of p for which the line intersects the curve at two distinct points.

Give your answer in exact form.

[5 marks]

13 Points P and Q lie on the curve with equation $y = x^4$

The x-coordinate of P is x
The x-coordinate of Q is $x + h$

13 (a) Expand $(x + h)^4$

[2 marks]

13 (b) Hence, find an expression, in terms of x and h, for the gradient of the line PQ

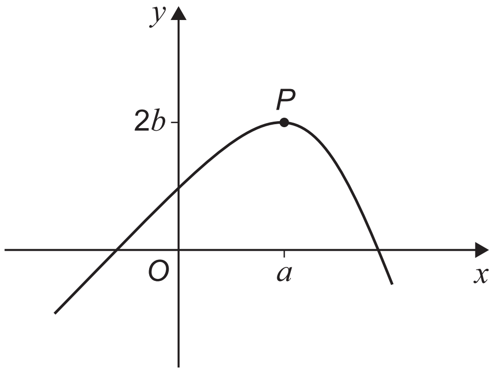
[1 mark]

13 (c) Explain how to use the answer from part **(b)** to obtain the gradient function of $y = x^4$

[2 marks]

14 The curve C has equation $y = \mathrm{f}(x)$

C has a maximum point at P with coordinates $(a, 2b)$ as shown in the diagram below.



14 (a) C is mapped by a single transformation onto curve C_1 with equation $y = \mathrm{f}(x + 2)$

State the coordinates of the maximum point on curve C_1

[1 mark]

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14 (b) C is mapped by a single transformation onto curve C_2 with equation $y = 4\mathrm{f}(x)$

State the coordinates of the maximum point on curve C_2

[1 mark]

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14 (c) C is mapped by a stretch in the x-direction onto curve C_3 with equation $y = \mathrm{f}(3x)$

State the scale factor of the stretch.

[1 mark]

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15 (a) Show that

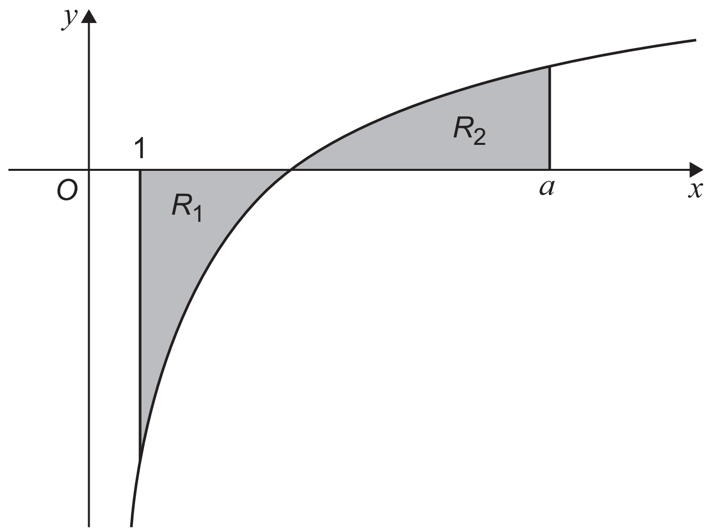
$$\int_1^a \left(6 - \frac{12}{\sqrt{x}}\right) \mathrm{d}x = 6a - 24\sqrt{a} + 18$$

[3 marks]

15 (b) The curve $y = 6 - \dfrac{12}{\sqrt{x}}$, the line $x = 1$ and the line $x = a$ are shown in the diagram below.

The shaded region R_1 is bounded by the curve, the line $x = 1$ and the x-axis.

The shaded region R_2 is bounded by the curve, the line $x = a$ and the x-axis.



It is given that the areas of R_1 and R_2 are equal.

Find the value of a

Fully justify your answer.

[4 marks]

16 Prove that the sum of the cubes of two consecutive odd numbers is always a multiple of 4.

[5 marks]

Do not write outside the box

Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

17 A continuous curve has equation $y = \mathrm{f}(x)$

The curve passes through the points *A*(2, 1), *B*(4, 5) and *C*(6, 1)

It is given that $\mathrm{f}'(4) = 0$

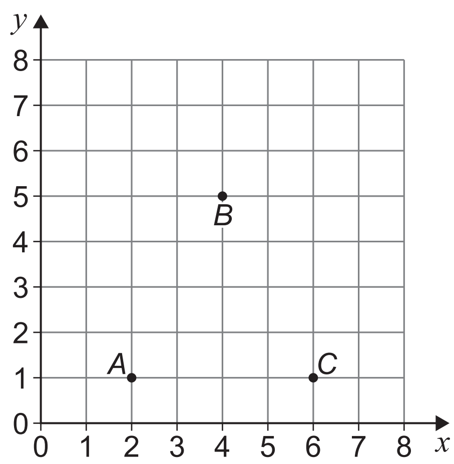
Jasmin made two statements about the nature of the curve $y = \mathrm{f}(x)$ at the point *B*:

Statement 1: There is a turning point at *B*

Statement 2: There is a maximum point at *B*

17 (a) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is correct and Statement 2 is correct.

[1 mark]



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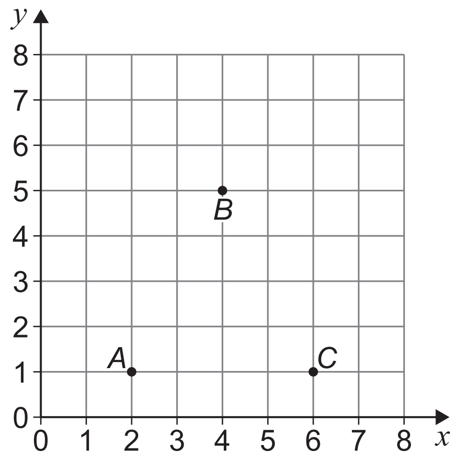
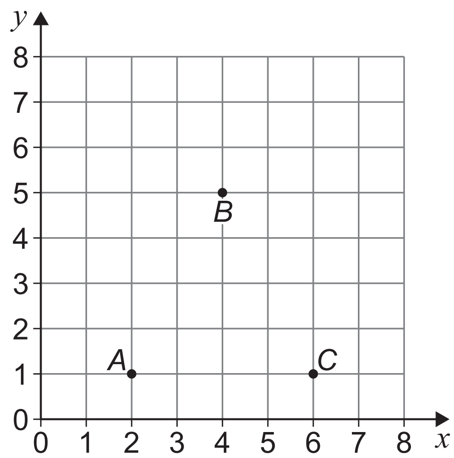
17 (b) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is correct and Statement 2 is **not** correct.

[1 mark]



17 (c) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is **not** correct and Statement 2 is **not** correct.

[1 mark]



18 Charlie buys a car for £18 000 on 1 January 2016.

The value of the car decreases exponentially.

The car has a value of £12 000 on 1 January 2018.

18 (a) Charlie says:

- because the car has lost £6000 after two years, after another two years it will be worth £6000.

Charlie's friend Kaya says:

- because the car has lost one third of its value after two years, after another two years it will be worth £8000.

Explain whose statement is correct, justifying the value they have stated.

[2 marks]

18 (b) The value of Charlie's car, £V, t years after 1 January 2016 may be modelled by the equation

$$V = A\mathrm{e}^{-kt}$$

where A and k are positive constants.

Find the value of t when the car has a value of £10 000, giving your answer to two significant figures.

[5 marks]

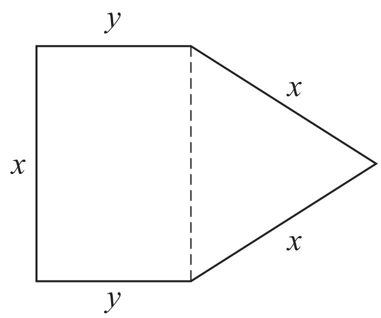
10 (c) Give a reason why the model, in this context, will not be suitable to calculate the value of the car when $t = 30$

[1 mark]

19 A piece of wire of length 66 cm is bent to form the five sides of a pentagon.

The pentagon consists of three sides of a rectangle and two sides of an equilateral triangle.

The sides of the rectangle measure x cm and y cm and the sides of the triangle measure x cm, as shown in the diagram below.



19 (a) (i) You are given that $\sin 60° = \dfrac{\sqrt{3}}{2}$

Explain why the area of the triangle is $\dfrac{\sqrt{3}}{4}x^2$

[1 mark]

19 (a) (ii) Show that the area enclosed by the wire, A cm^2, can be expressed by the formula

$$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$$

[3 marks]

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19 (b) Use calculus to find the value of x for which the wire encloses the maximum area.

Give your answer in the form $p + q\sqrt{3}$, where p and q are integers.

Fully justify your answer.

[7 marks]

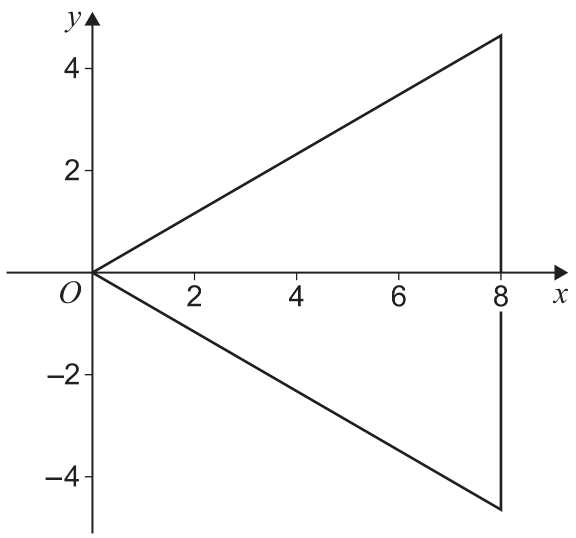
20 (a) A circle has equation

$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

[2 marks]

20 (b) An equilateral triangle has one vertex at the origin, and one side along the line $x = 8$, as shown in the diagram below.



20 (b) (i) Show that the vertex at the origin lies inside the circle $x^2 + y^2 - 10x - 6 = 0$

[1 mark]

20 (b) (ii) Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$

[4 marks]

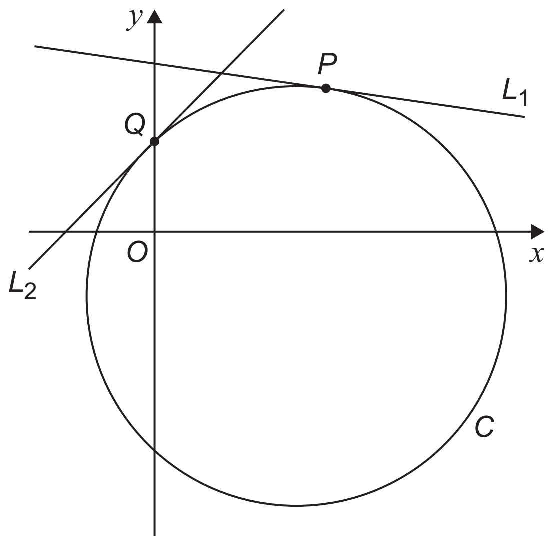
21 The line L_1 has equation $x + 7y - 41 = 0$

L_1 is a tangent to the circle C at the point $P(6, 5)$

The line L_2 has equation $y = x + 3$

L_2 is a tangent to the circle C at the point $Q(0, 3)$

The lines L_1 and L_2 and the circle C are shown in the diagram below.



21 (a) Show that the equation of the radius of the circle through P is $y = 7x - 37$

[3 marks]

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21 (b) Find the equation of C

[4 marks]