# EE338 : FIR Filter Design

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## 1 Student Details

**Name :** Joel Anto Paul
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**Filter Number :** 21

## 2 IIR Multi-Band pass Filter

### 2.1 Un-normalized Discrete Time Filter Specifications

Filter Number M = 21
M = 11Q + R
Q = Quotient when M is divided by 11 = 1
R = Remainder when M is divided by 11 = 10
**Passband 1 specifications:**
$B_L$(m) = 40 + 5Q = 40 + 5*1 = 45KHz
$B_H$(m) = 70 + 5Q = 70 + 5 = 75KHz
**Passband 2 specifications:**
$B_L$(m) = 170 + 5R = 170 + 5*10 = 220KHz
$B_H$(m) = 200 + 5R = 200 + 50 = 250KHz
Therefore the specifications of the **Multi-Band pass** Filter are:

- Passband : **45 - 75 KHz** and **220 - 250 KHz**
- Stopband : **0 - 40 KHz**, **80 - 215 KHz** and **255 - 300 KHz** (As **sampling rate** is **600 KHz**)
- Transition band : **5KHz** on either side of the passband and stopband
- Tolerance : **0.15** in **magnitude** for both passband and stopband

Sampling Rate = **600 KHz**

To design such a filter, we will cascade two filters, a Bandpass and a Bandstop filter, each of them being **FIR** filters. The specifications of these two filters are mentioned below:

# 3 Bandpass Filter

## 3.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Passband : **45 - 250 KHz**
- Stopband : **0 - 40 KHz** and **255 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.15** in **magnitude** for both passband and stopband

## 3.2 Normalized Digital Filter Specifications

Sampling rate = 600KHz
In the normalized frequency axis, sampling rate corresponds to $2\pi$

Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where $\Omega_s$ is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Passband : **0.15 - 0.833** $\pi$
- Stopband : **0 - 0.133** $\pi$ and **0.85 - 1** $\pi$
- Transition band : **0.0167** $\pi$ on either side of stopband

# 4 FIR Bandpass Filter

Both the passband and stopband tolerances are given to be 0.15
Therefore $\delta = 0.15$ and the minimum stopband attenuation A is given by :

$$A = -20log(\delta) = -20log(0.15) = 16.478$$

Since A $<$ 21, we get $\beta = 0$, where $\beta$ is the shape parameter of Kaiser window
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length

$$2N_{min} + 1 \geq 1 + \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the transition width which is the same on either side of the passband

$$\Delta\omega_T = \frac{5KHz * 2\pi}{600KHz} = 0.0167\pi$$

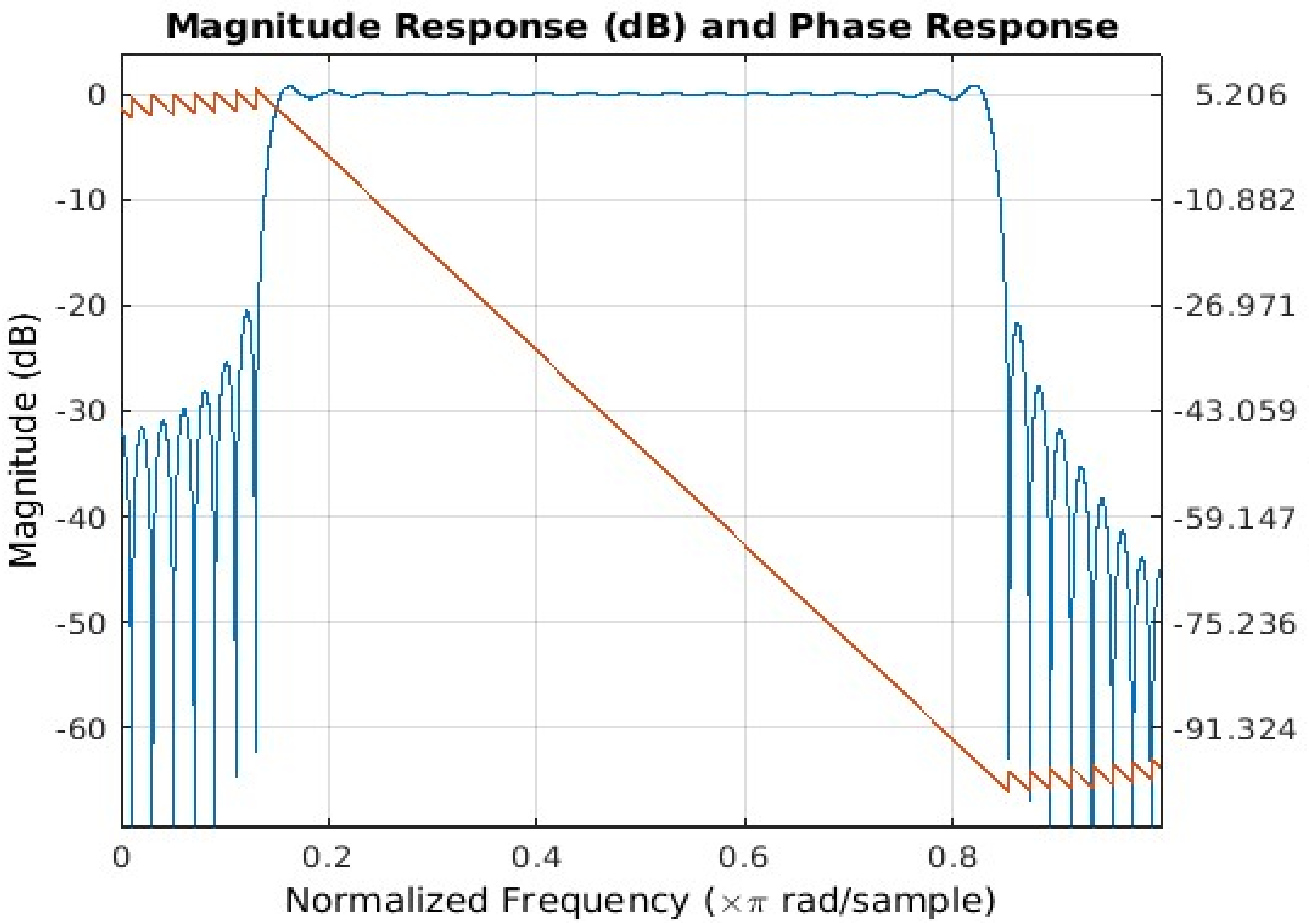
$$2N_{min} \geq 71.279$$

Hence we initially choose $N_{min} = 36$ ( $N_{min}$ is such that total number of samples is $2N_{min}$+1) Further for stringent tolerance and transition band specifications, we get $N_{total} = 2N_{min} + 23 = 95$ using trial and error.

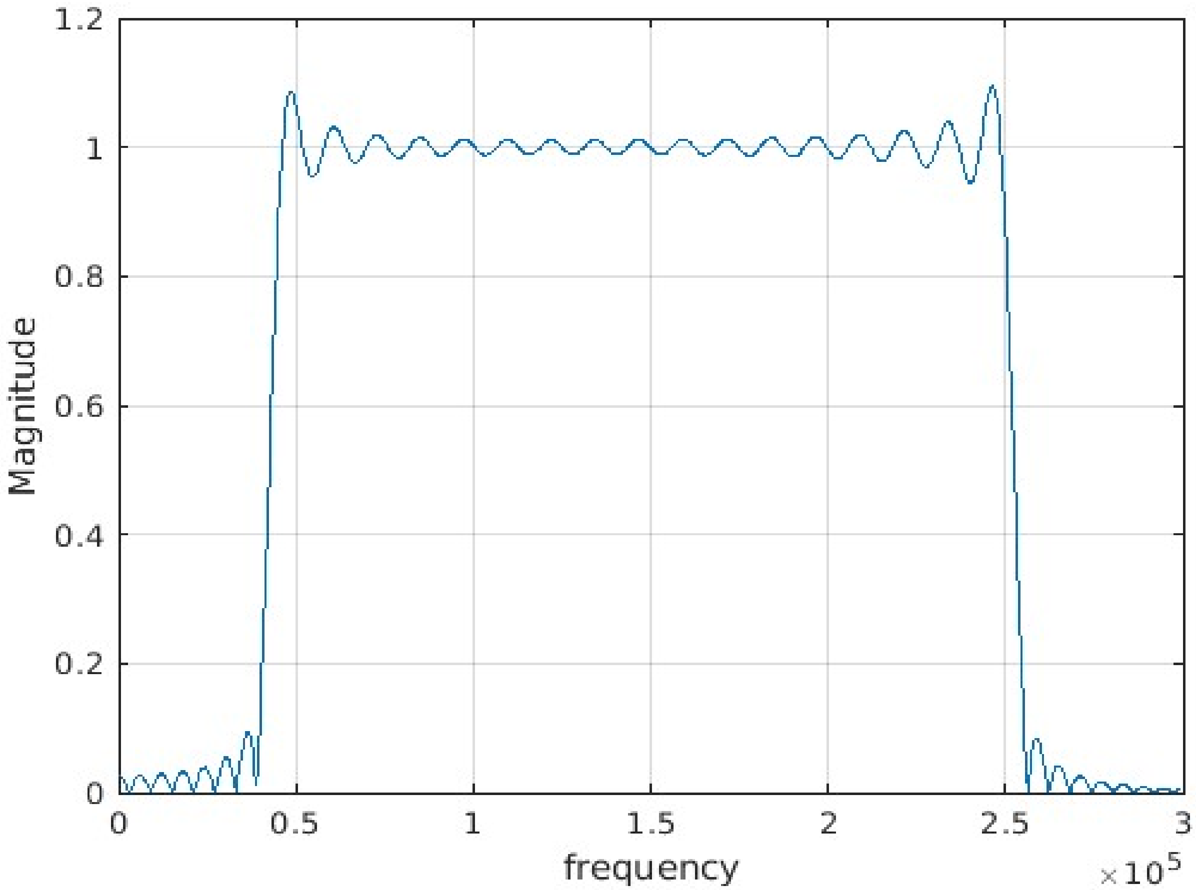
The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters with the cutoff frequencies being average of $\Omega_{s1}, \Omega_{p1}$ and $\Omega_{p2}, \Omega_{s2}$ respectively so that magnitude response reaches half of its peak value at the average of passband and stopband frequencies i.e. 0.475 $\pi$ and 0.7417 $\pi$

# 5 Matlab Plots

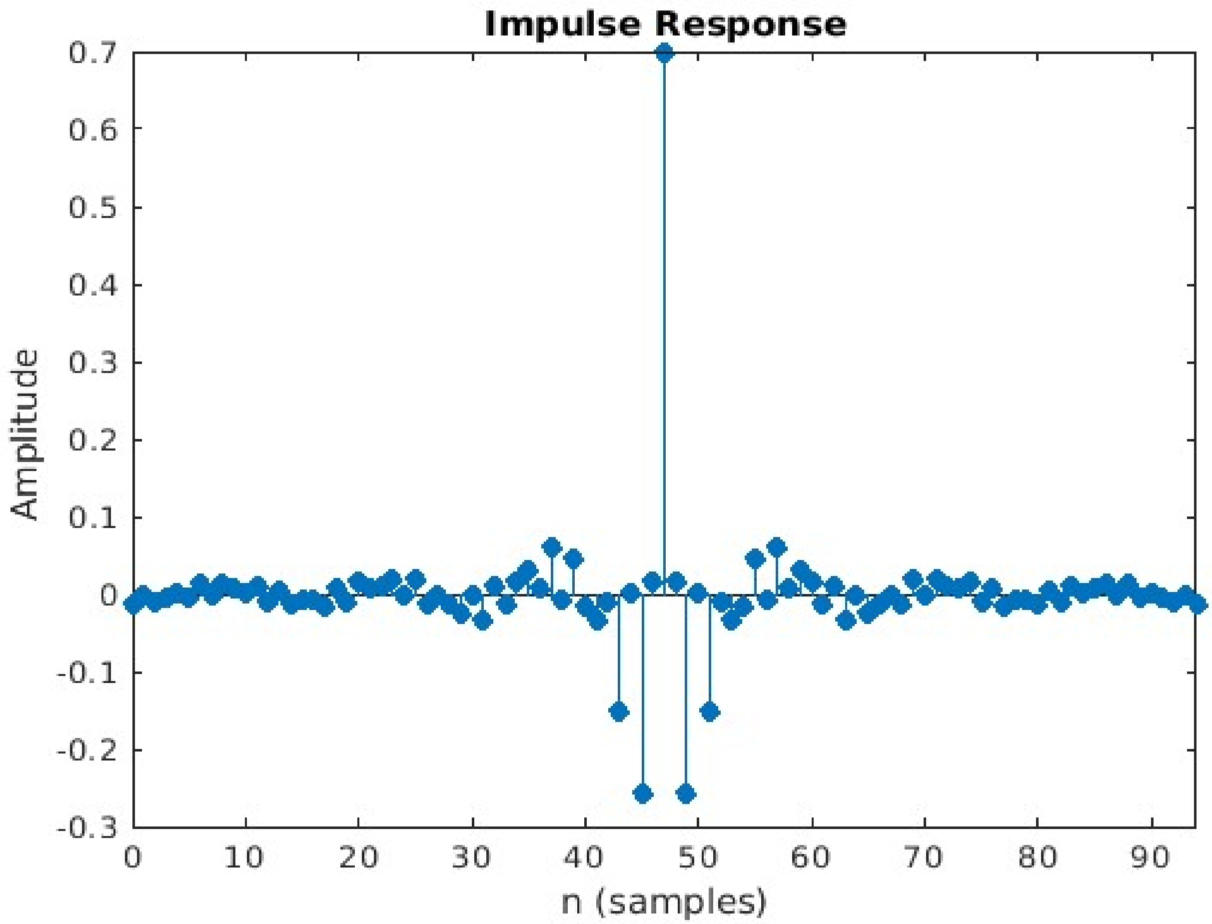
## 5.1 Frequency Response



### 5.2 Magnitude Response



### 5.3 Impulse Response



## 5.4 Coefficients

```
FIR_BandPass =

  Columns 1 through 14

   -0.0126   -0.0016   -0.0092   -0.0057    0.0021   -0.0056    0.0121   -0.0000    0.0124    0.0074    0.0022    0.0099   -0.0102    0.0037

  Columns 15 through 28

   -0.0144   -0.0079   -0.0067   -0.0151    0.0068   -0.0100    0.0151    0.0059    0.0109    0.0204   -0.0024    0.0197   -0.0141    0.0000

  Columns 29 through 42

   -0.0142   -0.0255   -0.0025   -0.0346    0.0114   -0.0131    0.0161    0.0297    0.0074    0.0615   -0.0075    0.0458   -0.0163   -0.0324

  Columns 43 through 56

   -0.0117   -0.1505    0.0026   -0.2572    0.0148    0.7000    0.0148   -0.2572    0.0026   -0.1505   -0.0117   -0.0324   -0.0163    0.0458

  Columns 57 through 70

   -0.0075    0.0615    0.0074    0.0297    0.0161   -0.0131    0.0114   -0.0346   -0.0025   -0.0255   -0.0142    0.0000   -0.0141    0.0197

  Columns 71 through 84

   -0.0024    0.0204    0.0109    0.0059    0.0151   -0.0100    0.0068   -0.0151   -0.0067   -0.0079   -0.0144    0.0037   -0.0102    0.0099

  Columns 85 through 95

    0.0022    0.0074    0.0124   -0.0000    0.0121   -0.0056    0.0021   -0.0057   -0.0092   -0.0016   -0.0126
```

# 6 Bandstop Filter

## 6.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Stopband : **80 - 215 KHz**
- Passband : **0 - 75 KHz** and **220 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.15** in **magnitude** for both passband and stopband

## 6.2 Normalized Digital Filter Specifications

Sampling rate = 600KHz
In the normalized frequency axis, sampling rate corresponds to $2\pi$
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where $\Omega_s$ is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Stopband : **0.267 - 0.717** $\pi$
- Passband : **0 - 0.25** $\pi$ and **0.733 - 1** $\pi$
- Transition band : **0.0167** $\pi$ on either side of stopband

# 7 FIR Bandstop Filter

Both the passband and stopband tolerances are given to be 0.15
Therefore $\delta = 0.15$ and the minimum stopband attenuation A is given by :

$$A = -20log(\delta) = -20log(0.15) = 16.478$$

Since A $<$ 21, we get $\beta = 0$, where $\beta$ is the shape parameter of Kaiser window
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length

$$2N_{min} + 1 \geq 1 + \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the transition width which is the same on either side of the passband

$$\Delta\omega_T = \frac{5KHz * 2\pi}{600KHz} = 0.0167\pi$$

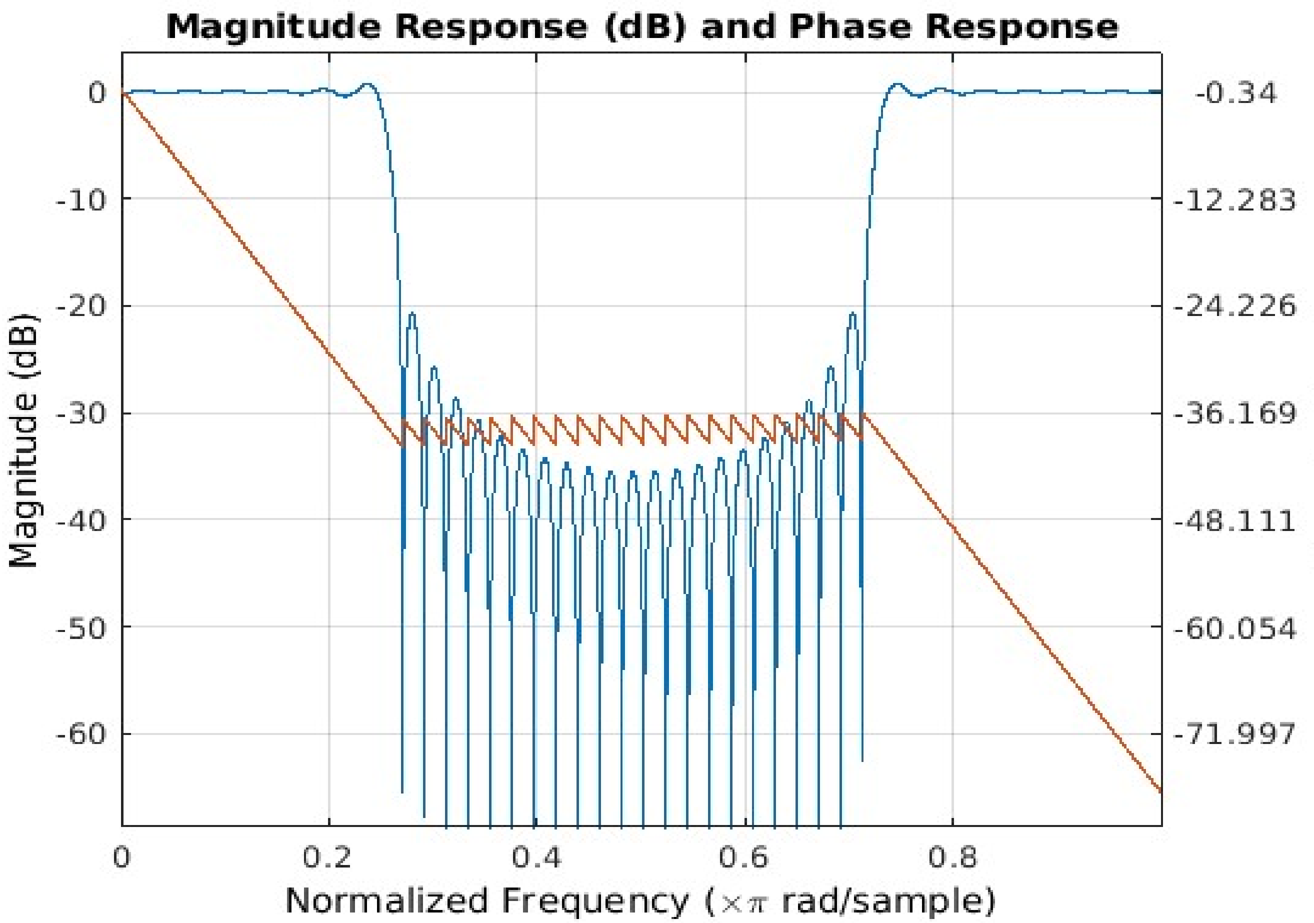
$$2N_{min} \geq 71.279$$

Hence we initially choose $N_{min} = 36$ ( $N_{min}$ is such that total number of samples is $2N_{min}$+1) Further for stringent tolerance and transition band specifications, we get $N_{total} = 2N_{min} + 23 = 95$ using trial and error.

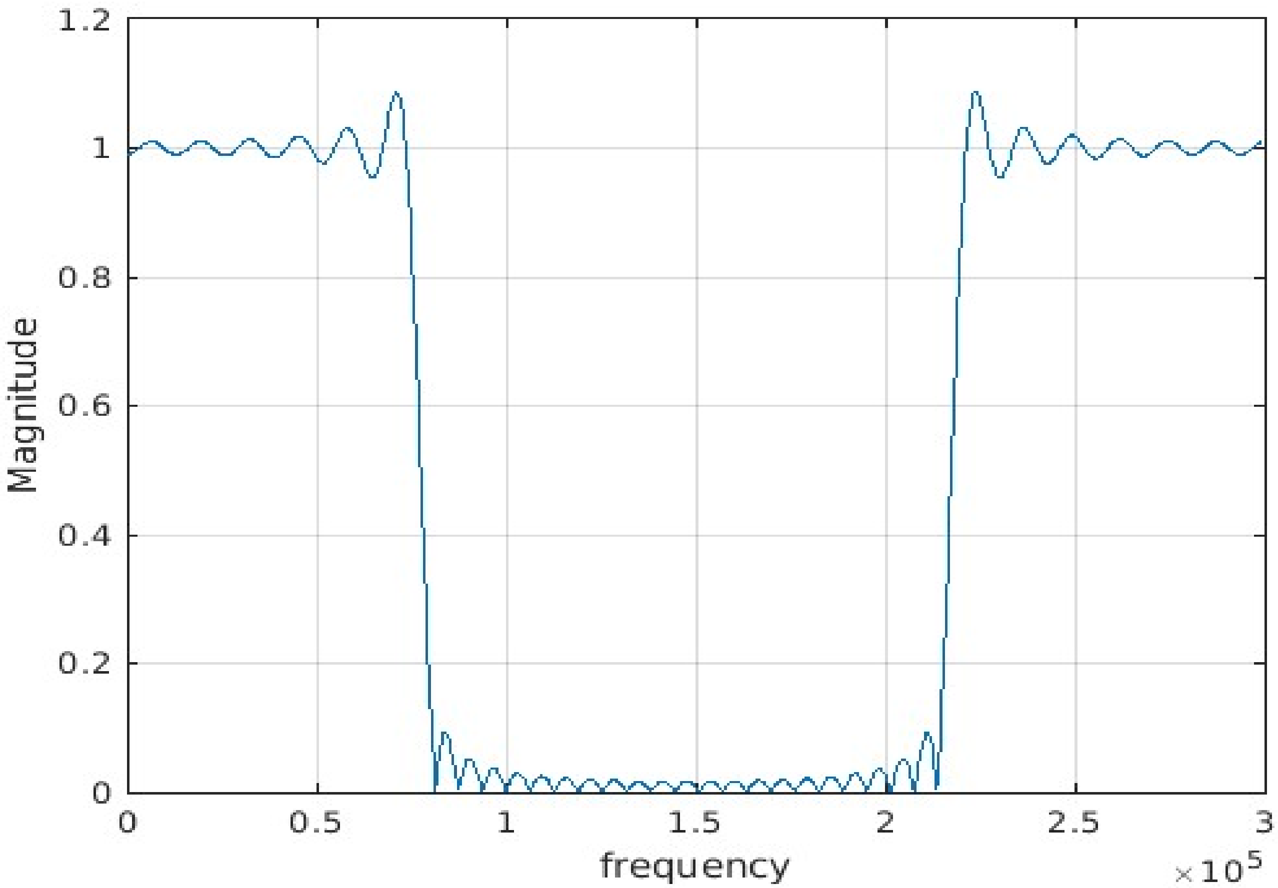
The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-stop impulse response samples were generated as the difference between an all pass filter and a band pass filter such that the cutoff frequencies are again at average of passband and stopband frequencies i.e. 0.2583 $\pi$ and 0.725 $\pi$

# 8 Matlab Plots

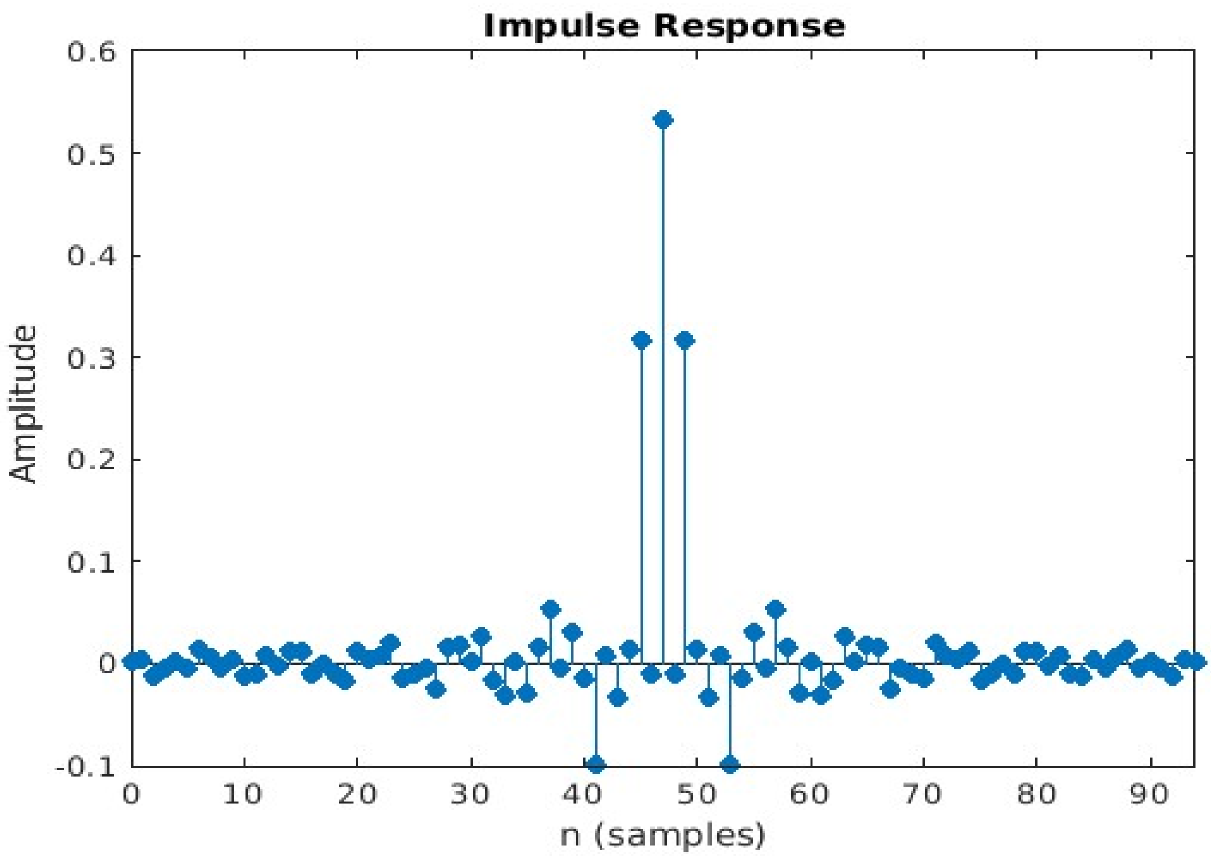
## 8.1 Frequency Response



### 8.2 Magnitude



### 8.3 Impulse Response



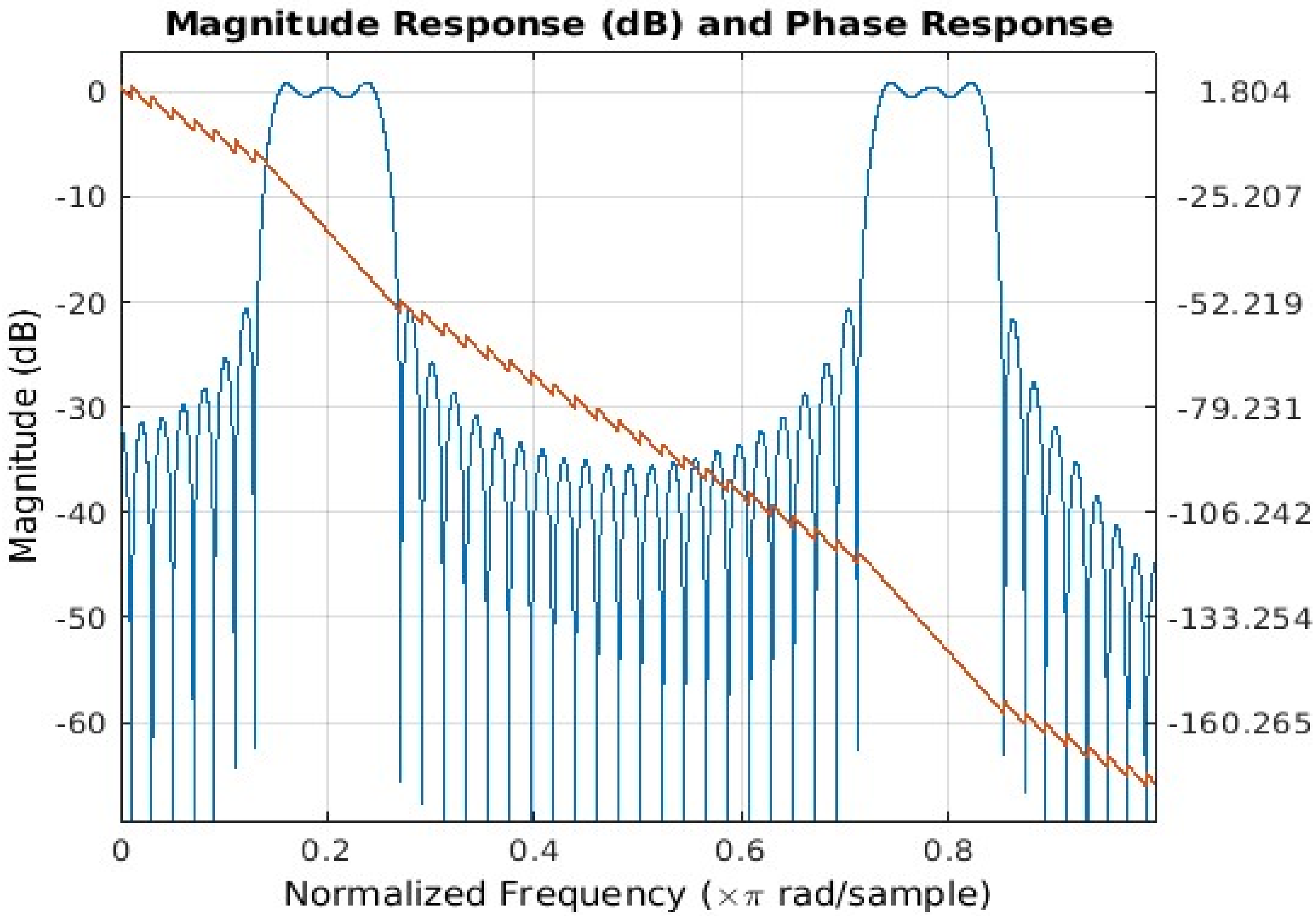
## 8.4 Coefficients

```
FIR_BandStop =

  Columns 1 through 14

    0.0013    0.0037   -0.0131   -0.0044    0.0014   -0.0040    0.0134    0.0069   -0.0043    0.0037   -0.0130   -0.0099    0.0072   -0.0024

  Columns 15 through 28

    0.0119    0.0132   -0.0100   -0.0000   -0.0101   -0.0168    0.0124    0.0040    0.0077    0.0204   -0.0144   -0.0099   -0.0049   -0.0238

  Columns 29 through 42

    0.0157    0.0185    0.0017    0.0270   -0.0163   -0.0316    0.0017   -0.0296    0.0161    0.0533   -0.0051    0.0316   -0.0152   -0.0997

  Columns 43 through 56

    0.0083   -0.0329    0.0135    0.3161   -0.0112    0.5333   -0.0112    0.3161    0.0135   -0.0329    0.0083   -0.0997   -0.0152    0.0316

  Columns 57 through 70

   -0.0051    0.0533    0.0161   -0.0296    0.0017   -0.0316   -0.0163    0.0270    0.0017    0.0185    0.0157   -0.0238   -0.0049   -0.0099

  Columns 71 through 84

   -0.0144    0.0204    0.0077    0.0040    0.0124   -0.0168   -0.0101   -0.0000   -0.0100    0.0132    0.0119   -0.0024    0.0072   -0.0099

  Columns 85 through 95

   -0.0130    0.0037   -0.0043    0.0069    0.0134   -0.0040    0.0014   -0.0044   -0.0131    0.0037    0.0013
```

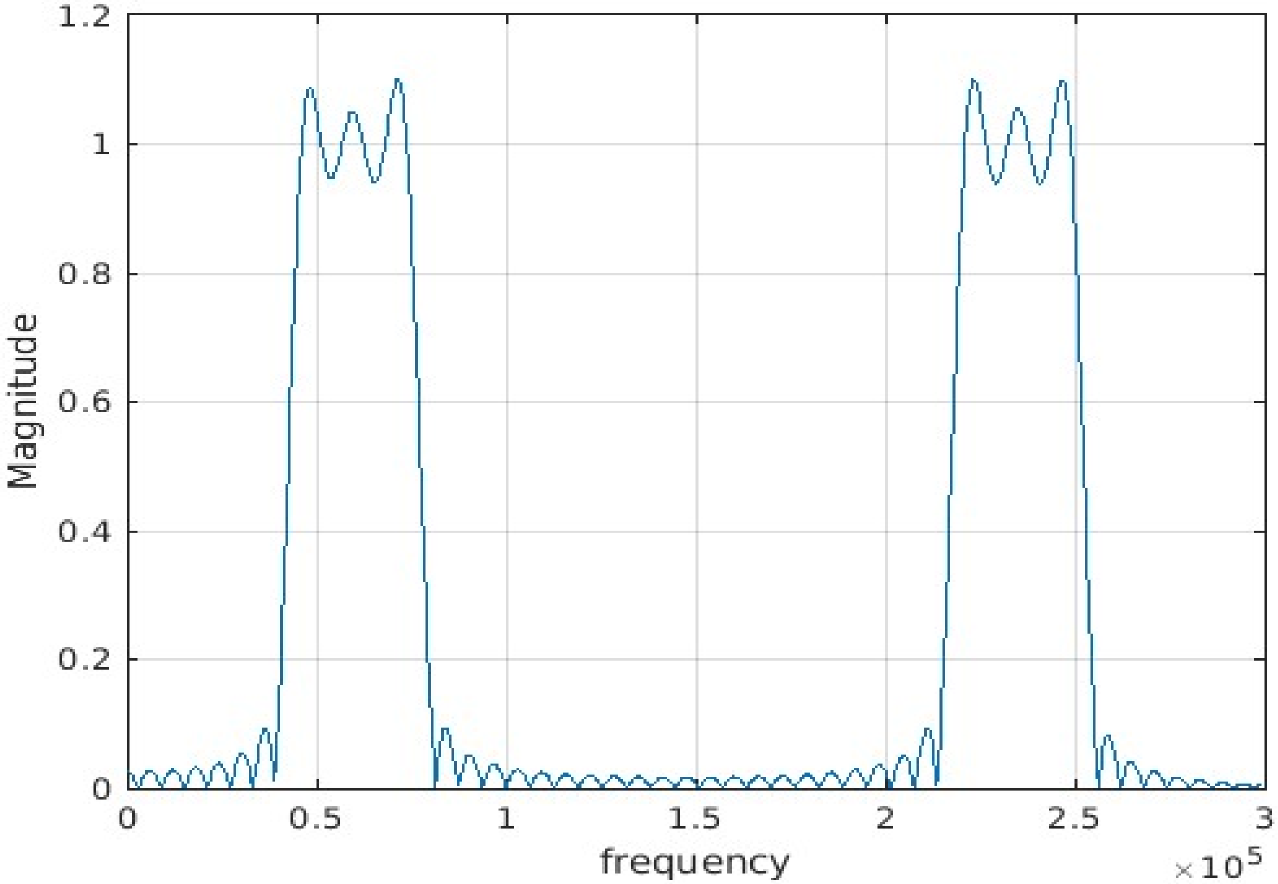
## 9 Final results after cascading the two filters

## 10 Matlab Plots

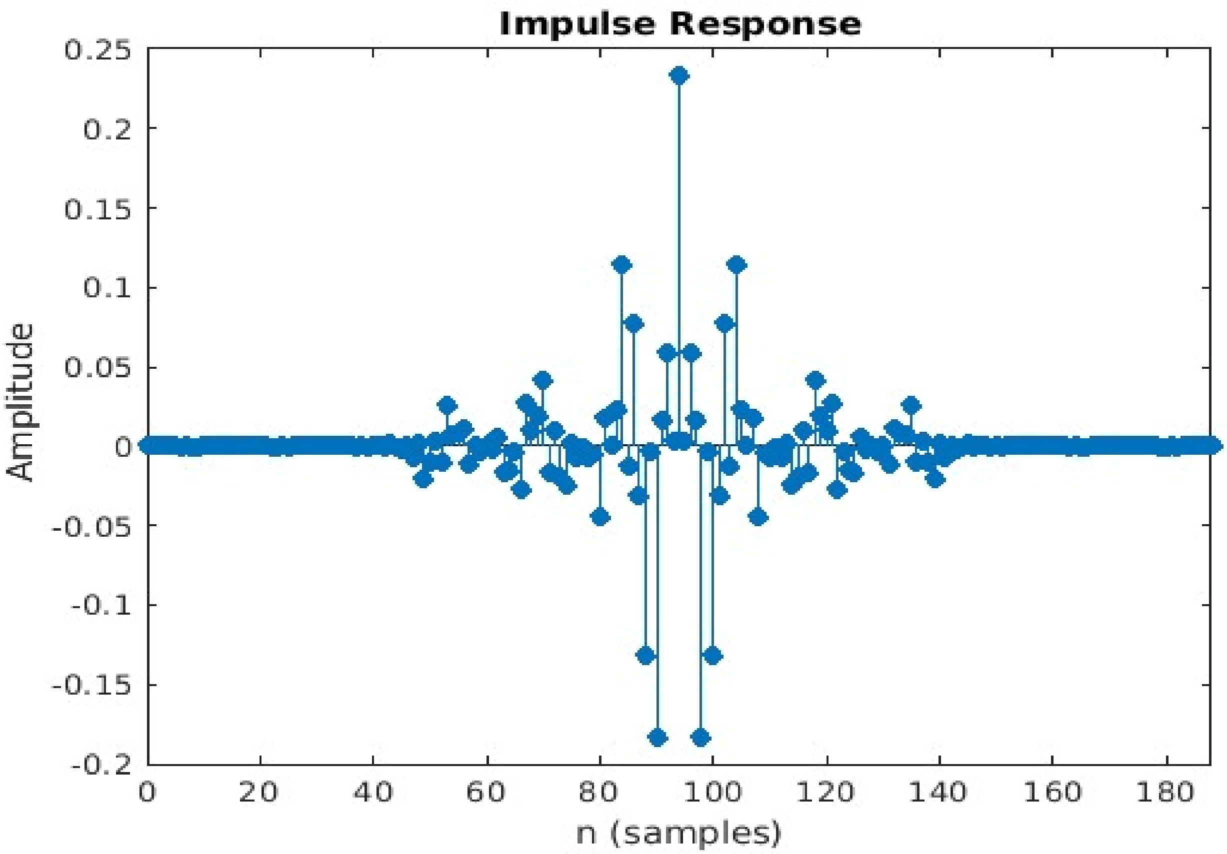
### 10.1 Frequency Response



### 10.2 Magnitude



### 10.3 Impulse Response



### 10.4 Coefficients

```
MULTI_BAND_FIR =

  Columns 1 through 14

   -0.0000   -0.0000    0.0001    0.0000    0.0001    0.0002   -0.0002    0.0000   -0.0002   -0.0002    0.0001   -0.0001    0.0001    0.0001

  Columns 15 through 28

   -0.0000    0.0000    0.0000   -0.0000    0.0001    0.0002   -0.0001    0.0001   -0.0002   -0.0004    0.0001   -0.0004    0.0002    0.0002

  Columns 29 through 42

   -0.0000    0.0003   -0.0001   -0.0001    0.0001    0.0001   -0.0000    0.0004   -0.0001   -0.0006    0.0001   -0.0013    0.0001    0.0005

  Columns 43 through 56

   -0.0002    0.0022   -0.0002   -0.0016   -0.0001   -0.0069    0.0018   -0.0212   -0.0095    0.0035   -0.0095    0.0260    0.0069    0.0080

  Columns 57 through 70

    0.0114   -0.0116   -0.0000   -0.0033    0.0008   -0.0021    0.0052   -0.0164   -0.0148   -0.0034   -0.0266    0.0275    0.0099    0.0187

  Columns 71 through 84

    0.0409   -0.0170    0.0098   -0.0191   -0.0241    0.0016   -0.0071   -0.0006   -0.0074   -0.0049   -0.0445    0.0177   -0.0000    0.0236

  Columns 85 through 98

    0.1147   -0.0126    0.0774   -0.0315   -0.1322   -0.0034   -0.1836    0.0162    0.0591    0.0036    0.2335    0.0036    0.0591    0.0162
```

```
Columns 99 through 112

 -0.1836   -0.0034   -0.1322   -0.0315    0.0774   -0.0126    0.1147    0.0236   -0.0000    0.0177   -0.0445   -0.0049   -0.0074   -0.0006

Columns 113 through 126

 -0.0071    0.0016   -0.0241   -0.0191    0.0098   -0.0170    0.0409    0.0187    0.0099    0.0275   -0.0266   -0.0034   -0.0148   -0.0164

Columns 127 through 140

  0.0052   -0.0021    0.0008   -0.0033   -0.0000   -0.0116    0.0114    0.0080    0.0069    0.0260   -0.0095    0.0035   -0.0095   -0.0212

Columns 141 through 154

  0.0018   -0.0069   -0.0001   -0.0016   -0.0002    0.0022   -0.0002    0.0005    0.0001   -0.0013    0.0001   -0.0006   -0.0001    0.0004

Columns 155 through 168

 -0.0000    0.0001    0.0001   -0.0001   -0.0001    0.0003   -0.0000    0.0002    0.0002   -0.0004    0.0001   -0.0004   -0.0002    0.0001

Columns 169 through 182

 -0.0001    0.0002    0.0001   -0.0000    0.0000    0.0000   -0.0000    0.0001    0.0001   -0.0001    0.0001   -0.0002   -0.0002    0.0000

Columns 183 through 189

 -0.0002    0.0002    0.0001    0.0000    0.0001   -0.0000   -0.0000
```

## 11 Comparison between FIR and IIR Filters :

- FIR filters are easier to design as we only need to truncate the ideal impulse response using a suitable window function, instead of applying the bilinear and frequency transformation, designing a low-pass filter and then converting it to bandpass/bandstop as per our requirement.
- We get a linear (or psuedo linear) phase response in FIR Filter which we don't get in IIR Filters.
- We can't control the passband and stopband tolerances individually in FIR Filters, nor can we change their nature (monotonic or equiripple), which was possible in IIR Filters.
- We usually need a lot more resources for FIR filters as compared to IIR filters, as we can see that the value of N for FIR filters is considerably large.

## 12 Review

- I have verified the filter design of my team-mate Tamojeet Roychowdhury (Roll No : 21D070079)