

Predictive Compression Dynamics: A Methodological Framework for Computable Information-Motivated Modeling

Mats Helander¹ and Jeeves¹

¹Independent Research

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Abstract

We present *Predictive Compression Dynamics* (PCD), a methodological recipe for constructing *computable*, local functionals $\hat{\Phi}$ and driving dynamics by gradient flow $\dot{x} = -\nabla\hat{\Phi}(x)$ with preregistered parameters. Two concrete instances are given: (i) a fixed-graph pair functional and (ii) a smooth compact-support kernel; both yield an attractive inverse-square two-body form (after calibration) and admit Lyapunov descent. These models serve as methodological demonstrations of computable, information-motivated dynamics. We make the MDL split $L_{\text{tot}} = L(M) + L(D \mid M)$ explicit, give a minimal coding scheme linking $-\hat{\Phi}$ descent to achievable ΔL_{tot} , address well-posedness (smooth-kernel variant), recommend robust integrators (BAOAB for Langevin), and provide a preregistration/model-card template and falsifiers for a chosen model instance. The goal is a reproducible toolbox for compression-driven dynamics across domains.

1 Positioning and Commitments

A disciplined workflow to construct and test *computable* local functionals $\hat{\Phi}$ whose gradients define dynamics, with explicit preregistration (domain, discretization, kernels, parameters) and sanity checks.

2 Operational Domain and Notation

We consider N point *particles* with positions $x_i \in \mathbb{R}^3$ and positive weights m_i . Computations use finite precision: lattice spacing a_{grid} and b bits/axis, stated *a priori*. A global calibration constant $G_{\text{eff}} > 0$ maps dimensionless forces to physical units. *Design choice*: we use the same m_i in the interaction and as inertial mass; this incidentally yields accelerations independent of m_i by construction.

3 Model–Data Decomposition and Coding Link

Following MDL, we split description length as

$$L_{\text{tot}} = L(M) + L(D \mid M), \quad (3.1)$$

where $L(M)$ encodes modeled regularities and $L(D \mid M)$ encodes residuals given M . A decrease $\Delta L_{\text{tot}} < 0$ corresponds to realized compression. PCD treats a computable, local $\hat{\Phi}$ as a proxy for (the negative of) an achievable ΔL_{tot} ; thus $\dot{x} = -\nabla\hat{\Phi}$ implements a descent in achievable codelength under the chosen surrogate. This identification is heuristic and intended only as a modelling analogy.

Minimal explicit coding scheme. Let (i, j) range over a symmetric set of “near” pairs. A two-part code describes (i) a shared pairwise template per distance bin and (ii) residual offsets:

- Partition distances into bins $\{B_k\}$ with centers r_k ; encode the histogram counts with a prefix-free code.
- For each pair (i, j) with $r_{ij} \in B_k$, encode a residual offset relative to a shared template; the expected residual codelength per pair is a decreasing function $\ell(r_{ij})$.

Then, for fixed binning overhead and under mild regularity,

$$L_{\text{tot}} \approx \text{const} + \sum_{(i,j)} \ell(r_{ij}). \quad (3.2)$$

Choosing $\hat{\Phi} \propto -\sum_{(i,j)} \ell(r_{ij})$ makes $-\nabla \hat{\Phi}$ a proxy for the gradient of achievable compression. A smooth choice $\ell(r) \approx (r^2 + a^2)^{-1/2}$ yields closed-form forces below.

Proposition 3.1 (Surrogate MDL Descent). *Suppose $L_{\text{tot}} = \text{const} + \sum_{(i,j)} \ell(r_{ij})$ with $\ell'(r) \leq 0$ and $\hat{\Phi} = -\kappa \sum_{(i,j)} \ell(r_{ij})$ for some $\kappa > 0$. Then along $\dot{x} = -\nabla \hat{\Phi}(x)$ we have $\frac{d}{dt} L_{\text{tot}}(x(t)) \leq 0$, with equality iff $\nabla \hat{\Phi}(x(t)) = 0$.*

Sketch. $\frac{d}{dt} L_{\text{tot}} = \langle \nabla L_{\text{tot}}, \dot{x} \rangle = -\kappa^{-1} \langle \nabla \hat{\Phi}, \nabla \hat{\Phi} \rangle \leq 0$.

4 Information-Motivated Functionals

4.1 Fixed-graph functional (corrected notation)

Let $E \subset \{(i, j) : 1 \leq i < j \leq N\}$ be a symmetric, degree-bounded edge set. Define

$$\hat{\Phi}_E(x) = - \sum_{(i,j) \in E} \frac{m_i m_j}{\sqrt{\|x_i - x_j\|^2 + a^2}}, \quad a > 0. \quad (4.1)$$

The minus sign ensures attraction under descent; a regularizes collisions. The force on i is

$$F_i^{(E)}(x) = -\nabla_{x_i} \hat{\Phi}_E(x) = - \sum_{\substack{j: \\ (i,j) \in E}} m_i m_j \frac{(x_i - x_j)}{(\|x_i - x_j\|^2 + a^2)^{3/2}}. \quad (4.2)$$

Two-body, $a \rightarrow 0$, $(i, j) \in E$ gives the attractive inverse-square form:

$$F_i^{(E)} \rightarrow -m_i m_j \frac{x_i - x_j}{\|x_i - x_j\|^3}. \quad (4.3)$$

4.2 Smooth-kernel functional (well-posedness)

To avoid neighbor-set discontinuities, choose a compactly supported, C^1 radial kernel $K_\sigma : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$ with support $\subset [0, R\sigma]$. Define

$$\hat{\Phi}_K(x) = - \sum_{i < j} m_i m_j K_\sigma(\|x_i - x_j\|), \quad (4.4)$$

so $F_i^{(K)}(x) = -\nabla_{x_i} \hat{\Phi}_K(x)$ is continuous and locally Lipschitz off collisions. If $K_\sigma(r) \sim (r^2 + a^2)^{-1/2}$ near $r = 0$, one recovers the regularized two-body form (4.3).

5 Dynamics and Integrators

With $\dot{x} = -\nabla\hat{\Phi}(x)$,

$$\frac{d}{dt}\hat{\Phi}(x(t)) = -\|\nabla\hat{\Phi}(x(t))\|^2 \leq 0, \quad (5.1)$$

so $\hat{\Phi}$ is a Lyapunov function. We preregister all parameters ($a_{\text{grid}}, b, a, \sigma, \Delta t, m_i, \gamma, T, \text{seeds}$).

Lemma 5.1 (Compression-Rate Identity). *Under $\dot{x} = -\nabla\hat{\Phi}(x)$ the quantity $\dot{C}_{\text{alg}}(t) := -\frac{d}{dt}\hat{\Phi}(x(t))$ equals $\|\nabla\hat{\Phi}(x(t))\|^2 \geq 0$. Hence $\hat{\Phi}$ is a Lyapunov function and \dot{C}_{alg} is the instantaneous achievable compression rate.*

Deterministic gradient flow. Explicit Euler:

$$x_i^{(t+\Delta t)} = x_i^{(t)} + \Delta t F_i(x^{(t)}), \quad F_i \in \{F_i^{(E)}, F_i^{(K)}\}. \quad (5.2)$$

For stability, use adaptive Δt or semi-implicit variants.

Underdamped Langevin (BAOAB recommended).

$$m_i \ddot{x}_i = F_i(x) - \gamma \dot{x}_i + \xi_i(t), \quad \langle \xi_i(t) \xi_j(t') \rangle = 2\gamma k_B T \delta_{ij} \delta(t - t'). \quad (5.3)$$

We recommend the BAOAB integrator with reported weak/strong orders.

6 Sanity Checks

With $a \rightarrow 0$ and a single pair, (4.3) holds (after one calibration G_{eff}). For $r \gg a$,

$$\frac{r}{(r^2 + a^2)^{3/2}} = \frac{1}{r^2} \left(1 - \frac{3a^2}{2r^2} + O\left(\frac{a^4}{r^4}\right) \right), \quad (6.1)$$

so

$$\|F_i^{(E)}\| = m_i m_j \frac{r}{(r^2 + a^2)^{3/2}} \approx m_i m_j \frac{1}{r^2} \left(1 - \frac{3a^2}{2r^2} \right). \quad (6.2)$$

These expansions serve purely as numerical consistency checks.

7 Well-posedness

For $a > 0$ and bounded degree, $\hat{\Phi}_E \in C^1(\mathbb{R}^{3N} \setminus \{x_i = x_j\})$ and $F^{(E)}$ is locally Lipschitz off collisions. For C^1 kernels with bounded K'_σ , $F^{(K)}$ is continuous and locally Lipschitz. Existence and uniqueness follow by Picard–Lindelöf on compact intervals. For dynamic k NN, forces are piecewise smooth; employ hysteresis or prefer the smooth kernel.

8 Preregistered Model Card (example)

Domain. $a_{\text{grid}} = 10 \mu\text{m}$, $b = 16$.

Functional. $\hat{\Phi}_K$ with Wendland C^2 kernel ($\sigma = 0.5 \text{ mm}$); softening $a = 50 \mu\text{m}$.

Dynamics. BAOAB underdamped with ($m_i \equiv 1, \gamma = 0.1, T = 300 \text{ K}$), $\Delta t = 1 \times 10^{-3} \text{ s}$.

Calibration. Single G_{eff} fit in a dilute two-body sandbox at $r \gg a$.

Sanity checks. Verify (4.3) and the far-field expansion; report seeds and residuals.

9 Falsifiers for a chosen instance

Given fixed $(\hat{\Phi}, \text{params})$, declare the instance falsified if:

- (F1) Two-body trajectories disagree with the calibrated inverse-square form beyond numerical error.
- (F2) Smooth-kernel vs fixed-graph variants differ systematically at small r beyond topology effects.
- (F3) The code-length proxy correlates poorly with realized compression in controlled tests (e.g. $-\hat{\Phi}$ vs measured L_{tot}).

10 Discussion and Scope

PCD supplies a reproducible route from *computable* information-motivated functionals to concrete dynamics. Future work will extend this framework to broader estimator families under the same preregistration discipline.

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