EE338 : Digital Signal Processing
Chebyschev Filter Design Assignment

1 Student Details

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2 IIR Multi-Band pass Filter

2.1 Un-normalized Discrete Time Filter Specifications

Filter Number M = 21
M = 11Q + R
Q = Quotient when M is divided by 11 = 1
R = Remainder when M is divided by 11 = 10
Passband 1 specifications:
B_L(m) = 40 + 5Q = 40 + 5*1 = 45KHz
B_H(m) = 70 + 5Q = 70 + 5 = 75KHz
Passband 2 specifications:
B_L(m) = 170 + 5R = 170 + 5*10 = 220KHz
B_H(m) = 200 + 5R = 200 + 50 = 250KHz
Therefore the specifications of the **Multi-Band pass** Filter are:

- Passband : **45 - 75 KHz** and **220 - 250 KHz**
- Stopband : **0 - 40 KHz**, **80 - 215 KHz** and **255 - 300 KHz** (As **sampling rate** is **600 KHz**)
- Transition band : **5KHz** on either side of the passband and stopband
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Passbands are **oscillatory** and stopbands are **monotonic**

Sampling Rate = **600 KHz**

To design such a filter, we will cascade two filters, a Bandpass and a Bandstop filter, each of them being **Chebyshev** filters. For the specifications to meet, the filters will have tolerances of 0.07 in magnitude. The specifications of these two filters are mentioned below:

2.2 Bandpass Chebyshev Filter

2.2.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Passband : **45 - 250 KHz**
- Stopband : **0 - 40 KHz** and **255 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.07** in **magnitude** for both passband and stopband
- Nature : Passbands are **oscillatory** and stopbands are **monotonic**

2.2.2 Normalized Digital Filter Specifications

In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Passband : **0.15 - 0.833** π
- Stopband : **0 - 0.133** π and **0.85 - 1** π
- Transition band : **0.0167** π on either side of stopband

2.2.3 Bilinear Transformation

To convert to analog domain, we use the following bilinear transformation :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\Omega_{analog} = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.133 π	0.213
0.15 π	0.24
0.833 π	3.732
0.85 π	4.165
π	∞

Therefore, the corresponding specifications are :

- Passband : **0.24** (Ω_{p1}) **- 3.732** (Ω_{p2})
- Transition band : Between the passband and stopband edges
- Stopband : **0 - 0.213**(Ω_{s1}) and **4.165** (Ω_{s2}) **-** ∞

2.2.4 Frequency Transformation and Relevant Parameters

We need to convert the Band - Pass filter into a Low - Pass analog filter as we are aware of it's frequency response in order to keep monotonic passband and stopband. For that purpose we use the following frequency transformation with two parameters B and Ω_o

$$\Omega_l = \frac{\Omega^2 - \Omega_o^2}{B\Omega}$$

If we follow the convention that the passband edges are mapped to +1 and -1, the parameters, in terms of the passband edges can be obtained by solving two equations and are given by :

$$\Omega_o = \sqrt{\Omega_{p1}\Omega_{p2}} = \sqrt{0.24 * 3.732} = 0.946$$

$$B = \Omega_{p2} - \Omega_{p1} = 3.492$$

Ω	Ω_L
0^+	- ∞
0.213 (Ω_{s1})	-1.142
0.24 (Ω_{p1})	-1
0.946 (Ω_o)	0
3.732 (Ω_{p2})	1
4.165 (Ω_{s2})	1.131
∞	∞

To make the filter as close to ideal as possible we choose the more stringent stopband for the lowpass filter i.e. $\Omega_{sL} = \min(\Omega_{sL1}$, - $\Omega_{sL2}) = 1.131$. (where Ω_{sL} stands for the stopband for the lowpass filter)

Therefore the analog lowpass filter specifications are as follows:

- Passband Edge : 1 (Ω_{pL})
- Stopband Edge : 1.131 (Ω_{sL})

2.2.5 Analog Lowpass Transfer Function

To keep the frequency response equiripple in passband and monotonic in stopband we use the Chebyschev approximation and let the transfer function of the analog lowpass filter to be of the form :

$$|H(j\Omega)|^2 = \frac{1}{1 + \epsilon^2 C_N^2(\frac{\Omega}{\Omega_p})}$$

where $\Omega_p = 1$ and C_N is the Chebyschev Polynomial given by

$$C_N(\Omega) = cos(N cos^{-1}(\Omega))$$

We define two new parameters D_1 and D_2 such that

$$D_1 = \frac{1}{(1 - \delta_1)^2} - 1 = 0.1562$$

$$D_2 = \frac{1}{(\delta_2)^2} - 1 = 203.0816 \tag{1}$$

where $\delta_1 = \delta_2 = 0.07$

We choose ϵ of the Chebyschev Filter to be $\sqrt{D_1}$ (to minimize N) and then upon applying the condition that the stopband must lie within the tolerances specified, we get a lower bound on the value of N which is given by:

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{\cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \right\rceil = 9$$

The poles of the transfer function can be obtained by solving the equation

$$1 + D_1 \cosh^2(N \cosh^{-1}(\frac{s}{j})) = 0$$

which is nothing but

$$1 + 0.1562(C_9^2(s/j)) = 0$$

where $C_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$

Using MATLAB, we plot the poles of the magnitude response of the analog low pass filter.

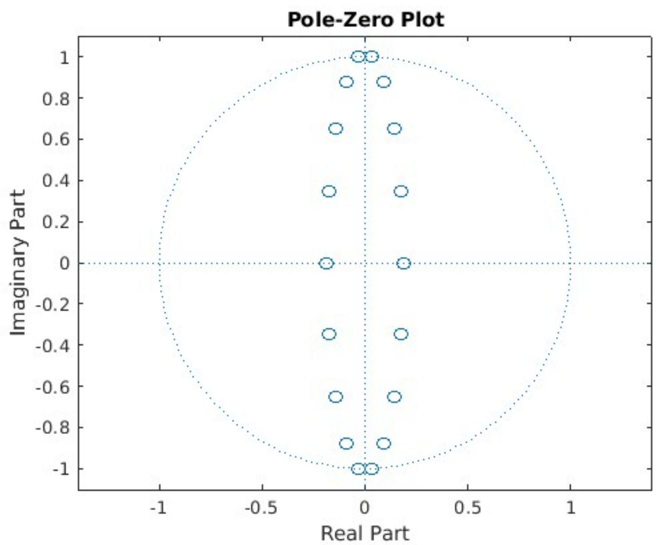


Figure 1:

In order to get a stable Analog Low Pass Filter we include the poles lying in the Left Half Plane in the Transfer Function. Thus, the required poles are :

p_1 = -0.0322 - 1.0016i
p_2 = -0.0927 - 0.8808i
p_3 = -0.1420 - 0.6537i
p_4 = -0.0322 + 1.0016i
p_5 = -0.0927 + 0.8808i
p_6 = -0.1420 + 0.6537i
p_7 = -0.1741 - 0.3478i
p_8 = -0.1741 + 0.3478i
p_9 = -0.1853 + 0.0000i

Using the above poles and the fact that N is odd (no need to normalize by a factor of $\sqrt{1+D_1}$) i.e. the DC gain is just the product of poles, we can write the analog low pass transfer function as :

$$H_{analog}(s) = \frac{\Pi_{i=1}^{9} p_i}{\Pi_{i=1}^{9}(s - p_i)}$$

The values of the coefficients of the corresponding Analog Lowpass Transfer Function is given as

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s_L^9	1.0000×10^0	s_L^8	1.0672×10^0	s_L^7	2.8194×10^0
s_L^6	2.2400×10^0	s_L^5	2.6346×10^0	s_L^4	1.4660×10^0
s_L^3	0.9108×10^0	s_L^2	0.3058×10^0	s_L^1	0.0853×10^0
s_L^0	0.9460099×10^0	-	-	-	-

Table 1: Denominator Coefficients

2.2.6 Analog Bandpass Transfer Function

The transformation is given by :

$$s_L = \frac{s^2 + \Omega_o^2}{Bs}$$

Substituting the values of B and Ω_o

$$s_L = \frac{s^2 + 0.8949}{3.492s}$$

After substituting it can be written in the form N(s)/D(s) where the coefficients of N(s) and D(s) are given below :

$$N(s) = 763.1035.s^9$$

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{18}	1.0000	s^{17}	3.7265	s^{16}	42.4344
s^{15}	122.0610	s^{14}	635.9561	s^{13}	1356.9490
s^{12}	4042.8322	s^{11}	5956.3007	s^{10}	10400.9982
s^9	9420.5746	s^8	9308.0197	s^7	4770.2502
s^6	2897.5607	s^5	870.3474	s^4	365.0383
s^3	62.7005	s^2	19.5071	s^1	1.5331
s^0	0.3682	-	-	-	-

Table 2: Denominator Coefficients

2.2.7 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use ofthe Bilinear Transformation which is given as :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}$(z) from $H_{analog,BPF}$(s).It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as :-

Degree	**Coefficient**	**Degree**	**Coefficient**	**Degree**	**Coefficient**
s^{18}	0.0152	s^{17}	0	s^{16}	-0.1366
s^{15}	0	s^{14}	0.5464	s^{13}	0
s^{12}	-1.2749	s^{11}	0	s^{10}	1.9124
s^{9}	0	s^{8}	-1.9124	s^{7}	0
s^{6}	1.2749	s^{5}	0	s^{4}	-0.5464
s^{3}	0	s^{2}	0.1366	s^{1}	0
s^{0}	-0.0152	-	-	-	-

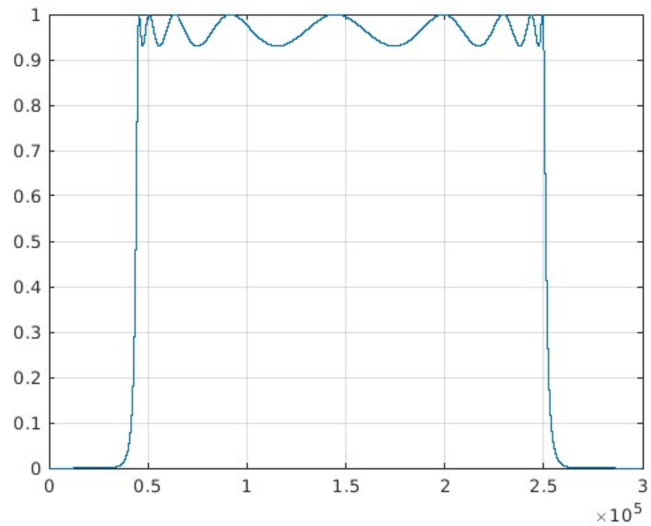
Table 3: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{18}	1.0000	s^{17}	-0.4273	s^{16}	-1.2084
s^{15}	0.2881	s^{14}	2.4685	s^{13}	-0.7523
s^{12}	-1.1718	s^{11}	0.0748	s^{10}	1.5745
s^{9}	-0.4211	s^{8}	0.0169	s^{7}	-0.1342
s^{6}	0.4522	s^{5}	-0.1686	s^{4}	0.2671
s^{3}	-0.0526	s^{2}	0.0653	s^{1}	-0.0540
s^{0}	0.1024	-	-	-	-

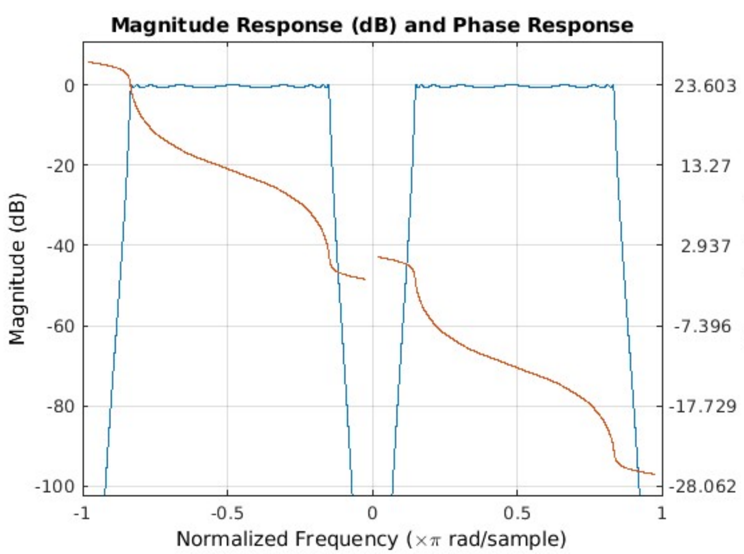
Table 4: Denominator Coefficients

2.2.8 Matlab Plots

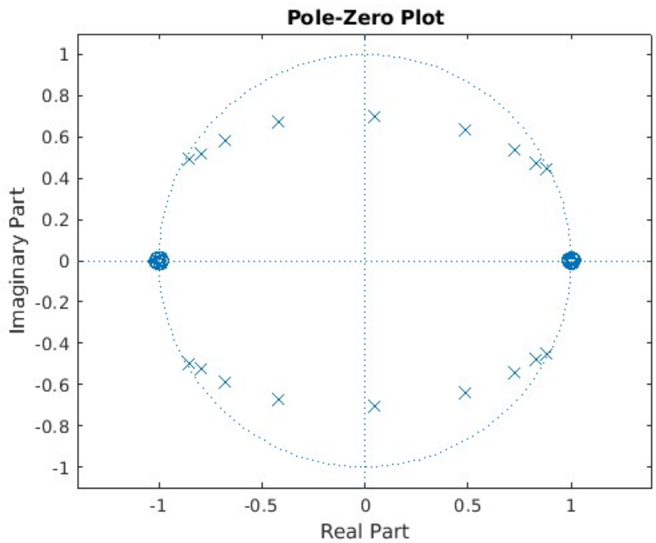
Magmitude Response



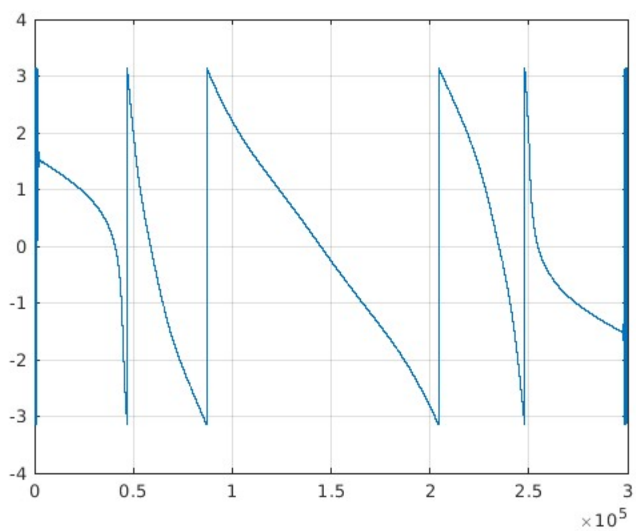
Frequency Response



Pole - Zero Plot



Unnormalized Phase response



2.3 Bandstop Chebyshev Filter

2.3.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Stopband : **80 - 215 KHz**

- Passband : **0 - 75 KHz** and **220 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.07** in **magnitude** for both passband and stopband
- Nature : Passbands are **oscillatory** and stopbands are **monotonic**

2.3.2 Normalized Digital Filter Specifications

In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Stopband : **0.267 - 0.717** π
- Passband : **0 - 0.25** π and **0.733 - 1** π
- Transition band : **0.0167** π on either side of stopband

2.3.3 Bilinear Transformation

To convert to analog domain, we use the following bilinear transformation :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\Omega_{analog} = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.25 π	0.414
0.267 π	0.445
0.717 π	2.097
0.733 π	2.246
π	∞

Therefore, the corresponding specifications are :

- Stopband : **0.445** (Ω_{s1}) **- 2.097** (Ω_{s2})
- Transition band : Between the passband and stopband edges
- Passband : **0 - 0.414**(Ω_{p1}) and **2.246** (Ω_{p2}) **-** ∞

2.3.4 Frequency Transformation and Relevant Parameters

We need to convert the Band - Stop filter into a Low - Pass analog filter as we are aware of it's frequency response. For that purpose we use the following frequency transformation with two parameters B and Ω_o

$$\Omega_l = \frac{B\Omega}{\Omega_o^2 - \Omega^2}$$

If we follow the convention that the passband edges are mapped to +1 and -1, the parameters, in terms of the passband edges can be obtained by solving two equations and are given by :

$$\Omega_o = \sqrt{\Omega_{p1}\Omega_{p2}} = \sqrt{0.414 * 2.246} = 0.964$$

$$B = \Omega_{p2} - \Omega_{p1} = 1.832$$

Ω	Ω_L
0^+	0^+
0.414 (Ω_{p1})	+1
0.445 (Ω_{s1})	1.115
0.946^- (Ω_o)	∞
0.946^+ (Ω_o)	$-\infty$
2.097 (Ω_{s2})	-1.108
2.246 (Ω_{p2})	-1
∞	0^-

To make the filter as close to ideal as possible we choose the more stringent stopband for the lowpass filter i.e. Ω_{sL} = min(Ω_{sL1} , - Ω_{sL2}) = 1.108. (where Ω_{sL} stands for the stopband for the lowpass filter)

Therefore the analog lowpass filter specifications are as follows:

- Passband Edge : 1 (Ω_{pL})
- Stopband Edge : 1.108 (Ω_{sL})

2.3.5 Analog Lowpass Transfer Function

To keep the frequency response monotonic in passband and stopband we use the Chebyshev approximation and let the transfer function of the analog lowpass filter to be of the form :

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 C_N^2(\frac{\Omega}{\Omega_p})}$$

where $\Omega_p = 1$ and C_N is the Chebyschev Polynomial given by

$$C_N(\Omega) = cos(Ncos^{-1}(\Omega))$$

We define two new parameters D_1 and D_2 such that

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.1562$$

$$D_2 = \frac{1}{(\delta_2)^2} - 1 = 203.0816 \tag{2}$$

where $\delta_1 = \delta_2 = 0.07$

We choose ϵ of the Chebyschev Filter to be $\sqrt{D_1}$ (to minimize N) and then upon applying the condition that the stopband must lie within the tolerances specified, we get a lower bound on the value of N which is given by:

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{\cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \right\rceil = 10$$

The poles of the transfer function can be obtained by solving the equation

$$1 + D_1 \cosh^2(N \cosh^{-1}(\frac{s}{j})) = 0$$

which is nothing but

$$1 + 0.1562(C_1^2 0(s/j)) = 0$$

where $C_{10}(x) = 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1$

Using MATLAB, we plot the poles of the magnitude response of the analog low pass filter.

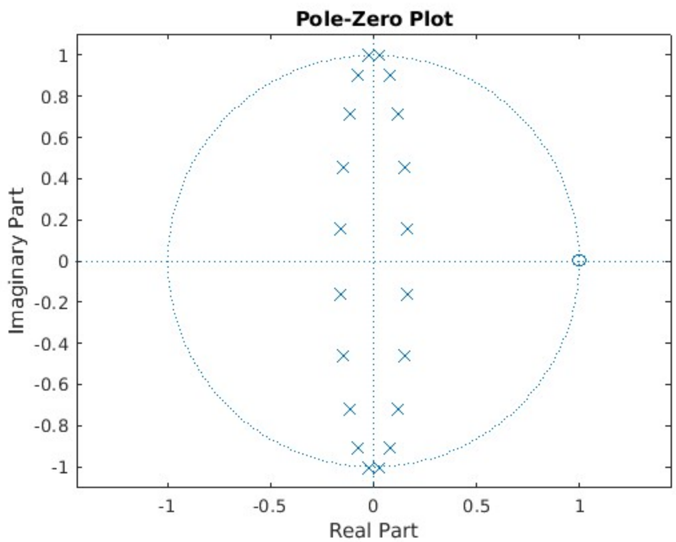


Figure 2:

In order to get a stable Analog Low Pass Filter we include the poles lying in the Left Half Plane in the Transfer Function. Thus, the required poles are :

$$
\begin{aligned}
p_1 &= \text{-}0.0261 + \text{j } 1.0013 \\
p_2 &= \text{-}0.0756 + \text{j } 0.9033 \\
p_3 &= \text{-}0.1178 + \text{j } 0.7169 \\
p_4 &= \text{-}0.1484 + \text{j } 0.4602 \\
p_5 &= \text{-}0.0261 - \text{j } 1.0013 \\
p_6 &= \text{-}0.0756 - \text{j } 0.9033 \\
p_7 &= \text{-}0.1178 - \text{j } 0.7169 \\
p_8 &= \text{-}0.1484 - \text{j } 0.4602 \\
p_9 &= \text{-}0.1645 + \text{j } 0.1586 \\
p_{10} &= \text{-}0.1645 - \text{j } 0.1586
\end{aligned}
$$

Using the above poles and the fact that N is even (we need to normalize by a factor of $\sqrt{1+D_1}$) i.e. the DC gain is just the product of poles times $\sqrt{1+D_1}$, we can write the analog low pass transfer function as :

$$H_{\text{analog}}(s) = \frac{\prod_{i=1}^{11} p_i}{\sqrt{1+D_1}\prod_{i=1}^{11}(s-p_i)}$$

The values of the coefficients of the corresponding Analog Lowpass Transfer Function is given as

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s_L^{10}	1.0650	s_L^9	3.0671	s_L^8	2.5024	s_L^7	3.2740	s_L^6	1.9568
s_L^5	1.4234	s_L^4	0.5652	s_L^3	0.2169	s_L^2	0.0439	s_L^1	0.0053

Table 5: Denominator Coefficients

2.3.6 Analog Bandstop Transfer Function

The transformation is given by :

$$s_L = \frac{Bs}{s^2 + \Omega_o^2}$$

Substituting the values of B and Ω_o

$$s_L = \frac{1.832s}{s^2 + 0.9293}$$

After substituting it can be written in the form N(s)/D(s) where the coefficients of N(s) and D(s) are given below :

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{20}	0.9300	s^{18}	8.6425	s^{16}	36.1413	s^{14}	89.5625	s^{12}	145.6527
s^{10}	162.4254	s^8	125.7844	s^6	66.7948	s^4	23.2771	s^2	4.8069
s^0	0.4467	-	-	-	-	-	-	-	-

Table 6: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{20}	1.0	s^{19}	15.1199	s^{18}	146.2702
s^{17}	780.4144	s^{16}	4074.5577	s^{15}	12323.4978
s^{14}	43525.5871	s^{13}	80805.5406	s^{12}	205220.0782
s^{11}	222934.7104	s^{10}	392707.4681	s^9	207172.3347
s^8	177226.2239	s^7	64849.0090	s^6	32460.9263
s^5	8540.9137	s^4	2624.2480	s^3	467.0934
s^2	81.3558	s^1	7.8151	s^0	0.4803

Table 7: Denominator Coefficients

2.3.7 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use ofthe Bilinear Transformation which is given as :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BSF}$(z) from $H_{analog,BSF}$(s).It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as :-

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{20}	4.5637×10^{-4}	s^{19}	-3.3449×10^{-4}	s^{18}	4.6741×10^{-3}
s^{17}	-3.0321×10^{-3}	s^{16}	2.1422×10^{-2}	s^{15}	-1.2193×10^{-2}
s^{14}	5.7871×10^{-2}	s^{13}	-2.8552×10^{-2}	s^{12}	1.0206×10^{-1}
s^{11}	-4.2904×10^{-2}	s^{10}	1.2278×10^{-1}	s^{9}	-4.2904×10^{-2}
s^{8}	1.0206×10^{-1}	s^{7}	-2.8552×10^{-2}	s^{6}	5.7871×10^{-2}
s^{5}	-1.2193×10^{-2}	s^{4}	2.1422×10^{-2}	s^{3}	-3.0321×10^{-3}
s^{2}	4.6741×10^{-3}	s^{1}	-3.3449×10^{-4}	s^{0}	4.5637×10^{-4}

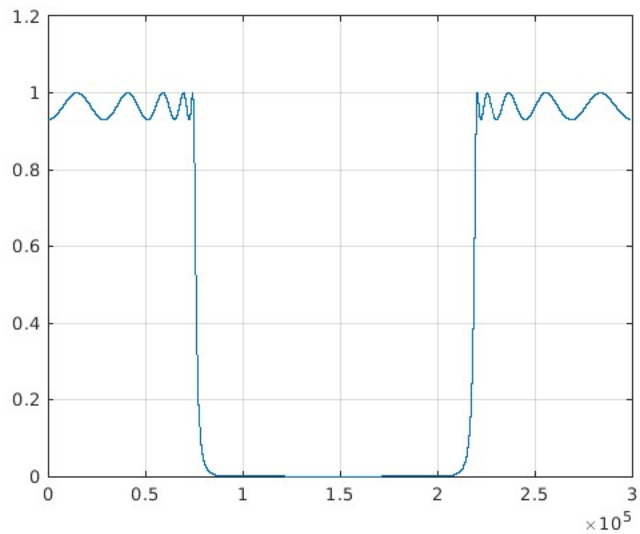
Table 8: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{20}	1.0	s^{19}	-0.2669	s^{18}	-2.6825
s^{17}	0.4631	s^{16}	5.8313	s^{15}	-0.7486
s^{14}	-8.7725	s^{13}	0.6917	s^{12}	10.6737
s^{11}	-0.4520	s^{10}	-10.3559	s^{9}	0.1059
s^{8}	8.1526	s^{7}	0.1181	s^{6}	-5.1175
s^{5}	-0.1661	s^{4}	2.4837	s^{3}	0.1015
s^{2}	-0.8586	s^{1}	-0.0339	s^{0}	0.1787

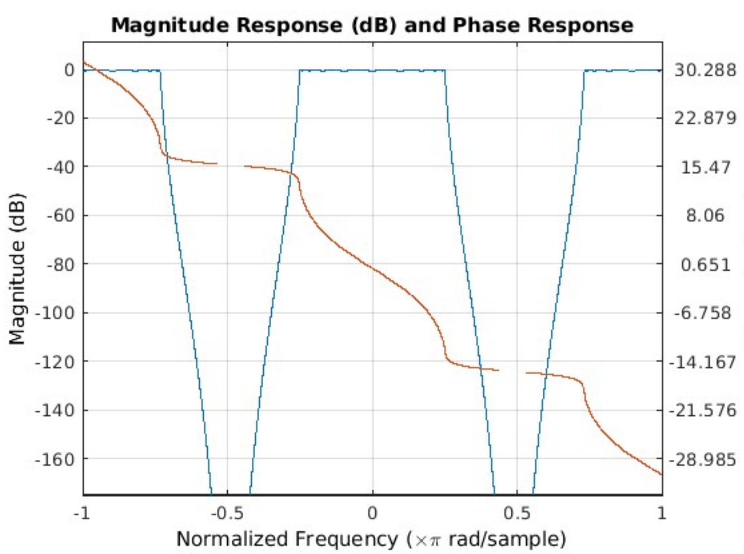
Table 9: Denominator Coefficients

2.3.8 Matlab Plots

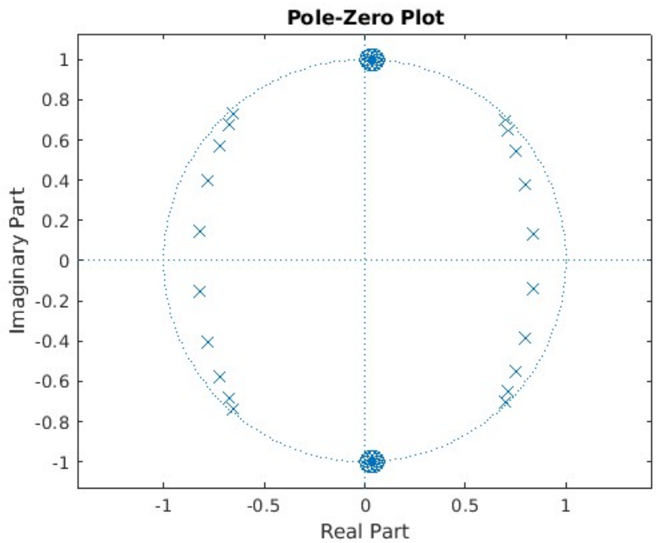
Magnitude Response



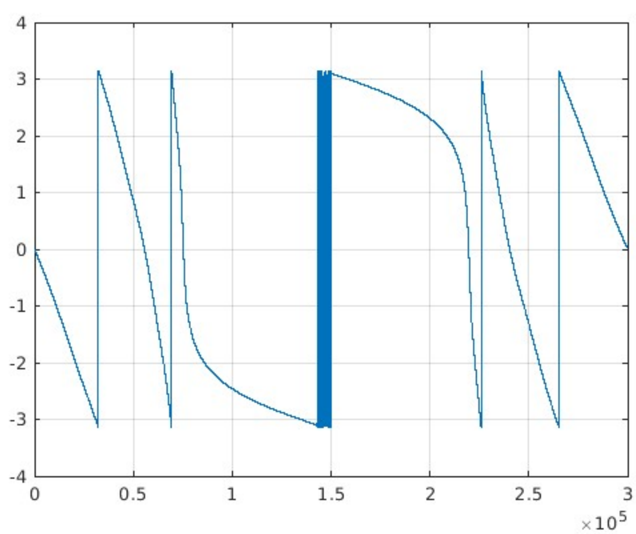
Frequency Response



Pole - Zero Plot



Unnormalized Phase response



2.4 Final Results

Using the above two designed Chebyschev filters, we can cascade them to get the desired Multiband IIR Filter. The resulting polynomial will have a degree of 20. The coefficients of the numerator and denominator of the transfer function are given below :

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{38}	6.9267×10^{-6}	s^{37}	-5.0769×10^{-6}	s^{36}	8.6012×10^{-6}
s^{35}	-3.2729×10^{-7}	s^{34}	-6.3973×10^{-5}	s^{33}	4.6343×10^{-5}
s^{32}	-7.5862×10^{-5}	s^{31}	1.9785×10^{-6}	s^{30}	0.0003
s^{29}	-0.0002	s^{28}	0.0003	s^{27}	-4.6412×10^{-6}
s^{26}	-0.0006	s^{25}	0.0004	s^{24}	-0.0007
s^{23}	4.6561×10^{-6}	s^{22}	0.001	s^{21}	-0.0007
s^{20}	0.001	s^{19}	-8.8752×10^{-19}	s^{18}	-0.001
s^{17}	0.0007	s^{16}	-0.001	s^{15}	-4.6561×10^{-6}
s^{14}	0.0007	s^{13}	-0.0004	s^{12}	0.0006
s^{11}	4.6412×10^{-6}	s^{10}	-0.0003	s^{9}	0.0002
s^{8}	-0.0003	s^{7}	-1.9785×10^{-6}	s^{6}	7.5862×10^{-5}
s^{5}	-4.6343×10^{-5}	s^{4}	6.3973×10^{-5}	s^{3}	3.2729×10^{-7}
s^{2}	-8.6012×10^{-6}	s^{1}	5.0769×10^{-6}	s^{0}	-6.9267×10^{-6}

Table 10: Numerator Coefficients

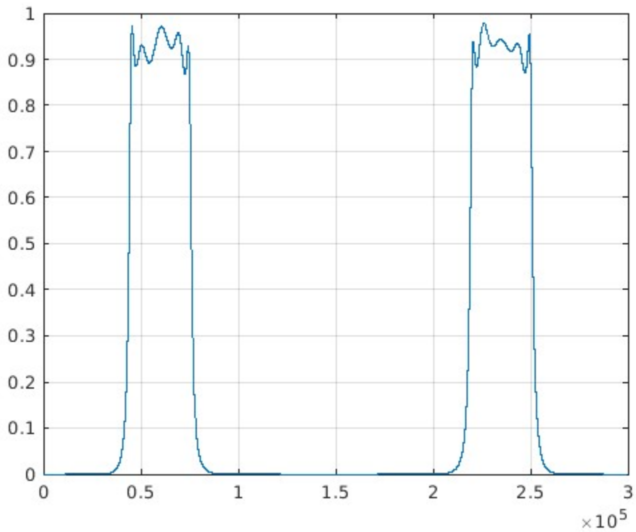
Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{38}	1	s^{37}	-0.6942	s^{36}	-3.7768
s^{35}	2.2198	s^{34}	11.2665	s^{33}	-5.9837
s^{32}	-22.9583	s^{31}	10.5732	s^{30}	39.5070
s^{29}	-16.1950	s^{28}	-54.8458	s^{27}	19.4920
s^{26}	65.9713	s^{25}	-20.6442	s^{24}	-67.0297
s^{23}	18.1318	s^{22}	59.3070	s^{21}	-14.1186
s^{20}	-44.6107	s^{19}	9.3569	s^{18}	28.6854
s^{17}	-5.6179	s^{16}	-15.0545	s^{15}	3.0354
s^{14}	6.0454	s^{13}	-1.6439	s^{12}	-1.2623
s^{11}	0.9000	s^{10}	-0.4888	s^{9}	-0.5303
s^{8}	0.7690	s^{7}	0.2795	s^{6}	-0.5012
s^{5}	-0.1385	s^{4}	0.2424	s^{3}	0.0451
s^{2}	-0.0744	s^{1}	-0.0131	s^{0}	0.0183

Table 11: Denominator Coefficients

The results of the cascaded Filters are as follows

2.4.1 Matlab Plots

Frequency Response



Magnitude Response

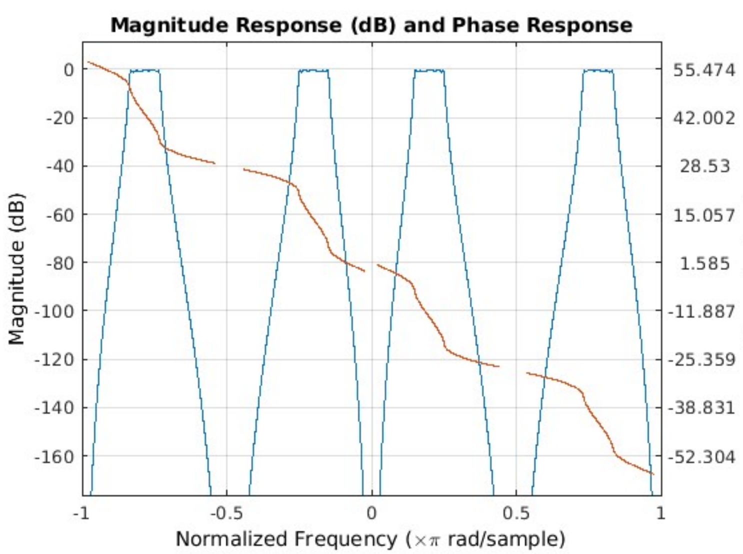


Figure 3:

Pole - Zero Plot

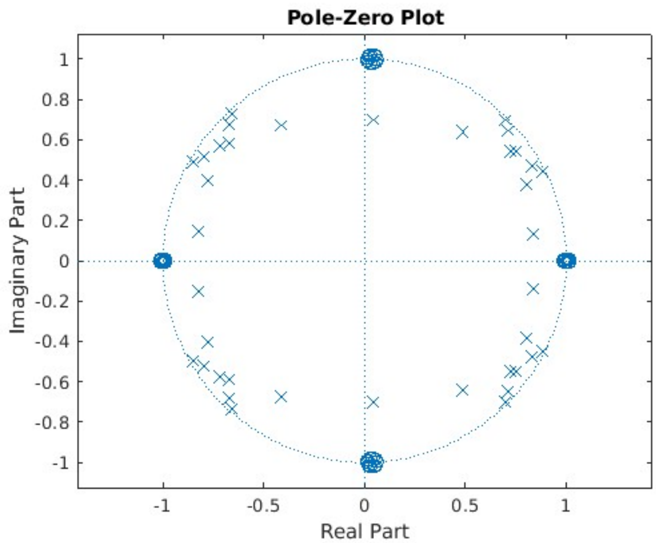


Figure 4:

Unnormalized Phase response