EE338 : Digital Signal Processing
Filter Design Assignment

1 Student Details

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Filter Number : 21

2 IIR Multi-Band pass Filter

2.1 Un-normalized Discrete Time Filter Specifications

Filter Number M = 21
M = 11Q + R
Q = Quotient when M is divided by 11 = 1
R = Remainder when M is divided by 11 = 10
Passband 1 specifications:
B_L(m) = 40 + 5Q = 40 + 5*1 = 45KHz
B_H(m) = 70 + 5Q = 70 + 5 = 75KHz
Passband 2 specifications:
B_L(m) = 170 + 5R = 170 + 5*10 = 220KHz
B_H(m) = 200 + 5R = 200 + 50 = 250KHz
Therefore the specifications of the **Multi-Band pass** Filter are:

- Passband : **45 - 75 KHz** and **220 - 250 KHz**
- Stopband : **0 - 40 KHz**, **80 - 215 KHz** and **255 - 300 KHz** (As **sampling rate** is **600 KHz**)
- Transition band : **5KHz** on either side of the passband and stopband
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Both passband and stopband are **monotonic**

Sampling Rate = **600 KHz**

To design such a filter, we will cascade two filters, a Bandpass and a Bandstop filter, each of them being butterworth filters. The specifications of these two filters are mentioned below:

2.2 Bandpass Butterworth Filter

2.2.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Passband : **45 - 250 KHz**
- Stopband : **0 - 40 KHz** and **255 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Both passband and stopband are **monotonic**

2.2.2 Normalized Digital Filter Specifications

In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Passband : **0.15 - 0.833** π
- Stopband : **0 - 0.133** π and **0.85 - 1** π
- Transition band : **0.0167** π on either side of stopband

2.2.3 Bilinear Transformation

To convert to analog domain, we use the following bilinear transformation :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\Omega_{analog} = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.133 π	0.213
0.15 π	0.24
0.833 π	3.732
0.85 π	4.165
π	∞

Therefore, the corresponding specifications are :

- Passband : **0.24** (Ω_{p1}) **- 3.732** (Ω_{p2})
- Transition band : Between the passband and stopband edges
- Stopband : **0 - 0.213**(Ω_{s1}) and **4.165** (Ω_{s2}) **-** ∞

2.2.4 Frequency Transformation and Relevant Parameters

We need to convert the Band - Pass filter into a Low - Pass analog filter as we are aware of it's frequency response in order to keep monotonic passband and stopband. For that purpose we use the following frequency transformation with two parameters B and Ω_o

$$\Omega_l = \frac{\Omega^2 - \Omega_o^2}{B\Omega}$$

If we follow the convention that the passband edges are mapped to +1 and -1, the parameters, in terms of the passband edges can be obtained by solving two equations and are given by :

$$\Omega_o = \sqrt{\Omega_{p1}\Omega_{p2}} = \sqrt{0.24 * 3.732} = 0.946$$

$$B = \Omega_{p2} - \Omega_{p1} = 3.492$$

Ω	Ω_L
0^+	- ∞
0.213 (Ω_{s1})	-1.142
0.24 (Ω_{p1})	-1
0.946 (Ω_o)	0
3.732 (Ω_{p2})	1
4.165 (Ω_{s2})	1.131
∞	∞

To make the filter as close to ideal as possible we choose the more stringent stopband for the lowpass filter i.e. $\Omega_{sL} = \min(\Omega_{sL1}$, - $\Omega_{sL2}) = 1.131$. (where Ω_{sL} stands for the stopband for the lowpass filter)

Therefore the analog lowpass filter specifications are as follows:

- Passband Edge : 1 (Ω_{pL})
- Stopband Edge : 1.131 (Ω_{sL})

Analog Lowpass Transfer Function To keep the frequency response monotonic in passband and stopband we use the Butterworth approximation and let the transfer function of the analog lowpass filter to be of the form :

$$|H(j\Omega)|^2 = \frac{1}{1 + (\frac{\Omega}{\Omega_c})^{2N}}$$

where Ω_c is the bandwidth frequency and N is the number of energy elements required to realize the filter.

We define two new parameters D_1 and D_2 such that

$$D_1 = \frac{1}{(1 - \delta_1)^2} - 1 = 0.384$$

$$D_2 = \frac{1}{(\delta_2)^2} - 1 = 43.44 \tag{1}$$

where $\delta_1 = \delta_2 = 0.15$

Upon applying the condition that at the passband and stopband edges the response must lie within the tolerances specified, we get a lower bound on the value of N which is given by:

$$N \geq \frac{\log(\frac{D_2}{D_1})}{2 * \log(\frac{\Omega_s}{\Omega_p})} = 19.2$$

As we would naturally use lesser number of elements, we choose N to be the least integer above 19.2 i.e. N = 20

Based on the bound on N, we also obtain the bounds on Ω_c for a fixed N :

$$\frac{\Omega_p}{(D_1)^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_s}{(D_2)^{\frac{1}{2N}}}$$

$$1.024 \leq \Omega_c \leq 1.029$$

We choose the value of Ω_c to be the average of the two bounds i.e. $\Omega_c = (1.024 + 1.029)/2 = 1.027$. Thus we obtained the parameters for our Butterworth Lowpass filter. Thus our analog filter becomes

$$|H_{analog}(s)|^2 = \frac{1}{1+(\frac{s}{1.027j})^{40}}$$

Using MATLAB, we plot the poles of the magnitude response of the analog low pass filter. In order to get a stable Analog Low Pass Filter we include the

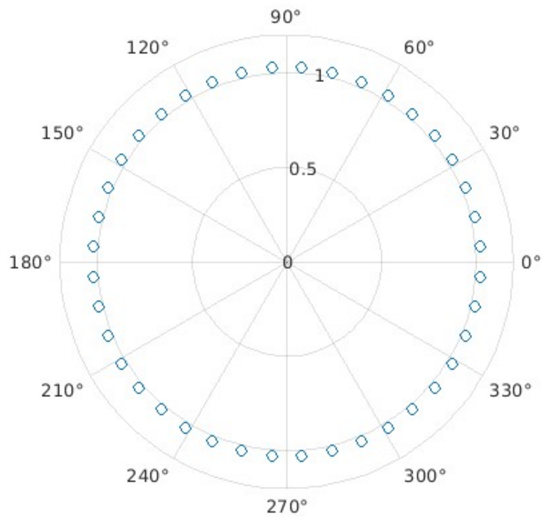


Figure 1:

poles lying in the Left Half Plane in the Transfer Function. Thus, the required poles are :

Columns 16 through 20

p_1 = -0.0806 + 1.0238i
p_2 = -0.2397 + 0.9986i
p_3 = -0.3930 + 0.9488i
p_4 = -0.5366 + 0.8757i
p_5 = -0.6670 + 0.7809i
p_6 = -0.7809 + 0.6670i
p_7 = -0.8757 + 0.5366i
p_8 = -0.9488 + 0.3930i
p_9 = -0.9986 + 0.2397i
p_{10} = -1.0238 + 0.0806i
p_{11} = -1.0238 - 0.0806i
p_{12} = -0.9986 - 0.2397i
p_{13} = -0.9488 - 0.3930i
p_{14} = -0.8757 - 0.5366i
p_{15} = -0.7809 - 0.6670i

p_{16} = -0.6670 - 0.7809i
p_{17} = -0.5366 - 0.8757i
p_{18} = -0.3930 - 0.9488i
p_{19} = -0.2397 - 0.9986i
p_{20} = -0.0806 - 1.0238i

Using the above poles, we can write the Analog Lowpass Transfer function as:

(The scaling of numerator is done to achieve a DC Gain of unity)

$$H_{analog}(s_L) = \frac{(\Omega_c)^N}{\Pi_{i=1}^{20}(s_L - p_i)}$$

The values of the coefficients of the corresponding Analog Lowpass Transfer Function is given as

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s_L^{20}	1.0×10^0	s_L^{13}	1.124×10^4	s_L^6	7.941×10^3
s_L^{19}	1.309×10^1	s_L^{12}	1.674×10^4	s_L^5	4.008×10^3
s_L^{18}	8.567×10^1	s_L^{11}	2.142×10^4	s_L^4	1.656×10^3
s_L^{17}	3.722×10^2	s_L^{10}	2.366×10^4	s_L^3	5.405×10^2
s_L^{16}	1.203×10^3	s_L^9	2.259×10^4	s_L^2	1.312×10^2
s_L^{15}	3.07×10^3	s_L^8	1.862×10^4	s_L^1	2.114×10^1
s_L^{14}	6.417×10^3	s_L^7	1.319×10^4	s_L^0	1.704×10^0

Table 1: Denominator Coefficients

2.2.5 Analog Bandpass Transfer Function

The transformation is given by :

$$s_L = \frac{s^2 + \Omega_o^2}{Bs}$$

Substituting the values of B and Ω_o

$$s_L = \frac{3.492s}{s^2 + 0.946}$$

After substituting it can be written in the form N(s)/D(s) where the coefficients of N(s) and D(s) are given below :

$$N(s) = 1.238 \times 10^{11}.s^{20}$$

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{40}	1.0×10^{0}	s^{26}	1.054×10^{12}	s^{12}	5.866×10^{10}
s^{39}	4.571×10^{1}	s^{25}	2.357×10^{12}	s^{11}	1.584×10^{10}
s^{38}	1.063×10^{3}	s^{24}	4.58×10^{12}	s^{10}	3.762×10^{9}
s^{37}	1.663×10^{4}	s^{23}	7.681×10^{12}	s^{9}	7.841×10^{8}
s^{36}	1.959×10^{5}	s^{22}	1.103×10^{13}	s^{8}	1.429×10^{8}
s^{35}	1.842×10^{6}	s^{21}	1.347×10^{13}	s^{7}	2.263×10^{7}
s^{34}	1.433×10^{7}	s^{20}	1.389×10^{13}	s^{6}	3.079×10^{6}
s^{33}	9.434×10^{7}	s^{19}	1.207×10^{13}	s^{5}	3.546×10^{5}
s^{32}	5.339×10^{8}	s^{18}	8.858×10^{12}	s^{4}	3.379×10^{4}
s^{31}	2.625×10^{9}	s^{17}	5.525×10^{12}	s^{3}	2.57×10^{3}
s^{30}	1.128×10^{10}	s^{16}	2.952×10^{12}	s^{2}	1.472×10^{2}
s^{29}	4.257×10^{10}	s^{15}	1.361×10^{12}	s^{1}	5.672×10^{0}
s^{28}	1.412×10^{11}	s^{14}	5.454×10^{11}	s^{0}	1.112×10^{-1}
s^{27}	4.12×10^{11}	s^{13}	1.91×10^{11}	-	-

Table 2: Denominator Coefficients

2.2.6 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use ofthe Bilinear Transformation which is given as :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}$(z) from $H_{analog,BPF}$(s).It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as :-

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{40}	1.436×10^{-3}	s^{26}	-1.113×10^{2}	s^{12}	5.564×10^{1}
s^{39}	0.0×10^{0}	s^{25}	0.0×10^{0}	s^{11}	0.0×10^{0}
s^{38}	-2.871×10^{-2}	s^{24}	1.808×10^{2}	s^{10}	-2.226×10^{1}
s^{37}	0.0×10^{0}	s^{23}	0.0×10^{0}	s^{9}	0.0×10^{0}
s^{36}	2.728×10^{-1}	s^{22}	-2.411×10^{2}	s^{8}	6.955×10^{0}
s^{35}	0.0×10^{0}	s^{21}	0.0×10^{0}	s^{7}	0.0×10^{0}
s^{34}	-1.637×10^{0}	s^{20}	2.652×10^{2}	s^{6}	-1.637×10^{0}
s^{33}	0.0×10^{0}	s^{19}	0.0×10^{0}	s^{5}	0.0×10^{0}
s^{32}	6.955×10^{0}	s^{18}	-2.411×10^{2}	s^{4}	2.728×10^{-1}
s^{31}	0.0×10^{0}	s^{17}	0.0×10^{0}	s^{3}	0.0×10^{0}
s^{30}	-2.226×10^{1}	s^{16}	1.808×10^{2}	s^{2}	-2.871×10^{-2}
s^{29}	0.0×10^{0}	s^{15}	0.0×10^{0}	s^{1}	0.0×10^{0}
s^{28}	5.564×10^{1}	s^{14}	-1.113×10^{2}	s^{0}	1.436×10^{-3}
s^{27}	0.0×10^{0}	s^{13}	0.0×10^{0}	-	-

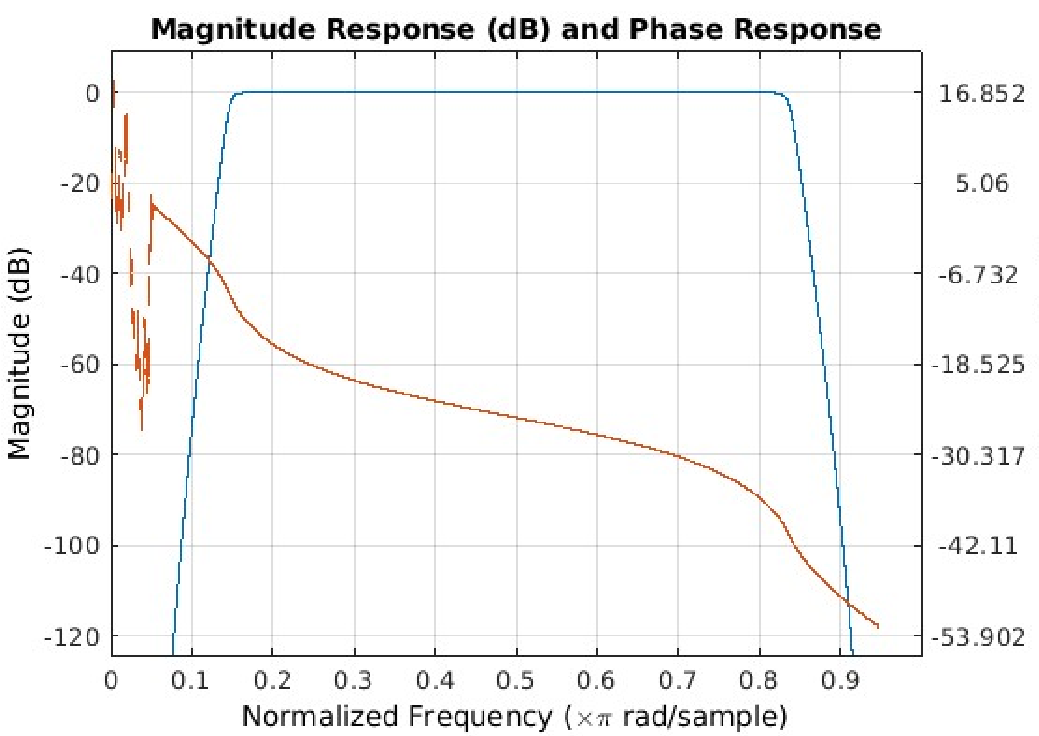
Table 3: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{40}	1.0×10^{0}	s^{26}	-2.387×10^{2}	s^{12}	1.66×10^{0}
s^{39}	-6.797×10^{-1}	s^{25}	8.806×10^{1}	s^{11}	-2.368×10^{-1}
s^{38}	-7.384×10^{0}	s^{24}	1.974×10^{2}	s^{10}	-3.873×10^{-1}
s^{37}	4.65×10^{0}	s^{23}	-6.55×10^{1}	s^{9}	4.494×10^{-2}
s^{36}	2.807×10^{1}	s^{22}	-1.373×10^{2}	s^{8}	7.163×10^{-2}
s^{35}	-1.633×10^{1}	s^{21}	4.07×10^{1}	s^{7}	-6.492×10^{-3}
s^{34}	-7.15×10^{1}	s^{20}	8.06×10^{1}	s^{6}	-1.011×10^{-2}
s^{33}	3.828×10^{1}	s^{19}	-2.117×10^{1}	s^{5}	6.713×10^{-4}
s^{32}	1.353×10^{2}	s^{18}	-3.991×10^{1}	s^{4}	1.024×10^{-3}
s^{31}	-6.644×10^{1}	s^{17}	9.199×10^{0}	s^{3}	-4.428×10^{-5}
s^{30}	-2.008×10^{2}	s^{16}	1.662×10^{1}	s^{2}	-6.626×10^{-5}
s^{29}	9.004×10^{1}	s^{15}	-3.32×10^{0}	s^{1}	1.401×10^{-6}
s^{28}	2.41×10^{2}	s^{14}	-5.779×10^{0}	s^{0}	2.061×10^{-6}
s^{27}	-9.828×10^{1}	s^{13}	9.856×10^{-1}	s^{-1}	1.0×10^{0}

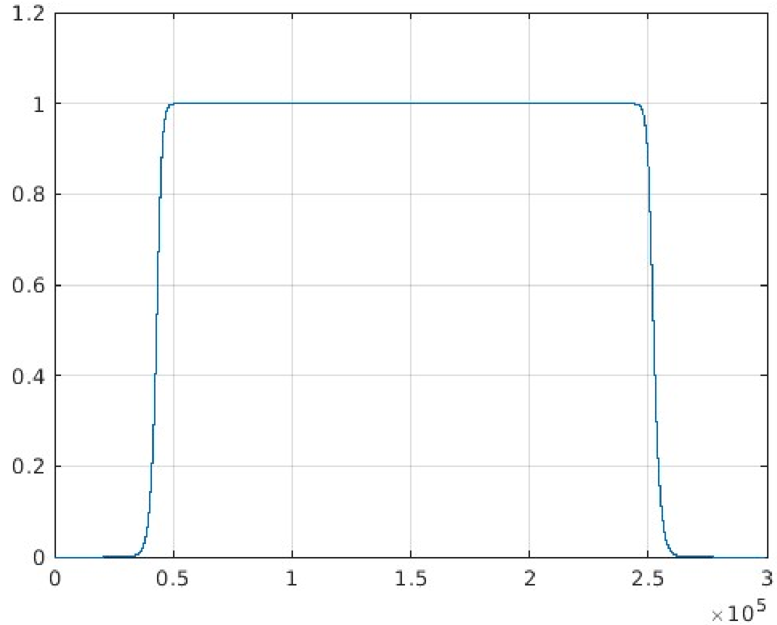
Table 4: Denominator Coefficients

2.2.7 Matlab Plots

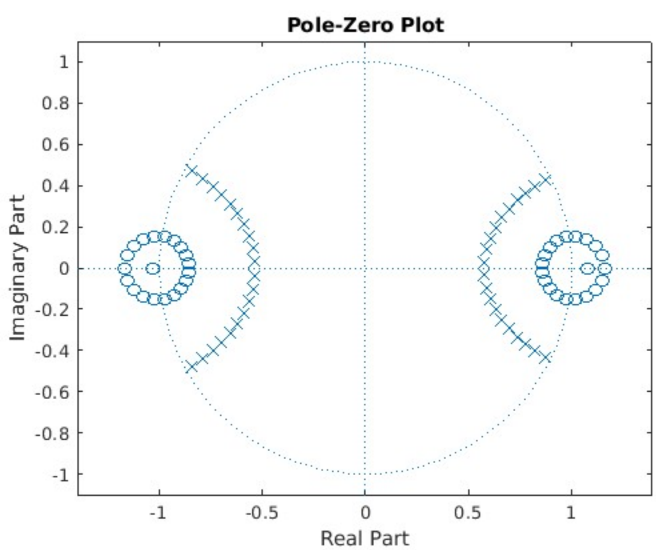
Frequency Response



Magnitude Response



Pole - Zero Plot



2.3 Bandstop Butterworth Filter

2.3.1 Un-normalized Discrete Time Filter Specifications

The specifications of this filter are :

- Stopband : **80 - 215 KHz**
- Passband : **0 - 75 KHz** and **220 - 300 KHz**
- Transition band : **5KHz** on either side of passband
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Both passband and stopband are **monotonic**

2.3.2 Normalized Digital Filter Specifications

In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Stopband : **0.267 - 0.717** π
- Passband : **0 - 0.25** π and **0.733 - 1** π
- Transition band : **0.0167** π on either side of stopband

2.3.3 Bilinear Transformation

To convert to analog domain, we use the following bilinear transformation :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\Omega_{analog} = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.25 π	0.414
0.267 π	0.445
0.717 π	2.097
0.733 π	2.246
π	∞

Therefore, the corresponding specifications are :

- Stopband : **0.445** (Ω_{s1}) **- 2.097** (Ω_{s2})
- Transition band : Between the passband and stopband edges
- Passband : **0 - 0.414**(Ω_{p1}) and **2.246** (Ω_{p2}) **-** ∞

2.3.4 Frequency Transformation and Relevant Parameters

We need to convert the Band - Stop filter into a Low - Pass analog filter as we are aware of it's frequency response in order to keep monotonic passband and stopband. For that purpose we use the following frequency transformation with two parameters B and Ω_o

$$\Omega_l = \frac{B\Omega}{\Omega_o^2 - \Omega^2}$$

If we follow the convention that the passband edges are mapped to +1 and -1, the parameters, in terms of the passband edges can be obtained by solving two equations and are given by :

$$\Omega_o = \sqrt{\Omega_{p1}\Omega_{p2}} = \sqrt{0.414 * 2.246} = 0.964$$

$$B = \Omega_{p2} - \Omega_{p1} = 1.832$$

Ω	Ω_L
0^+	0^+
0.414 (Ω_{p1})	+1
0.445 (Ω_{s1})	1.115
0.946^- (Ω_o)	∞
0.946^+ (Ω_o)	$-\infty$
2.097 (Ω_{s2})	-1.108
2.246 (Ω_{p2})	-1
∞	0^-

To make the filter as close to ideal as possible we choose the more stringent stopband for the lowpass filter i.e. $\Omega_{sL} = \min(\Omega_{sL1}$, - $\Omega_{sL2}) = 1.108$. (where Ω_{sL} stands for the stopband for the lowpass filter)

Therefore the analog lowpass filter specifications are as follows:

- Passband Edge : 1 (Ω_{pL})
- Stopband Edge : 1.108 (Ω_{sL})

Analog Lowpass Transfer Function To keep the frequency response monotonic in passband and stopband we use the Butterworth approximation and let the transfer function of the analog lowpass filter to be of the form :

$$|H(j\Omega)|^2 = \frac{1}{1 + (\frac{\Omega}{\Omega_c})^{2N}}$$

where Ω_c is the bandwidth frequency and N is the number of energy elements required to realize the filter.

We define two new parameters D_1 and D_2 such that

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$$

$$D_2 = \frac{1}{(\delta_2)^2} - 1 = 43.44 \qquad (2)$$

where $\delta_1 = \delta_2 = 0.15$

Upon applying the condition that at the passband and stopband edges the response must lie within the tolerances specified, we get a lower bound on the value of N which is given by:

$$N \geq \frac{\log(\frac{D_2}{D_1})}{2 * \log(\frac{\Omega_s}{\Omega_p})} = 23.05$$

As we would naturally use lesser number of elements, we choose N to be the least integer above 23.05 i.e. N = 24

Based on the bound on N, we also obtain the bounds on Ω_c for a fixed N :

$$\frac{\Omega_p}{(D_1)^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_s}{(D_2)^{\frac{1}{2N}}}$$

$$1.020 \leq \Omega_c \leq 1.024$$

We choose the value of Ω_c to be the average of the two bounds i.e. $\Omega_c = (1.020 + 1.024)/2 = 1.022$. Thus we obtained the parameters for our Butterworth Lowpass filter. Thus our analog filter becomes

$$|H_{analog}(s)|^2 = \frac{1}{1 + (\frac{s}{1.022j})^{48}}$$

Using MATLAB, we plot the poles of the magnitude response of the analog low pass filter. In order to get a stable Analog Low Pass Filter we include the

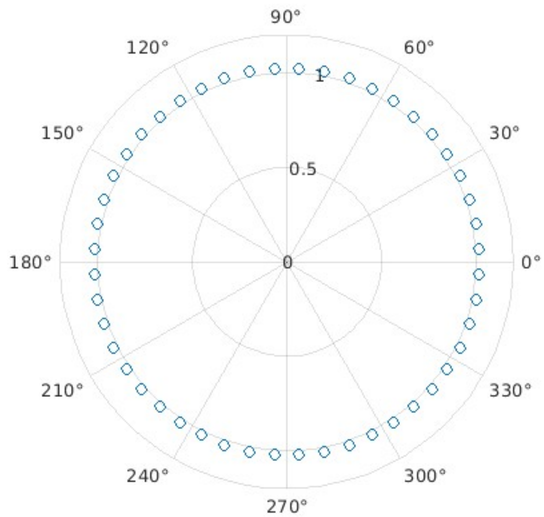


Figure 2:

poles lying in the Left Half Plane in the Transfer Function. Thus, the required poles are :

Columns 16 through 20

$$p_0 = \text{-0.0668 + j 1.0198}$$
$$p_1 = \text{-0.1994 + j 1.0024}$$
$$p_2 = \text{-0.3285 + j 0.9678}$$
$$p_3 = \text{-0.452 + j 0.9166}$$
$$p_4 = \text{-0.5678 + j 0.8498}$$
$$p_5 = \text{-0.6739 + j 0.7684}$$
$$p_6 = \text{-0.7684 + j 0.6739}$$
$$p_7 = \text{-0.8498 + j 0.5678}$$
$$p_8 = \text{-0.9166 + j 0.452}$$
$$p_9 = \text{-0.9678 + j 0.3285}$$
$$p_{10} = \text{-1.0024 + j 0.1994}$$
$$p_{11} = \text{-1.0198 + j 0.0668}$$
$$p_{12} = \text{-1.0198 + j 0.0668}$$
$$p_{13} = \text{-1.0024 + j 0.1994}$$
$$p_{14} = \text{-0.9678 + j 0.3285}$$

$$p_{15} = \text{-0.9166 + j 0.452}$$
$$p_{16} = \text{-0.8498 + j 0.5678}$$
$$p_{17} = \text{-0.7684 + j 0.6739}$$
$$p_{18} = \text{-0.6739 + j 0.7684}$$
$$p_{19} = \text{-0.5678 + j 0.8498}$$
$$p_{20} = \text{-0.452 + j 0.9166}$$
$$p_{21} = \text{-0.3285 + j 0.9678}$$
$$p_{22} = \text{-0.1994 + j 1.0024}$$
$$p_{23} = \text{-0.0668 + j 1.0198}$$

Using the above poles, we can write the Analog Lowpass Transfer function as:

(The scaling of numerator is done to achieve a DC Gain of unity)

$$H_{analog}(s_L) = \frac{(\Omega_c)^N}{\Pi_{i=1}^{24}(s_L - p_i)}$$

The values of the coefficients of the corresponding Analog Lowpass Transfer Function is given as

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s_L^{24}	1.0×10^0	s_L^{20}	7.542×10^3	s_L^{15}	1.66×10^5	s_L^{10}	1.355×10^5	s_L^5	3.479×10^3
s_L^{23}	1.563×10^1	s_L^{19}	1.907×10^4	s_L^{14}	2.041×10^5	s_L^9	8.884×10^4	s_L^4	9.382×10^2
s_L^{22}	1.221×10^2	s_L^{18}	4.072×10^4	s_L^{13}	2.218×10^5	s_L^8	5.062×10^4	s_L^3	1.887×10^2
s_L^{21}	6.341×10^2	s_L^{17}	7.464×10^4	s_L^{12}	2.132×10^5	s_L^7	2.476×10^4	s_L^2	2.522×10^1
s_L^{20}	2.456×10^3	s_L^{16}	1.189×10^5	s_L^{11}	1.811×10^5	s_L^6	1.023×10^4	s_L^1	1.686×10^0

Table 5: Denominator Coefficients

2.3.5 Analog Bandpass Transfer Function

The transformation is given by :

$$s_L = \frac{Bs}{s^2 + \Omega_o^2}$$

Substituting the values of B and Ω_o

$$s_L = \frac{1.832s}{s^2 + 0.964}$$

After substituting it can be written in the form N(s)/D(s) where the coefficients of N(s) and D(s) are given below :

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{48}	1.0×10^{0}	s^{38}	2.962×10^{4}	s^{28}	9.527×10^{5}	s^{18}	4.427×10^{5}	s^{8}	2.507×10^{3}
s^{47}	0.0×10^{0}	s^{37}	0.0×10^{0}	s^{27}	0.0×10^{0}	s^{17}	0.0×10^{0}	s^{7}	0.0×10^{0}
s^{46}	2.233×10^{1}	s^{36}	8.727×10^{4}	s^{26}	1.128×10^{6}	s^{16}	2.316×10^{5}	s^{6}	4.443×10^{2}
s^{45}	0.0×10^{0}	s^{35}	0.0×10^{0}	s^{25}	0.0×10^{0}	s^{15}	0.0×10^{0}	s^{5}	0.0×10^{0}
s^{44}	2.389×10^{2}	s^{34}	2.088×10^{5}	s^{24}	1.137×10^{6}	s^{14}	1.014×10^{5}	s^{4}	5.637×10^{1}
s^{43}	0.0×10^{0}	s^{33}	0.0×10^{0}	s^{23}	0.0×10^{0}	s^{13}	0.0×10^{0}	s^{3}	0.0×10^{0}
s^{42}	1.63×10^{3}	s^{32}	4.128×10^{5}	s^{22}	9.763×10^{5}	s^{12}	3.669×10^{4}	s^{2}	4.56×10^{0}
s^{41}	0.0×10^{0}	s^{31}	0.0×10^{0}	s^{21}	0.0×10^{0}	s^{11}	0.0×10^{0}	s^{1}	0.0×10^{0}
s^{40}	7.96×10^{3}	s^{30}	6.827×10^{5}	s^{20}	7.137×10^{5}	s^{10}	1.078×10^{4}	s^{0}	1.768×10^{-1}
s^{39}	0.0×10^{0}	s^{29}	0.0×10^{0}	s^{19}	0.0×10^{0}	s^{9}	0.0×10^{0}	-	-

Table 6: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{48}	1.0×10^{0}	s^{38}	2.421×10^{8}	s^{28}	2.47×10^{11}	s^{18}	7.78×10^{10}	s^{8}	6.384×10^{6}
s^{47}	2.741×10^{1}	s^{37}	7.173×10^{8}	s^{27}	3.145×10^{11}	s^{17}	4.474×10^{10}	s^{7}	1.451×10^{6}
s^{46}	3.979×10^{2}	s^{36}	1.933×10^{9}	s^{26}	3.698×10^{11}	s^{16}	2.372×10^{10}	s^{6}	2.901×10^{5}
s^{45}	4.007×10^{3}	s^{35}	4.757×10^{9}	s^{25}	4.017×10^{11}	s^{15}	1.159×10^{10}	s^{5}	5.021×10^{4}
s^{44}	3.116×10^{4}	s^{34}	1.072×10^{10}	s^{24}	4.031×10^{11}	s^{14}	5.208×10^{9}	s^{4}	7.352×10^{3}
s^{43}	1.98×10^{5}	s^{33}	2.22×10^{10}	s^{23}	3.737×10^{11}	s^{13}	2.15×10^{9}	s^{3}	8.796×10^{2}
s^{42}	1.064×10^{6}	s^{32}	4.227×10^{10}	s^{22}	3.201×10^{11}	s^{12}	8.127×10^{8}	s^{2}	8.125×10^{1}
s^{41}	4.951×10^{6}	s^{31}	7.416×10^{10}	s^{21}	2.532×10^{11}	s^{11}	2.806×10^{8}	s^{1}	5.207×10^{0}
s^{40}	2.027×10^{7}	s^{30}	1.2×10^{11}	s^{20}	1.85×10^{11}	s^{10}	8.808×10^{7}	s^{0}	1.768×10^{-1}
s^{39}	7.39×10^{7}	s^{29}	1.792×10^{11}	s^{19}	1.249×10^{11}	s^{9}	2.502×10^{7}	-	-

Table 7: Denominator Coefficients

2.3.6 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use ofthe Bilinear Transformation which is given as :

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}$(z) from $H_{analog,BPF}$(s).It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as :-

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{48}	1.982×10^{-6}	s^{38}	1.057×10^{-1}	s^{28}	5.377×10^{0}	s^{18}	3.547×10^{0}	s^{8}	2.555×10^{-2}
s^{47}	-3.433×10^{-6}	s^{37}	-1.247×10^{-1}	s^{27}	-4.383×10^{0}	s^{17}	-1.863×10^{0}	s^{7}	-6.399×10^{-3}
s^{46}	5.041×10^{-5}	s^{36}	3.445×10^{-1}	s^{26}	6.886×10^{0}	s^{16}	1.966×10^{0}	s^{6}	4.68×10^{-3}
s^{45}	-8.046×10^{-5}	s^{35}	-3.778×10^{-1}	s^{25}	-5.188×10^{0}	s^{15}	-9.253×10^{-1}	s^{5}	-9.003×10^{-4}
s^{44}	6.102×10^{-4}	s^{34}	9.074×10^{-1}	s^{24}	7.476×10^{0}	s^{14}	9.074×10^{-1}	s^{4}	6.102×10^{-4}
s^{43}	-9.003×10^{-4}	s^{33}	-9.253×10^{-1}	s^{23}	-5.188×10^{0}	s^{13}	-3.778×10^{-1}	s^{3}	-8.046×10^{-5}
s^{42}	4.68×10^{-3}	s^{32}	1.966×10^{0}	s^{22}	6.886×10^{0}	s^{12}	3.445×10^{-1}	s^{2}	5.041×10^{-5}
s^{41}	-6.399×10^{-3}	s^{31}	-1.863×10^{0}	s^{21}	-4.383×10^{0}	s^{11}	-1.247×10^{-1}	s^{1}	-3.433×10^{-6}
s^{40}	2.555×10^{-2}	s^{30}	3.547×10^{0}	s^{20}	5.377×10^{0}	s^{10}	1.057×10^{-1}	s^{0}	1.982×10^{-6}
s^{39}	-3.243×10^{-2}	s^{29}	-3.12×10^{0}	s^{19}	-3.12×10^{0}	s^{9}	-3.243×10^{-2}	s^{-1}	1.0×10^{0}

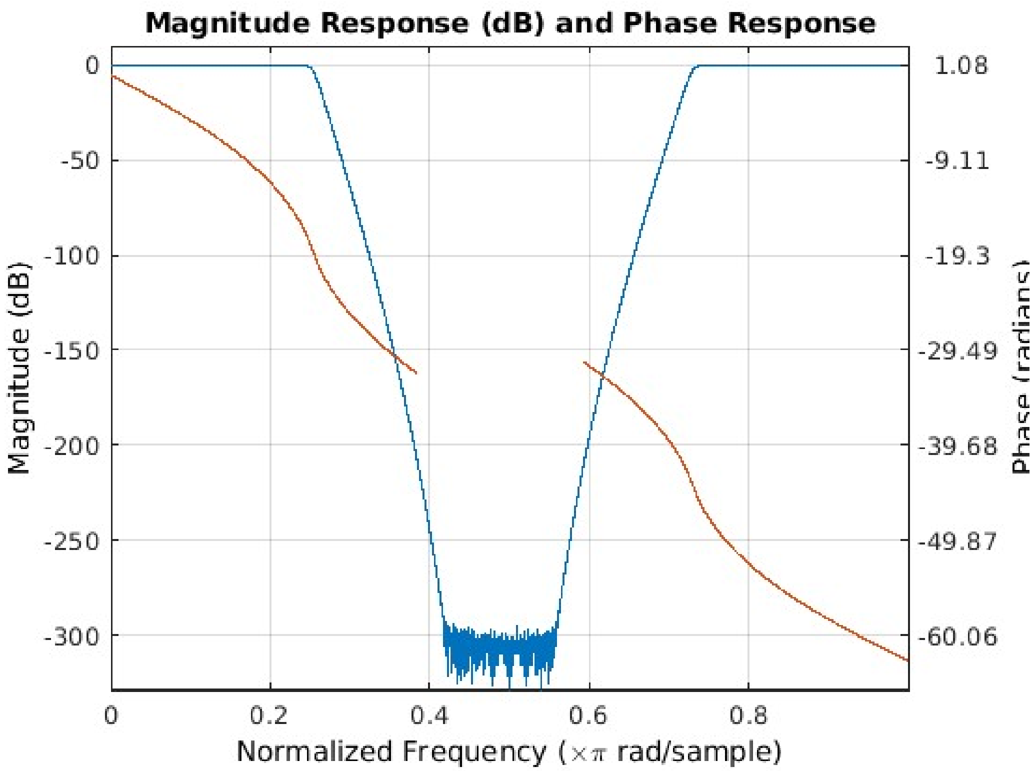
Table 8: Numerator Coefficients

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
s^{48}	1.0×10^{0}	s^{38}	6.746×10^{0}	s^{28}	1.115×10^{0}	s^{18}	6.13×10^{-3}	s^{8}	8.657×10^{-7}
s^{47}	-9.069×10^{-1}	s^{37}	-4.768×10^{0}	s^{27}	-5.797×10^{-1}	s^{17}	-2.25×10^{-3}	s^{7}	-1.555×10^{-7}
s^{46}	1.529×10^{0}	s^{36}	6.983×10^{0}	s^{26}	4.767×10^{-1}	s^{16}	1.545×10^{-3}	s^{6}	7.233×10^{-8}
s^{45}	-1.331×10^{0}	s^{35}	-4.571×10^{0}	s^{25}	-2.382×10^{-1}	s^{15}	-5.007×10^{-4}	s^{5}	-1.04×10^{-8}
s^{44}	4.497×10^{0}	s^{34}	4.749×10^{0}	s^{24}	2.101×10^{-1}	s^{14}	2.962×10^{-4}	s^{4}	4.887×10^{-9}
s^{43}	-3.643×10^{0}	s^{33}	-3.015×10^{0}	s^{23}	-9.567×10^{-2}	s^{13}	-8.823×10^{-5}	s^{3}	-4.711×10^{-10}
s^{42}	5.059×10^{0}	s^{32}	3.598×10^{0}	s^{22}	7.053×10^{-2}	s^{12}	5.364×10^{-5}	s^{2}	1.91×10^{-10}
s^{41}	-3.968×10^{0}	s^{31}	-2.108×10^{0}	s^{21}	-3.059×10^{-2}	s^{11}	-1.365×10^{-5}	s^{1}	-9.941×10^{-12}
s^{40}	7.798×10^{0}	s^{30}	1.941×10^{0}	s^{20}	2.374×10^{-2}	s^{10}	7.192×10^{-6}	s^{0}	4.038×10^{-12}
s^{39}	-5.679×10^{0}	s^{29}	-1.099×10^{0}	s^{19}	-9.271×10^{-3}	s^{9}	-1.614×10^{-6}	s^{-1}	1.0×10^{0}

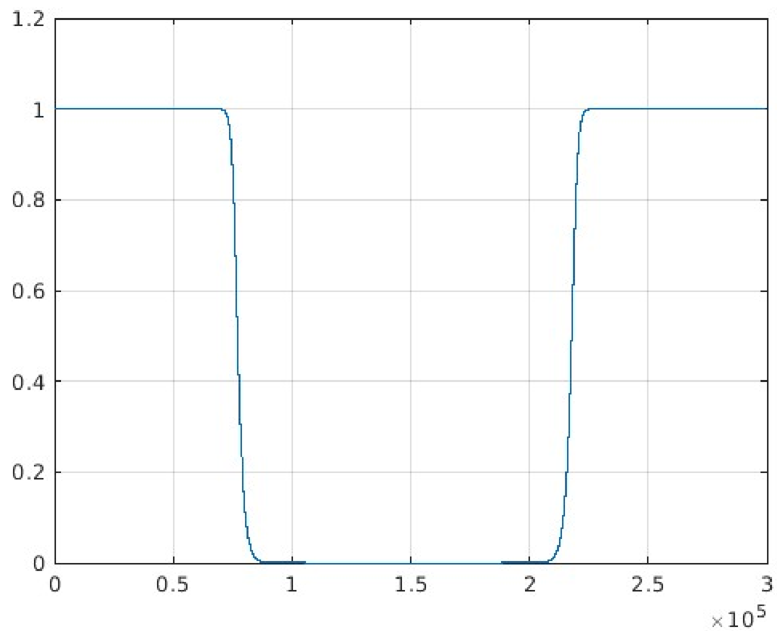
Table 9: Denominator Coefficients

2.3.7 Matlab Plots

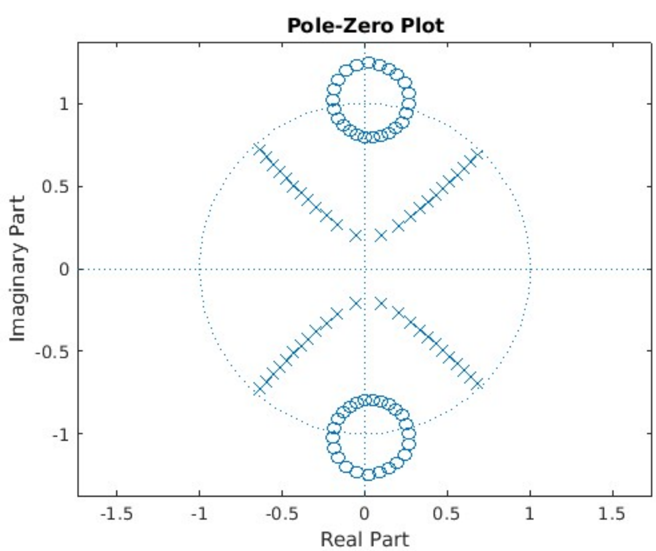
Frequency Response



Magnitude Response



Pole - Zero Plot

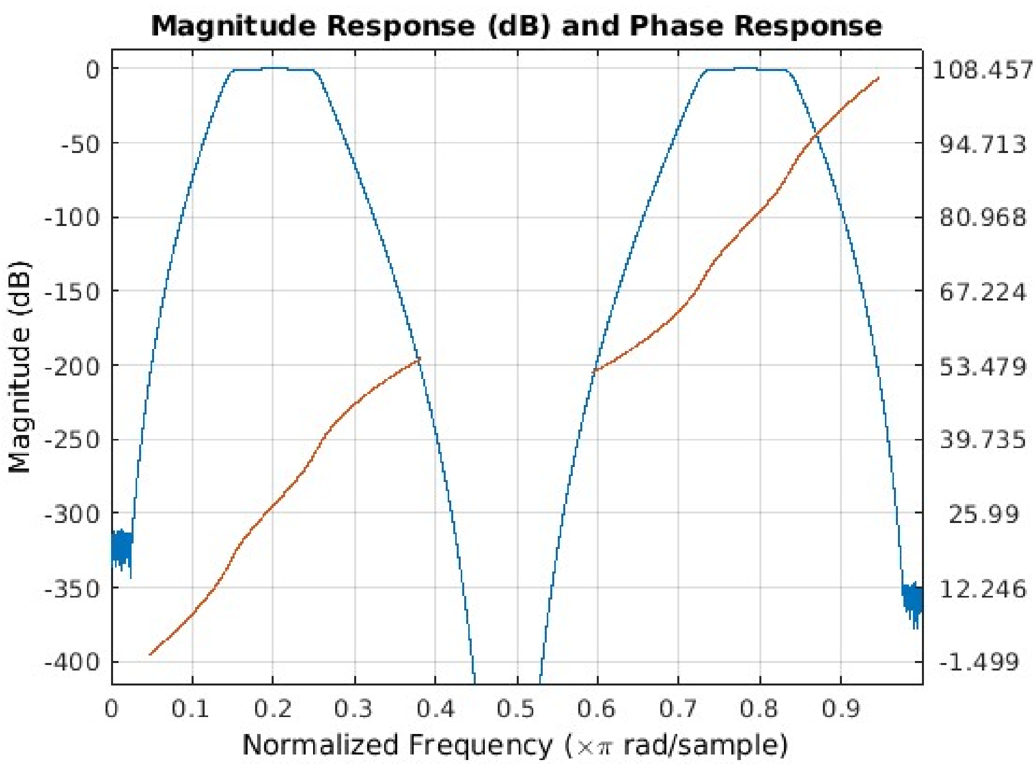


2.4 Final Results

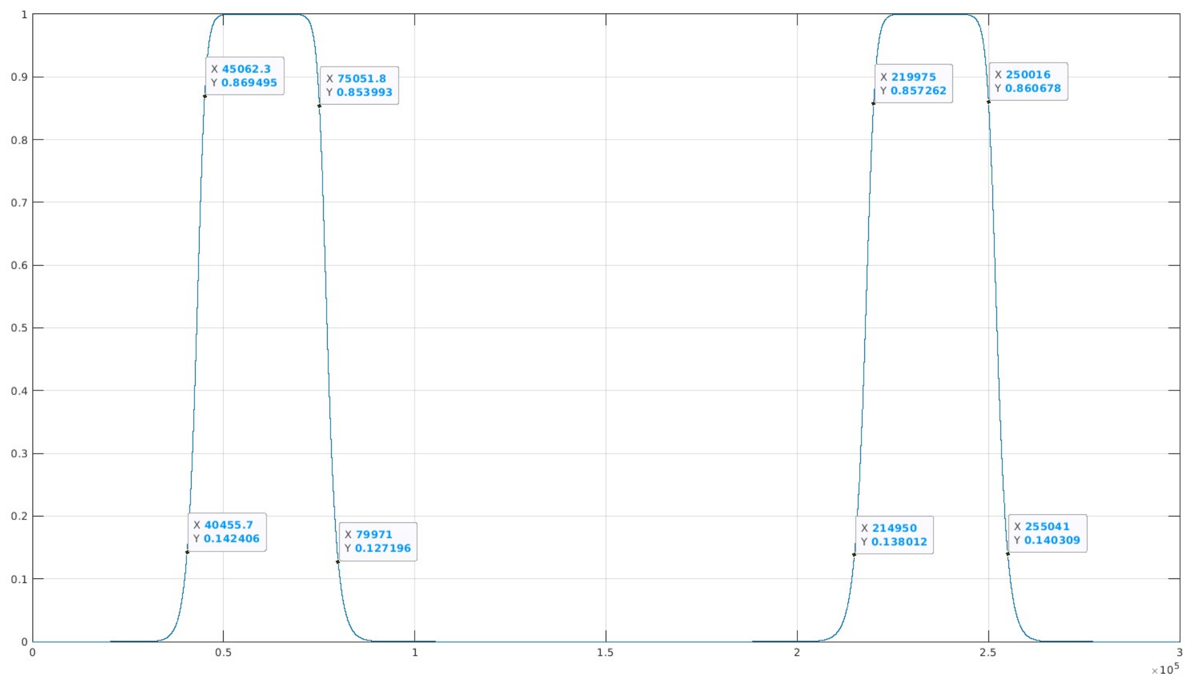
Using the above two designed butterworth filters, we can cascade them to get the desired Multiband IIR Filter. The discrete Transfer function resulting has s^{89} as the highest order in numerator and the denominator, therefore I am omitting their coefficients. The results of the cascaded Filters are as follows

2.4.1 Matlab Plots

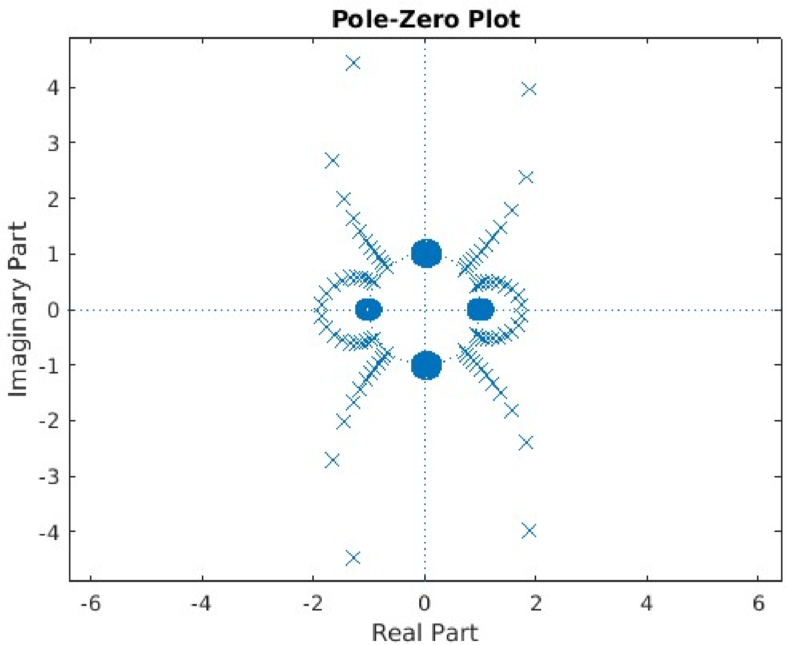
Frequency Response



Magnitude Response



Pole - Zero Plot


Pole-Zero Plot
Imaginary Part
Real Part
4
3
2
1
0
-1
-2
-3
-4
-6
-4
-2
0
2
4
6